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Is inert matter from indecomposable positive energy "infinite spin" representations the much sought-after dark matter?

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Indecomposable positive energy quantum matter comes in form of three families (one massive and two massless) of which the massless so called "infinite spin" family is either not mentioned at all or, if it is presented for reasons of completeness, it is immediately dismissed as "unphysical" without pinpointing at a violated principle.

Using novel methods which are particularly suited for problems of localization, it was shown that these representations cannot be generated by pointlke localized fields but rather require the introduction of spacelike semiinfinite stringlike generators which leads to their invisibility and makes them ideal candidates for dark matter.

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POSITIVE ENERGY MATTER OF THE THIRD KIND, HISTORY AND PRESENT UNDERSTANDING

Partially invisible quantum matter in the sense of this note is quantum matter which has no coupling to photons but whose weak interaction with visible quantum matter may still permit an indirect counter registration as the various proposals for the astrophysical dark matter in the form of WIMP.

A more extreme case, to which we want to direct the reader's attention in the sequel, is *completely invisible matter*. As all positive energy matter, this quantum matter has a gravitational manifestation, but it permits no compact localization and consequently cannot be registered in laboratory counters. Although such objects did not yet enter the present hunt for the physical identification of dark matter, they existed in a dorment incompletely understood form ever since Wigner in 1939 wrote his famous paper on unitary irreducible ray-representations [1] of the Poincaré group. He found that there are precisely three families of indecomposible positive energy representations. They are distinguished by the nature of the little group and its representation theory. Besides the best studied massive representation, for which the little group is the invariance group of a timelike vector and hence isomorphic to $SO(3)$, there exist two massless families whose little group of a lightlike vector is isomorphic to a noncompact euclidean subgroup group of the Lorentz group $E(2) \subset L(3,1)$, and since the representation of the P-group is induced from $E(2)$, this property is passed on to the P-representation. What distinguishes the two massless families is the *nature of the* $E(2)$ *representations*; whereas the finite helicity family which contains the known zero mass particles is a degenerate representation in which the euclidean translation is represented trivially (which compactifies the representation despite the noncompactness of the group), the third family results from a faithful $E(2)$ representation which preserves the group theoretic noncompactness and comes with unusual and conceptually challenging properties. The little Hilbert space is now an infinite dimensional space of Fourier components which describe an $E(2)$-irreducible infinite intrinsic abelian angular momentum tower; this is why we prefer "infinite spin" over Wigner's "continuous spin" (which refers to the continuous values of the Casimir invariant). The appearance of this infinite tower prevents the extension of the P-group to the conformal group despite the vanishing of the mass.

Only recently [2] it became clear why the more than 60 year struggle to understand the quantum field theoretic content of this huge family of indecomposable positive energy representations resisted attempts of incorporation into a Lagrangian quantization setting. It turned out that this third kind of matter is generated by noncompact extended singular objects which are spacelike semiinfinite covariant string-like localized fields, not to be confused with the objects of string theory (comments on this distinction can be found in [3]).

Already Wigner was fascinated by these extreme quantum objects for which apparently his intrinsic (independent of any quantization) representation-theoretical setting was the *only* access since any subsequent attempt to understand them in terms of a classical-quantum quantization-parallelism led him nowhere. When he found out in 1948 [4] that there were apparent problems with placing such objects into a thermal Gibbs state, he begun to have doubts about their physical utility. This difficulty is however solved by noticing that KMS states of indecomposable semiinfinite strings cannot be approximated by Gibbs states [2]. The subsequent investigation of localization properties which unfortunately consisted in trying to force this family into the standard Lagrangian quantization scheme was pursued by several generations of particle physicists and ended in inconclusive results [5].

The most laconic way to excorcise the apparent conceptual nuisance posed by this third third kind of matter can be found in Weinberg's 1995 excellent first volume [6]. In constrast to most other textbooks he does present these representations, but only to dismiss them afterwards with the remark that nature has apparently no use for them. This left the reader without a clue if any principle of nature was possibly violated by this matter.

Actually there was an unheeded early hint indicating a new direction in a 1970 paper [7] in which a mathematically precise no-go theorem was derived, proving that the infinite spin representation cannot be obtained within the setting of covariant pointlike local free fields (the Wightman framework). But only by the end of the 90, when the conceptual-mathematical tools were in place, a good part of their physical properties, in particular about their precise localization status, begun to unravel. The mathematical framework of the relevant quantum localization concept is fairly new (but not revolutionary in the sense string theorists use this word) and goes under the name of *modular localization* [8][9]. Since in the deafening noise of present particle physics fashions probably none of the readers has taken notice about significant conceptual progress in QFT, I will at least sketch the main idea without proof in the simplest spinless case (where also traditional methods would be sufficient) and only quote the results for the case at hand.

Intuitively modular localization results from the causal localization, which is inherent in relativistic QFT, after one liberates it from the use of particular field coordinatizations and localization in the standard formulation of QFT is a special case of modular localization. Starting from a Wigner representation space of wave functions of a scalar particle

$$H_{Wig} = \left\{ \psi(p) \mid \int |\psi(p)|^2 \, d\mu(p) < \infty \right\} \quad (1)$$
$$(\mathfrak{u}(\Lambda, a)\psi)(p) = e^{ipa}\psi(\Lambda^{-1}p),$$
$$(\mathfrak{u}(j_{W_0})\psi)(p) = \overline{\psi(-j_{W_0}p)}$$

one first defines two commuting operators which are associated to the $t-x$ wedge $W_0 = \{x \mid x_1 > |x_0|\}$ namely the unitary representers $\mathfrak{u}$ of the wedge-preserving Lorentz boost $\Lambda_{W_0}(\chi)$ and commutes with the antiunitary representers of the wedge-reversing reflection j_{W_0} across the edge of the wedge (third line). One then forms the [15] "analytic continuation" in the rapidity $\mathfrak{u}(\chi \to -i\pi)$ which leads to unbounded positive operators. Using the customary notation in modular theory, we define the following unbounded closed antilinear involutive operators in H_{Wig}

$$\mathfrak{s}(W_0) := \mathfrak{j}_{W_0}\delta_{W_0}^{\frac{1}{2}}, \; \delta_{W_0}^{it} := \mathfrak{u}_{W_0}(\chi = -2\pi t) \quad (2)$$
$$(\mathfrak{s}(W_0)\psi)(p) = \psi(-p)^*, \; dom\, \mathfrak{s}(W_0) = dom\, \delta_{W_0}^{\frac{1}{2}}$$

where the analytic properties of the domain of this unbounded modular involution $\mathfrak{s}(W_0)$ with $\mathfrak{s}^2(W_0) \subset \mathbf{1}$ consists precisely of that subspace of Wigner wave functions which permit that analytic continuation on the complex mass shell which is necessary in order to get from the forward to the backward mass shell ($\chi \to \chi - \pi i$). The main assertion of modular localization is that the ± 1 eigenspaces (real since $\mathfrak{s}(W_0)$ is antiunitary) are the real closed component of the dense $dom\, \mathfrak{s}(W_0)$

$$\mathfrak{K}(W_0) = \{\psi \mid \mathfrak{s}(W_0)\psi = \psi\}, \; \mathfrak{s}(W_0)i\psi = -i\psi \quad (3)$$
$$dom\, \mathfrak{s}(W_0) = \mathfrak{K}(W_0) + i\mathfrak{K}(W_0)$$
$$\mathfrak{s}(W_0)(\psi + i\varphi) = \psi - i\varphi$$

The dense subspace $dom\, \mathfrak{s}(W_0)$ (i.e. $\overline{dom\, \mathfrak{s}(W_0)} = H_{Wig}$) is precisely the one-particle component of the W_0 localization space associated with a scalar free field $A(x)$, or in terms of the real subspace[16]

$$\mathfrak{K}(W_0) = clos\,\{(A(f) + A(f)^*)\Omega \mid \sup pf \subset W_0\} \quad (4)$$

but the modular construction of localized subspaces avoids the use of singular field coordinatizations smeared with classically localized test functions and relies instead on the more intrinsic description in terms of domains of distinguished unbounded operators in the unique[17] Wigner space associated with the representation $(m, s = 0)$. The second line is the defining relation of what mathematicians call a *standard real subspace.* The standardness property is equivalent to the existence of an abstract (nongeometric) modular involution.

Applying Poincaré transformations one generates from $\mathfrak{s}(W_0)$ and $\mathfrak{K}(W_0)$ to the W-indexed families $\{\mathfrak{s}(W)\}_{W\in\mathcal{W}}$, $\{\mathfrak{K}(W)\}_{W\in\mathcal{W}}$. The localization spaces for smaller causally complete spacetime regions $\mathcal{O}$ (which could be trivial) are obtained by intersections $\mathfrak{K}(\mathcal{O}) = \cap_{W\supset\mathcal{O}}\mathfrak{K}(W)$. A remarkable property of all these spaces resulting from Wigner´s positive energy representation setting is the validity of *Haag duality*

$$\mathfrak{K}(\mathcal{O}') = \mathfrak{K}(\mathcal{O})' \quad (5)$$

where the dash on the region denotes the causal complement and that on the K-space stands for its simplectic complement within H_{Wig} i. e. $Im(K,\varphi)=0$ for all $\varphi\in\mathcal{K}(\mathcal{O})'=j_{\mathcal{O}}\mathfrak{K}(\mathcal{O})$

The final step is the functorial ascend to the net of spacetime localized operator algebras in the Wigner-Fock space (with creation/annihilation operators $a^*(p),a(p)$)

$$Weyl(\psi)=\exp i(a(\psi)+a^*(\psi)),\ \psi\in\mathfrak{K}(\mathcal{O})$$
$$\mathcal{A}(\mathcal{O}):=alg\left\{Weyl(\psi)\mid\psi\in\mathfrak{K}(\mathcal{O})\right\},\ \mathcal{A}:=\cup_{\mathcal{O}}\mathcal{A}(\mathcal{O})$$
$$K(\mathcal{O})=\overline{\{(A+A^*)\,\Omega\mid A\in\mathcal{A}(\mathcal{O})\}},\ \mathfrak{K}(\mathcal{O})=P_1K(\mathcal{O})$$

where alg denotes the operator (von Neumann) algebra generated by the unitary Weyl operators in the Wigner-Weyl space and P_1 is the projection of the Wigner-Fock space onto the Wigner one particle space. Note that there are no spacetime dependent field coordinates, the construction is as intrinsic and unique as the Wigner representation theory.

This modular construction exists for all three Wigner representation families. The $\mathfrak{K}(\mathcal{O})+i\mathfrak{K}(\mathcal{O})$ spaces for $\mathcal{O}=\mathcal{D}=$ double cone (the prototype of a simly connected causally complete compact region) for the first 2 families are dense in H_{Wig} whereas the third kind of Wigner matter yields a vanishing $\mathfrak{K}(\mathcal{D})$.for double cones. In that case the nontrivial space with the tightest localization $\mathfrak{K}(\mathcal{C})$ is associated with an (arbitrarily thin) noncompact spacelike cone $\mathcal{C}=x+\mathbb{R}_+\mathcal{D}$ with apex x and an opening angle which is determined by $\mathcal{D}$. All relations about $\mathfrak{K}$ pass to the K's in Wigner-Fock space.

There is no problem in adapting the modular setting to the presence of interactions; however there are no one-particle creators in compactly localized algebras (for details see [3]).

All the steps explained above in the spinless context can be carried out for the first two families with the help of intertwiners. These can also be constructed without modular theory by standard group theoretical techniques as explained in Weinberg's first volume of [6]. In that case there are are intertwiners from the unique Wigner representation to the denumerable infinite set of $(2A+1)\,(2\dot{B}+1)$ component spinorial fields indexed by $r=(A,\dot{B})$

$$\Phi_r(x)=\sum_{k=-s}^{s}\int d\mu(p)\{e^{ipx}u_{k,r}(p)a^*(p,r)+ \\ +e^{-ipx}u_c(p)_{k,r}b(p,r)\},\ \ \left|A-\dot{B}\right|\leq s\leq A+\dot{B}: \tag{6}$$

The covariantization of the $(m=0,s)$ family leads to stronger restrictions on $(A,\dot{B})$; In particular there is no covariant vector potential for $s=1$.On the other hand a covariant semiinfinite string-localized vector potential $A_\mu(x,e)$ poses no problems i.e. missing possibilies the gaps in the spinorial formalism can be filled with string-localized field generators. These covariant string-localized "potentials" associated to pointlike "field strengths" possess mild short distance property (scale dimension = one) and are certainly more intrinsic objects than the contrived BRST ghost extension whose advantage is that they upholding the pointlike formalism [3].

For the third kind of matter the only systematic construction is one which determines a continuous (α-dependent) family of intertwiners $u^\alpha(p,e)$ using their modular localization properties [2][10]. In this way one obtains a continuous set which depend in addition to the momentum p on a spacelike unit vector $e,\ e^2=-1$. It intertwines the Wigner transformation, which involves the representation D_κ of the noncompact $E(2)$ little group with the covariance transformation law in p and e and leads to a string field whose intrinsic stringlike extension can be seen by the appearance of a nontrivial commutator if one string enters the causal influence region of the other

$$D_\kappa(R(\Lambda,p))u^\alpha(\Lambda^{-1}p,e)=u^\alpha(p,\Lambda e) \tag{7}$$
$$\Psi(x,e)=\left(\frac{1}{2\pi}\right)^{\frac{3}{2}}\int_{\partial V_+}d\mu(p)(e^{ipx}u^\alpha(p,e)\circ a^*(p)+ \\ e^{-ipx}\overline{u^{\bar{\alpha}}(p,e)}\circ a(p))$$
$$[\Psi(x,e),\Psi(x',e')]=0,\ x+\mathbb{R}_+e\ \ ><\ \ x'+\mathbb{R}e'$$

That certain objects do not admit a presentation in terms of pointlike fields is not a speciality of these infinite spin representation. The d=1+2 "plektons" (particle associated to braid group statistics) are particles whose field theoretic description requires spacelike strings [11]. By forming charge-neutral bilinear composites one descends to compactly localizable observables. Another case is that of vector potentials in zero mass $s=1$ representation mentioned before. The string fields of the third kind of matter are however neither potentials associated with pointlike field strengths [7] nor do their algebras contain any compactly localizable subobservables; they are string-like in a very radical sense. The absence of pointlike localized composites can be supported by the following calculation. The most general covariant bilinear scalar object in the Wigner infinite spin creation/annihilation operators is of the form [10]

$$B(x) = \int\int_{\partial V} d\nu(k)d\nu(l)d\mu(p)d\mu(q) \\ e^{i(p+q)x}u_2(p,q)(k,l)a^*(p,k)a^*(q,l) \quad (8)$$

$$u_2(p,q)(k,l) = \int d^2zd^2we^{i(kz+lw)}F(B_p\zeta(z)\cdot B_q\zeta(w))$$

where $\zeta(z) = (\frac{1}{2}(z^2+1), z_1, z_2, \frac{1}{2}(z^2-1))$ and F is any smooth sufficiently decreasing function so that u_2 is square integrable in k,l for fixed p,q. This function is so constructed that u_2 absorbes the little group Wigner transformations (involving the little group with and the net result is a scalar field. The momentum integration is over both light cones $\partial V = \partial V_+ \cup\ \partial V_-$, and the we used the notation $a^*(-p) \equiv a(p)$. According to the Kallen-Lehmann representation its two-point function is automatically causal, but this only means that the distribution-valued *vector* $B(x)\Omega$ is point-localized and implies nothing about the localization of the operator. The string generated algebra would have compactly localized subalgebras in case of existence of tensor fields which are relatively local to the string. In case of our scalar binilear field B (8) the answer to the question:

$$\exists\, B\ s.t.\ \langle q,l|\,[B(x),\Psi(y,e)]\,|0\rangle = 0,\ x >< y+\mathbb{R}_+e\ ? \quad (9)$$

is negative and this is best understood by comparing the contraction functions with those for standard matter. By splitting off a plane wave exponential the matrixelement in (9) only depends on the x-y difference. The Fourier transform of this function is then polynomial in the Fourier momentum and this leads to the spacelike vanishing. The presence of the z, w little-group Fourier transforms in (8) as well as in the definition of $\Psi(x,e)$ indicates a more complicated non-polynomial dependence which after Fourier transform to the relative distance variable $x-y$ has no support properties at all. A more predestrian way to see this is to restrict the string and B to equal times. This situation cannot be improved by going from bilinear scalars to tensors, or by generalizing from bilinear to 2n-linear expressions in the $a^{\#}$. The best one can do is forming composite local strings which at least maintain the original string localization.

COMPLETE INVISIBILITY, NON-GRAVITATIONAL INERTNESS

The existence of local observables is a prerequisite for measuring properties of quantum matter. There are two notions of localization, the Born-localization of wave functions which in the relativistic context becomes frame-dependent Newton-Wigner localization and the above explained covariant modular localization[18]. It is only the first which comes with a (Born) probability interpretation and projection operators which are only in an macrocausal asymptotic sense of large time like separation between two such Born-localized events consistent with a luminal-bounded propagation whereas the strictly causal modular localization has nothing to do with projectors and localization probabilities but rather with domains of modular involutions. In the absence of interactions B-N-W and modular localized states are, although conceptually totally different, in the effective FAPP sense the same; the difference consists in an exponential tail which in case of massive matter is characterized by the Compton wave length of the particle. The idealization of a counter as a sharp modular localizator would lead to vacuum-polarization caused activation in the vacuum state even if no particle is around. To avoid this zero effect we follow [12] and identify counters with members of the quasi-local observable C^*-algebra $\mathcal{A}_{qua}$ which is the algebra whose operators can be approximated rapidly (faster than any inverse Euclidean power) by local observables; this somewhat larger C*-algebra contains observables which annihilate the vacuum and localize one-particle states.

The vacuum polarization at the causal boundary may appear as a conceptual nuisanse in the measurement process, but it is of crucial importance in the understanding of astrophysical manifestations of "localization thermality" (Unruh, Hawking temperature) and the use of holographic projections onto the causal boundary for the computation of the leading $c\ln\varepsilon$ behavior of the localization entropy in the attenuation size ε of the vacuum polarization cloud [13]. One of the marvelous conceptual achievments of modular theory is that it exposes a basic difference beween the quantum mechanical Born localization and its relation to an information theoretical kind of entanglement and the quantum field theoretical modular localization for which the restriction of pure global states to modular localized algebras creates a completely different type of thermal entanglement [14].

Semiinfinite strings of the infinite spin kind are not measurable by any counter since counters are at least quasilocal and to register a finite piece of an indecomposable semi-infinite object is a contradiction in terms. This leaves in principle the possibility of an indirect evidence via interaction with standard matter; in this case the third kind of matter has a chance of being detected with the planned underground dark matter detecting devices. The problems one faces to formulate interactions with standard matter may indicate that the third kind of matter is inert, apart from gravitational manifestation. But the final clarification on inertness is left to further research.

As we have seen in the previous section, the change from Wigner's first kind of perfectly localized massive matter to the third kind of massless infinite spin matter with noncompact localization is not quite that abrupt as it appears at first sight. between the perfectly localizable (visible) massive fields and the not cpmpactly localizable (invisible) third kind of quantum matter there is the small (countably infinite) but important class of finite helicity fields. In that case not all covariant $(A,\dot{B})$ pointlike fields fulfilling (6) exist, but they do exist as semiinfinitely localized covariant strings and there are many reasons why their introduction is indispensable in the presence of interactions. Surprisingly there are also two entirely intrinsic arguments which indicate their presence in the interaction-free Wigner setting. On the one hand one needs the potentials in order to express the inner product in Wigner space as an integral over a local bilinear conserved "current" and on the other hand there is violation of Haag duality 5 for non simply connected spacetime region in Minkowski space which does not occur in the massive case and whose presence is explained in terms of the existence of stringlike potentials [3]. The third kind of quantum matter radically extends this tendency of weakening of localization so that there are no "visible" field strength or composites at all.

There are reasons to believe that the confinement of gluons in the setting of gauge theory and perhaps even quark confinement have their deeper explanation in terms of partial *invisibility* so that a better insight of the issues raised in this note may not only be beneficial to understand how things work in heaven, but also may bring a totally new perspective on earthly LHC kind of standard model physics.

An object which carries energy cannot hide from the influence of gravitation and hence there is a deep paradigmatic problem here: how does gravity interact with a substance which is presumably totally inert relative to any compactly localizable matter? Since the infinite spin matter has no classical Lagrangian of which it can be considered to arise by quantization, it is tempting to think that the understanding of *quantum gravity* may inexorably linked to that of the third kind of positive energy matter.

The motor behind these investigations was not only their conceptual appeal but also the historical charm resulting from the possibility that the discoverer of the DM Fritz Zwicky and his contemporary, the protagonist of symmetriy and of the first intrinsic classification of particles theory Eugene Wigner, may have more in common than was hitherto expected. As a theoretical physicist interested in conceptual problems, I always admired Wigner's strict insistence in exploring known principles *before* doing mind games. Whereas the traditional way of valuating observations essentially did not change since the time of Zwicky, the same cannot be said about modern particle theory where the number of researchers following the intrinsic logic of theoretical principles a la Wigner unfortunately has decreased in favor of those who prefer mind games.

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[15] The unboundedness of the $\mathfrak{s}$ involution is of crucial importance for the encoding of geometry into domain properties of unbounded operators.
[16] The closedness of K does not imply that of $K+iK$.
[17] It was precisely this uniqueness which was Wigner's main motivation for bypassing the confusing plurality of the quantization setting (many different equations of motion with the same physical content) in favor of an intrinsic description. However the adaptation of the Born particle localization (the Newton-Wigner localization) was taking him away from covariant causal locality.
[18] This difference in localization also leads to a significant distinction in the information theoretic entanglement of QM (Born-localization) and the KMS thermal manifestation of causally restricted states (in particular the vacuum state) whose nonobservance leads to the black hole information paradox [14].