ON THE PSEUDOSCALAR MESON THEORY OF THE DEUTERON*

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We report on an attempt to find a mathematically valid consequence of the Yukawa pseudoscalar meson theory which disagrees with experiment. As is well known, the symmetrical theory with gradient coupling predicts in first order a nuclear potential:

$$V = g^2 \mu (\tau_1 \tau_2) \left\{ \frac{1}{3}(\sigma_1 \sigma_2) + S_{12} \left(\frac{1}{3} + \frac{1}{x} + \frac{1}{x^2} \right) \right\} \frac{e^{-x}}{x} \tag{1}$$

where $x = \mu r$ and μ is the reciprocal Compton wave-lenght of the π-meson; S_{12} is the tensor force. But this is not a true mathematical consequence of the theory because (a) the nucleons are treated non-relativistically, and (b) only the term of first order in g^2 has been included. Nevertheless, at first sight we might expect that a complete mathematical theory would not essentially alter the potential (1) outside some small value of x. The relativistic corrections ought to come at the Compton nuclear wave lenght $x \sim 0{,}15$ and should be small for x large relative to this. The corrections of order g^4 come from the exchange of two mesons. They represent intermediate states of energy 2μ and produce exponentials like e^{-2x}. Hence they might be expected to have effects at $x \sim 0{,}5$ and here only of order g^2 or 20% of the first order term (1) (however, see below).

We have tried to see whether the present data on n-p forces (the deuteron properties in particular) could be consistent with potential (1) outside some small $x = x_0$. What the potential is inside of this rough limit x_0 we do not know. The procedure is illustrated by imagining for a moment we were dealing only with a central potential, which is the case of (1) for the singlet state:

$$V_s = -g^2 \mu \frac{e^{-x}}{x} \tag{2}$$

We solve the wave equation for $u = x\psi$ for S-wave scattering for all x in this potential, but do impose the usual boundary condition that u vanishes at the origin. Starting from the outside and proceeding inward this ought to give the correct wave function at least until x_0 - that is over the greater part of the interaction range. At x_0 it should be fitted in slope and value to a completely unknown function coming from inside x_0. But the solution from the inside is probably energy insensitive until momenta of order x_0^{-1} are involved. Assuming the slope/value unknown but constant at x_0 would permit thereby a determination of the scattering cross-section for much higher energies than the present approximations of the theory of effective range and scattering length would permit. Instead of choosing an x_0 and giving the slope there, since there is really only one unknown constant, we have for convenience carried the solution all the way to $x = 0$. The behavior of the solution extrapolated in this manner to the origin is a new effective constant to describe the inadequacy of the potential (2) for short distances. One expects such things as the scattering at energies below momenta x_0^{-1} and properties depending on the wave function not too close to the origin to be correctly described by (2) with its new boundary condition. Actually, we have two accurate data on the singlet interaction, the scattering length and the effective range, The first determines the asymptotic behavior $1 - x/a_0$ of the solution as $x \to \infty$. We used it, $a_0^{-1} = -0{,}06$, to start the numerical solution of the radial wave equation from the outside. The effective range r_0 depends on an integral, $\int_0^\infty dx\left[(1 - x/a_0)^2 - u_0^2\right]$ where u_0 is the zero energy u, but we needed the experimental value $r_0 = 2{,}76 \cdot 10^{-13}$ cm. to determine the as yet unknown g^2. By trial and error we determined $g = 0{,}18$. The wave function at the origin behaved like a series invol-

-ving Bessel functions. We did not go on to calculate phase shifts from potential (2) and boundary conditions for higher energies but turned instead to the triplet state potential. If χ and φ represent the S and D parts of the wave function

$$x\psi = \chi + \frac{\sqrt{2}}{4} S_{12}\varphi$$

the Schrödinger equation becomes for the deuteron ground state:

$$\ddot{\chi} = A\chi - \sqrt{2}B\varphi \quad ; \quad \ddot{\varphi} = \left(A + B + \frac{6}{x^2}\right)\varphi - \sqrt{2}B\chi \tag{3}$$

where $A = \epsilon^2 - C\dfrac{e^{-x}}{x}$, $\epsilon^2 = -\dfrac{ME}{\mu^2}$, $C = \dfrac{M}{\mu}g^2$

$$B = 6Ce^{-x}\left(\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{3x}\right)$$

using the g^2 value obtained from the singlet state. Of course these equations have a singularity, but although we shall carry the solution to the origin, we do not imagine (3) to be valid below some x. The equations have an explosive exponential solution (of the form k exp($(\alpha/\sqrt{x})$) near the origin. This exponential rises on decreasing x so soon and so rapidly that for appreciable k it does not seem possible that any modification of the potential for small x could bring the solution within a reasonable size again. Therefore, we have assumed that our solution has as x tends to zero an asymptotic form with k = 0. It varies then as $\cos\left(\lambda + \alpha/\sqrt{x}\right)$. This boundary condition has been suggested by Von Neumann and Pauli and discussed by Case.[1] We consider λ as another unknown constant whose value depends on the true potential. We used binding energy of the deuteron as the experimental quantity determining this constant. With the solutions obtained in this way numerically we have three quantities which we can compare to experiment. (Table 1).

TABLE 1

QUANTITY		EXPERIMENT	THEORY
Quadrup. moment	$\int_0^\infty (\chi\varphi - 0{,}353\varphi^2)x^2dx$	$2{,}73\ 10^{-27}$ cm^2	$3{,}53\ 10^{-27}$ cm^2
Effect. range	$\frac{1}{2\epsilon} - \int_0^\infty (\chi^2 + \varphi^2)dx$	$1{,}65\ 10^{-13}$ cm	$2{,}32 \cdot 10^{-13}$ cm
% D state	$\int_0^\infty \varphi^2 dx \Big/ \int_0^\infty (\varphi^2 + \chi^2)dx$	4 ± 2%	10,2 %

The second columh tells how these quantities depend on the properties of the ground state wave function. The last two show a serious disagreement between theory and experiment. There is too much tensor force. Estimates show that the potential must be wrong by its own order of magnitude at least as far out as $x = 0.7$.

If the argument about the validity of (1) for x large given above were valid we would have here evidence that the Yukawa pseudoscalar theory with gradient coupling would not give correct results even if the potential were calculated accurately. Unfortunately the argument about the g^4 corrections is not valid. A direct calculation of the g^4 static correction by Taketani, Machida an Onuma[2] (and confirmed independently by us) showa that large combinatorial coefficients come in so that the g^4 order corrections are larger than the g^2 potential (1) for $g^2 = 0.2$ even as far out as $x = 1$. Therefore, the potential (1) has neither theoretical nor experimental justification. This conclusion has also been obtained much earlier by Taketani, Nakamura and Sasaki[3] in much the same way.

It is unlikely that any calculation by a perturbation expansion (for example, using the diagrams and methods of electrodynamics) has value for nucleon-π-meson problems. At least such a calculation should be accompanied with a good argument establishing its validity.

Incidentally, the g^4 potential was carried out to first order in v/c as well. It shows a spin-orbit force too large but of the right sign for the nuclear shell model of Haxel, Jensen and Suess and of Mayer. It arises from the fact that the non-relativistic nucleon-meson coupling (σ, q)-where q is the meson momentum-must, for galilean invariance, be replaced by $(\sigma, q - p\,E/M)$ where E is the meson energy, p the average momentum before and after interaction, and M the proton mass. A velocity v increases q by Ev and p by Mv so tho torm is now Galilean invariant. Hence we apparently cannot conclude from the existence of the spin-orbit force that the pi-meson theory of nuclear forces is incorrect.

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[1] Phys. Rev. 80, 797 (1950)

[2] Progr. Theor. Phys. 6, 638 (1951)

[3] Progr. Theor. Phys. 6, 581 (1951)