CBPF-NF-001/01

Quantum Geometry of Fermionic Strings-Revisited

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Abstract

We revisited the famous Polyakov's paper Quantum Geometry of Fermionic Strings (Phys. Lett. 103B, 211, 1989).

Key-words: String; Supersymmetry; Bozonization.

We review [as a Brief note] the original paper by A.M. Polyakov (Quantum Geometry of Fermionic Strings (Phys. Lett. 103B, 211, 1989) with *corrections and improvements* on the concepts exposed there and following as closely as possible to the original A.M. Polyakov's *paper writing* since our knowledge of English remains to this day very primitive and we apologize for our cumbersome use of a foreing language.

In this previous CBPF–Notas de Física (CBPF-NF-042/99) (ref. [1]), we have corrected and improved the Polyakov's procedure for quantizing Bosonic strings as 2D quantum gravity models. It is, thus, very urgent to extend these results to fermionic strings because, as was shown in refs. [2] its Kaluza-Klein reduction to certain manifolds are expected to represent from a Q.F.T. point of view the $N = 4$ extended Super-Yang Mills Field Theory (The Maldecena suggestion).

Let us begin from the supersymmetric extension of the Bose string (quantum gravity!) lagrangean (see ref. [1]).

$$S = \frac{1}{2\pi\alpha'}\left\{\int d^2\xi\left[\frac{1}{2}\sqrt{g}\; g^{\alpha\beta}\partial_\alpha X^A\partial_\beta X_A + \frac{1}{2}\overline{\psi}^A(i\gamma^\alpha\partial_\alpha)\psi_A \right.\right.$$
$$\left.\left. \overline{\chi}_\alpha\gamma^\beta\gamma^\alpha(\partial_\beta\chi^A + \frac{1}{2}\chi_\beta\psi^A)\psi_A\right]\right\} + \mu\int_D d^2\xi\sqrt{g}(\xi) \qquad (1)$$

Here, the surface is parametrized by $X_A = X_A(\xi)$, $(A = 1\cdots D)$; ψ^A is a ξ-two component Majorana spinor, $g_{\alpha\beta}(\xi)$ is a metric tensor and χ_α is a spinor gravitino field. The Polyakov's strategy as exposed in the previous paper, will be to integrate out the χ^A and ψ^A fields firstly and, then, he has examined the resulting theory of "induced ξ-supergravity". By choosing the "super-conformal" gauge

$$g_{\alpha\beta}(\xi) = \rho^2(\xi)\delta_{\alpha\beta}\ ;\ \chi_\alpha(\xi) = (\gamma_\alpha\chi)(\xi) \qquad (2)$$

Polyakov has showed that the only expression which satisfies all ξ-supersymmetries not destroyed by the super-confirmed gauge eq. (2) is the direct supersymmetric extension of the Bosonic action written in ref. [1], eq. (2)

$$e^{-W} = \int D\psi^A DX_a e^{-S} \qquad (3)$$

In terms of the original fields $\rho(\xi)$ and $\chi(\xi)$, the component form of eq. (3) can be (correctly) rewritten as (with $2\pi\alpha' = 1$)

$$W[\rho,\chi]=\frac{10-D}{8\pi}\int[\frac{1}{2}\left(\frac{\partial_\xi\rho}{\rho}\right)^2+\left[\frac{1}{2}i\overline{\chi}(\gamma\partial)\chi+\frac{1}{2}\mu(\overline{\chi}\gamma_5\chi)\rho\right.$$
$$\left.+\frac{1}{2}\mu^2\rho^2\right](\xi)d^2\xi \tag{4}$$

Note that in the usual Liouville field parametrization the induced 2D-supergravity is written as ($\rho=e^W$)

$$W[e,\chi]=\frac{10-D}{8\pi}\int_\xi\left[\frac{1}{2}(\partial W)^2+\frac{1}{2}i\overline{\chi}(\gamma\partial)\chi+\frac{1}{2}\mu(\overline{\chi}\gamma_5\chi)e^W+\frac{1}{2}\mu^2\ e^{2W}\right](\xi) \tag{5}$$

At this point it is worth remark that the intrinsic fermionic degrees of freedom in eq. (5) may be easily integrated out with the following result: [if one considers $\chi(\xi)$ as an usual 2D-Dirac fermion field]

$$\int D^F[\chi(\xi)D^F[\overline{\chi}(\xi)]exp\left\{-\left(\frac{10-D}{8\pi}\right)\int_\xi d^2\xi\left[\frac{1}{2}i\overline{\chi}(\gamma\partial)\chi+\frac{1}{2}\mu(\overline{\chi}\gamma_5\chi)\rho\right]\right\}$$
$$=det\left[i\gamma\partial+\frac{1}{2}\mu\gamma_5\rho\right]=I(\rho) \tag{6}$$

At this point, we note that (after introducing the notation $\sigma_+=\beta\left(\frac{1+\gamma_5}{2}\right)\overline{\beta}$ and $\sigma_-=\beta\left(\frac{1-\gamma_5}{2}\right)\overline{\beta}$, we have the μ-expansion ($\mu<<1$)

$$I(\rho)=\sum_{n=0}^{\infty}\frac{(-\frac{1}{2}\mu)^n}{n!}\int d^2\xi_1\cdots\ d^2\xi_n$$
$$\int D\beta D\overline{\beta}\ exp\left[-\frac{1}{2}\int_\xi(\overline{\beta}(i\gamma\partial)\beta)\right]$$
$$(\sigma_+\rho-\sigma_-\rho)(\xi_1)\cdots(\sigma_+\rho-\sigma_-\rho)(\xi_n) \tag{7}$$

and it is a result of a well-known theorem on 2D-Fermionic model's that the only non zero terms of eq. (6) are those with equal number of σ'_+s and σ'_-s. We get, thus, that eq. (6) becomes the bosonized path-integral below written

$$I(\rho)=\int D^F[a(\xi)]exp\left(-\frac{1}{2}\int d^2\xi(\partial a)^2(\xi)\right)exp\left(-\int d^2\xi\left[\frac{1}{2}\mu\ e^{\Delta(0)}\ \frac{1}{4\pi}\ sen(\sqrt{4\pi}a+\rho)\right](\xi)\right) \tag{8}$$

where the (bare) ξ-cosmological constant μ (gets a multiplicative ultraviolet) renormalization $\mu_R=\frac{1}{2}\mu(\varepsilon)^{-\frac{1}{2\pi}}$.

As a final comment let us use as dynamical degrees of freedom the Polyakov's original conformal factor $W(\xi) = lg\ \rho(\xi)$. In terms of this variable the bosonized theory's path integral is written as

$$Z = \int D^F[e^{W(\xi)}].exp\left\{ -\frac{1}{2}\int_\xi \left[(\partial W)^2(\xi) + \mu^2 \left(\frac{10-D}{8\pi}\right) e^{2\sqrt{\frac{8\pi}{10-D}}W} \right] \right\}$$
$$\times \left\{ \int D^F[a(\xi)]\ exp\left(-\frac{1}{2}\int d^2\xi(\partial a)^2(\xi)\right) \right.$$
$$\left. exp\left(-\int d^2\xi \left[\frac{1}{8\pi}\mu_R sin(\sqrt{4\pi a})e^{\sqrt{\frac{8\pi}{10-D}}W} \right] \right) \right\} \quad (9)$$

It is worth to note that the one must use as the Feynman product measure that written in eq. (8) $\prod_\xi (e^{W(\xi)} dW(\xi))$ since the associated functional (ξ-covariant) functional metric is given by

$$\|\delta g_{ab}\|^2 = \int_\xi \left(e^{2W(\xi)}\delta W \cdot \delta W\right)(\xi) d^2\xi = \int_\xi [\delta(e^W)\delta(e^W)](\xi) d^2\xi \quad (10)$$

Note that only for weak intrinsic metric fluctuations (or for $D = 10 - \varepsilon$) $e^{W(\xi)}$ may be replaced by $W(\xi)$ inside the Feynman product measure as it was supposed in Polyakov's original paper.

Acknowledgments

Luiz C.L. Botelho is grateful to Prof. Helayël-Neto from DCP-CBPF, Brasil for warm hospitality. Luiz C.L. Botelho wants to state here that he agree completely with the co-author of ref. [1] in his "protest-joke" (exposed on the acknowledgments of the above cited ref. [1]) and against to the present philosophy on Physics: Write papers to fill up "WEB, Journals and pockets" as best sellers novels writers, without making real attempts to solve the very difficult still largelly unsolved problems of present day physics: Superconductivity, Turbulence, Strong Forces, Higgs Mesons, Infrared Divergences in Perturbative Quantum-Mechanical Calculations, Open Quantum Systems, Random Physics, 3D-Ising Models, Mathematics of Path-Integrals, *and specially how to quantize correctly*

the Nambu area string action in the present einsteinian universe of four-dimensional (3 spatial dimensions plus the one time coordinate) ([5]).

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