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Algebraic Characterization of Vector Supersymmetry in Topological Field Theories

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Abstract

An algebraic cohomological characterization of a class of linearly broken Ward identities is provided. The examples of the topological vector supersymmetry and of the Landau ghost equation are discussed in detail. The existence of such a linearly broken Ward identities turns out to be related to BRST exact antifield dependent cocycles with negative ghost number, according to the cohomological reformulation of the Noether theorem given by M. Henneaux et al. [1].

1 Introduction

The topological theories [1] are known to be characterized, besides their BRST simmetry, by the so called *topological vector supersymmetry* [2, 3, 4, 5]; an additional invariance possessing rather interesting properties.

The first property is that the generators of the topological susy carry a Lorentz index and, together with the BRST operator, give rise to an algebra of the Wess-Zumino type which, closing on-shell on the space-time translations, allows for a supersymmetric interpretation [2, 3, 4, 5].

The second feature of the topological susy is that it is present only after the introduction off all the ghost fields needed in order to quantize the model, *i.e.* it is an invariance of the fully quantized action and not only of its classical part as one can check, for instance, in the case of the Schwartz type topological BF models [4]. It should also be remarked that this last feature is unavoidably related to the choice of the gauge fixing term. In other words, the vector susy can exist only for certain values of the gauge parameters present in the gauge fixing condition. As an example of this feature let us mention the three dimensional Chern-Simons model for which, among the class of the linear covariant gauge fixings, the vector susy turns out to select the Landau gauge [2, 3].

The third interesting aspect of the vector susy is that, after the introduction of the antifields (or BRST external sources), the algebra between the vector susy Ward operator and the BRST operator closes off-shell on the space-time translations, without making use of the equations of motion. Moreover, the vector susy loses now the property of being an exact invariance of the fully quantized action. Rather, it yields a broken Ward identity [3, 4, 5]. It is a remarkable feature, however, that the corresponding breaking term is in fact a classical breaking, *i.e.* a breaking which is purely linear in the quantum fields. It is known that such a kind of breaking does not get renormalized by the quantum corrections and does not spoil the usefulness of the corresponding Ward identity [6]. The latter turns out not only to be free from anomalies at the quantum level, but it plays a crucial role in order to establish the ultraviolet finiteness of the topological models [3, 4, 5].

Let us also mention that the vector susy Ward operator can be introduced on a more abstract geometrical way for a large class of gauge models [7, 8, 9, 10, 11], independently from the fact that it is or not related to a (linearly broken) symmetry of the action. In this case the vector Ward operator plays the role of an algebraic operator which, thanks to the fact that it decomposes the space-time derivative as a BRST anticommutator, turns out to be very useful in order to solve the descent equations associated to the BRST cohomology classes for the anomalies and the invariant counterterms. In addition, it allows to encode all the relevant informations (BRST transformations of the fields, BRST cohomology classes, solutions of the descent equations,...) into a unique equation which takes the suggestive form of a generalized zero curvature condition [12, 13].

All these properties, if on the one hand make the vector susy quite interesting, on the other hand motivate further investigations about its origin. For instance, in the case of the Witten four dimensional topological Yang-Mills theory one may wonder about the possibility of performing a twist [14] of the generators of the corresponding N=2 supersymmetric Yang-Mills theory in order to obtain the vector susy. The situation is less clear for other topological models, especially for those belonging to the Schwartz class which are not manifestly related to an extended supersymmetric algebra. For these models the vector susy has been introduced essentially by hand [2, 3, 4] and later on has been related to the existence of a conserved current related to the fact that the energy-momentum tensor of a topological theory can be expressed as a pure BRST variation [15, 6].

Moreover, a general set up accounting for all the features displayed by the vector susy has not yet been completely worked out. This is the aim of this paper, *i.e.* to provide a purely algebraic cohomological characterization of the existence of the vector susy and of the related linear classical breaking term. We shall also see that it is precisely the requirement of linearity in the quantum fields of the breaking term which selects a particular set of gauge parameters, clarifying thus the relationship between the vector susy and the gauge fixing condition.

In particular, we shall be able to prove that the existence of the topological vector susy turns out to be deeply related to a vector BRST invariant antifield dependent cocycle with ghost number -1. The existence or not of a vector supersymmetry depends purely on the fact that such an antifield cocycle is cohomological trivial or not. When it is, the vector susy Ward identity is present and turns out to be necessarily accompanied by a breaking term linear in the quantum fields. On the other hand, when such an antifield cocycle in not BRST exact, the vector susy cannot be established and we are left with an example of a nontrivial antifield dependent BRST cohomology class.

This algebraic framework is very related to the cohomological reformulation of the Noether theorem given by M. Henneaux et al. [16]. We shall see in fact that the aforementioned vector cocycle can be related to a set of currents among which one can identify the BRST invariant energy-momentum tensor, showing then that this antifield cocycle is related to the Poincaré transformations, according to the analysis of Henneaux et al. [16]. Let us also mention that, recently, this vector antifield cocycle has been considered in connection with the problem of including into a unique extended Slavnov-Taylor identity additional global invariances of the action [17].

The paper is organized as follows. In the Sect.2 we introduce the general algebraic set up, we present the relevant properties of the vector antifield cocycle and we discuss its relation with the BRST invariant energy-momentum tensor. In the Sect.3 the pure Yang-Mills model is considered. We shall prove that in this case the vector cocycle is not trivial, identifying then a BRST cohomology class. The Sect.4 is devoted to a detailed analysis of several examples of topological models. It will be proven that in these cases the BRST triviality of the vector cocycle is at the origin of the existence of the linearly broken topological vector susy Ward identities. Finally, in the Sect.5 we will present another example of a linearly broken Ward identity whose contact terms are generated, in complete analogy with the case of the topological models, by the BRST exactness of an antifield dependent cocycle possessing a group index. Such a Ward identity, known as the Landau ghost equation [18], turns out to be related to the rigid gauge invariance.

2 General Notations and the Vector Cocycle

In order to present the general algebraic set up let us begin by fixing the notations. We shall work in a flat D-dimensional space-time equipped with a set of fields generically denoted by $\{\varphi^i\}$, i labelling the different kinds of fields needed in order to properly quantize the model, *i.e.* gauge fields, ghosts, ghosts for ghosts, etc... Following the standard quantization procedure we introduce for each field φ^i of ghost number $\mathcal{N}_{\varphi\rangle}$ and dimension d_{φ^i}, the corresponding antifield φ^{i*} with ghost number $-\left(1+\mathcal{N}_{\varphi^i}\right)$ and dimension $\left(D-d_{\varphi^i}\right)$. We shall also assume that the set of fields $\{\varphi^i\}$ do not explicitely contains the antighosts and their corresponding Lagrange multipliers which, being grouped in BRST doublets, do not contribute to the BRST cohomology [19]. Accordingly, some of the antifields φ^{i*} have to be understood as shifted antifields [6] which already take into account the antighosts.

In order to define the model it remains now to introduce the classical gauge fixed *reduced* [1] action [6] $\Sigma(\varphi,\varphi^*)$. This is done by requiring that Σ is power counting renormalizable and that it is the solution of the classical homogeneous Slavnov-Taylor (or master equation) identity [20]

$$\int d^Dx \frac{\delta\Sigma}{\delta\varphi^i}\frac{\delta\Sigma}{\delta\varphi^{i*}} \;=\; \frac{1}{2}\mathcal{B}_\Sigma\Sigma \;=\; 0 \quad , \tag{2.1}$$

where $\mathcal{B}_\Sigma$ denotes the nilpotent linearized Slavnov-Taylor operator

$$\mathcal{B}_\Sigma \;=\int d^Dx \left(\frac{\delta\Sigma}{\delta\varphi^i}\frac{\delta}{\delta\varphi^{i*}} \;+\; \frac{\delta\Sigma}{\delta\varphi^{i*}}\frac{\delta}{\delta\varphi^i}\right) \quad , \tag{2.2}$$

$$\mathcal{B}_\Sigma\,\mathcal{B}_\Sigma \;=\; 0 \;. \tag{2.3}$$

As usual, the classical action $\Sigma(\varphi,\varphi^*)$ will be assumed to be invariant under the space-time translations, *i.e.*

$$\mathcal{P}_\mu\Sigma \;=\; \int d^Dx \,\left(\partial_\mu\varphi^i\frac{\delta\Sigma}{\delta\varphi^i} \;+\; \partial_\mu\varphi^{i*}\frac{\delta\Sigma}{\delta\varphi^{i*}}\right) \;=\; 0 \;. \tag{2.4}$$

The classical Slavnov-Taylor identity (2.1) and the translation invariance (2.4) will be taken therefore as the basic starting points for the characterization of the classical action $\Sigma(\varphi,\varphi^*)$ and for the algebraic analysis which will be carried out in the next sections. Let us also remark that the requirement of the translation invariance, as expressed by the equation (2.4), does not imply any further restriction on Σ than those that are tacetely assumed in any local field theory.

Let us introduce now the following integrated local polynomial, linear in the antifields φ^{i*}, of ghost number -1, dimension $(D+1)$, and of the vector type

[1]The name *reduced* action is used to denote the part of the complete gauge fixed action which does not depend from the Lagrangian multipliers and which depends from the antighosts only through the shifted antifields. As it will be discussed in the next Sections, the dependence of the complete action from the Lagrangian multipliers as well as from the gauge parameters present in the gauge fixing condition will be determined by requiring that the breaking term associated to the topological vector susy Ward identity is at most linear in the quantum fields.

$$\Omega_\nu^{-1} = \int d^D x \ \left[\omega_\nu^{-1}\right]_{D+1} \equiv \int d^D x \ (-)^{\left(1+\mathcal{N}_{\varphi^i}\right)} \varphi^i \ \partial_\nu \varphi^{i*} \ . \tag{2.5}$$

It is easily proven that the above expression is $\mathcal{B}_\Sigma-$invariant. In fact, one has

$$\mathcal{B}_\Sigma \ \Omega_\nu^{-1} \ = \int d^D x \ \left(\partial_\nu \varphi^i \frac{\delta \Sigma}{\delta \varphi^i} \ + \ \partial_\nu \varphi^{i*} \frac{\delta \Sigma}{\delta \varphi^{i*}} \right) = \mathcal{P}_\nu \Sigma \ = 0 \ \ , \tag{2.6}$$

due to the translation invariance of the calssical action $\Sigma\left(\varphi, \varphi^*\right)$. Having proven the invariance of Ω_ν^{-1} we have now to establish if it eventually identifies a cohomology class of the operator $\mathcal{B}_\Sigma$. We have in fact the following two possibilities, namely

1) Ω_ν^{-1} is $\mathcal{B}_\Sigma-$exact, *i.e.*

$$\Omega_\nu^{-1} = \mathcal{B}_\Sigma \Xi_\nu^{-2} \ \ , \tag{2.7}$$

Ξ_ν^{-2} being an integrated local polynomial of ghost number -2 and dimension $(D+1)$.

2) Ω_ν^{-1} belongs to the integrated cohomology of $\mathcal{B}_\Sigma$,

$$\Omega_\nu^{-1} \neq \mathcal{B}_\Sigma \Xi_\nu^{-2} \ . \tag{2.8}$$

The detailed analysis of these two possibilities will be the main subject of the next Sections. Let us limit here to underline that while the first possibility (2.7) turns out to be a feature of the topological theories, the second one (2.8) is typical of a Yang-Mills theory and, more generally, of any model possessing an invariant energy-momentum tensor $T_{\mu\nu}$ which cannot be written as a pure $\mathcal{B}_\Sigma-$variation, *i.e.*

$$\mathcal{B}_\Sigma T_{\mu\nu} = 0 \ \ , \ \ T_{\mu\nu} \neq \mathcal{B}_\Sigma \Lambda_{\mu\nu} \ \ . \tag{2.9}$$

In this case Ω_ν^{-1} provides an example of a nontrivial antifield dependent cocycle of the operator $\mathcal{B}_\Sigma$, giving thus an explicit realization of the general results of [16].

Concerning now the first possibility (2.7), it is worthwhile to recall that one of the peculiar property of the topological models is precisely that of having a nonphysical energy momentum tensor [14, 1], *i.e.* the energy momentum tensor of a topological field theory $T_{\mu\nu}^{top}$ can be written as a pure $\mathcal{B}_\Sigma-$variation

$$T_{\mu\nu}^{top} = \mathcal{B}_\Sigma \Lambda_{\mu\nu} \ \ , \tag{2.10}$$

for some local $\Lambda_{\mu\nu}$, yielding thus a more direct indication of the $\mathcal{B}_\Sigma-$exactness of the vector cocycle Ω_ν^{-1}.

For a better understanding of the relation between Ω_ν^{-1} and the energy-momentum tensor, let us write down the descent equations [19, 16, 6] corresponding to the integrated invariance condition (2.6), *i.e.*

$$\mathcal{B}_\Sigma \ \left[\omega_\nu^{-1}\right]_{D+1} \quad = \quad \partial^{\mu_1} \left[\omega_{\mu_1 \nu}^0\right]_D \ \ , \tag{2.11}$$

$$
\begin{aligned}
\mathcal{B}_\Sigma \left[\omega^0_{\mu_1\nu}\right]_D &= \partial^{\mu_2}\left[\omega^1_{[\mu_1\mu_2]\nu}\right]_{D-1} \ , \\
\mathcal{B}_\Sigma \left[\omega^1_{[\mu_1\mu_2]\nu}\right]_{D-1} &= \partial^{\mu_3}\left[\omega^2_{[\mu_1\mu_2\mu_3]\nu}\right]_{D-2} \ , \\
&\ \ \ .... \ , \\
&\ \ \ .... \ , \\
\mathcal{B}_\Sigma \left[\omega^{D-2}_{[\mu_1\mu_2...\mu_{D-1}]\nu}\right]_2 &= \partial^{\mu_D}\left[\omega^{D-1}_{[\mu_1\mu_2...\mu_{D-1}\mu_D]\nu}\right]_1 \ , \\
\mathcal{B}_\Sigma \left[\omega^{D-1}_{[\mu_1\mu_2...\mu_{D-1}\mu_D]\nu}\right]_1 &= 0 \ ,
\end{aligned}
\tag{2.12}
$$

where the $\left[\omega^{j-1}_{[\mu_1\mu_2...\mu_j]\nu}\right]_{D-j+1}$ with $(j=0,...,D)$ are local currents of ghost number $(j-1)$, antisymmetric in the lower indices $(\mu_1,\mu_2,...,\mu_j)$, and of dimension $(D-j+1)$. The usefulness of working with the system (2.11) is due to the fact that these equations relate the local cocycle $[\omega^{-1}_\nu]_{D+1}$ with currents of lower dimension which can provide a more easy and transparent interpretation of the physical meaning of Ω^{-1}_ν. To this purpose it should be remarked that the cocycle $\left[\omega^0_{\mu_1\nu}\right]_D$ entering the second equation of the system (2.11) has the same quantum numbers of the energy momentum tensor, being of dimension D, of ghost number 0, and possessing two Lorentz indices. In particular, from the general results on the BRST cohomology [19] it follows that, independently from the fact that the cohomology of the operator $\mathcal{B}_\Sigma$ is empty or not in the sectors with dimension lower than D, the existence of a nontrivial local cohomology in the sector of dimension D and with two free Lorentz indices necessarily implies the nontriviality of the upper level of dimension $D+1$. It becomes apparent thus that the nontriviality of the BRST invariant energy-momentum tensor is deeply related to the existence of a nontrivial integrated cohomology in the sector of dimension $D+1$ and negative ghost-number.

In fact, as we shall see explicitily in the case of the Yang-Mills theory, the current $\left[\omega^0_{\mu_1\nu}\right]_D$ turns out to be precisely the improved BRST invariant energy-momentum tensor. On the other hand, the use of the descent equations (2.11) provides a simple demonstration of the triviality of the vector cocycle Ω^{-1}_ν in the case of the topological theories. This is actually due to the fact that the field content of the topological models gives rise to BRST cohomology classes which are empty in the various sectors appearing in the descent equations (2.11), implying in particular that the corresponding energy momentum tensor is BRST trivial, according to eq.(2.10). Moreover, as we shall see in detail later on, the right hand side of the equation (2.7) expressing the BRST triviality of Ω^{-1}_ν will provide the contact terms of the vector susy Ward identity of the topological theories. In this latter case we shall also check that the left hand side of eq.(2.7) reduces to a classical breaking, *i.e.* to a breaking purely linear in the quantum fields, showing then that the topological vector susy Ward identity is always linearly broken.

3 Pure Yang-Mills Theory

As a first application of the general algebraic set up discussed in the previous Section, let us now prove that in the case of pure Yang-Mills action the vector cocycle Ω^{-1}_ν cannot be written as an exact $\mathcal{B}_\Sigma-$term. Let us begin by considering the complete fully quantized gauge fixed Yang-Mills action which, choosing a Feynman gauge, reads

$$\mathcal{S} = \int d^4x \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + b^a \partial A^a + \frac{\alpha}{2} b^a b^a + \partial^\mu \overline{c}^a (D_\mu c)^a \right) \qquad (3.13)$$
$$+ \int d^4x \ \hat{A}^{a*}_\mu (D^\mu c)^a - \frac{1}{2} f_{abc} C^{*a} c^b c^c \ ,$$

where $(c, \overline{c}, b)$ denote respectively the ghost, the antighost and the Lagrange multiplier fields and

$$(D_\mu c)^a = \partial_\mu c^a + f^a_{\ bc} A^b_\mu c^c \ , \qquad (3.14)$$

is the covariant derivative with f_{abc} the totally antisymmetric structure constants of a compact semisimple Lie group G. The two antifields $\left(\hat{A}^*, C^*\right)$ are introduced in order to properly define the nonlinear BRST transformations of the gauge field A and of the Faddeev-Popov ghost c. The quantum numbers, *i.e.* the dimensions and the ghost numbers, of all the fields and antifields are assigned as in the following table

	A	$\hat{A}^*$	c	C^*	$\bar{c}$	b
dim	1	3	0	4	2	2
$gh-numb$	0	-1	1	-2	-1	0

Table 1: dimension and ghost number

As it is well known [6], the complete action $\mathcal{S}$ is characterized by the classical Slavnov-Taylor identity, expressing the invariance of (3.13) under the BRST transformations, *i.e.*

$$\int d^4x \left(\frac{\delta \mathcal{S}}{\delta A^a_\mu} \frac{\delta \mathcal{S}}{\delta \hat{A}^{*a\mu}} + \frac{\delta \mathcal{S}}{\delta c^a} \frac{\delta \mathcal{S}}{\delta C^{*a}} + b^a \frac{\delta \mathcal{S}}{\delta \overline{c}^a} \right) = 0 \ , \qquad (3.15)$$

and by the linear gauge fixing condition [6]

$$\frac{\delta \mathcal{S}}{\delta b^a} = \partial A^a + \alpha b^a \ . \qquad (3.16)$$

A further identity, the antighost equation [6]

$$\frac{\delta \mathcal{S}}{\delta \overline{c}^a} + \partial^\mu \frac{\delta \mathcal{S}}{\delta \hat{A}^{*a\mu}} = 0 \qquad (3.17)$$

follows from the gauge fixing condition (3.16) and from the Slavnov-Taylor identity (3.15), implying that the antighost $\overline{c}$ and the antifield $\hat{A}^*$ can enter only through the shifted antifield $A^{*a\mu}$ of ghost number -1 and dimension 3

$$A^{*a\mu} = \hat{A}^{*a\mu} + \partial^\mu \overline{c}^a \ . \qquad (3.18)$$

Introducing now the reduced Yang-Mills action $\Sigma\,(A, A^*, c, C^*)$ defined by the identities (3.16) and (3.17)

$$\mathcal{S} = \Sigma + \int d^4x \ \left(b^a \partial A^a + \frac{\alpha}{2} b^a b^a \right) \ , \qquad (3.19)$$

i.e.

$$\Sigma \ = \int d^4x \ \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} \ + \ A^{*a\mu} (D_\mu c)^a - \frac{1}{2} \, f_{abc} C^{*a} c^b c^c \right) \ \ , \tag{3.20}$$

it is easily verified that Σ obeys the homogeneous Slavnov-Taylor identity

$$\int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu} \frac{\delta\Sigma}{\delta A^{*a\mu}} \ + \frac{\delta\Sigma}{\delta c^a} \frac{\delta\Sigma}{\delta C^{*a}} \right) = \ \frac{1}{2} \mathcal{B}_\Sigma \Sigma \ = \ 0 \ \ , \tag{3.21}$$

with

$$\mathcal{B}_\Sigma \ = \ \int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu} \frac{\delta}{\delta A^{*a\mu}} + \frac{\delta\Sigma}{\delta A^{*a\mu}} \frac{\delta}{\delta A^{a\mu}} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta C^{*a}} \ + \frac{\delta\Sigma}{\delta C^{*a}} \frac{\delta}{\delta c^a} \right) \ \ , \tag{3.22}$$

and

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma \ = \ 0 \ \ . \tag{3.23}$$

For the vector cocycle Ω^{-1}_ν of eq. (2.5) we have now

$$\Omega^{-1}_\nu = \ \int d^4x \ \ \left[\omega^{-1}_\nu \right]_5 \ \equiv \ \int d^4x \ \left(c^a \partial_\nu C^{*a} - A^a_\mu \partial_\nu A^{*a\mu} \right) \ \ , \tag{3.24}$$

which, according to the equation (2.6), turns out to be $\mathcal{B}_\Sigma-$invariant

$$\mathcal{B}_\Sigma \ \Omega^{-1}_\nu = \ \mathcal{P}_\nu \Sigma \ = 0 \ \ . \tag{3.25}$$

We are now ready to prove that in the case of Yang-Mills theory the vector cocycle Ω^{-1}_ν of eq.(3.24) is nontrivial. Let us proceed by assuming the converse, *i.e.* let us suppose that Ω^{-1}_ν can be written as an exact $\mathcal{B}_\Sigma-$term

$$\Omega^{-1}_\nu = \mathcal{B}_\Sigma \Xi^{-2}_\nu \ \ , \tag{3.26}$$

for some integrated local polynomial Ξ^{-2}_ν of ghost number -2 and dimension 5. From the Table 1 it follows that the most general form for Ξ^{-2}_ν is given by

$$\Xi^{-2}_\nu \ = \beta \int d^4x \ C^{*a} A^a_\nu \ \ , \tag{3.27}$$

where β is an arbitrary free parameter which has to be fixed by the exactness condition (3.26). Thus, from eq.(3.26), we should have the algebraic equality

$$\int d^4x \left(c^a \partial_\nu C^{*a} - A^a_\mu \partial_\nu A^{*a\mu} \right) = \beta \int d^4x \left(C^{*a} \partial_\nu c^a + A^a_\nu \left(D_\mu A^{*\mu} \right)^a \ \right) \ \ . \tag{3.28}$$

However it is almost immediate to check that the above equation has no solution for β, showing then that Ω^{-1}_νcannot be written as a pure $\mathcal{B}_\Sigma-$variation,

$$\Omega^{-1}_\nu \neq \mathcal{B}_\Sigma \Xi^{-2}_\nu \ \ . \tag{3.29}$$

Therefore, in the case of Yang-Mills theory the vector cocycle Ω_ν^{-1} identifies a cohomology class of the operator $\mathcal{B}_\Sigma$ in the sector of the integrated local polynomials of ghost number -1 and dimension 5, providing thus an explicit example of an antifield dependent cohomology class of $\mathcal{B}_\Sigma$.

For a better understanding of the nontriviality of the vector cocycle Ω_ν^{-1} it remains now to discuss its relation with the energy-momentum tensor of the model. To this purpose we analyse the descent equations corresponding to the local polynomial $[\omega_\nu^{-1}]_5$, *i.e.*

$$\begin{aligned}
\mathcal{B}_\Sigma \left[\omega_\nu^{-1}\right]_5 &= \partial^{\mu_1}\left[\omega^0_{\mu_1\nu}\right]_4 \ , \\
\mathcal{B}_\Sigma \left[\omega^0_{\mu_1\nu}\right]_4 &= \partial^{\mu_2}\left[\omega^1_{[\mu_1\mu_2]\nu}\right]_3 \ , \\
\mathcal{B}_\Sigma \left[\omega^1_{[\mu_1\mu_2]\nu}\right]_3 &= \partial^{\mu_3}\left[\omega^2_{[\mu_1\mu_2\mu_3]\nu}\right]_2 \ , \\
\mathcal{B}_\Sigma \left[\omega^2_{[\mu_1\mu_2\mu_3]\nu}\right]_2 &= \partial^{\mu_4}\left[\omega^3_{[\mu_1\mu_2\mu_3\mu_4]\nu}\right]_1 \ , \\
\mathcal{B}_\Sigma \left[\omega^3_{[\mu_1\mu_2\mu_3\mu_4]\nu}\right]_1 &= 0 \ .
\end{aligned} \tag{3.30}$$

After some straightforward algebraic manipulations, for the local currents $\left(\omega^{j-1}_{[\mu_1..\mu_j]\nu},\ j=1,..4\right)$ one finds

$$\left[\omega^0_{\mu_1\nu}\right]_4 = \frac{1}{g^2}\left(F^a_{\mu_1\sigma}F^{a\,\sigma}_{\nu} - \frac{1}{4}g_{\mu_1\nu}F^a_{\rho\sigma}F^{a\rho\sigma}\right) + \mathcal{B}_\Sigma\left(A^{*a}_{\mu_1}A^a_\nu\right) \ , \tag{3.31}$$

$$\left[\omega^1_{[\mu_1\mu_2]\nu}\right]_3 = \varepsilon_{\mu_1\mu_2}{}^{\mu_3\mu_4}\,\partial_{\mu_3}\left[\tilde{\omega}^1_{\mu_4\nu}\right]_2 \ , \tag{3.32}$$

$$\left[\omega^2_{[\mu_1\mu_2\mu_3]\nu}\right]_2 = \varepsilon_{\mu_1\mu_2\mu_3}{}^{\mu_4}\mathcal{B}_\Sigma\left[\tilde{\omega}^1_{\mu_4\nu}\right]_2 + \varepsilon_{\mu_1\mu_2\mu_3}{}^{\mu_4}\partial_{\mu_4}\left[\tilde{\omega}^2_\nu\right]_1 \ , \tag{3.33}$$

$$\left[\omega^3_{[\mu_1\mu_2\mu_3\mu_4]\nu}\right]_1 = \varepsilon_{\mu_1\mu_2\mu_3\mu_4}\mathcal{B}_\Sigma\left[\tilde{\omega}^2_\nu\right]_1 \ , \tag{3.34}$$

with $\left[\tilde{\omega}^1_{\mu_4\nu}\right]_2$ and $[\tilde{\omega}^2_\nu]_1$ local arbitrary polynomials. From the above expressions we observe that while the last three currents are trivial solutions of the descent equations (3.30), the first one, *i.e.* $\left[\omega^0_{\mu_1\nu}\right]_4$, yields the familiar expression of the BRST invariant improved energy-momentum tensor which, as it is well known , belongs to the cohomology of $\mathcal{B}_\Sigma$ [19]. One sees thus that, as already remarked in Sect.1, the existence of a nontrivial invariant energy-momentum tensor is at the origin of the nontriviality of the vector cocycle Ω_ν^{-1}. Let us conclude this Section by remarking that the exact term $\mathcal{B}_\Sigma\left(A^{*a}_{\mu_1}A^a_\nu\right)$ which naturally appears in the right hand side of the equation (3.31) is needed in order to ensure the off-shell conservation of the improved Yang-Mills energy-momentum tensor $T^{YM}_{\mu\nu} = F^a_{\mu\sigma}F^{a\,\sigma}_{\nu} - \frac{1}{4}g_{\mu\nu}F^a_{\rho\sigma}F^{a\rho\sigma}$. Of course, the same conclusions hold if the pure Yang-Mills action is supplemented with the introduction of matter fields.

4 The Case of the Topological Field Theories

Having proven the nontriviality of Ω_ν^{-1} for Yang-Mills type theories, let us now turn to analyse the vector cocycle (2.5) in the context of the topological models. In this case, as already mentioned in the Sect.2, it turns out that Ω^{-1}_{ν} is always BRST exact, *i.e.*

$$\Omega^{-1}_{\nu} = \mathcal{B}_{\Sigma}\Xi^{-2}_{\nu} \ , \tag{4.35}$$

for some integrated local field polynomial Ξ^{-2}_{ν} with ghost number -2. The reason of the exactness of Ω^{-1}_{ν} for the topological theories relies on their field content which, as proven by several authors [21, 22, 23, 24, 4, 5, 6], does not allow for nontrivial BRST cohomology classes with free Lorentz indices, as the ones needed in order to have nontrivial solutions of the descent equations (2.11). As it is well known, the topological models can be basically divided in two classes [1], yielding respectively the so called cohomological and Schwartz type theories, both having a BRST trivial energy-momentum tensor. The models belonging to the first class are identified by the fact that the gauge fixed classical action can be expressed as a pure BRST variation and that it is invariant under the so called topological shift symmetry [1, 21, 22, 23, 24, 25, 26]. In the second case the invariant action is not a pure BRST variation, although it depends on the metric tensor only through the unphysical gauge fixing term. Examples of theories belonging to the first class are given by the Witten's topological Yang-Mills theory [14] and by the topological sigma model [27]. The three-dimensional Chern-Simons model [1, 2, 3] and the BF systems [1, 4] provide examples of Schwartz type topological theories.

Without entering into the details, let us limit here to mention that in the case of the cohomological models the topological shift symmetry implies the emptiness of the BRST cohomology[2] [1, 5, 21, 22, 23] and therefore the triviality of the vector cocycle Ω^{-1}_{ν}. Concerning now the Schwartz type models, it can be proven that the field content of these theories allows for nontrivial BRST cohomology classes [3, 4, 6]. Moreover, as observed in [12], these models can be formulated in a pure geometrical way due to the fact that all the fields (gauge fields, ghosts, ghosts for ghosts, etc...) can be viewed as being the components of a generalized gauge connection obeying a zero curvature condition. This structure, also called complete ladder structure, implies that all nontrivial BRST cohomology classes can be identified with invariant polynomials built up with the undifferentiated dimensionless scalar ghosts present in the model [3, 4, 6]. In particular, from this result it follows that the BRST cohomology classes entering the descent equations (2.11) are empty, meaning thus that also in the case of the Schwartz type topological theories the vector cocycle Ω^{-1}_{ν} is BRST exact .

Although the equation (4.35) could seem empty of any interesting information because of its BRST exactness, it turns out however that its content is far from being irrelevant. In fact, as we shall see explicitely in the models considered below, the right hand side of the eq.(4.35), due to the form (2.2) of the operator $\mathcal{B}_{\Sigma}$, is easily seen to provide the contact terms of a Ward identity. This identity, due to the presence of the vector cocycle Ω^{-1}_{ν} in the left hand side of (4.35), cannot express an exact invariance of the theory. Instead, eq.(4.35) will have the meaning of a broken Ward identity. Moreover, it is not difficult to convince oneself that such a breaking term is in fact a classical breaking, *i.e.*

[2] Let us remark here that, as discussed by R. Stora et al.[21, 22, 23, 24], the relevant cohomology for the topological theories of the Witten's type is the so called equivariant cohomology. The latter is the restriction of the BRST cohomology to gauge invariant local field polynomials which do not depend from the Faddee-Popov ghost field c. Contrary to the BRST cohomology, the equivariant cohomology is not empty and turns out to provide a consistent definition of the Witten's observables.

a breaking which is linear in the quantum fields. Therefore it does not get renormalized by the radiative corrections and does not spoil the usefulness of the corresponding broken Ward identity. As one can easily understand, the unavoidable existence of this classical breaking term is a consequence of the fact that the general expression of the vector cocycle Ω_{ν}^{-1} given in eq.(2.5) is linear in the quantum fields, apart possible quadratic terms coming from the fact that some of the antifields φ^{i*} have been shifted in order to take into account the antighosts. However we shall check that these quadratic terms turn out to be handled in a very simple way, being reinterpreted as pure contact terms by means of an appropriate choice of the nonphysical gauge parameters present in the gauge condition. This means that the requirement of having breaking terms at most linear in the quantum fields will fix the gauge parameters, implying therefore that the corresponding Ward identity is consistent with the Quantum Action Principles [28] only for certain classes of gauge fixings. We shall give explicit examples of this feature later on in the cases of the three dimensional Chern-Simons model and of the Landau ghost equation derived in the Sect.5.

The equation (4.35) has thus the meaning of a purely algebraic cohomological characterization of a Ward identity. In fact, once one is able to prove that the vector cocycle Ω_{ν}^{-1} is trivial, one knows authomatically that the equation (4.35) has to be necessarily satisfied for some local polynomial Ξ_{ν}^{-2}. The strategy to be followed now is almost trivial. One writes down the most general expression for Ξ_{ν}^{-2} compatible with the dimension of the space-time and with the field content of the model under consideration. Such an expression will, in general, depend on a set of free global coefficients. These coefficients are fixed by demanding that eq.(4.35) is valid. Moreover, recalling that the functional form of the operator $\mathcal{B}_{\Sigma}$ (see eq. (2.2)) depends on the reduced action Σ, it is apparent to see that the coefficients of Ξ_{ν}^{-2} as well as the contact terms of the corresponding Ward identity, are uniquely determined by the form of the classical action Σ. This means that, whenever the cocycle Ω_{ν}^{-1} is trivial, the equation (4.35) tells us that the classical action Σ, in addition to the Slavnov-Taylor identity, obeys a further Ward identity whose contact terms are precisely given by the equation (4.35), providing thus a simple mechanism for an algebraic characterization of a new linearly broken symmetry of the action. Of course such an additional Ward identity will have the same quantum numbers of the vector cocycle Ω_{ν}^{-1}, *i.e.* it will carry a Lorentz index and will have a negative ghost number. This vector identity, always present in the topological field theories, is known in the litterature as the topological vector supersymmetry [2, 3, 4, 5].

It is important at this point to spend some words on the possibility of generalizing this purely algebraic mechanism in order to find other kinds of unknown symmetries eventually present in the model. What seems to emerge from the previous analysis is that the antifield dependent cocycles which are relevant for the existence of a *class of linearly broken* new Ward identities are those which are linear in the fields $\{\varphi^{i}\}$ and in the antifields $\{\varphi^{i*}\}$ and that are BRST exact. It is in fact this last property which when cast in the form of the equation (4.35) allows us to identify the contact terms of the corresponding Ward identity thanks to the form of the linearized Slavnov-Taylor operator. It is worthwhile to remark that an equation of the kind of (4.35) implies that the associate Ward identity is always broken, due to the existence of a nonvanishing left hand side term. Moreover, being this term linear in the quantum fields, the resulting breaking term is classical.

In the next chapter we will discuss another interesting example of a classically linearly broken Ward identity, associated to the rigid gauge invariance, whose contact terms can be characterized in a purely algebraic way by means of an equation of the type of (4.35).

Let us focus, for the time being, on the discussion of the vector susy Ward identity in the case of the topological field theories. In order to present in an explicit way the previous algebraic mechanism, we shall analyse in detail the examples of the three dimensional Chern-Simons gauge model, of the four dimensional BF model and of the so called two dimensional b-c ghost system. The argument can be easily repeated and adapetd to other kinds of topological models, such as the Witten's cohomological theories and the higher dimensional BF models.

4.1 The three dimensional Chern-Simons model

Using the same notations of the Sect.3, for the complete three dimensional Chern-Simons action [1] quantized in the Landau gauge we have

$$\mathcal{S} = \mathcal{S}_{cs} + \int d^3x \, \left(b^a \partial A^a + \partial^\mu \overline{c}^a (D_\mu c)^a + \hat{A}^*_\mu (D^\mu c)^a - \frac{1}{2} f_{abc} C^{*a} c^b c^c \right) , \tag{4.36}$$

where $\mathcal{S}_{cs}$ is given by

$$\mathcal{S}_{cs} = -\frac{k}{4\pi} \int d^3x \, \varepsilon^{\mu\nu\rho} \left(A^a_\mu \partial_\nu A^a_\rho + \frac{1}{3} f_{abc} A^a_\mu A^b_\nu A^c_\rho \right) , \tag{4.37}$$

k identifying the inverse of the coupling constant. The fields and the antifields have now the following quantum numbers

	A	$\hat{A}^*$	c	C^*	$\bar{c}$	b
dim	1	2	0	3	1	1
$gh-numb$	0	-1	1	-2	-1	0

Table 2: dimension and ghost number

For the classical Slavnov-Taylor identity one has

$$\int d^3x \left(\frac{\delta\mathcal{S}}{\delta A^a_\mu} \frac{\delta\mathcal{S}}{\delta \hat{A}^{*a\mu}} + \frac{\delta\mathcal{S}}{\delta c^a} \frac{\delta\mathcal{S}}{\delta C^{*a}} + b^a \frac{\delta\mathcal{S}}{\delta \overline{c}^a} \right) = 0 \ . \tag{4.38}$$

As before, making use of the Landau gauge condition

$$\frac{\delta\mathcal{S}}{\delta b^a} = \partial A^a \ , \tag{4.39}$$

and of the antighost equation

$$\frac{\delta\mathcal{S}}{\delta \overline{c}^a} + \partial^\mu \frac{\delta\mathcal{S}}{\delta \hat{A}^{*a\mu}} = 0 \ , \tag{4.40}$$

for the reduced Chern-Simons action $\Sigma\,(A, A^*, c, C^*)$

$$\mathcal{S} = \Sigma + \int d^3x \; b^a \partial A^a \ , \tag{4.41}$$

we obtain

$$\Sigma \ = \mathcal{S}_{cs} + \int d^3x \ \left(A^{a*}_\mu (D^\mu c)^a - \frac{1}{2} \ f_{abc} C^{*a} c^b c^c \right) \ , \tag{4.42}$$

where, as usual, A^{a*}_μ is the shifted antifield

$$A^{*a\mu} \ = \ \hat{A}^{*a\mu} \ + \ \partial^\mu \overline{c}^a \ . \tag{4.43}$$

Finally, taking into account the eqs. (4.39), (4.40) and the expression (4.42), the Slavnov-Taylor identity (4.38) becomes

$$\int d^3x \left(\frac{\delta\Sigma}{\delta A^a_\mu} \frac{\delta\Sigma}{\delta A^{*a\mu}} \ + \frac{\delta\Sigma}{\delta c^a} \frac{\delta\Sigma}{\delta C^{*a}} \right) \ = \ \frac{1}{2} \mathcal{B}_\Sigma \Sigma \ = \ 0 \ , \tag{4.44}$$

with

$$\mathcal{B}_\Sigma \ = \ \int d^3x \left(\frac{\delta\Sigma}{\delta A^a_\mu} \frac{\delta}{\delta A^{*a\mu}} + \frac{\delta\Sigma}{\delta A^{*a\mu}} \frac{\delta}{\delta A^{a\mu}} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta C^{*a}} \ + \frac{\delta\Sigma}{\delta C^{*a}} \frac{\delta}{\delta c^a} \right) \ , \tag{4.45}$$

and

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma \ = \ 0 \ . \tag{4.46}$$

Repeating the same procedure done in the case of Yang-Mills theory, for the vector cocycle Ω^{-1}_ν we can write

$$\Omega^{-1}_\nu = \ \int d^3x \ \ \left[\omega^{-1}_\nu \right]_4 \ \equiv \ \int d^3x \ \left(c^a \partial_\nu C^{*a} - A^a_\mu \partial_\nu A^{*a\mu} \right) \ . \tag{4.47}$$

and

$$\mathcal{B}_\Sigma \ \Omega^{-1}_\nu = \mathcal{P}_\nu \Sigma \ = 0 \ . \tag{4.48}$$

In this case, as already mentioned, Ω^{-1}_ν turns out to be exact

$$\Omega^{-1}_\nu = \mathcal{B}_\Sigma \Xi^{-2}_\nu \ , \tag{4.49}$$

where Ξ^{-2}_ν is an integrated local polynomial of dimension 4 and ghost number -2. From the Table 2 it follows that the most general expression for Ξ^{-2}_ν is given by

$$\Xi^{-2}_\nu = \int d^3x \ \left(\gamma C^{*a} A^a_\nu + \ \frac{\beta}{2} \varepsilon_{\nu\sigma\tau} A^{*a\sigma} A^{*a\tau} \right) \ , \tag{4.50}$$

where γ and β are free arbitrary parameters. Using now the expressions of the linearized Slavnov-Taylor operator (4.45) and of the reduced Chern-Simons action (4.42), for the right hand side of eq. (4.49) we have

$$\mathcal{B}_\Sigma \Xi^{-2}_\nu \ = \int d^3x \ \left(\gamma A^a_\nu \frac{\delta\Sigma}{\delta c^a} + \gamma C^{*a} \frac{\delta\Sigma}{\delta A^{*a\nu}} - \beta \varepsilon_{\nu\sigma\tau} A^{*a\sigma} \frac{\delta\Sigma}{\delta A^a_\tau} \right) \ . \tag{4.51}$$

Comparing then both sides of eq.(4.49), for the coeffiecients γ and β we get

$$\gamma = -1 \; , \qquad \beta = \frac{2\pi}{k} \; , \tag{4.52}$$

so that eq.(4.49) can be rewritten as

$$\int d^3x \; \left(A^a_\nu \frac{\delta \Sigma}{\delta c^a} + C^{*a} \frac{\delta \Sigma}{\delta A^{*a\nu}} + \frac{2\pi}{k} \varepsilon_{\nu\sigma\tau} A^{*a\sigma} \frac{\delta \Sigma}{\delta A^a_\tau} + c^a \partial_\nu C^{*a} - A^a_\mu \partial_\nu A^{*a\mu} \right) = 0 \; . \tag{4.53}$$

Finally, moving from the reduced action Σ to the complete action $\mathcal{S}$ of eq. (4.36) and making use of the gauge condition (4.39), the identity (4.53) can be cast in the form of a linearly broken Ward identity, namely

$$\mathcal{W}_\nu \mathcal{S} = \Delta^{cl}_\nu \; , \tag{4.54}$$

with

$$\mathcal{W}_\nu = \int d^3x \left(A^a_\nu \frac{\delta}{\delta c^a} + C^{*a} \frac{\delta}{\delta \hat{A}^{*a\nu}} + \frac{2\pi}{k} \varepsilon_{\nu\sigma\tau} \left(\hat{A}^{*a\sigma} + \partial^\sigma \bar{c}^a \right) \frac{\delta}{\delta A^a_\tau} + \partial_\nu \bar{c}^a \frac{\delta}{\delta b^a} \right) \; , \tag{4.55}$$

and

$$\Delta^{cl}_\nu = \int d^3x \; \left(C^{*a} \partial_\nu c^a - \hat{A}^{*a\mu} \partial_\nu A^a_\mu \; - \frac{2\pi}{k} \varepsilon_{\nu\sigma\tau} \hat{A}^{*a\sigma} \partial^\tau b^a \right) \; . \tag{4.56}$$

The equation (4.54) is easily recognized to be the well known topological vector susy Ward identity of the three dimensional Chern-Simons theory [2, 3]. Let us conclude this section by observing that in the case in which instead of a Landau gauge fixing condition we had adopted a Feynman gauge (see eq.(3.16)), the right hand side of the eq. (4.54) would have been modified by the additional term

$$\alpha \int d^3x \; b^a \partial_\nu \bar{c}^a \; , \tag{4.57}$$

which, being quadratic in the quantum fields, would have spoiled the usefulness of the identity (4.54). We see therefore that, as already remarked, the requirement that the breaking term Δ^{cl}_ν is a classical breaking, *i.e.* at most linear in the quantum fields, forces the gauge parameter α to vanish, *i.e.* $\alpha = 0$, picking up thus the Landau gauge.

4.2 The four dimensional BF model

As the second example, we shall present the case of the four dimensional BF model [1] whose classical invariant action is given by

$$\mathcal{S}_{BF} = -\frac{1}{4} \int d^4x \; \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} B^a_{\rho\sigma} \; . \tag{4.58}$$

The quantization of this model requires the Batalin-Vilkoviski procedure [20] due to the presence of ghosts for ghosts. We shall limit here only to report the final result, reminding the reader to the numerous references [1] for the technical details. In particular, using the same notations of ref. [4], in order to gauge fix the invariant action (4.58) we introduce a set of lagrangian multipliers $\left(b^a, h^a_\mu, \omega^a, \lambda^a\right)$, a set of antighosts $\left(\overline{c}^a, \overline{\xi}^a_\mu, \overline{\phi}^a, e^a\right)$, and a triple of ghosts $\left(c^a, \xi^a_\mu, \phi^a\right)$. For the gauge fixing action $\mathcal{S}_{gf}$ we have

$$
\begin{aligned}
\mathcal{S}_{gf} \;=\int d^4x \quad & \Big(b^a\partial A^a - \partial^\mu\overline{c}^a(D_\mu c)^a + h^a_\nu\partial_\mu B^{a\mu\nu} + \omega^a\partial\xi^a + h^{a\mu}\partial_\mu e^a + \omega^a\lambda^a \\
& -\partial^\mu\overline{\phi}^a\left((D_\mu\phi)^a + f_{abc}c^b\xi^c_\mu\right) + \frac{1}{2}f_{abc}\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu\overline{\xi}^a_\nu) \\
& -\partial^\mu\overline{\xi}^{a\nu}\left((D_\mu\xi_\nu)^a - (D_\nu\xi_\mu)^a + f_{abc}B^b_{\mu\nu}c^c\right) - \lambda^a\partial\overline{\xi}^a\ \Big) \quad .
\end{aligned}
\tag{4.59}
$$

The ghost numbers and the dimensions of all the fields and ghosts are assigned as follows

	A	B	c	ξ	ϕ	$\bar{c}$	$\bar{\xi}$	$\bar{\phi}$	e	b	h	ω	λ
dim	1	2	0	1	0	2	1	2	2	2	1	2	2
$gh-numb$	0	0	1	1	2	-1	-1	-2	0	0	0	-1	1

Table 3: dimension and ghost number

Introducing now a set of antifields $\left(\hat{A}^{*a\mu}, \hat{B}^{*a\mu\nu}, C^{*a}, \phi^{*a}, \hat{\xi}^{*a\mu}\right)$ associated respectively to the fields $(A^{a\mu}, B^{a\mu\nu}, c^a, \phi^a, \xi^{a\mu})$, *i.e.*

$$
\begin{aligned}
\mathcal{S}_{ext} = \int d^4x \quad & \Big(\frac{1}{2}\hat{B}^{*a\mu\nu}\left((D_\nu\xi_\mu)^a - (D_\mu\xi_\nu)^a - f_{abc}B^b_{\mu\nu}c^c + f_{abc}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho\overline{\xi}^{b\sigma}\phi^c\right) \\
& +\hat{\xi}^{*a\mu}\left((D_\mu\phi)^a + f_{abc}c^b\xi^c_\mu\right) + \frac{1}{8}f_{abc}\varepsilon_{\mu\nu\rho\sigma}\hat{B}^{*a\mu\nu}\hat{B}^{*b\rho\sigma}\phi^c \\
& -\hat{A}^{*a\mu}(D_\mu c)^a + \frac{1}{2}f_{abc}C^{*a}c^bc^c + f_{abc}\phi^{*a}c^b\phi^c\Big) \quad ,
\end{aligned}
\tag{4.60}
$$

	$\hat{A}^*$	$\hat{B}^*$	C^*	ϕ^*	$\hat{\xi}^*$
dim	3	2	4	4	3
$gh-numb$	-1	-1	-2	-3	-2

Table 4: dimension and ghost number

we have that the complete action $\mathcal{S}$

$$
\mathcal{S} = \mathcal{S}_{BF} + \mathcal{S}_{gf} + \mathcal{S}_{ext} \quad , \tag{4.61}
$$

obeys the following Slavnov-Taylor identity

$$
\begin{aligned}
\int d^4x \quad & \Big(\frac{\delta\mathcal{S}}{\delta A^a_\mu}\frac{\delta\mathcal{S}}{\delta\hat{A}^{*a\mu}} + \frac{\delta\mathcal{S}}{\delta c^a}\frac{\delta\mathcal{S}}{\delta C^{*a}} + \frac{1}{2}\frac{\delta\mathcal{S}}{\delta B^a_{\mu\nu}}\frac{\delta\mathcal{S}}{\delta\hat{B}^{*a\mu\nu}} + \frac{\delta\mathcal{S}}{\delta\phi^a}\frac{\delta\mathcal{S}}{\delta\phi^{*a}} \\
& +\frac{\delta\mathcal{S}}{\delta\xi^a_\mu}\frac{\delta\mathcal{S}}{\delta\hat{\xi}^{*a\mu}} + h^a_\mu\frac{\delta\mathcal{S}}{\delta\overline{\xi}^a_\mu} + b^a\frac{\delta\mathcal{S}}{\delta\overline{c}^a} + \omega^a\frac{\delta\mathcal{S}}{\delta\overline{\phi}^a} + \lambda^a\frac{\delta\mathcal{S}}{\delta e^a}\Big) \;=\; 0 \; .
\end{aligned}
\tag{4.62}
$$

In order to define the reduced action for the BF model let us write down the gauge-fixing conditions

$$
\begin{aligned}
\frac{\delta\mathcal{S}}{\delta b^a} \;&=\; \partial A^a \;, \qquad \frac{\delta\mathcal{S}}{\delta h^{a\mu}} = \partial_\mu e^a \;+\; \partial^\nu B^a_{\nu\mu} \;, \\
\frac{\delta\mathcal{S}}{\delta\omega^a} \;&=\; \lambda^a \;+\; \partial\xi^a \;, \quad \frac{\delta\mathcal{S}}{\delta\lambda^a} \;=\; -\partial\overline{\xi}^a \;-\; \omega^a \; .
\end{aligned}
\tag{4.63}
$$

Commuting now the above conditions with the Slavnov-Taylor identity (4.62) we get the antighost equations

$$
\begin{array}{ll}
\dfrac{\delta\mathcal{S}}{\delta\overline{c}^a} + \partial^\mu \dfrac{\delta\mathcal{S}}{\delta\hat{A}^{*a\mu}} = 0 \ , & \dfrac{\delta\mathcal{S}}{\delta\overline{\phi}^a} - \partial^\mu \dfrac{\delta\mathcal{S}}{\delta\hat{\xi}^{*a\mu}} = 0 \ , \\
\dfrac{\delta\mathcal{S}}{\delta e^a} = -\partial h^a \ , & \dfrac{\delta\mathcal{S}}{\delta\overline{\xi}^{a\nu}} + \partial^\mu \dfrac{\delta\mathcal{S}}{\delta\hat{B}^{*a\mu\nu}} = -\partial_\nu\lambda^a \ ,
\end{array}
\tag{4.64}
$$

so that, introducing the following shifted antifields

$$
\begin{aligned}
A^{*a\mu} &= \hat{A}^{*a\mu} + \partial^\mu\overline{c}^a \ , \ \ \xi^{*a\mu} = \hat{\xi}^{*a\mu} - \partial^\mu\overline{\phi}^a \ , \\
B^{*a\mu\nu} &= \hat{B}^{*a\mu\nu} + \left(\partial^\mu\overline{\xi}^{a\nu} - \partial^\nu\overline{\xi}^{a\mu}\right) \ ,
\end{aligned}
\tag{4.65}
$$

for the reduced BF action Σ we get

$$
\mathcal{S} = \Sigma + \int d^4x \left(b^a\partial A^a + h^a_\nu\partial_\mu B^{a\mu\nu} + \omega^a\partial\xi^a + h^{a\mu}\partial_\mu e^a + \omega^a\lambda^a + -\lambda^a\partial\overline{\xi}^a \right) \ , \tag{4.66}
$$

and

$$
\begin{aligned}
\Sigma = \int d^4x \ \ & \Big(-\frac{1}{4}\,\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}B^a_{\rho\sigma} \\
& +\frac{1}{2}B^{*a\mu\nu}\left((D_\nu\xi_\mu)^a - (D_\mu\xi_\nu)^a - f_{abc}B^b_{\mu\nu}c^c + f_{abc}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho\overline{\xi}^{b\sigma}\phi^c \right) \\
& +\xi^{*a\mu}\left((D_\mu\phi)^a + f_{abc}c^b\xi^c_\mu \right) + \frac{1}{8}f_{abc}\varepsilon_{\mu\nu\rho\sigma}B^{*a\mu\nu}B^{*b\rho\sigma}\phi^c \\
& -A^{*a\mu}(D_\mu c)^a + \frac{1}{2}f_{abc}C^{*a}c^bc^c + f_{abc}\phi^{*a}c^b\phi^c \Big) \ .
\end{aligned}
\tag{4.67}
$$

As usual, the reduced action Σ obeys the homogeneous Slavnov-Taylor

$$
\begin{aligned}
\tfrac{1}{2}\mathcal{B}_\Sigma\Sigma = 0 = \int d^4x \ \ & \Big(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta\Sigma}{\delta A^{*a\mu}} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta\Sigma}{\delta C^{*a}} + \frac{1}{2}\frac{\delta\Sigma}{\delta B^a_{\mu\nu}}\frac{\delta\Sigma}{\delta B^{*a\mu\nu}} \\
& + \frac{\delta\Sigma}{\delta\phi^a}\frac{\delta\Sigma}{\delta\phi^{*a}} + \frac{\delta\Sigma}{\delta\xi^a_\mu}\frac{\delta\Sigma}{\delta\xi^{*a\mu}} \Big) \ ,
\end{aligned}
\tag{4.68}
$$

with

$$
\begin{aligned}
\mathcal{B}_\Sigma = \int d^4x \ \ & \Big(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta A^{*a\mu}} + \frac{\delta\Sigma}{\delta A^{*a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta C^{*a}} + \frac{\delta\Sigma}{\delta C^{*a}}\frac{\delta}{\delta c^a} \\
& + \frac{1}{2}\frac{\delta\Sigma}{\delta B^a_{\mu\nu}}\frac{\delta}{\delta B^{*a\mu\nu}} + \frac{1}{2}\frac{\delta\Sigma}{\delta B^{*a\mu\nu}}\frac{\delta}{\delta B^a_{\mu\nu}} + \frac{\delta\Sigma}{\delta\phi^a}\frac{\delta}{\delta\phi^{*a}} \\
& + \frac{\delta\Sigma}{\delta\phi^{*a}}\frac{\delta}{\delta\phi^a} + \frac{\delta\Sigma}{\delta\xi^a_\mu}\frac{\delta}{\delta\xi^{*a\mu}} + \frac{\delta\Sigma}{\delta\xi^{*a\mu}}\frac{\delta}{\delta\xi^a_\mu} \Big) \ ,
\end{aligned}
\tag{4.69}
$$

and

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0 \ . \tag{4.70}$$

Let us turn now to the invariant vector cocycle (2.5), which in the present case takes the form

$$\Omega_\nu^{-1} \equiv \int d^4x \left(c^a \partial_\nu C^{*a} - A_\mu^a \partial_\nu A^{*a\mu} + \xi_\mu^a \partial_\nu \xi^{*a\mu} - \phi^a \partial_\nu \phi^{*a} - \frac{1}{2} \, B_{\mu\tau}^a \partial_\nu B^{*a\mu\tau} \right) \ , \tag{4.71}$$

and

$$\mathcal{B}_\Sigma \Omega_\nu^{-1} = P_\nu \Sigma = 0 \ . \tag{4.72}$$

Again, the vanishing of the cohomology of the operator $\mathcal{B}_\Sigma$ [4, 6] in the sector of the integrated local polynomials with ghost number -1 and with a free Lorentz index implies that, as in the case of the Chern-Simons model, Ω_ν^{-1} is an exact $\mathcal{B}_\Sigma$-cocycle

$$\Omega_\nu^{-1} = \mathcal{B}_\Sigma \Xi_\nu^{-2} \ , \tag{4.73}$$

for some local integrated polynomial Ξ_ν^{-2} of dimension 5 and ghost number -2. In fact, repeating the same procedure done in the previous exemple, Ξ_ν^{-2} is easily found to be

$$\Xi_\nu^{-2} = \int d^4x \ \left(C^{*a} A_\nu^a + \ \frac{1}{2} \varepsilon_{\sigma\tau\mu\nu} A^{*a\sigma} B^{*a\tau\mu} - \phi^{*a} \xi_\nu^a - \xi^{*a\mu} B_{\mu\nu}^a \right) \ . \tag{4.74}$$

Finally, converting the equation (4.73) into contact terms by means of the expression (4.69) and moving from the reduced action (4.67) to the complete one (4.61), we get the linearly broken vector Ward identity

$$\mathcal{W}_\nu \mathcal{S} = \Delta_\nu^{cl} \ , \tag{4.75}$$

with

$$\begin{aligned} \mathcal{W}_\nu = \int d^4x \ \ & \Bigg(\frac{1}{2} \varepsilon_{\sigma\tau\mu\nu} \left(\hat{B}^{*a\tau\mu} + \partial^\tau \overline{\xi}^{a\mu} - \partial^\mu \overline{\xi}^{a\tau} \right) \frac{\delta}{\delta A_\sigma^a} + A_\nu^a \frac{\delta}{\delta c^a} - \partial_\nu \overline{c}^a \frac{\delta}{\delta b^a} \\ & - \frac{1}{2} \varepsilon_{\sigma\tau\mu\nu} \left(\hat{A}^{*a\sigma} + \partial^\sigma \overline{c}^a \right) \frac{\delta}{\delta B_{\tau\mu}^a} - B_{\mu\nu}^a \frac{\delta}{\delta \xi_\mu^a} - \xi_\nu^a \frac{\delta}{\delta \phi^a} + \overline{\phi}^a \frac{\delta}{\delta \overline{\xi}^{a\mu}} \\ & - \partial_\nu \overline{\phi}^a \frac{\delta}{\delta \omega^a} + \partial_\nu e^a \frac{\delta}{\delta \lambda^a} - \left(\omega^a \delta_\nu^\tau + \partial_\nu \overline{\xi}^{a\tau} \right) \frac{\delta}{\delta h^{a\tau}} + C^{*a} \frac{\delta}{\delta \hat{A}^{*a\nu}} \\ & + \phi^{*a} \frac{\delta}{\delta \hat{\xi}^{*a\nu}} - \hat{\xi}^{*a\mu} \frac{\delta}{\delta \hat{B}^{*a\mu\nu}} \Bigg) \ , \end{aligned} \tag{4.76}$$

and the classical breaking Δ_ν^{cl} given by

$$\begin{aligned} \Delta_\nu^{cl} = \int d^4x \ \ & \Bigg(\hat{A}^{*a\mu} \partial_\nu A_\mu^a - C^{*a} \partial_\nu c^a - \hat{\xi}^{*a\mu} \partial_\nu \xi_\mu^a + \phi^{*a} \partial_\nu \phi^a + \frac{1}{2} \hat{B}^{*a\tau\mu} \partial_\nu B_{\tau\mu}^a \\ & - \frac{1}{2} \varepsilon_{\sigma\tau\mu\nu} \hat{B}^{*a\tau\mu} \partial^\sigma b^a + \frac{1}{2} \varepsilon_{\sigma\tau\mu\nu} \hat{A}^{*a\sigma} \partial^\tau h^{a\mu} \Bigg) \ . \end{aligned} \tag{4.77}$$

The equation (4.75) is recognized to be the well known topological vector susy Ward identity of the four dimensional BF systems [4]. Let us conclude by remarking that the above construction can be easily generalized to the BF systems in higher space-time dimensions, reproducing then the results of [4, 6]. In particular it is not difficult to check that, as it happens in the case of the three dimensional Chern-Simons model, the requirement that the breaking term (4.77) is at most linear in the quantum fields completely fixes the gauge parameters that one could introduce in the gauge fixing term (4.63) and which would be left free by the Slavnov-taylor identity [4]. In other words, the relative coefficients of the lagrangian multiplier part of the gauge fixing are uniquely determined by the vector susy Ward identity (4.75).

4.3 The b-c ghost system

We present here, as the last example of a topological model, the two dimensional b-c ghost system whose action reads

$$\mathcal{S}_{bc} = \int dz d\overline{z} b \overline{\partial} c \ , \tag{4.78}$$

where the fields $b = b_{zz}$ and $c = c^z$ are anticommuting and carry respectively ghost number -1 and +1. The action (4.78) is the ghost part of the quantized bosonic string action [1, 29] and, as it is well known, is invariant under the following nonlinear nilpotent BRST transformations

$$\begin{aligned} sc &= c\partial c \ , \\ sb &= -(\partial b)c - 2b\partial c \ . \end{aligned} \tag{4.79}$$

In particular, the right hand-side of the BRST transformation of the field b is easily identified with the component T_{zz} of the energy-momentum tensor corresponding to the action (4.78). This property allows for the topological interpretation of the b-c ghost system.

Transformations (4.79) being nonlinear, one needs to introduce two antifields ($b^* = b^{z*}_{\overline{z}},\ c^* = c^*_{zz\overline{z}}$) of ghost number 0 and -2

$$\mathcal{S}_{ext} = \int dz d\overline{z} \left(c^* c\partial c + b^*(c\partial b - 2b\partial c) \right) \ . \tag{4.80}$$

The complete action

$$\mathcal{S} = \mathcal{S}_{bc} + \mathcal{S}_{ext} \ , \tag{4.81}$$

obeys thus the Slavnov-Taylor identity

$$\int dz d\overline{z} \left(\frac{\delta \mathcal{S}}{\delta b} \frac{\delta \mathcal{S}}{\delta b^*} + \frac{\delta \mathcal{S}}{\delta c} \frac{\delta \mathcal{S}}{\delta c^*} \right) = \frac{1}{2} \mathcal{B}_{\mathcal{S}} \mathcal{S} = 0 \ , \tag{4.82}$$

$\mathcal{B}_{\mathcal{S}}$ denoting the linearized operator

$$\mathcal{B}_{\mathcal{S}} = \int dz d\overline{z} \left(\frac{\delta \mathcal{S}}{\delta b} \frac{\delta}{\delta b^*} + \frac{\delta \mathcal{S}}{\delta b^*} \frac{\delta}{\delta b} + \frac{\delta \mathcal{S}}{\delta c} \frac{\delta}{\delta c^*} + \frac{\delta \mathcal{S}}{\delta c^*} \frac{\delta \mathcal{S}}{\delta c} \right) \ , \tag{4.83}$$

and

$$\mathcal{B}_{\mathcal{S}} \mathcal{B}_{\mathcal{S}} = 0 \ . \tag{4.84}$$

	c	b	c^*	b^*
dim	0	1	1	0
$gh-numb$	1	-1	-2	0

Table 5: dimension and ghost number

Concerning now the vector cocycle (2.5), here written in components, we have

$$\begin{aligned} \Omega_z^{-1} &= \int dzd\overline{z}\,(b\partial b^* + c\partial c^*) \ , \\ \Omega_{\overline{z}}^{-1} &= \int dzd\overline{z}\,\left(b\overline{\partial} b^* + c\overline{\partial} c^*\right) \ , \end{aligned} \tag{4.85}$$

and

$$\begin{aligned} \mathcal{B}_{\mathcal{S}}\Omega_z^{-1} &= P_z\mathcal{S} = 0 \ , \\ \mathcal{B}_{\mathcal{S}}\Omega_{\overline{z}}^{-1} &= P_{\overline{z}}\mathcal{S} = 0 \ . \end{aligned} \tag{4.86}$$

As done before, we look then at the solution of the BRST exact conditions

$$\begin{aligned} \Omega_z^{-1} &= \mathcal{B}_{\mathcal{S}}\Xi_z^{-2} \ , \\ \Omega_{\overline{z}}^{-1} &= \mathcal{B}_{\mathcal{S}}\Xi_{\overline{z}}^{-2} \ , \end{aligned} \tag{4.87}$$

for some local integrated polynomials $\left(\Xi_z^{-2}, \Xi_{\overline{z}}^{-2}\right)$ of ghost number -2 and dimension 1. After some almost trivial algebraic manipulations one easily find

$$\begin{aligned} \Xi_z^{-2} &= -\int dzd\overline{z} c^* \ , \\ \Xi_{\overline{z}}^{-2} &= -\int dzd\overline{z} c^* b^* \ . \end{aligned} \tag{4.88}$$

Converting then the equations (4.87) into contact terms, we get the two linearly broken Ward identities

$$\begin{aligned} &\int dzd\overline{z}\frac{\delta\mathcal{S}}{\delta c} = -\int dzd\overline{z}\,(b\partial b^* + c\partial c^*) \ , \\ &\int dzd\overline{z}\left(b^*\frac{\delta\mathcal{S}}{\delta c} + c^*\frac{\delta\mathcal{S}}{\delta b}\right) = -\int dzd\overline{z}\left(b\overline{\partial} b^* + c\overline{\partial} c^*\right) \ , \end{aligned} \tag{4.89}$$

which are nothing but the topological vector susy Ward identities of the b-c ghost system [9, 12, 6].

5 The case of the rigid gauge invariance: the Landau Ghost Equation

In this section we shall analyse another interesting example of a classically linearly broken Ward identity whose contact terms can be characterized in the same purely algebraic cohomological way of the topological vector susy. This Ward identity is not related to a specific gauge model, being present in all the cases in which the rigid gauge invariance is an exact symmetry of the action. However in order to present its derivation in a detailed way we shall consider, as explicit example, the four dimensional pure Yang-Mills theory of the Sect.3, the generalization to other models being straightforward.

As it is well known the rigid gauge invariance is an exact symmetry of the Yang-Mills action (3.20), expressing the simple fact that all the fields and antifields belong to the adjoint representation of the gauge group, *i.e.*

$$\mathcal{R}_a^{rig}\Sigma = \int d^4x f_{abc}\left(A^b_\mu \frac{\delta\Sigma}{\delta A^c_\mu} + A^{*b}_\mu \frac{\delta\Sigma}{\delta A^{*c}_\mu} + c^b\frac{\delta\Sigma}{\delta c^c} + C^{*b}\frac{\delta\Sigma}{\delta C^{*c}} \right) = 0 \ , \tag{5.90}$$

Let us consider now the following integrated local polynomial, linear in the (shifted) antifields $\left(A^*_\mu,\ C^*\right)$, of ghost number -1, dimension four, and possessing a group index belonging to the adjoint representation

$$\Omega_a^{-1} = \int d^4x f_{abc}\left(A^{b\mu}A^{*c}_\mu - c^bC^{*c}\right) \ . \tag{5.91}$$

As in the case of the vector cocycle of the eq.(2.5), the above cocycle turns out to be BRST invariant. In fact we have

$$\mathcal{B}_\Sigma\Omega_a^{-1} = \mathcal{R}_a^{rig}\Sigma = 0 \ , \tag{5.92}$$

due to the rigid invariance (5.90). The operator $\mathcal{B}_\Sigma$ appearing in the above equation is the usual linearized Slavnov-Taylor operator defined in eq.(3.22). However, unlike the vector cocycle Ω_ν^{-1}, the coloured cocycle Ω_a^{-1} turns out to be always $\mathcal{B}_\Sigma$-exact, due to a very well known result of the BRST cohomolgy stating that there are no nontrivial cohomology classes with one free group index [19]. Therefore we can write

$$\Omega_a^{-1} = \mathcal{B}_\Sigma\Xi_a^{-2} \ , \tag{5.93}$$

for some local integrated polynomial Ξ_a^{-2} of ghost number -2 and dimension four. From the Table 1 it follows that the most general form for Ξ_a^{-2} can be written as

$$\Xi_a^{-2} = \beta\int d^4x C^*_a \ , \tag{5.94}$$

β being an arbitrary free parameters. Using the expression of the operator $\mathcal{B}_\Sigma$ of eq.(3.22) and of the reduced Yang-Mills action Σ given in eq.(3.20), the condition (5.93) becomes

$$\int d^4x f_{abc}\left(A^{b\mu}A^{*c}_\mu - c^bC^{*c}\right) = \beta\int d^4x \frac{\delta\Sigma}{\delta c^a} = \beta\int d^4x \ f_{abc}\left(A^{b\mu}A^{*c}_\mu - c^bC^{*c}\right) \ , \tag{5.95}$$

which gives $\beta = 1$. Moving now from the reduced action Σ to the complete Yang-Mills action $\mathcal{S}$ (3.20) and making use of the gauge condition (3.16) and of the definition (3.18), the equation (5.95) takes the form

$$\int d^4x \left(\frac{\delta\mathcal{S}}{\delta c^a} - f_{abc}\bar{c}^b\frac{\delta\mathcal{S}}{\delta b^c}\right) = \int d^4x \ f_{abc}\left(A^{b\mu}\hat{A}^{*c}_\mu - c^bC^{*c}\right) + \alpha\int d^4x \ f_{abc}b^b\bar{c}^c \ . \tag{5.96}$$

It should be remarked that the left hand side of this equation, besides a pure linear breaking, contains a term which is quadratic in the quantum fields, *i.e.* $\alpha f_{abc} b^b \overline{c}^c$. This term, being subject to renormalization, would have been defined as an insertion, spoiling then the usefulness of the eq.(5.96). Therefore we see that the requirement that the breaking term is at most linear in the quantum fields implies the vanishing of the gauge parameter, *i.e.* $\alpha = 0$, selecting thus the Landau gauge as the gauge fixing condition. Finally, setting $\alpha = 0$ in the gauge condition (3.16), we obtain the linearly broken identity

$$\int d^4x \left(\frac{\delta \mathcal{S}}{\delta c^a} - f_{abc} \overline{c}^b \frac{\delta \mathcal{S}}{\delta b^c} \right) \ = \ \int d^4x \ f_{abc} \left(A^{b\mu} \hat{A}^{*c}_{\mu} - c^b C^{*c} \right) \ , \tag{5.97}$$

which is recognized to be the so-called *ghost equation* Ward identity [18], always present in the Landau gauge. Let us conclude by recalling that the ghost equation (5.97), although valid only in the Landau gauge, turns out to be a very powerful tool in order to study the ultraviolet finiteness properties of a large class of gauge invariant local field polynomials belonging to the BRST cohomology [30]. These gauge invariant polynomials can be promoted at the quantum level to local insertions whose anomalous dimensions are independent from the gauge fixing parameter α. Therefore they can be studied without loss of generality in the Landau gauge. In particular, the ghost equation (5.97) allows to prove the vanishing of the anomalous dimensions of the invariant ghost monomials[3] $tr(c^{2n+1})$, which are deeply related to the gauge anomalies and to the generalized Chern-Simons terms [6]. This result is of great importance in order to prove the Adler-Bardeen nonrenormalization theorem for the gauge and the $U(1)$ axial anomalies [30, 31, 6]. Let us mention, finally, that the ghost equation (5.97) has been proven to be renormalizable [18, 6] and that, recently, has been extended to the $N = 1$ supersymmetric gauge theories in superspace [32].

6 Conclusion

A purely algebraic characterization of the topological vector susy and of the Landau ghost equation Ward identities has been given. These Ward identities, always linearly broken, are obtained by exploiting the BRST exactness condition of antifield dependent cocycles with ghost number -1. Applications to other kinds of linearly broken Ward identities as well as to other topological theories and to superspace supersymmetric models are under investigation.

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[3] We have used here the matrix notation $c = c^a T_a$, T_a being the generators of the gauge group, i.e. $[T_a, T_b] = i f_{ab}{}^c T_c$.

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