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GAMMA RADIATION EMITTED IN THE PI-MU DECAY*

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One of the fundamental properties of the π-meson which led to its discovery by Lattes, Occhialini and Powell was the possibility [1] of disintegration with the omission of a μ-meson with a definite momentum. Indeed the histogram of the ranges of the μ-meson in the $\pi \rightarrow \mu$ decay gave a distribution with a sharp maximum at a range of 594 microns, the width of the peak being consistent with the straggling, experimental errors and possible decay in flight. This implied that the $\pi \rightarrow \mu$ decay is a two particle decay, a neutral particle being produced together with the μ-meson. The experimental values of the mass of π and μ-mesons showed that the mass of the neutral particle

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1. C.M.G. Lattes, C.P.S. Occhialini, C.F. Powell, Nature, 60, 453, 486, 1947

could not be much larger than of an electron and could even be zero. It has been assumed that such a neutral particle is a neutrino:

$$\pi \longrightarrow \mu + \nu \qquad (1)$$

Recently Fry and others [2,3] found a few cases of very short μ-tracks in the $\pi \rightarrow \mu$ decay ($\sim$200 microns) which could not be explained by straggling or decays in flight. The frequency of such low energy μ-mesons in the $\pi \longrightarrow \mu$ decay, say one in a thousand to ten thousand, strongly suggests a radiative process with emission of a gamma-ray:

$$\pi \longrightarrow \mu + \nu + h\nu \,. \qquad (2)$$

This is a second order process that exists whenever the first order process (1) is possible, in view of the interaction of the charged μ-meson with the electro-magnetic field.

In this paper we examine the frequency of the radiative process (2) as compared to (1) and the energy distribution curve for the emitted μ-meson with the usual assumption that this particle has spin 1/2. One should keep in mind, however, that possibilities of spin zero [4] or three halves [5] for the μ-meson is assumed to be described by a pseudo-scalar wave operator $\phi(x)$ as shown by recent experiments on production and absorption by nuclei.

If we represent the μ-meson and neutrino fields by the spinors $\Psi(x)$ and $\varphi(x)$ respectively, the Hamiltonian density of inter

2. W.F.Fry, Phys. Rev, 83, 1268, 1951, Bull. Am. Phys. Soc. 27, No. 3, 43, 1952.

3. W.F.Fry and G.R. White, Bull. Am. Phys. Soc. 27, No. 3, 43, 1952.

4. J.Tiomno, Phys. Rev. 76, 856, 1948. In this case the neutral particle in process (1) cannot be a neutrino as it should have an integer spin.

5. E.R. Caianiello, Phys. Rev. 83, 735, 1952.

action with the π-meson field is

$$g(\bar{\psi}\gamma_5\,\varphi)\phi + \text{complex conj.} \tag{3}$$

where $\bar{\psi} = -\,i\,\psi^*\beta$.

The interaction of the μ-meson field with the electromagnetic field $A_\mu(x)$ is taken as

$$e\,\psi\,\gamma^\mu\psi\,A_\mu \tag{4}$$

We use natural units: $\hbar = c = 1$.

By the usual perturbation methods we find for the probability per unit time of process (1), the π-meson being at rest [6]

$$W_o = g^2(M^2 - \mu^2)^2 \,/\, 2M^3 \tag{5}$$

where M and μ are the masses of the π and μ mesons, respectively. For the probability per unit time fo the radiative process (2), the μ-meson coming out with an energy between E and E + dE meson being between θ and $\theta + d\theta$ we find

$$dW = \frac{g^2c^2P}{\pi\,M}\,(f_c + f_m)\,dE\,d(\cos\theta)\;; \tag{6}$$

where P is the momentum of the μ-meson and

$$f_c = \frac{P^2\sin^2\theta(M^2 - \mu^2)}{(E - P\cos\theta)^2(M^2 + \mu^2 - 2ME)}, \tag{6a}$$

$$f_m = W/(E - P\cos\theta) \tag{6b}$$

The term f_c gives the electric current (dipole) contribution to the γ-ray emission (it is proportional to $P^2\sin^2\theta$ for small P) and f_m gives the contribution from the "intrinsic" magnetic dipole of the μ-meson. In formula (6b) ω is the energy of the emitted pho

6. J. Leite Lopes, Rev. 74, 1722, 1948; see also C. Marty and J. Prestki, J. de Phys. et Rad. 9, 147, 1948.

ton, given by

$$\omega = (M^2 + \mu^2 - 2ME)/2(M - E + P\cos\theta)$$

After integrating over the angle θ we obtain for the probability per unit energy range

$$W(E) = \frac{e^2}{2\pi}\,\frac{PW_o}{P_o^2}\,(F_c + F_m) \qquad (7)$$

with

$$F_c = \frac{4P_o}{E-E_o}\,\frac{E}{P}\left[\log\frac{E+P}{\mu} - 1\right], \qquad (7a)$$

$$F_m = \frac{E_o-E}{P}\,\log\frac{(E+P)M-\mu^2}{(E-P)M-\mu^2} \qquad (7b)$$

Here $P_o = (M^2 - \mu^2)/2M$ and $E_o = (M^2 + \mu^2)/2M$ are, respectively, the maximum momentum and maximum energy of the emitted μ-meson, say, those corresponding to the radiationless decay.

We should point out that, as the μ-meson energies are practically non relativistic in this case, formula (7) can be in first approximation generalized to the case of a magnetic moment different from $e/2\mu$ by the substitution:

$$F_m \longrightarrow \alpha F_m \qquad (8)$$

where $\sqrt{\alpha}$ measures the magnetic moment in units $e/2\mu$. If the μ-meson has zero spin, $\alpha = 0$. We see that an experimental determination of the energy spectrum of the emitted μ-meson may decide among the possibilities of zero or non zero spin for the μ-meson [7].

As the present experimental results on the frequency of short-range μ-meson from $\pi \longrightarrow \mu$ are very incomplete, we only make a comparison with the theoretical predictions for the case computed

7. It cannot, however, decide among the several possibilities of finite spin as in these cases anamolous magnetic moment is possible.

in this paper, as given by expression (7). So the experimental value for the ratio of the number of μ-mesons with energy smaller than 3.5 Mev to the total number of decays [2] is $(2.8 \pm 1.2)\ 10^{-4}$, excluding events which can be decay in flight. The theoretical value for this ratio, computed from expression (7) is $1.3 \times 10-4$. This result is in reasonable agreement with the referred experimental value and indicates that the short range μ tracks may be explained by radiative $\pi \longrightarrow \mu$ decay.

After the present work was accomplished two papers came to our attention which dealt with the same subject, say, by Eguchi [8] and by Nakano et al.[9] Although our formulae are not very easily comparable with theirs, we verified that the frequency curve computed from expression (7) was identical with the one given by Eguchi [8] and smaller by a factor 1.5 than that of Nakano [9].

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8. T. Eguchi, Phys. Rev. 85, 943, 1952.

9. T. Nakano, J. Nishimura, Y. Yamaguchi, Prog. Theor. Phys., 6, 1028, (1951); see also H. Primakoff, Phys. Rev. 84, 1255, 1951.