ON THE LOW ENERGY MU-MESONS FROM PI-MESON DECAY*

G.E.A.Fialho**
Centro Brasileiro de Pesquisas Fisicas
Rio de Janeiro, D.F.

It has been recently proposed[1] that the short μ-mesons tracks observed in the decay of π-mesons[2] should be explained as radiative decay:

$$\pi \rightarrow \mu + \nu + h\nu \tag{1}$$

in competition with radiationless decay

$$\pi \rightarrow \mu + \nu \tag{2}$$

The agreement of the theoretical value for the frequency of μ-mesons with energy smaller then 3.5 Mev., computed under the assumption that the μ-meson has spin 1/2 and no anomalous magnetic moment, whith the experimental result is satifactory. The fact that the theoretical result is somewhat smaller than the experimental one lead us to examine the possibility that the μ-meson has an anomalous magnetic moment ($je\hbar/2\mu c$) as suggested in the paper by Fialho and Tiomno[1] (here referred as I). We hope that these results will be helpful to decide on the properties of the μ-mesons when more precise results will be knowm about the frequency and energy distribution of "anomalous" π-decays.

We make the same assumptions as in Paper I, except that we take for the Hamiltonian of interaction of the μ-meson field with the electromagnetic field A_μ:

$$e\bar{\psi}\gamma^\mu\psi A_\mu + (je\hbar/2\mu c)\bar{\psi}\gamma^\mu\gamma^\nu\psi F_{\mu\nu} \tag{3}$$

The probability per unit time per decay of the radiative process I, is readily found by Feynmn's Method (from now on we take $\hbar=c=1$):

$$dW = (\pi/8\rho\mu^4 E_\nu E\omega)\sum|M|^2 D \tag{4}$$

where μ is the mass of the μ-meson, $\mu\rho$ is the mass of the π-meson, assumed at rest; μE_ν, μE and $\mu\omega$ are the energies of the neutrino, μ-meson and photon, respectively; the summation is to be extended to the polarizations of the photon and spins of the μ-meson and neutrino. Also

$$D dp\, d(\cos\theta) = E_\nu \omega^3 p^2/8\pi(\mathbf{k},p_\nu)\, dp\, d(\cos\theta) \tag{5}$$

is the number of states where the μ-meson has a mometum between μp and $\mu(p+dp)$ and an angle between θ and $\theta + d\theta$ with the mometum $\mathbf{k}$ of the photon $(\mathbf{k},p_\nu)$ is the scalar product $k_\lambda p_\nu^\lambda$

Finally

$$M = \frac{2\pi e G}{\mu}\left(\psi_\nu\middle|\gamma_5 \frac{\not p+\not k+1}{(p+k)^2-1}\left[j(\not e\not k - \not k\not e) + 2\not e\right]\middle|\psi_\mu\right) \tag{6}$$

is the matrix element for the transition expressed in the momentum space, where

$$\not p = p_\mu\gamma^\mu \text{ and } (p+k)^2 = (p_\lambda+k_\lambda)(p^\lambda+k^\lambda)$$

We find:

$$W(P,\theta)\, dp\, d(\cos\theta) = G^2e^2P/\pi E\rho \cdot \left\{f_c+(1+j)^2 f_m + j^2 f_1\right\} dp\, d(\cos\theta) \tag{7}$$

where f_c and f_m are the same as in expressions (6a), (6b) of I, and

$$f_1 = (E_o-E)\rho^3 p^2 \sin^2\theta \Big/ \left\{(\rho-E+P\cos\theta)^3 (E-P\cos\theta)\right\} \tag{7a}$$

Here $\mu P_o = (\rho^2-1)\mu/2\rho$ and $\mu E_o = (\rho^2+1)\mu/2\rho$ are respectively the maximum momentum and energy of the emmited μ-meson.

Integrating over the angle θ we obtain for the probability per unit range of P/P_o:

$$w(P/P_o) = w_c+(1+j)^2 w_m + j^2 w_1 \tag{8}$$

with

$$(w_c;w_m;w_1) = W_o e^2\ P^2/2\pi E\ P_o\ (F_c;F_m;F_1) \tag{9}$$

where
$$W_o = G^2/2\mu(\rho^2-1)^2/\rho^3 \tag{10}$$

is the probability per unit time of process (2); F_c and F_m are the same as in expressions 7a, 7b, of I and

$$F_1 = (\rho E-1) - (\frac{E_o-E}{P}) \log\left\{ (E+P-1/\rho) \ / \ (E-P-1/\rho)\right\} \tag{11}$$

The probability that the μ-meson will have a momentum less than P was found by graphical integration:

$$W(P) = W_c+(1+j)^2\ W_m+j^2\ W_1 \tag{12}$$

with

$$(W_c;W_m;W_1) = \int_0^{P/P_o} (w_c;w_m;w_1)\ dP/P_o \tag{13}$$

In Fig.1 (upper left corner), we give the values of W_c, W_m and W_1 plotted against P/P_o. In the same figure (lower part), the curves for W(P) are given for j=0,1,2; The experimental point obtained by Fry[2] is also indicated. As we can see, the magnitude of the experimental error do not permit yet to decide which is the value of the magnetic moment of the μ meson Others experimental points will be helpful to decide which curve correspond to the actual case.

The author is indebted to Prof. J.Tiomno for the sugestion and orientation of the present work, and to Prof.R.P.Feynman for helpfull discussions.

*This paper will be published in the Proceedings of the Symposium on New Techniques in Physics, Brazil, 1952

**This work was supported by the Conselho Nacional de Pesquisas

[1]H.Primakoff,Phys.Rev.,84, 1255,1951; T.Nakano, J.Nisimura and Y.Yamaguchi,Prog. Theor.Phys.,6,1028,1951; T.Eguchi,Phys.Rev.,85,943,1952; G.E.A.Fialho and J.Tiomno,

Notas Fis. No.I ,1952 (to be referred as I); Anais Ac.Bras.Ci., In press.

[2]W.F.Fry, Bull.Am.Phys.Soc., 27, No.3, 43, 1952. (For more references see I).