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Non Trivial Critical Exponents for Finite Temperature Chiral Transitions at Fixed Total Fermion Number

by

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Abstract

We analyze the finite temperature chiral restoration transition of the $(D = d + 1)$-dimensional Gross-Neveu model when the total fermion number is constrained to be fixed. This leads to the study of the model with a nonzero imaginary chemical potential. In this formulation of the theory, we have obtained that, in the transition region, the model is described by a chiral conformal field theory where the concepts of dimensional reduction and universality do apply and we have the realization of a "universal" scenario, in which the reduced theory belongs to the same universality class of the original one exhibiting the same symmetry breaking pattern.

Key-words: Dynamical symmetry breaking; Finite temperature QFT; Dimensional reduction; Phase transitions; Chemical potential.

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1 Introduction

Recently, Kocić and Kogut [1] have shown that, in the limit of large number of flavors (N), the universality class of the finite temperature chiral symmetry restoration transition in the $3D$ Gross-Neveu model is mean field theory, in contrast to the "standard" sigma model scenario which predicts the $2D$ Ising model universality class. The responsible for the breakdown of the "standard" scenario, dimensional reduction plus universality [2], would be the absence of canonical scalar fields in the model, since the σ auxiliary field has a composite nature. Indeed, as explained in [1], both the density and the size of the σ meson increase with the temperature in such a way that close to the restoration temperature the system is densely populated with overlapping composites. The constituent fermions are essential degrees of freedom, even in the scaling region. Thus, we conclude that, for this model, the effect of compactifying the Euclidean time direction is simply to regulate the infrared behavior and supress fluctuations. This fermionic model was first analyzed in [3] and it was further shown that the results are not artifacts of the large-N limit [4]. (In addition, it was shown how the Ising point is recovered in 4D Yukawa models beyond the leading order in the $1/N$ expansion and why this does not happen in Gross-Neveu models.) Lattice simulations of this 3D model have verified the predictions of the large-N expansion at zero temperature, at nonzero temperature, and at nonzero chemical potential separatedly [5]. The results for the critical indices have been checked and improved by larger scale simulations enhanced by histogram methods [6]. The conclusion is that the data are in excellent agreement with mean field theory and rule out the Ising model values.

In this letter we show that, despite the breakdown of the "standard" scenario, it is still possible to make use of the concepts of dimensional reduction connected to universality in Gross-Neveu models in what we call the "universal" scenario. According to the "standard" scenario, if dimensional reduction plus universality arguments hold, the finite temperature transition of the D-dimensional model would lie in the same universality class of the $(d = D - 1)$-dimensional Ising model. This is not what is found and the reason why is easy to understand in the finite temperature Matsubara formalism: fermions do not have a zero frequency Matsubara mode. On the other hand, in the "universal" scenario, the statement is slightly different. It says that if dimensional reduction plus universality arguments hold, the finite temperature transition of the D-dimensional theory would lie in the same universality class of the zero temperature $(d = D - 1)$-dimensionally reduced model. This is not new and, indeed, it was shown already in [1] that, for the $O(N)$ linear σ model at either zero and finite temperature, the rotational symmetry restoration transition is described by scalar conformal field theories. Similarly, we will show that for Gross-Neveu models at fixed total fermion number at either zero and finite temperature, the chiral symmetry restoration transition is described by chiral conformal field theories. The basic idea is to introduce a pure imaginary chemical potential in the partition function of the model. "Physically", this is equivalent to the imposition of a constraint which fixes the total fermion number. In this new "physical" system, we do have a zero frequency Matsubara mode associated to fermions affecting the infrared sensitiveness of thermodynamic quantities and, consequently, changing its critical indices. We observed that the new critical exponents are exactly the same as those corresponding to a zero temperature dimensionally reduced effective theory. In other words, after the decoupling of nonzero frequency Matsubara modes (dimensional reduction) the effective theory for the light degrees of freedom has a set of critical indices identical to that of the original zero temperature one (universality), where one simply has to replace $D = d + 1$ by d.

The motivation for considering an imaginary fermion density was that, on general grounds, the (bulk) behavior of the system at a fixed (real) chemical potential μ is identical to that at a fixed fermion number B, provided B is the mean fermion number for the system at chemical potential μ. In particular, the critical behavior of the two systems is expected to be the same [7]. This is not the first time one tries to relate an imaginary chemical potential to the phase structure of a quantum field theory. When considering the confining-deconfining phase transition in gauge theories with fermions one concludes that there is a relation between the order parameter of the transition (the Wilson line $< L >$) and the imaginary chemical potential. The high temperature theory has a first order phase transition as a function of the imaginary chemical potential [8]. Moreover, it is expected that the study of the properties of QCD at full complex chemical potential may shed some light on the problem of deconfining transition at zero temperature and high density. In this paper we use $\frac{h}{2\pi} = c = 1$.

2 Introducing the imaginary chemical potential

We start by considering a $(D = d + 1)$-dimensional fermionic model with fixed total fermion number B described by the canonical partition function

$$Z^B = Tr\left\{e^{-\beta H}\delta(\hat{N} - B)\right\}, \tag{1}$$

where β is the inverse of the temperature, H is some Hermitian Hamiltonian and

$$\hat{N} = \int d^d\mathbf{x}(\psi^\dagger\psi) \tag{2}$$

is the fermion number operator. Due to the discreteness of the baryon number B, we can use a periodic Fourier representation to the δ-function and rewrite eq. (1) as

$$Z^B = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{-i\theta B}\, Tr\left\{e^{-\beta H + i\theta\hat{N}}\right\}. \tag{3}$$

This is equivalent to introducting an imaginary chemical potential θ into the system. Z^B can be realized as an "integral" over systems with nonzero imaginary chemical potential θ

$$Z^B = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{-i\theta B} Z(\theta), \tag{4}$$

where

$$Z(\theta) = Tr\left\{e^{-\beta H + i\theta\hat{N}}\right\}, \tag{5}$$

is the grand canonical partition function for a system with unstable baryon number. It is remarkable that all thermodynamic properties of $Z(\theta)$ should, in the infinite volume limit, be the same as those obtained at a fixed (real) chemical potential which is chosen so that the mean fermion number is B. Thus, the properties of the theory at imaginary chemical potential θ determines its properties at non zero fermion density. Although this reasoning seems to be quite natural it has, as it will become clear soon, severe consequences like making possible a zero frequency Matsubara mode to appear.

3 Scaling properties at fixed total fermion number

We have chosen to develop our ideas using the Gross-Neveu model [9] because it is the simplest four-fermi model which concentrates all the features we want to analyze. The functional integral representation for $Z(\theta)$ in the case of the Gross-Neveu model is given by

$$Z(\theta) = \int_{B.C.} \mathcal{D}\sigma \mathcal{D}\psi \mathcal{D}\psi^\dagger \, e^{-S_E(\psi,\psi^\dagger,\sigma,\theta)}, \tag{6}$$

where the $(D = d + 1)$-dimensional Euclidean functional action is

$$S_E(\psi,\psi^\dagger,\sigma,\theta) = \int_0^\beta d\tau \int d^d\mathbf{x} \left\{ \bar{\psi} \left(-\gamma^0 \frac{\partial}{\partial \tau} + i\gamma . \bigtriangledown + m + g\sigma + i\frac{\theta}{\beta}\gamma^0 \right) \psi + \frac{1}{2}\sigma^2 \right\}, \tag{7}$$

and, as usual, the compact time interval runs from $\tau = 0$ to $\tau = \beta$ and we impose anti-periodic boundary conditions (B.C.) for the fermionic fields $\psi(\mathbf{x},\tau) = -\psi(\mathbf{x},\tau \pm \beta)$. For convenience, we use the Dirac representation of the γ matrices where, for odd dimensions, the role of γ_s is played by $-\mathbf{I}$, the identity matrix.

For bare fermion mass $m = 0$, there is a discrete chiral symmetry $\psi \to \gamma_s \psi$, $\bar{\psi} \to -\bar{\psi}\gamma_s$ which is spontaneously broken whenever a non-vanishing condensate $< \bar{\psi}\psi >$ is generated. The condensate

$$\langle \bar{\psi}(x)\psi(x) \rangle \propto \int \mathcal{D}\sigma \mathcal{D}\bar{\psi} \mathcal{D}\psi \left[\bar{\psi}(x)\psi(x) \right] \, e^{S_E(\psi,\psi^\dagger,\sigma,\theta)}, \tag{8}$$

serves as an order parameter of the transition. It is well known that this is the role played by the auxiliary field in (7). (Simply solve the equations of motion obtained from the original action). Thus, we will study the phase structure of the chiral symmetry restoration transition with the aid of the order parameter σ. The $1/N$ expansion can be conveniently developed by integrating over the ψ field using the path integral formalism [10]. At finite temperature the integration over the time direction is replaced by a sum over Matsubara frequencies. The lowest order in the $1/N$ expansion is simply the steepest descent approximation to the exponential. As usual, this involves an expansion of the action about its extremum which is itself determined by a gap equation. If, for simplicity, we define $\rho = \theta/\beta$ the gap equation reads

$$\Sigma = m + \frac{4g^2}{\beta} \sum_{n=-\infty}^{\infty} \int^\Lambda \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{\Sigma}{\mathbf{q}^2 + (\rho - \bar{\omega}_n)^2 + \Sigma^2}, \tag{9}$$

where $\bar{\omega}_n = \frac{2\pi}{\beta}(n + \frac{1}{2})$, and we have used the fact that, to the leading order, the fermion self-energy Σ comes from the σ auxiliary field tadpole: $\Sigma = m - g^2 < \bar{\psi}\psi >$. Here Λ is an ultraviolet cutoff. To find the temperature dependence of the order parameter σ one has to solve the gap equation near criticality. The critical temperature is determined by

$$1 = \frac{4g^2}{\beta_c} \sum_{n=-\infty}^{\infty} \int^{\Lambda} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{1}{\mathbf{q}^2 + (\rho_c - \bar{\omega}_{nc})^2}, \tag{10}$$

with $\bar{\omega}_{nc} = \frac{2\pi}{\beta_c}(n + \frac{1}{2})$ and $\rho_c = \theta/\beta_c$, so that the gap equation can be rewritten as

$$\frac{m}{\Sigma} + \frac{4g^2}{\beta_c} \sum_{n=-\infty}^{\infty} \int^{\Lambda} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{(\mathbf{q}^2 + (\rho_c - \bar{\omega}_{nc})^2)\left(\frac{\beta_c}{\beta}\right) - (\mathbf{q}^2 + (\rho - \bar{\omega}_n)^2 + \Sigma^2)}{(\mathbf{q}^2 + (\rho_c - \bar{\omega}_{nc})^2)(\mathbf{q}^2 + (\rho - \bar{\omega}_n)^2 + \Sigma^2)} = 0. \tag{11}$$

This form of the gap equation is particularly well suited for extracting critical indices since the problem reduces to the power counting of the infrared divergences in the integral over $\mathbf{q}$ [11]. We see that this integral is infrared finite in any dimension except when $\rho_c = \bar{\omega}_{nc}$ for some Matsubara frequency k. In this special case we do have a zero mode associated to fermions and this is exactly

$$k = \frac{1}{2}\left(\frac{\theta}{\pi} - 1\right). \tag{12}$$

The k mode goes soft at the transition temperature and becomes the only relevant degree of freedom in the scaling region. At low energies, all $n \neq k$ decouple and we can split the sum over n into two parts

$$\frac{m}{\Sigma} - \frac{4g^2}{\beta_c} \int^{\Lambda} \frac{d^d\mathbf{q}}{(2\pi)^d} \frac{\mathbf{q}^2 t + (\bar{\omega}_{nc}^2 t^2 + \Sigma^2)}{\mathbf{q}^2(\mathbf{q}^2 + (\bar{\omega}_{nc})^2 t^2 + \Sigma^2)} + \sum_{n \neq k} ... = 0, \tag{13}$$

with $(\bar{\omega}_{nc} - \bar{\omega}_n)^2 = t^2 \bar{\omega}_{nc}^2$ and $t \equiv \left(1 - \frac{\beta_c}{\beta}\right)$ being the deviation from the critical temperature. The first integral in eq. (13) is clearly infrared sensitive in the limit of vanishing Σ, while all the other integrals in the sum over $n \neq k$ are infrared finite. Hence, the critical exponents we read from it should be clearly non trivial.

The critical indices are defined by $\langle\bar{\psi}\psi\rangle\big|_{m\to 0} \sim t^{\beta}$, $\langle\bar{\psi}\psi\rangle\big|_{t\to 0} \sim m^{1/\delta}$, $\langle\Sigma\rangle|_{m\to 0} \sim t^{\nu}$ etc and, since $\Sigma \sim <\bar{\psi}\psi>$, $\beta = \nu$ to the leading order. Above four dimensions the integral is infrared finite and the scaling is mean field. Below four dimensions, however, the $\Sigma \to 0$ limit is singular and the integral scales as $(\bar{\omega}_{nc}^2 t^2 + \Sigma^2)^{\frac{d-2}{2}}$. At a critical point $t = 0$ away from the chiral limit we have $m \sim \Sigma^{d-1}$. Thus, we can easily see that, below four dimensions the β and δ exponents are

$$\beta = \frac{1}{d-2},\ \delta = d - 1. \tag{14}$$

The remaining exponents are easily obtained aswell, $\eta = 4 - d$ and $\gamma = 1$, and one can check that they obey hyperscaling. If we compare these indices with those obtained from the zero temperature case: $\beta = \frac{1}{D-2}, \delta = D - 1, \eta = 4 - D, \gamma = 1$ (for a recent review see [12]), we conclude that both sets define a chiral conformal field theory in d and $(D = d+1)$ dimensions respectively. We should also note that the exponents (14) are the same as for a zero temperature dimensionally reduced Gross-Neveu model in which the chiral symmetry restoration transition will occur as we approach a thermally renormalized coupling constant obtained during the reducing procedure [13]. This is the realization of the "universal" scenario.

Some final comments are in order. First, since we have been using the saddle point method, we still have to apply a stationary condition to Z^B. This will give another mean field equation, which comes from the condition

$$\frac{\partial}{\partial\theta}\left(S_E + \int_0^\beta d\tau \frac{i\theta}{\beta}\, B\right) = 0. \tag{15}$$

Note that the stationary condition over θ is very important because it gives an equation for the mean number of fermions ($<\hat{N}>= B$) which is the link between Z^B and $Z(\theta)$ in the infinite volume limit. Second, since the mean number of fermions $<\hat{N}>$ depends, in principle, on the temperature, we can solve eq. (15) for θ and find a temperature dependent $\theta(\beta)$, or equivalently $\rho(\beta)$ [14]. This feature also states that the in the definition of the zero frequency Matsubara mode k in eq. (12) we should replace θ by $\theta_c \equiv \theta(\beta_c)$. This is to say, the Matsubara mode which is left as the relevant degree of freedom in the reduced theory changes as the critical temperature changes. For each different critical temperature β_c^{-1} such that θ_c/π is an odd number, we will have a zero frequency fermionic Matsubara mode associated given by eq. (12).

4 Conclusions

We have shown that, even in the absence of canonical scalar fields, we do obtain non mean field critical exponents for the finite temperature chiral restoration transition in a four-fermi theory provided we introduce a pure imaginary chemical potential. This procedure gives rise to a fermionic zero frequency Matsubara mode which is interpreted, after the limit of high temperatures, as the only relevant degree of freedom of the reduced theory. The reduced theory is still a four-fermi theory at zero temperature and in a lower dimension with thermally renormalized parameters [15]. It also exhibits dynamical breaking of discrete chiral symmetry which is restored near the critical thermally renormalized coupling constant, in such a way that the universality class is the same as the original model.

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