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NEUTRINOS IN ANTIPODAL UNIVERSES: PARITY TRANSFORMATIONS AND ASYMMETRIES

by

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ABSTRACT

The members of the class of rotating universes considered in the preceding paper and their antipodals, obtained by an inversion of the vorticity of the matter, are indistinguishable by gravitational observations. It is shown that massless neutrinos produced by weak interaction processes can be used as a probe to distinguish physically such antipodal universes. This is so because the transformation of a universe into its antipodal is not a symmetry of the system universe-plus-neutrinos of a given helicity.

Key-words: Neutrinos and rotating universes; Parity asymmetry; Neutrinos in antipodal universes.

1 INTRODUCTION

In a companion paper we have introduced on the group manifolds S^3xR and H^3xR a two parameter family of left-invariant g_L and rigth-invariant g_R metrics, and we have exhibited a coordinate system where left-invariant metrics and right-invariant metrics are connected by a coordinate inversion $\chi \rightarrow - \chi$ or $\eta \rightarrow - \eta$. We have denoted these universes *antipodal*. From the point of view of pure gravitational interaction the two geometries should be indistinguishable *unless* improper transformations are excluded from the covariance group of gravitation theory (general coordinate transformations).

The aim of the present note is to show that improper transformations are no longer symmetries of the system when we include neutrinos of a given helicity as test particles in these universes. In other words the system universe-plus-neutrinos of a given helicity is not symmetric under the exchange of g_L and $\bar{g}_R$. Therefore neutrinos could be used as a probe to distinguish antipodal universes physically. This idea is due to the fact that - as prescribed by weak interaction processes - a massless neutrino is an absolute left-handed screw and can be used to define an absolute sense of rotation, about a given direction. In our discussion we formulate Dirac's equation on the space-times with left-invariant and right-invariant metrics and a careful analysis of its transformation properties under parity transformations is made. Our result follows immediately from this analysis.

2 DIRAC'S EQUATION FOR NEUTRINOS AND INVARIANT MODES OF NEUTRINO FIELDS

Let us introduce neutrinos as test particles in these antipodal universes. Neutrinos in interaction with gravitational fields are described by spinor fields in the curved space-time. For a general review of spinors on a Riemannian space-time see Refs. [1,2]. Here we use four-component spinors from the point of view of the tetrad formalism [3]. We choose a tetrad field $e^{(A)}_{\alpha}(x)$ such that the line element is expressed as [4].

$$g = \eta_{AB}\theta^{A}\theta^{B} \quad , \tag{2.1}$$

where $\theta^{A} = e^{(A)}_{\alpha}dx^{\alpha}$. The neutrino wave function ψ provides a spinorial representation of the local Lorentz group

$$\theta'^{A} = L^{A}_{B}(x)\,\theta^{B} \quad , \tag{2.2}$$

with

$$L^{A}_{D}(x)\,\eta_{AB}L^{B}_{F}(x) = \eta_{DF} \quad , \tag{2.3}$$

Under (2.2)-(2.3) spinors ψ transform as

$$\psi'(x) = S(x)\psi(x) \tag{2.4}$$

where the 4 x 4 matrix S(x) must satisfy [5]

$$(L^{-1})^{A}{}_{B}(x)\gamma^{B} = S(x)\gamma^{A}S^{-1}(x)$$

On the other hand spinors ψ transform as scalar functions with respect to general coordinate transformations of the space-time. Dirac's equation for neutrinos coupled to gravitation is expressed as

$$\gamma^{A}(e^{\alpha}_{(A)}\partial_{\alpha} - \Gamma_{A})\psi = 0 \quad , \tag{2.5}$$

where the Γ_A are the Fock-Ivanenko coefficients [6,7] given by $\Gamma_A = \frac{1}{4}\, e^{\alpha}_{(B)}{}_{||\beta}\, e_{\alpha(c)} e^{\beta}_{(A)} \gamma^{B}\gamma^{C}$. Introducing a new parameter λ, and coordinates $x^{\alpha} = (\bar{\chi},\bar{\eta},r,z)$ defined by

$$A = \frac{\alpha^2 - \varepsilon^2\beta^2}{\beta^2} \quad , \quad \bar{\chi} = \beta\chi \quad , \quad \bar{\eta} = \beta\eta \quad ,$$

we choose for the left- and right-invariant metrics (cf. Eq. (2.16) of the preceding paper) the tetrad basis

$$\theta^0 = \sqrt{\varepsilon^2 + \lambda\cosh^2(\tfrac{\varepsilon r}{2})}\cosh(\tfrac{\varepsilon r}{2})\,d\bar{\chi} + \frac{e\,\lambda\sinh^2(\varepsilon r)}{4\sqrt{\varepsilon^2 + \lambda\cosh^2(\frac{\varepsilon r}{2})}\,\cos(\frac{\varepsilon r}{2})}\,d\bar{\eta} \,.$$

$$\theta^1 = \frac{\sqrt{\varepsilon^2+\lambda}\,\sinh(\frac{\varepsilon r}{2})}{\varepsilon\sqrt{\varepsilon^2+\lambda\cosh^2(\varepsilon r/2)}}\,d\bar{\eta} \tag{2.6}$$

$$\theta^2 = \beta/2\; dr$$

$$\theta^3 = dz \;,$$

where $g(e=-1) = g_R$ and $g(e=+1) = g_L$. The main advantage of the choice (2.6) is to have a tetrad basis $e_\alpha^{(A)}(x)$ diagonal whenever $\lambda = 0$. In the basis (2.6) the Fock-Ivanenko coefficients are given by

$$\Gamma_o = \bar{A}_1(r)\gamma^2\gamma^0 + e\bar{A}_2(r)\gamma^2\gamma^1 \ ,$$

$$\Gamma_1 = \bar{A}_3(r)\gamma^2\gamma^1 + e\bar{A}_2(r)\gamma^2\gamma^0 \ , \tag{2.7}$$

$$\Gamma_2 = e\bar{A}_2(r)\gamma^0\gamma^1 \quad , \quad \Gamma_3 = 0 \ ,$$

where the functions $\bar{A}_i(r)$, $i = 1,2,3$ are given explicitly in [8].

Dirac's equation for neutrinos assume then the form

$$i\partial_\chi\psi = i\{-M\gamma^0\gamma^1\partial_\eta - N\gamma^0\gamma^2\partial_\chi - eP\gamma^0\gamma^1\partial_\lambda - Q\gamma^0\gamma^3\partial_z +$$

$$+ (A_3 - A_1)\gamma^0\gamma^2 - eA_2\gamma^2\gamma^1\}\psi \tag{2.8}$$

where, for the sake of notation simplicity, we have introduced

$$A_i = \bar{A}_i/e^0_{(0)} \ ,$$

$$P = e\, e^0_{(1)}/e^0_{(0)} \ , \qquad Q = 1/e^0_{(0)} \ , \tag{2.9}$$

$$M = e^1_{(1)}/e^0_{(0)} \ , \qquad N = e^2_{(2)}/e^0_{(0)} \ .$$

The matrix $(e^{\alpha}_{(A)}(x))$ is the inverse of $(e^{(A)}_{\alpha}(x)$ defined by (2.6) and the quantities (2.9) are functions of r only, and do not depend on the parameter e [9].

Let us restrict our analysis to the cases $\alpha^2 \geq \beta^2$, for the hyperbolic family. In other cases the analysis still applies for the regions of the space-times where the coordinate lines associated to the vector fields $\partial/\partial\chi$ and $\partial/\partial\eta$ have time-like and space-like character, respectively. Since the metrics g(e) admit the Killing vectors $\partial/\partial\chi$, $\partial/\partial\eta$ and $\partial/\partial z$ we consider neutrino fields in the invariant global modes [10,11]

$$\psi(\chi,\eta,r,z) = \phi(r)e^{-iE\chi}e^{-ik\eta}e^{-i\nu z} , \tag{2.10}$$

where $\phi(r)$ is a four-spinor depending on r only. These modes can be interpreted [10] as energy - and momentum invariant modes with energy eigenvalue E and momenta eigenvalues k and ν. For the modes (2.10), equation (2.8) can be reexpressed as

$$E\phi = H\phi , \tag{2.11}$$

where

$$H = \{i\gamma^2\gamma^0(A_1-A_3 + N\partial/\partial r) - \gamma^0\gamma^1(eEP + kM) - ie\gamma^2\gamma^1 A_2 + \nu Q\gamma^0\gamma^3\} . \tag{2.12}$$

H can be identified with the Hamiltonian operator, acting on the space of neutrino wave functions (2.10), in the sense that the time development of any operator acting on the space of neutrino functions (2.10) is proportional to the commutator of H and the operator. Since γ^5 commutes with H we choose from (5.10) simultaneous eigenstates of H and γ^5, namely

$$H\phi(r,L) = E\phi(r,L) \; ,$$

$$\gamma^5\phi(r,L) = L\phi(r,L) \; , \tag{2.13}$$

with $L^2 = 1$. The operator γ^5 is proportional to the helicity operator for neutrinos [12], in the local Lorentz frame determined by (5.6), and the eigenvalue L is associated to the helicity of the neutrino field.

3 SYMMETRY TRANSFORMATIONS OF THE SYSTEM NEUTRINO-UNIVERSE. PARITY AND ASYMMETRIES

To begin with, let us define a symmetry transformation of a system from the point of view of passive and active transformations: we say that a coordinate transformation (*passive* transformation) is a symmetry of the system if there exists a corresponding *active* transformation of the system into another system equivalent to it [13]. In the present case the passive transformation $\eta \to -\eta$ (or $\chi \to -\chi$) is a symmetry of the gravitational field (as far as pure gravitational interaction is concerned) in the sense that it is equivalent to inverting the rotation of the universe. Indeed, as shown in the Appendix of the preceding paper, the solutions g_L and g_R can be interpreted as describing universes whose matter content has opposite vorticities relative to the local compass of inertia (the axes of the compass of inertia being determined, for instance, by gyroscopes) and are indistinguishable in the context of gravitational interaction only.

Let us consider now the neutrino fields

$$\psi(E,k,\nu,L) = \phi(r,L)e^{-iE\chi}e^{-ik\eta}e^{-i\nu z} \tag{2.14}$$

eigenstates of energy E, momenta k and ν, and helicity L in the space-times of (2.6). On the system *universe with geometry* $g_R = g(e=-1)$ *plus neutrinos* let us perform the following active (physical) transformations P:

(i) inversion of the rotation of the matter content of the universe (cf. Appendix of the preceding paper);

(ii) inversion of the momentum of neutrinos, associated to the Killing symmetry $\partial/\partial\eta$.

In Dirac's equation for g_R the effect of inverting the rotation is obtained making $e=+1$ in (2.11)-(2.12), which corresponds to transforming the metric g_R into the metric g_L (as already discussed g_R and g_L describe universes with opposite vorticity). On the other hand, the inversion of the momentum of neutrinos is obtained by the substitution $k\to -k$ in (2.12). The resulting Dirac's equation after the operations P is given by

$$E\phi'(L') = \{i\gamma^2\gamma^0(A_1 - A_3 + N\partial/\partial r) + \gamma^0\gamma^1(-EP + kM) - iA_2\gamma^2\gamma^1 + \nu Q\gamma^0\gamma^3\}\phi'(L') \tag{2.15}$$

The new spinor $\phi'(L')$ is related to $\phi(L)$ (cf. (2.12) for $e=-1$) by

$$\phi'(L',k) = \gamma^1 \phi(L,-k) \qquad (2.16)$$

Since γ^1 anticommutes with γ^5, the neutrino states $\phi'(L')$ and $\phi(L)$ have opposite helicity. Therefore, the physical operations consisting in inverting the momentum k of neutrinos and the rotation of the matter of the universe (transforming g_L into g_R and vice-versa) also transform neutrino states $\phi(L)$ in g_R into neutrino states with opposite helicity in g_L. These active transformations correspond to the passive transformation $\eta \to -\eta$ over the system universe with metric g_L or g_R plus neutrinos. If neutrinos in each of these universes can have both types of helicity, the transformation is a symmetry of the system universe-plus-neutrinos, and the universes with metric g_L and g_R are physically indistinguishable. If however neutrinos are assumed to have only one type of helicity (as prescribed by weak interactions experiments) the transformations discussed above are no longer a symmetry of the system. Indeed the configuration g_L plus left-handed neutrinos is transformed under the active transformations P, into the configuration g_R plus right-handed neutrinos, which is forbidden. The systems g_L plus left-handed neutrinos and g_R plus left-handed neutrinos are therefore distinguishable. Of course a passive (coordinate transformation) $\eta \to -\eta$ is a mere change of labels and can always be performed (even in the presence of weak interactions processes). It corresponds just to changing conventions as, for instance, the definition of the signal of the neutrino helicity, and does not produce any physically distinct situation.

We must finally remark that, although the hyperbolic class of metrics with $\alpha^2 < \beta^2$ are solutions of Einstein-Cartan theory only [14], the preceding analysis is also valid in this case because it can be easily verified that the Fock-Ivanenko coefficients calculated for these cases in the context of Einstein-Cartan theory [15] differ from (2.7) by constant terms added to $\tilde{A}_2(r)$ only, corresponding to the associated torsion fields of the solutions.

5 CONCLUSIONS

In the present paper we have shown that neutrino can be used to distinguish physically the antipodal universes. We define active transformations corresponding to the passive operation of coordinate inversion and show that these active transformations are not a symmetry of the system universe-plus-left-handed neutrinos (neutrinos considered as test particles) because a universe with metric g_L plus neutrinos of a given helicity is related to a universe with metric g_R plus neutrinos of *opposite* helicity. Weak interaction processes then allow us to distinguish these universes physically.

REFERENCES

1) D.R. Brill and J.A. Wheeler, Rev. Mod, Phys. 29, 465 (1957)

2) R. Penrose and W. Rindler, Spinors and Space-Time, Vol.1, Cambridge University Press (Cambridge 1984).

3) P.A.M. Dirac in Recent Developments in General Relativity (Pergamon, New York 1962) pp. 191-200.

4) Capital Latin indices are tetrad indices and run from 0 to 3; they are raised and lowered with the Minkowski metric $\eta^{AB}, \eta_{AB} = \mathrm{diag}(+1,-1,-1,-1)$. Throughout the paper we use units such that $\hbar = c = 1$. A double bar denotes covariant derivative.

5) γ^A are the constant Dirac matrices; we use a representation such that $(\gamma^A)^T = \gamma^0\gamma^A\gamma^0$, with $(\gamma^0)^2 = -1$, k =1,2,3, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

6) V. Fock and D.Ivanenko, Comp. Rend. 188, 1470 (1929).

7) D.R. Brill and J.M. Cohen, J. Math. Phys. 7, 238 (1966).

8) The explicit expressions of the $\tilde{A}_i$'S are

$$\bar{A}_1 = -\frac{\varepsilon}{2}\,\frac{(2\lambda\cosh^2\frac{\varepsilon r}{2} + \varepsilon^2)\,\sinh\frac{\varepsilon r}{2}}{(\lambda\cosh^2\frac{\varepsilon r}{2} + \varepsilon^2)\beta\,\cosh\frac{\varepsilon r}{2}}$$

$$\bar{A}_2 = \frac{\lambda}{2}\sqrt{\lambda+\varepsilon^2}\;\frac{\cosh^2\frac{\varepsilon r}{2}}{(1+\varepsilon^2\lambda\cosh^2\frac{\varepsilon r}{2})\beta}$$

$$\bar{A}_3 = -\frac{\varepsilon(\lambda+\varepsilon^2)}{2}\;\frac{\cosh\frac{\varepsilon r}{2}}{(\lambda\cosh^2\frac{\varepsilon r}{2} + \varepsilon^2)\beta\,\sinh\frac{\varepsilon r}{2}}$$

9) Since our only concern here are the symmetries of Dirac's equation, the explicit expression of the functions (2.9) are not important for our purposes, and will not be given here. They can be immediately obtained from (2.6), (2.9) and Ref. [8].

10) G. Gibbons, Comm. Math. Phys. 44, 245(1975).

11) M. Henneaux, GR G 9,1031 (1978).

12) In fact Dirac's equation (2.8) for energy eigenstates can be rewritten

$$E\psi = \gamma^5 \vec{\Sigma}.\vec{\pi}\psi$$

where $\vec{\pi}$ is the generalized local momentum.

$(\vec{\pi})^j = (-iM\partial_\eta - ieP\partial_\chi, -iN\partial_r + iA_3 - iA_1, eA_2\gamma^5 + iQ\partial_z)$, and

$\Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$ is the spin matrix.

It is then obvious that eigenstates of γ^5 are also eigenstates of the helicity operator $\vec{\Sigma}.\vec{\pi}$ defined in the local Lorentz frame (2.6).

13) Equivalent in the sense that no physical experiment is able to distinguish one system from the other.

14) J. Duarte de Oliveira, A.F.F. Teixeira and J. Tiomno, Phys. Rev. D34, 3661 (1986).

15) F. Hehl and B.K. Datta, J. Math. Phys. 12, 1334 (1971).