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The Fermion Boson Interaction Within the Linear Sigma Model at Finite Temperature

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We study the interaction of massless fermions with massless bosons at finite temperature. Specifically, we calculate the self-energy of massless fermions due the interaction with massless bosons at high temperature, which is the region where thermal effects are maximal. The calculations are concentrated in the limit of vanishing fermion three momentum and after considering the effective boson dressed mass, we obtain the damping rate of the fermion. It is shown that in the limit $k_0 \ll T$ the fermion acquire a thermal mass of order gT and the leading term of the fermion damping rate is of order $g^2T + g^3T$.

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I. INTRODUCTION

The fermion dispersion relation to one-loop order at high temperature has been studied several times in the literature [1,2]. The dispersion relation of a fermion, which gives its energy ω as it propagate through the medium as a function of its momentum k, is very important in different kinds of physical situations [3–5] and has been focused on various approximations and limits [6,7]. During the last few years, there has been some controversy if the damping rate is gauge dependent or not and other problems [8]. It has been proposed that a proper resummation cures such problems found in most one-loop calculations [9]. In this article we reinvestigate the self-energy of massless fermions interacting with massless bosons at high temperature in the framework of the linear sigma model. This is the simpler, but instructive situation of fermions interacting with scalar and pseudoscalar fields. We calculate the fermion dispersion relation for massless bosons and fermions in the limit $k_0, k \ll T$ and compute the fermion damping rate at rest, $k = 0$, for the case where the effective dressed boson mass is considered.

In a recent paper [10] we proposed a modified self-consistent resummation (MSCR) which resums higher-order terms in a non-perturbative way in order to cure the problem of breakdown of the perturbative expansion at finite temperature up to one-loop order in the perturbative expansion. We have shown that the MSCR, when applied to the study of the chiral fermion meson model, has the essential features which lead to the satisfaction of Goldstone's theorem and renormalization of the UV divergences, in the low and high temperature regions. We have explicitly shown that the scheme breaks down around T_c i.e., in the region of intermediate temperatures, since quantum fluctuations are known to play a major role there. In this region higher-order terms in the perturbative expansion are required.

It is well known that at high temperature the perturbative expansion can also be broken in theories with spontaneous symmetry breaking (SSB) or in massless field theories because powers of the temperature can compensate for powers of the coupling constant, even if the strength of the coupling is small [11,12]. Infrared divergence appears. When a set of infrared-divergent diagrams is summed up one gets an infrared-finite result. This is implemented in the MSCR by the recalculation of the self-energy. So, a motivation to study the fermion dispersion relation and damping rate at high temperature is the fact that the MSCR showed itself an efficient method to execute the resummation in a divergence-free way in this region. Thus, in the computation of the damping rate of the fermion, we shall use the dressed by interactions boson mass obtained in [10] rather than the zero mass parameter of the Lagrangian. In this way, this calculation is interesting since it provides another simple example for the application of the MSCR method.

This paper is organized as follows. In Section II we study the fermion self-energy and obtain the dispersion relation of the fermion with some of its interesting limits. In Section III we compute the fermion damping rate at rest. Section IV is devoted to conclusions.

II. THE FERMION SELF-ENERGY

We describe the fermion-bosons vertex by the interaction Lagrangian extracted from the linear sigma model [13]

$$\mathcal{L}^{int.} = -g\overline{\psi}\left[\sigma + i\gamma^5\vec{\pi}\cdot\vec{\tau}\right]\psi, \tag{1}$$

where ψ, σ, and π represent the quark, sigma and pion fields, respectively, and g is a non-dimensional positive coupling constant.

The fermion self-energy is defined by

$$\mathcal{D}(\omega_n,\mathbf{k})^{-1} = \mathcal{D}_0(\omega_n,\mathbf{k})^{-1} + \Sigma(\omega_n,\mathbf{k}), \tag{2}$$

where $\mathcal{D}_0(\omega_n,\mathbf{p})$ is the tree-level fermion propagator, expressed as

$$\mathcal{D}_0(\omega_n,\mathbf{k})^{-1} = \gamma_\mu k^\mu - m_\psi, \tag{3}$$

and $\Sigma = \Sigma_s + \gamma^\mu\Sigma_\mu$, with Σ_s, Σ_0 and $\vec{\Sigma}$ being the contributions proportional to the unit, γ^0 and $\vec{\gamma}$ matrices respectively.

To one-loop order the fermion self-energy expression is given [14] by

$$\begin{aligned}\Sigma(k_0,\vec{k}) = \left(\frac{\delta \ln Z_I^{2-loop}}{\delta\mathcal{D}_{0\psi}}\right)_{1PI} &= \\ -g^2 T\sum_n\int\frac{d^3p}{(2\pi)^3}\mathbf{D}_{0\sigma}(\omega_{n+l},\mathbf{p}+\mathbf{k})\mathcal{D}_{0\psi}(\omega_n,\mathbf{p}) &+ \\ -3g^2 T\sum_n\int\frac{d^3p}{(2\pi)^3}\mathbf{D}_{0\pi}(\omega_{n+l},\mathbf{p}+\mathbf{k})\mathcal{D}_{0\psi}(\omega_n,\mathbf{p}),\end{aligned} \tag{4}$$

since the logarithm of the two-loop interaction partition function is found to be [10]:

$$\ln Z_I^{2-loop} = \frac{1}{2}g^2\int_0^\beta d\tau_1 d\tau_2\int d^3x_1 d^3x_2\frac{\int[d\phi]e^{S_0}[(\overline{\psi}\sigma\psi)^2 + (\overline{\psi}i\gamma^5\vec{\pi}\cdot\vec{\tau}\psi)^2]}{\int[d\phi]e^{S_0}}. \tag{5}$$

In eq.(4) ω_n are the Matsubara frequencies, defined as $\omega_n = 2n\pi T$ for bosons and $\omega_n = (2n+1)\pi T$ for fermions. The one-loop fermion self-energy is shown in Fig.1.

An evaluation of eq.(4) at zero three momentum gives

$$\Sigma(k_0,|\mathbf{k}|=0) = (\Sigma_0+\Sigma_s)_\sigma + 3(\Sigma_0+\Sigma_s)_\pi = \tag{6}$$
$$\frac{g^2}{2}\int_0^\infty \frac{dpp^2}{\pi^2}\frac{n_\sigma}{\omega_\sigma}\frac{[k_0(-k_0^2+\omega_\sigma^2+\omega_\psi^2)+m_\psi(-k_0^2-\omega_\sigma^2+\omega_\psi^2)]}{[k_0^2-(\omega_\psi-\omega_\sigma)^2][k_0^2-(\omega_\psi+\omega_\sigma)^2]} +$$
$$\frac{g^2}{2}\int_0^\infty \frac{dpp^2}{\pi^2}\frac{n_\psi}{\omega_\psi}\frac{[2k_0\omega_\psi^2+m_\psi(k_0^2-\omega_\sigma^2+\omega_\psi^2)]}{[k_0^2-(\omega_\psi-\omega_\sigma)^2][k_0^2-(\omega_\psi+\omega_\sigma)^2]} + 3(m_\sigma\leftrightarrow m_\pi),$$

where $\omega_{\sigma,\pi}^2 \equiv \mathbf{p}^2 + m_{\sigma,\pi}^2$ and $\omega_\psi^2 \equiv \mathbf{p}^2 + m_\psi^2$.

Based on the facts that the linear sigma model serves as an effective model for the low energy phase of QCD and we are working in the high temperature limit ($T > 200MeV$), we shall consider in the calculations through this paper massless fermions ($m_\psi = 0$), which may be thought as the up and down quarks.

A. The Dispersion Relation Of Massless Fermions Interacting With Massless Bosons

As a first approximation, in this subsection by considering the interaction of massless fermions with massless bosons we get an effective thermal fermion mass and also calculate the dispersion relation of the fermions.

The poles of the massless fermion propagator ($m_\psi = \Sigma_s = 0$) gives the dispersion relation which occurs at the positive-energy root of

$$[k_0(k)+\Sigma_0(k_0,k)]^2 = \left|\vec{k}+\vec{\Sigma}(k_0,\vec{k})\right|^2, \tag{7}$$

where $k \equiv |\vec{k}|$.

For our intentions, it is sufficient to evaluate eq.(4) in the limit $k_0, k \ll T$, and consider the interaction of massless bosons and fermions in order to obtain an effective fermion thermal mass and dispersion relation. Thus,

$$\Sigma_0(k_0,k) = -\frac{1}{8}g^2\frac{T^2}{k}\ln\left|\frac{k_0+k}{k_0-k}\right|, \tag{8}$$

$$\vec{\Sigma}(k_0,k) = -\frac{1}{4}g^2\frac{T^2}{k^2}\left[\frac{k_0}{2k}\ln\left|\frac{k_0+k}{k_0-k}\right|-1\right]\vec{k} \equiv -\widetilde{\Sigma}\vec{k}, \tag{9}$$

where we have defined $\frac{1}{4}g^2\frac{T^2}{k^2}\left[\frac{k_0}{2k}\ln\left|\frac{k_0+k}{k_0-k}\right|-1\right] \equiv \widetilde{\Sigma}$.

Some limits of expressions (8) and (9) are [15]:

$$\begin{aligned}\Sigma_0(k_0=0,k)&=0,\\ \Sigma_0(k_0,k=0)&=-\frac{g^2T^2}{4k_0}.\end{aligned}\tag{10}$$

$$\begin{aligned}\vec{\Sigma}(k_0,k=0)&=0,\\ \vec{\Sigma}(k_0=0,k)&=\frac{g^2T^2}{4k}\hat{k}.\end{aligned}\tag{11}$$

Defining the fermion mass as the location of the pole in the limit $k=0$, we have $k_0+\Sigma_0(k_0,k\to 0)=0$, which implies

$$M_\psi^2=\frac{g^2T^2}{4}.\tag{12}$$

From (7), we see that the fermion dispersion relation is given by $k_0+\Sigma_0=k(1+\widetilde{\Sigma})$, that is

$$k_0-\frac{1}{2}\frac{M_\psi^2}{k}\ln\left|\frac{k_0+k}{k_0-k}\right|=k-\frac{M_\psi^2}{k}\left[\frac{k_0}{2k}\ln\left|\frac{k_0+k}{k_0-k}\right|-1\right],\tag{13}$$

which has the following well known form in the low momentum expansion [1,15]

$$k_0=M_\psi+\frac{1}{3}k+\frac{k^2}{3M_\psi}.\tag{14}$$

III. THE FERMION DAMPING RATE

Let us now proceed with the computation of the damping rate at rest ($k=0$). These calculations will be done considering the dressed boson mass in the internal lines of the fermion self-energy rather than the zero boson mass parameter of the Lagrangian. In the high temperature region, the bosons dressed masses (given by the MSCR) to be used in internal lines of the fermion self-energy read

$$m_\pi^2=m_\sigma^2\equiv M_B^2=\frac{g^2}{3}T^2.\tag{15}$$

So, eq.(6) may be written as

$$\begin{aligned}\Sigma_0(k_0,|\mathbf{k}|=0)=&-\frac{2g^2k_0}{\pi^2}\int_0^\infty dp p^2\frac{n_B}{\omega_B}\frac{k_0^2-M_B^2-2p^2}{[k_0^2-M_B^2]^2-4k_0^2p^2}+\\ &\frac{4g^2k_0}{\pi^2}\int_0^\infty dp p^4\frac{n_\psi}{\omega_\psi}\frac{1}{[k_0^2-M_B^2]^2-4k_0^2p^2},\end{aligned}\tag{16}$$

where n_B and n_ψ are the usual distribution functions for bosons and fermions given respectively by

$$n_B(\omega_B; T) = \frac{1}{e^{\beta\omega_B} - 1}, \tag{17}$$

$$n_\psi(\omega_\psi; T) = \frac{1}{e^{\beta\omega_\psi} + 1}. \tag{18}$$

with $\omega_B \equiv \sqrt{\mathbf{p}^2 + M_B^2}$ and $\omega_\psi \equiv |\mathbf{p}|$.

It is worth to note that for $k_0 \approx M_B$ in eq.(16), it is easy to see that $\Sigma_0(k_0, |\mathbf{k}| = 0)$ reduces to eq.(10-b) which is a stable state without singularities and we have that there is no decay.

An explicit evaluation of eq.(16) furnishes

$$\Sigma_0 = -\frac{g^2}{4\pi^2}\alpha \int_0^\infty dx \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B}\left[1 + \frac{\beta\alpha}{4}\left(\frac{1}{x - \beta\frac{\alpha}{2}} - \frac{1}{x + \beta\frac{\alpha}{2}}\right)\right] + \quad (1$$

$$-\frac{3g^2}{8\pi^2}\alpha \int_0^\infty dx n_\psi(\widetilde{\omega}_\psi)\left(\frac{1}{x - \beta\frac{\alpha}{2}} + \frac{1}{x + \beta\frac{\alpha}{2}}\right) - \frac{3g^2T^2}{\pi^2\alpha}\int_0^\infty dx x^2 \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} - \frac{3g^2T^2}{\pi^2\alpha}\int_0^\infty dx x^2 \frac{n_\psi(\widetilde{\omega}_\psi)}{\widetilde{\omega}_\psi},$$

with the definitions $\alpha \equiv k_0 - \frac{M_B^2}{k_0}$, $\widetilde{\omega}_B \equiv \sqrt{\beta^2\mathbf{p}^2 + \beta^2 M_B^2}$, $\widetilde{\omega}_\psi \equiv |\beta\mathbf{p}|$ and $\beta p \equiv x$. The interesting physics happens when $k_0 > M_B$. Otherwise (if $k_0 < M_B$) one would get imaginary (forbidden) frequency.

The expression for Σ_0 in (19) has singularities, and now we adopt the prescription $\alpha = \omega - i\gamma$, since in general k_0 is complex, where ω is the real frequency and γ is the real damping constant. With this assumption for α, eq.(19) is expressed as

$$\begin{aligned}
\Sigma_0 = -\frac{g^2}{4\pi^2}\alpha \int_0^\infty dx \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} &\left[1 + \frac{\alpha}{2}\left(\frac{\frac{2x}{\beta} - \omega}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} - \frac{\frac{2x}{\beta} + \omega}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}\right)\right] + \\
& i\frac{g^2}{8\pi^2}\alpha^2 \int_0^\infty dx \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B}\left[\frac{\gamma}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} + \frac{\gamma}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}\right] + \\
& -\frac{3g^2}{4\pi^2}\frac{\alpha}{\beta}\int_0^\infty dx n_\psi(\widetilde{\omega}_\psi)\left[\frac{\frac{2x}{\beta} - \omega}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} + \frac{\frac{2x}{\beta} + \omega}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}\right] + \\
& -i\frac{3g^2}{4\pi^2}\frac{\alpha}{\beta}\int_0^\infty dx n_\psi(\widetilde{\omega}_\psi)\left[-\frac{\gamma}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} + \frac{\gamma}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}\right] + \\
& -\frac{3g^2T^2}{\pi^2\alpha}\int_0^\infty dx x^2 \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} - \frac{3g^2T^2}{\pi^2\alpha}\int_0^\infty dx x^2 \frac{n_\psi(\widetilde{\omega}_\psi)}{\widetilde{\omega}_\psi}.
\end{aligned} \tag{20}$$

Now making use of the definition of the delta function

$$\delta(y) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{y^2 + \epsilon^2}, \tag{21}$$

and the definitions

$$F(x,\omega) \equiv \frac{\frac{2x}{\beta} - \omega}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} - \frac{\frac{2x}{\beta} + \omega}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}, \tag{22}$$

and

$$G(x,\omega) \equiv \frac{\frac{2x}{\beta} - \omega}{\left(\frac{2x}{\beta} - \omega\right)^2 + \gamma^2} + \frac{\frac{2x}{\beta} + \omega}{\left(\frac{2x}{\beta} + \omega\right)^2 + \gamma^2}, \tag{23}$$

we get

$$\begin{aligned} \Sigma_0 = &-\frac{g^2}{4\pi^2}\alpha \int_0^\infty dx \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B}\left[1 + \frac{\alpha}{2} F(x,\omega)\right] + i\frac{g^2}{8\pi}\left[\frac{\omega^2}{\sqrt{\frac{\omega^2}{4} + M_B^2}} \frac{1}{e^{\beta\sqrt{\frac{\omega^2}{4} + M_B^2}} - 1}\right] + \\ &-\frac{3g^2}{4\pi^2}\frac{\alpha}{\beta}\int_0^\infty dx n_\psi(\widetilde{\omega}_\psi) G(x,\omega) + i\frac{3g^2}{4\pi}\frac{|\omega|}{e^{\frac{\beta|\omega|}{2}} + 1} + \\ &-\frac{3g^2 T^2}{\pi^2 \alpha}\int_0^\infty dx x^2 \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} - \frac{3g^2 T^2}{\pi^2\alpha}\int_0^\infty dx x^2 \frac{n_\psi(\widetilde{\omega}_\psi)}{\widetilde{\omega}_\psi}. \end{aligned} \tag{24}$$

Here we use some results from high temperature expansion of one-loop integrals derived by Dolan and Jackiw in [12]:

$$\int_0^\infty dx \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} = \frac{\pi T}{2M_B} + \frac{1}{2}\ln\left(\frac{M_B}{4\pi T}\right) + O\left(\frac{M_B^2}{T^2}\right) \simeq \frac{\pi\sqrt{3}}{2g}, \tag{25}$$

where in eq.(25) we have used from (15) that $M_B = \frac{gT}{\sqrt{3}}$. This means that the first term in the r.h.s. of eq.(24) is $\frac{g\sqrt{3}}{8\pi} \ll 1$. On the other hand, for the last two terms in the r.h.s. of this same expression, we have

$$\frac{T^2}{\pi^2}\int_0^\infty dx x^2 \frac{n_B(\widetilde{\omega}_B)}{\widetilde{\omega}_B} + \frac{T^2}{\pi^2}\int_0^\infty dx x^2 \frac{n_\psi(\widetilde{\omega}_\psi)}{\widetilde{\omega}_\psi} = \left(\frac{T^2}{6} - \frac{M_B T}{2\pi} + O(g^2)\right) + \left(\frac{T^2}{12}\right) \to \frac{T^2}{4}. \tag{26}$$

The parts involving $F(x,\omega)$ and $G(x,\omega)$ are less important contributions in comparison to the dominant term that is proportional to $\frac{g^2T^2}{\omega}$, mainly for small ω. So, in a first glance one can neglect them. Putting these results in eq.(24) and assuming weak damping ($\gamma \ll \omega$), the leading terms of the real frequency and damping rate can be written respectively as

$$\omega^2 = \frac{9g^2T^2}{4}, \tag{27}$$

$$\gamma = \frac{3g^2}{32\pi}\left[\frac{\omega^2}{\sqrt{\frac{\omega^2}{4}+M_B^2}}\frac{1}{e^{\beta\sqrt{\frac{\omega^2}{4}+M_B^2}}-1}\right] + \frac{9g^2}{16\pi}\frac{\omega}{e^{\frac{\beta\omega}{2}}+1} \simeq \frac{3g^2}{32\pi}\frac{\omega}{e^{\frac{\beta\omega}{2}}-1} + \frac{9g^2}{16\pi}\frac{\omega}{e^{\frac{\beta\omega}{2}}+1}. \tag{28}$$

Equations (27) and (28) has the following interpretation: The real frequency ω is of order gT, in concordance with (12). The damping is proportional to the probability $n_\psi(\frac{1}{2}\omega)$ of having a fermion with energy $\frac{1}{2}\omega$ and a probability $n_B(\frac{1}{2}\omega)$ of having a boson with energy $\frac{1}{2}\omega$. These probabilities are weighted by numerical factors and the available phase space ω [14]. The distribution functions can be expanded for low energies, $n_B(\frac{1}{2}\omega) \simeq 2T/\omega$ and $n_\psi(\frac{1}{2}\omega) \simeq 1/2$ and the damping rate reduces to

$$\gamma \simeq \frac{3g^2T}{16\pi} + \frac{27g^3T}{128\pi}. \tag{29}$$

IV. CONCLUDING REMARKS

In this paper, we have considered the fermion boson interaction at finite temperature. First, we have calculated the self-energy of the fermions due the interaction with scalar-bosons and pseudoscalar-bosons in the framework of the linear sigma model.

Next, we have calculated the fermion dispersion relation in the limit $k_0, k \ll T$ of massless fermions interacting with massless bosons and some of its limits. Also, we have obtained the thermal fermion mass which is of order gT.

Finally, we have computed the frequency and the damping rate of the fermion at rest, considering the dressed boson masses in the internal lines of the fermion self-energy rather than the zero mass parameter of the Lagrangian. The damping rate of the fermion was found to be of order g^2T from the boson internal line of the fermion self-energy plus a part which is of order g^3T from the fermion internal line of the self-energy.

The calculation of the fermion damping rate at rest constitutes another simple but instructive application of the MSCR method.

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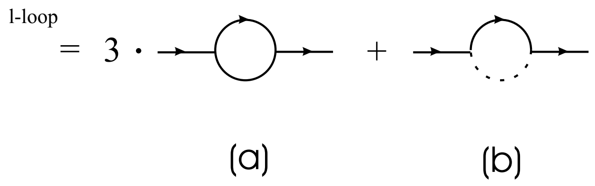

FIG. 1. The one-loop fermion self-energy. The pseudoscalar-boson contribution (a) and the scalar-boson contribution (b).