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A Short Note on Simply Connecteness of the Einstein Space-Time

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We give "reasonable", conjecture that a compact space-time generated by matter fields content in classical Einstein gravitation theory should be simply connected.

One of the most interesting aspects of Einstein gravitation theory is the question of the non-existence of "holes" in the space-time C^2-manifold from the view point of a mathematical observer situated on the Euclidean space R^9 associated to the "minimal" Whitney imbedding theorem of M on Euclidean spaces ([1]).

In order to conjecture the validity of such topological space-time property, let us suppose that M is a C^2-compact manifold and the analitically continued (Euclidean) matter distribution tensor generating the (Euclidean) gravitation field an M allows a well-defined Euclidean metric tensor (solution of Euclidean Einstein equation) ([2]).

At this point we note that M must be always *orientable* in order to have a well-defined theory of integration on M and, thus, the validity of the rule of integration by parts: Stoke's-theorem is always need in order to construct matter tensor energy momentum. Since Euclidean Einstein's equation says simply that the sum of sectional curvatures is a measure of the (classical) matter energy density generating gravity, which must be always considered positive, it will be natural to conjecture that the positivity of the Euclidean Energy-Momentum of the matter content leads to the result that the associated positive sectional curvatures are positive individually. Since M is even-dimensional (four), the famous Synge's theorem ([4]) leads to the result that M is simply connected (note that this topological property is obviously independent of the metric structure being Lorentzian or Euclidean!) and as a direct consequence of this result, any physical geodesic (particles trajectory) on M can be topologically deformed to a point, and, thus, M does not posseses "holes" from the point of view of the Whitney imbedding extrinsic minimal space R^9.

Finally, let us point out that the existence of a (symmetric) energy-momentum tensor on M is associated to the "General Relativity" description of the space-time manifold M by means of charts (the Physics is invariant under the action of the difeomorphism group of M) which by its turn; leads to the existence of the matter energy-momentum tensor by means of Noether theorem (a metric-independent result) applied to the matter distribution Lagrangean (a scalar function defined on the tangent bundle of M.

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