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Anisotropic Correlated Electron Model Associated With the Temperley-Lieb Algebra

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Abstract

We present an anisotropic correlated electron model on a periodic lattice, constructed from an R-matrix associated with the Temperley-Lieb algebra. By modification of the coupling of the first and last sites we obtain a model with quantum algebra invariance.

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Since the discovery of high T_c superconductivity there has been a great interest in the area of integrable highly correlated electron systems. Notably for many years the Hubbard and the supersymmetric $t-J$ models, which are both exactly solvable in one dimension, stood as the prototypes for such types of models. Subsequently other correlated electron models have been formulated [1–3].

In this letter we present a new q-deformed integrable electronic model following the approach of [4], which consists of using the Temperley-Lieb (TL) algebra to obtain solutions of the Yang-Baxter equation. In [3] this was achieved utilizing a 4-dimensional module of Lie superalgebra $g\ell(2/1)$, here we proceed along the same lines by investigating the corresponding module of $U_q(g\ell(2/1))$ in order to obtain a representation of the TL algebra.

The procedure adopted in [3] involved a symmetry breaking transformation so that the resulting Hamiltonian, defined on a one-dimensional periodic lattice, was not invariant with respect to $g\ell(2/1)$ but rather its even subalgebra $g\ell(2)\otimes u(1)$. For the present situation, the usual impositon of periodic boundary conditions has the effect of also breaking the $U_q(g\ell(2))\otimes u(1)$ symmetry due to the non-cocommutativity of the quantum algebra generators. However we will show, following the methodology of [5–8] that a quantum algebra invariant closed system can be defined by the introduction of an operator coupling the first and last sites into the expression for the Hamiltonian.

Let $\{|x\rangle\}_{x=1}^4$ be an orthonormal basis for a four-dimensional $U_q(g\ell(2/1))$ module V. The quantum superalgebra $U_q(g\ell(2/1))$ obtained by deforming $g\ell(2/1)$ has simple generators $\{E_i^i\}_{i=1}^3 U\{E_{i+1}^i, E_i^{i+1}\}_{i=1}^2$ which act on this module according to

$$\begin{aligned}
E_i^i|j\rangle &= -\left(\delta_j^i+\delta_4^j\right)|j\rangle \;;\; i=1,2\;,\; j=1,2,3,4\\
E_3^3|j\rangle &= \left(\frac{1}{2}+\delta_4^j-\delta_1^j\right)|j\rangle \;;\; j=1,2,3,4\\
E_2^1|j\rangle &= \delta_3^j|2\rangle \;\;;\;\; j=1,2,3,4\\
E_1^2|j\rangle &= \delta_2^j|3\rangle \;\;;\;\; j=1,2,3,4\\
E_3^2|1\rangle &= E_3^2|3\rangle = E_2^3|2\rangle = E_2^3|4\rangle = 0\\
E_3^2|2\rangle &= \left[\frac{1}{2}\right]_q^{1/2}|1\rangle \;\;,\;\; E_3^2|4\rangle = \left[\frac{1}{2}\right]_q^{1/2}|3\rangle\\
E_2^3|1\rangle &= -\left[\frac{1}{2}\right]_q^{1/2}|2\rangle \;\;,\;\; E_2^3|3\rangle = \left[\frac{1}{2}\right]_q^{1/2}|4\rangle
\end{aligned} \tag{1}$$

where

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}} \ , \ x \in C \ .$$

The quantum superalgebra $U_q(g\ell(2/1))$ carries the following parity

$$[E^i_j] = ([i] + [j]) \ (mod.2) \ , \tag{2}$$

where $[1] = [2] = 0, \ [3] = 1$, and by consistency the parity of the basis vectors is chosen to be

$$[|1\rangle] = [|4\rangle] = 0 \qquad [|2\rangle] = [|3\rangle] = 1 \tag{3}$$

Associated to $U_q(g\ell(2/1))$ there is also a co-product structure

$$\Delta : U_q(g\ell(2/1)) \rightarrow U_q(g\ell(2/1)) \otimes U_q(g\ell(2/1))$$

given by

$$\begin{aligned} \Delta(E^i_i) &= I \otimes E^i_i + E^i_i \otimes I \quad i = 1,2,3 \\ \Delta(E^i_j) &= E^i_j \otimes q^{(E^1_1 - E^2_2)/2} + q^{-(E^1_1 - E^2_2)/2} \otimes E^i_j \quad i,j = 1,2 \\ \Delta(E^k_\ell) &= E^k_\ell \otimes q^{(E^2_2 + E^3_3)/2} + q^{-(E^2_2 + E^3_3)/2} \otimes E^k_\ell \quad k,\ell = 2,3 \end{aligned} \tag{4}$$

Everywhere we shall use the graded-tensor product law, defined by.

$$(a \otimes b)(c \otimes d) = (-1)^{[b][c]} \ \ (ac \otimes bd)$$

Following a strategy analogous to the one employed in ref. [3] to contruct an hermitian Hamiltonian, we now consider the operator

$$T = |\psi\rangle\langle\psi|$$

where $|\psi\rangle$ in an unnormalized vector of $V \otimes V$ defined by

$$\begin{aligned} |\psi\rangle &= \left(q^{-1/2}|4\rangle \otimes |1\rangle + q^{+1/2}|1\rangle \otimes |4\rangle\right) \\ &+ \left(q^{-1/2}|3\rangle \otimes |2\rangle - q^{1/2}|2\rangle \otimes |3\rangle\right) \end{aligned} \tag{5}$$

and

$$\begin{aligned} \langle\psi| &= \left(q^{-1/2}\langle 4| \otimes \langle 1| + q^{+1/2}\langle 1| \otimes \langle 4|\right) \\ &+ \left(-q^{-1/2}\langle 3| \otimes \langle 2| + q^{+1/2}\langle 2| \otimes \langle 3|\right) \end{aligned} \tag{6}$$

A straightforward calculation shows that

$$T^2 = [2(q + q^{-1})]T \tag{7}$$

and, using the fact that $|\psi\rangle$ spans a 1-dimensional submodule of $U_q(g\ell(2/1))$ (see [3, 4])

$$\begin{aligned}(T\otimes I)(I\otimes T)(T\otimes I) &= T\otimes I \\ (I\otimes T)(T\otimes I)(I\otimes T) &= I\otimes T ,\end{aligned} \tag{8}$$

such that T provides a representation of the TL algebra. This can be used to obtain an R-matrix by the transformation [4]

$$\check{R}(u) = PR(u) = I + \frac{\sinh(u)}{\sinh(\eta - u)}\, T , \tag{9}$$

where $cosh(\eta) = (q + q^{-1})$ and P is the Z_2-graded permutation operator defined by $P(|x\rangle \otimes |y\rangle) = (-1)^{[|x\rangle][|y\rangle]}|y\rangle \otimes |x\rangle$, $\forall\ 1 \leq x$, $y \leq 4$.

It is easy to check that it satisfies the Yang-Baxter equation

$$(I\otimes \check{R}(u)(\check{R}(u+v)\otimes I)(I\otimes \check{R}(v)) = (\check{R}(v)\otimes I)(I\otimes \check{R}(u+v))(\check{R}(u)\otimes I) \tag{10}$$

A local Hamiltonian can be defined by [9]

$$H_{i,i+1} = sinh(\eta)\ \frac{d}{du}\ \check{R}(u)_{i,i+1}\bigg|_{u=1} = T_{i,i+1} , \tag{11}$$

where on the N-fold tensor product space we denoted

$$\check{R}(u)_{i,i+1} = I^{\otimes(i-1)} \otimes \check{R}(u) \otimes I^{\otimes(N-i-1)} .$$

Finally in view of the grading the basis vectors of the module V can be identified with the eletronic states as follows

$$|1\rangle \equiv |+-\rangle = c_+^\dagger c_-^\dagger|0\rangle\ ,\ |2\rangle \equiv |-\rangle = c_-^\dagger|0\rangle\ ,\ |3\rangle \equiv |+\rangle = c_+^\dagger|0\rangle\ ,\ |4\rangle \equiv |0\rangle$$

allowing $H_{i,i+1}$ to be expressed in terms of the canonical fermion operators as

$$\begin{aligned} H_{i,i+1} &= qn_{i,+}n_{i,-}(1-n_{i+1,+})(1-n_{i+1,-}) + q^{-1}(1-n_{i,+})(1-n_{i,-})n_{i+1,+}n_{i+1,-} \\ &+ q^{-1}n_{i,+}(1-n_{i,-})n_{i+1,-}(1-n_{i+1,+}) + qn_{i,-}(1-n_{i,+})n_{i+1,+}(1-n_{i+1,-}) \\ &- S_i^+S_{i+1}^- - S_i^-S_{i+1}^+ + c_{i,+}^\dagger c_{i,-}^\dagger c_{i+1,-}c_{i+1,+} + c_{i+1,+}^\dagger c_{i+1,-}^\dagger c_{i-}c_{i+} \\ &+ qc_{i,+}^\dagger c_{i+1,+}n_{i,-}(1-n_{i+1,-}) + h.c. - c_{i,-}^\dagger c_{i+1,-}n_{i,+}(1-n_{i+1,+}) + h.c. \\ &+ c_{i,-}c_{i+1,-}^\dagger n_{i+1,+}(1-n_{i,+}) + h.c. - q^{-1}c_{i,+}c_{i+1,+}^\dagger n_{i+1,-}(1-n_{i,-}) + h.c. \end{aligned} \tag{12}$$

where the $c_{i\pm}^{(+)}$ are spin up or down annihilation (creation) operators, the $S_i' s$ spin matrices and the $n_i' s$ occupation numbers of electrons at lattice site i.

Thus we obtain a Hamiltonian describing electron pair hopping, correlated hopping and generalized spin interactions. We notice that in the limit $q \to 1$ we recover the isotropic Hamiltonian discussed in [3].

By means of the quantum inverse scattering method [9, 10] it is possible to show that the model is integrable. Basically, by this procedure, the Hamiltonian (12) is related to the transfer matrix of a graded vertex model [11] (see also [12]), constructed from the spectral parameter dependent R-matrix. The associated Yang-Baxter algebra implies the commutativity of the transfer matrix for different spectral parameters, which reflects the integrability of the model.

The global Hamiltonian takes the form

$$H = \sum_{i=1}^{N-1} H_{i,i+1} + H_{N1} \ . \tag{13}$$

It is not invariant with respect to $U_q(g\ell(2)) \otimes u(1)$ since $H_{N1} \neq H_{1N}$ reflecting the non-cocommutativity of the co-product. However, by modifying the above Hamiltonian we can obtain a quantum algebra invariant model as follows [c.f. 5- 8].

Let σ denote the braid generator defined by

$$\begin{aligned} \sigma &= \lim_{u\to\infty} \check{\mathrm{R}}(u) \\ &= I - e^{-\eta} T \end{aligned} \tag{14}$$

which satisfies the braid relations

$$(\sigma \otimes I)(I \otimes \sigma)(\sigma \otimes I) = (I \otimes \sigma)(\sigma \otimes I)(I \otimes \sigma) \ . \tag{15}$$

Setting $G = \sigma_{12}\sigma_{23} \cdots \sigma_{N-1N}$, then it follows from the TL relations that

$$G H_{i,i+1} G^{-1} = H_{i+1,i+2} \quad , \quad i = 1, \cdots N-2$$

We now define

$$H_o = G H_{N-1,N} G^{-1} \tag{16}$$

which can be shown to satisfy

$$G H_o G^{-1} = H_{12}$$

and set our new Hamiltonian to be

$$H = \sum_{i=1}^{N-1} H_{i,i+1} + H_o \tag{17}$$

satisfying $[H, G] = 0$ and additionally invariance with respect to the quantum algebra $U_q(g\ell(2)) \otimes u(1)$.

The details about the Bethe ansatz solution of the present model will be developed elsewhere.

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