MONOGRAFIAS DE FÍSICA

VII

HIGH ENERGY INTERACTIONS OF ELEMENTARY PARTICLES

by

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1 - Introduction

I have been asked to give you a course on high energy physics, or more precisely on high energy interactions between elementary particles.

The experimental situation we have in mind so to say is a particle of high kinetic energy shot against a particle at rest. This is the most general case we like to refer to, although, of course, you can imagine more complicated experimental situations where the target is not really at rest.

Why we study high energy interactions ? There is clearly an obvious answer; we like to extend our knowledge of the physical phenomena in a field not yet explored, but this is not enough.

The point is that we always thought that in the high energy phenomena we could find the answer to few fundamental questions and essentially to the problem of the structure of elementary particles.

The original idea was quite simple: because we believe that elementary particles are small in extension, say of the order of $\sim 10^{-13}$ cm, and because quantum mechanics tell us that in order to explore spacial regions of linear dimension d we must use objects with wave length $\lambda \ll d$ then we have to use high energy particles, i.e., particles with $p \gg \frac{\hbar}{d}$.

This very naive idea does not have a simple answer because the increase in relative momentum of 2 colliding particles does not simply mean that you observe the same phenomena as in the low energy range where we have greater λ. As far as you increase the energy in general new phenomena occur, so rich and so complicated and un-expected that the original idea looks inadequate and your interest is immediately shifted to the problem of understanding the new phenomena.

Probably the only case in which the original plan has been consequently developped is in the electron-proton scattering where the extension toward high energy has really meant the possibility to understand the electromagnetic structure of the proton.

I am not going to discuss this last problem but the more complicated things that occur when strongly interacting particles collide at high energy giving rise to a variety of phenomena like multiple meson production, production of strange particles, antiparticles and so on.

Of these high energy collisions the most familiar is probably the pion-nucleon (π - N) collision and probably the best known up to now.

Let me work few general remarks on it; these remarks, although definitely pertinent to π - N collisions can be adapted to other high energy encounters with little change.

Here we can distinguish 3 energy regions characterized by different phenomena.

a) <u>A low energy region</u>, covered by what we call now the "classical pion physics" in which the dominant phenomenon is the elastic scattering

$$\pi + N \longrightarrow \pi + N \qquad (1.1)$$

This region extends from zero K.E. up to the energy of the well known (3/2, 3/2) resonance. (0 to $\sim$ 300 Mev incident pion kinetic energy in the lab. system).

We have learnt from this region the importance of the spin orbit coupling and the notion of isotopic spin because the π - N interaction is strongly selective in this region for what concerns total angular momentum, parity and total I-spin.

Among the many channels open for the reaction (1.1) and identified by (J, P, I) (total angular momentum, parity and isotopic spin) one dominates and the dominant channel is precisely the (3/2, + 3/2) channel.

The intrinsic analysis (uses only conservation laws as of energy-momentum, parity, angular momentum and total I-spin) of the scattering phenomena can be performed, as Fermi told us, in this region on the basis of these 3 quantum numbers only.

As far as the theoretical situation is concerned one starts with the constatation of the inadequacy of any approach based on the perturbation theory, mainly the so called weak coupling approach.

Further Chew and Low developped a phenomenological theory (all the theories are in a certain sense phenomenological up to now), the so called fixed source theory in which they developped some kind of strong coupling approach, assuming the nucleon at rest and infinitely heavy (as compared to the pion mass), with an extension, and coupled to the meson field by the P. V. interaction:

$$\mathcal{H}_{int} = \sqrt{4\pi}\left(\frac{f}{\mu}\right)\int d\vec{r}\,\rho(\vec{r})\left(\sum_{\lambda=1}^{3} \tau_\lambda\,\vec{\sigma}\cdot\vec{\nabla}\,\phi_\lambda(\vec{r})\right)$$

where $\rho(\vec{r})$ is a form-factor normalized to $\int \rho(\vec{r})\,d\vec{r} = 1$, and whose Fourier transform: $V(k) = \int d\vec{r}\, e^{-i\vec{k}\cdot\vec{r}}\, \rho(\vec{r})$ is approximated by a step function

$$v(k) = \begin{cases} 1 & k \leqslant k_{max} \\ 0 & k > k_{max} \end{cases}$$

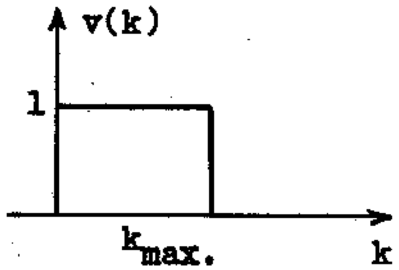


Fig. 1

The theory has clearly 2 parameters f and k_{max}. The theory although very limited in his possibilities (giving only P-waves) succeeded in giving agreement with the experiment or at least in giving a way to handle the experiments and the connection between them.

In spite of the limitations of the theory itself we must be grateful to Chew and Low who gives us a machinery to calculate physical quantities and to have a first determination of f^2 - the strength of the coupling between meson and nucleon.

The inability of Chew-Low theory to describe the existing ~~S-wave contribution~~, the nucleon ~~recoil~~ and the ~~existence of~~ the other resonances made it necessary to find new methods, f.i., Dispersion Relations Theory. But unfortunately the dispersion relations give only a small amount of information about the dynamical behaviour of the system, just because they are based in very general concepts such as microcausality and Lorentz invariance. Up to now some few results were obtained by the use of dispersion relations, f.i., they are able to correlate well some different experiments and it was possible to get information on f^2, the coupling constant, by the use of "polology" arguments.

b) An intermediate region extending from 300 Mev to few Bev.

In this region, which is the region we will be mostly concerned with in these lectures the π - N interaction still conserves his selective character for angular momentum and isospin but the appearance of the inelastic processes complicates the situation. Next the most interesting things, i.e., the threshold for strange particles production fall in this region and contribute to this interest.

In this region we do not have theories, and the only possible ordering is still on the basis of conservation principles and of few quite crude models.

Anderson theory and polology serve to solve few particular questions but are not yet in a position to describe the phenomena.

c) A true high energy region extending from few Bev on, of which very little is known and in which we are going to play with the biggest accelerator like the CERN machine now in operation and the Brookhaven machine expected to start quite soon.

This region seems to be characterized at his begining by the complete unusefulness of concepts like angular momenta and isospin due to the fact that all the total cross sections seem to levell of and to be therefore independent on the energy and on the charge of the particles. If this is only apparent and due to our very poor information is a matter of future experiment. But despite of this, this region will tell us many new things because we know that within it are located the threshold for antiparticles generation. If the knowledge of these processes will tell us a lot of things or not is an open question about which we will heard certainly in the near future.

2 - Analysis of experimental information in the low energy region.

We shall enter a little more deeply now into the analysis of scattering experiments in the low energy region. As we have already announced, we shall deal only with π - N interactions and in particular in this low energy region the cleanest experimental information can be obtained from scattering of π's on protons. These experiments can be easily performed because it is fairly simple to get good H_2 targets and π beams of well defined energy are available nowadays.

We shall then observe what happens in the following type of reactions:

$$\pi^+ + p \longrightarrow \pi^+ + p \qquad (2.1)$$

which is the only important process one can obtain with π^+ on protons, and:

$$\pi^- + p \longrightarrow \pi^- + p \quad \text{(~~elastic~~ scattering)} \qquad (2.2)$$

$$\pi^- + p \longrightarrow \pi^0 + n \quad \text{(charge exchange)} \qquad (2.3)$$

and also

$$\pi^- + p \longrightarrow \gamma + n \quad \text{(radiative capture)} \qquad (2.4)$$

This last case is quite interesting because if it exists, as it does, the inverse process:

$$\gamma + n \longrightarrow \pi^- + p \qquad (2.5)$$

should also exist ans its analysis will provide information about the conections between scattering and photo-production by detailed balance arguments.

One could imagine also processes with π^0 as bombarding particle, but simple experiments are impossible since π^0 beams are not available because of the short lifetime of the π^0, neither are free neutron targets. Of course one could use deuterons or more complicated nuclei but then one has bounded neutrons and this complicates the situation. Nevertheless, some information about other processes can be obtained assuming charge symmetry, by which the following simultaneous substitutions can be done:

$$\pi^+ \longleftrightarrow \pi^-$$
$$\pi^0 \longleftrightarrow \pi^0$$
$$p \longleftrightarrow n$$

and then, information on the following processes can be obtained:

$$\pi^- + n \longrightarrow \pi^- + n \qquad (2.6)$$
$$\pi^+ + n \longrightarrow \pi^+ + n \qquad (2.7)$$
$$\pi^+ + n \longrightarrow \pi^0 + p \qquad (2.8)$$

and finally:

$$\pi^+ + n \longrightarrow \gamma + p \qquad (2.9)$$

We can readily see that the cross section for (2.9) will be substantially different from that for (2.4), since the γ-ray is able to distinguish between different charge states and magnetic moments.

Finally assuming not only charge symmetry but also charge independence to be valid, the processes:

$$\pi^0 + p \longrightarrow \pi^0 + p \qquad (2.10)$$

$$\pi^0 + n \longrightarrow \pi^0 + n \qquad (2.11)$$

which are also possible, turn out to be of the same type than the former and it is possible to calculate them.

Now, we will try to see which are the main facts in all these processes. Firstly, there will be of course the different total cross sections:

σ_{++} for (2.1)

σ_{--} for (2.2)

σ_{-0} for (2.3)

For the moment we shall neglect reactions (2.4) and (2.9) for two reasons: 1) They are not pure nuclear interactions since a γ-ray is involved and 2) the cross sections for them are much lower than the other three (except at zero energy; at higher energies it represents only about 1% of the others).

If we plot the total cross section versus the kinetic energy (lab. system) of the incident pion, the main features of the low energy region will appear (Fig. 2).

The outstanding fact being the strikingly high peak in the σ_{++} which reaches a value of 200 mb at about 200 Mev. This extraordinarily high value of the cross section (about 3 times a geometri-

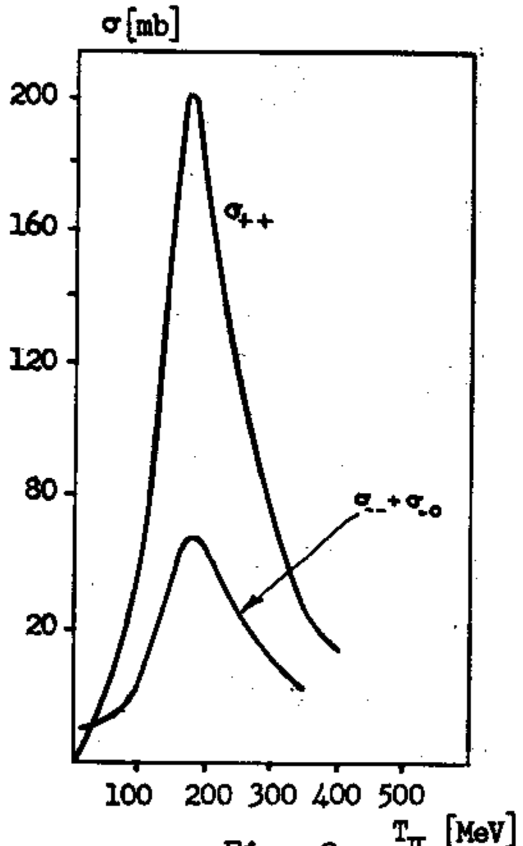


Fig. 2

cal cross section for cosmic rays) together with the shape of the curve, made people to think immediatly of a resonance.

Summing up the other two cross-sections, the resulting curve has approximately the same behavior, except for the very low energy region, reaching a maximum which is a third of the value of σ_{++} at exactly the same energy than the maximum in this latter. The detailed ratio between the three cross sections, is: σ_{++}: σ_{-o}:σ_{--} = = 9:2:1.

Another source of information will be the angular distributions of the pions. Plotting the angles of the pion in the center of mass system, one gets approximately the curves represented in Fig.3.

At very low energy, the pions begin to scatter isotropically (a). At 200 Mev, the distribution is parabolic and symmetric with respect to 90° (b), the curve being fitted by a polynomial $1 + 3\cos^2\theta$.

Below 200 Mev the distribution is again parabolic but asymmetric with respect to 90° (c), while above 200 Mev, the curve is also pa-

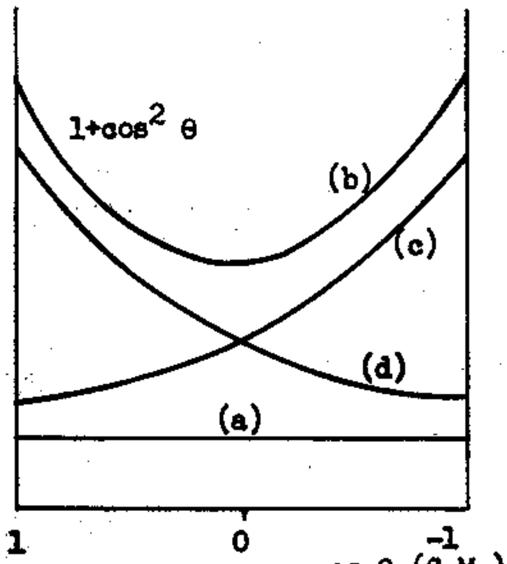


Fig. 3

rabolic and asymmetric (d), only inverted with respect to the former, passing from backward peaked to forward peaked asymmetry.

All this is of course an oversimplified description of facts; there are small differences, for instance, the polynomial $1+3\cos^2\theta$ does not describe exactly the angular distribution at 200 Mev, neither the ratio between cross sections is exactly 9 : 2 : 1. Nevertheless it is quite a good description of what really happens.

Next thing is to try a scheme on the basis of the three conservation laws mentioned in the first lecture namely: conservation of energy and momentum, conservation of parity and conservation of I-spin. This is in fact, the experimentalists way of working: to observe first reliable experimental facts, and on the basis of some general principles, to try to get a reasonable description of those facts, with the smallest number of parameters.

We shall begin trying to say something about differential cross--sections. These are expressed by:

$$\frac{d\sigma}{d\Omega} = |T|^2$$

the square of the scattering amplitude. If we knew the interaction, and were able to solve the Schroedinger equation we could get the scattering amplitude but we do not know them. Fermi tried to see if it is possible to describe the phenomena neglecting the charge states of the particles, i.e., assuming that the interaction depends only on the total isotopic spins.

Now, if reactions (2.1) (2.2) and (2.3) are essentially the same, then the chosen method must give results in agreement with the experimental facts.

If one takes into account only the total I-spin forgetting for the moment its projections, then the combination of the I-spin 1 of the pion and 1/2 of the nucleon, gives two possible states for the π - N system, namely T = 3/2, T = 1/2.

Then it is easy to see, treating vectors as ordinarily and using Clebsch - Gordon coefficients, that:

$$\left|\frac{d\sigma}{d\Omega}\right|_{++} = \left| T_{3/2} \right|^2$$

$$\left|\frac{d\sigma}{d\Omega}\right|_{-o} = \frac{2}{9} \left| T_{3/2} - T_{1/2} \right|^2 \qquad (2.12)$$

$$\left|\frac{d\sigma}{d\Omega}\right|_{--} = \frac{1}{9} \left| T_{3/2} + 2\, T_{1/2} \right|^2$$

In T one specifies only I-spin, but it includes also spin, angular momenta, energy and angle $T_{(J,P,I)}(E,\theta)$.

Developing those expressions one gets:

$$\left|\frac{d\sigma}{d\Omega}\right|_{-o} = \frac{2}{9}\left\{T_{3/2}^{2} + T_{1/2}^{2} - 2\,\mathrm{Re}\,T_{3/2}^{*}\,T_{1/2}\right\}$$

$$\left|\frac{d\sigma}{d\Omega}\right|_{--} = \frac{1}{9}\left\{T_{3/2}^{2} + 4\,T_{1/2}^{2} + 4\,\mathrm{Re}\,T_{3/2}^{*}\,T_{1/2}\right\}$$

Summing up those expressions:

$$\left|\frac{d\sigma}{d\Omega}\right|_{-o} + \left|\frac{d\sigma}{d\Omega}\right|_{--} = \frac{1}{3}\,T_{3/2}^{2} + \frac{2}{3}\,T_{1/2}^{2}$$

Next step will be to discover which are the states of I-spin and angular momentum which are important to account for the experimental facts. If one neglects the contribution of the T = 1/2 state, at resonance, that is assuming that only T = 3/2 is important a good agreement with the experimental data is obtained, regarding the ratio $\sigma_{++} : \sigma_{-o} : \sigma_{--} = 9 : 2 : 1$

Therefore we shall continue the analysis by looking only into the π^{+} induced reactions, which are pure $T_{3/2}$, and trying to see which are the states of angular momentum to which the shape of the angular distribution at resonance energy can be attributed. One can immediatly see that it cannot be pure s wave; since in this case the angular distribution would be isotropic. Only p or d waves are able to account for the parabolic form of the curve. Because of energetic reasons one can eliminate d waves whose contribution if existing should be very small. Furthermore, because the interference changes sign passing through the resonance, this interference must take place between waves of opposite parities such as s^- and p^+ while d is again of the same parity as s.

Next we can try to see if from the two possible p waves, only one of them can account for the observed total cross section. For elastic processes, the maximum σ which can be built up with a pure state of total angular momentum J is:

$$\sigma_{max} = \frac{4\pi}{K^2} (J + \tfrac{1}{2})$$

At about 200 Mev, $\frac{1}{K^2} \sim 8 \times 10^{-27} cm^2$. Thus, if one takes only $J = 3/2$ the total cross section turns out to be $\sim$ 200 mb.

Now, the contribution of a partial wave to the total cross section is:

$$\sigma = \frac{4\pi}{K^2} (J + \tfrac{1}{2}) \sin^2 \delta$$

and then, the assumed ${}_{3/2}P_{3/2}$ wave would give a maximum contribution as required in order to account for the 200 mb of the cross section, when δ passes through 90^o, and that means a resonance as the general aspect of the σ_{++} curve indicated. Of course all this is a first approach which simply indicates that assuming a single dominant ${}_{3/2}P^+_{3/2}$ state, one can obtain agreement with experimental facts, both with regard to the absolute value of the cross sections, to the ratios between them and to the shape of the angular distribution.

A more complete analysis in terms of I-spin, permits one to gain something. Supose one begins to perform experiments in the very low energy region. Thus, only s wave would be present and the angular distribution would result: $\sigma(\theta) = \text{const.}$ Then, for each one of the three reactions (2.1) (2.2) and (2.3) one gets from the experiment one number: a_+, a_-, a_o, the three being described by two parameters, namely $a_{3/2}$ and $a_{1/2}$.

Going towards higher energies, p wave would begin to contribute and one gets for reactions (2.1) (2.2) and (2.3), curves which are accounted for by polynomials:

$$a_+ + b_+ \cos\theta + c_+ \cos^2\theta$$

$$a_- + b_- \cos\theta + c_- \cos^2\theta$$

$$a_o + b_o \cos\theta + c_o \cos^2\theta$$

Then the experiment would give 9 numbers, which, within the framework of the present scheme, must be described by 6 parameters:

$$\begin{array}{cccc} & a_{3/2} & a_{1/2} & \\ \overset{+}{b}_{3/2} & \overset{-}{b}_{3/2} & \overset{+}{b}_{1/2} & \overset{-}{b}_{1/2} \end{array}$$

where the + and - represent $\ell + \frac{1}{2}$ and $\ell - \frac{1}{2}$ respectively. In general, the experiments would provide n numbers to be accounted for by $\frac{2}{3}$ n parameters.

The superabundance of experimental numbers with respect to the parameters which are necessary to describe them, provides an interst ing check for the goodness of the theory.

3 - Phase shift analysis.

Up to now, we have analysed the esperimental results on elastic scattering of pions on protons in terms of the 2 possible I-spin states of the system, namely $T_{3/2}$ and $T_{1/2}$ and we saw that in a first approximation, those processes are dominated in the low energy region by only one channel defined by ${}_{3/2}P^{+}_{3/2}$.

In order to perform a more refined analysis one must expand in partial waves both amplitudes. When one does that, each amplitude is expressed by:

$$T_I = \frac{1}{2ik} \sum \left[\ell \left(e^{2i\delta^-_{I,\ell}} - 1 \right) + (\ell+1) \left(e^{2i\delta^+_{I,\ell}} - 1 \right) \right] P_\ell \,(\cos\theta) + \qquad (3.1)$$
$$+ \frac{1}{2ik} \sum \ell(\ell+1) \left[e^{2i\delta^-_{\ell,I}} - e^{2i\delta^+_{\ell,I}} \right] \sin\theta \, P'_\ell \,(\cos\theta)$$

where the signs + and - on the δ's indicate states $\ell + \frac{1}{2}$ and $\ell - \frac{1}{2}$ respectively.

In this decomposition, the first term represents non spin flip amplitude and the second, the spin flip amplitude. Of course the second term would be zero if forces were not dependent on spin orientation. But nuclear forces are in fact dependent on spin orientation. If spin were not important, then it would not be possible to distinguish between $P_{3/2}$ and $P_{1/2}$ and we saw that the experimental facts indicate the necessity of dominant $P_{3/2}$.

Now, if we try to describe σ_{++} we must take into account only

$T_{3/2}$. On the contrary, when one has $\pi^- + p$, then a proper combination of both states $T_{3/2}$ and $T_{1/2}$ must be taken, as we saw before in (2.12).

With regard to the number of terms one must take in the expansion, the method is to make a gess on how many waves will enter in the process on the basis of what one sees of the form of the curve giving the experimental angular distribution. We must remember that the square of the modulus of expression (3.1) will give the differential cross-section. Thus, once one has got from the experiment the angular distribution one must try to fit the curve with a polynomial by means of the least squares method for instance. One can try the fit with polynomials of increasing degree, stoping as soon as one sees that the fit is not improved by taking polynomials of higher order.

The notation for the phase shifts is well known, and is reminded in the following table.

	$S^-_{1/2}$	$P^+_{1/2}$	$P^+_{3/2}$	$D^-_{3/2}$	$D^-_{5/2}$
$T_{3/2}$	α_3	α_{31}	α_{33}	δ_{33}	δ_{35}
$T_{1/2}$	α_1	α_{11}	α_{13}	δ_{13}	δ_{15}

It is very difficult to go any further because the calculations become too complicated and the experimental determination turns out to be nearly impossible because of the smallness of the phase shifts.

On the other hand, we already know that classical pion physics can be described in terms of the three first pairs of phase shifts.

The behavior of these six phase shifts are sketched below: in Fig. 4 for the 3/2 state and in Fig. 5 for the 1/2 state.

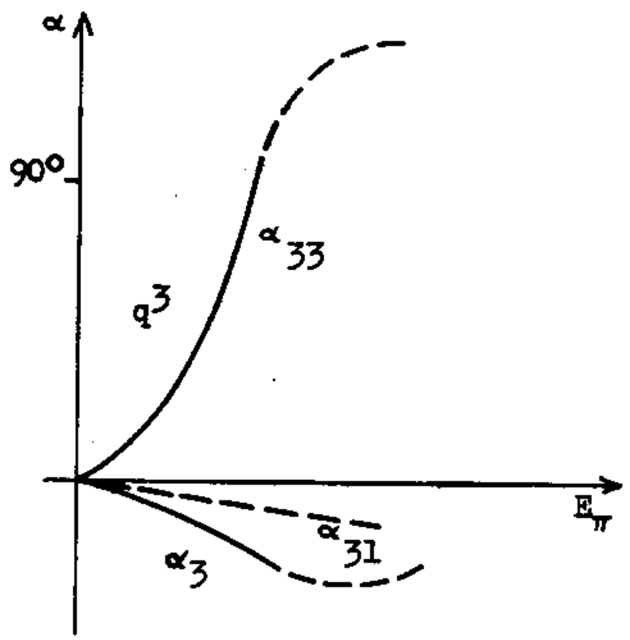


Fig. 4

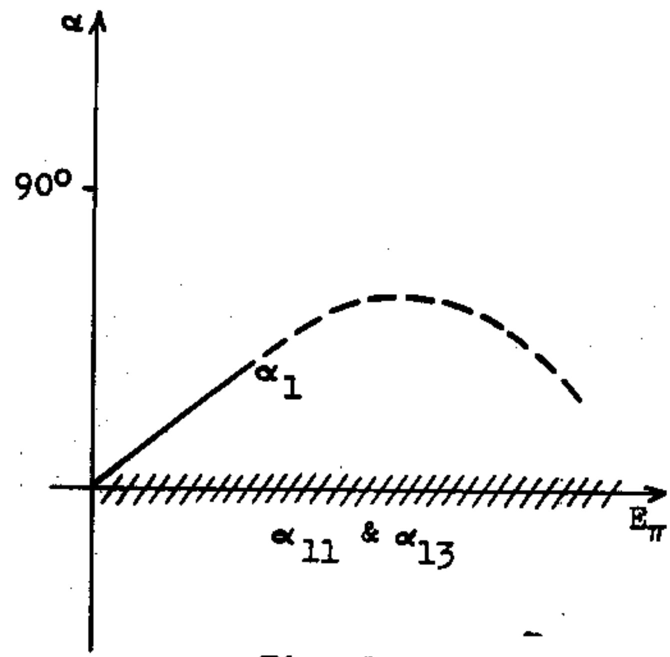


Fig. 5

For the $P_{3/2}$, α_{33} is rapidily increasing, nearly as q^3 where q is the momentum of the pion in the center of mass system. For higher energies its values have not been determined, but it seems to vary according to the dotted curve. As α_{33} is always positive, it reveals a strong attraction between the pion and the nucleon. One cannot obtain directly from scattering experiments the sign of α_{33} but it has been determined by observing the interference between coulomb and nuclear forces in $\pi^+ + p \longrightarrow \pi^+ + p$. In this kind of experiment, if the nuclear potential were repulsive, a curve of type A (Fig. 6) would be obtained, while if attractive, type B curve is expected and this is in fact what comes out from the experiment.

Once one has taken into account the influence of the coulomb scattering, the corrected curve C comes out revealing the pure nuclear scattering.

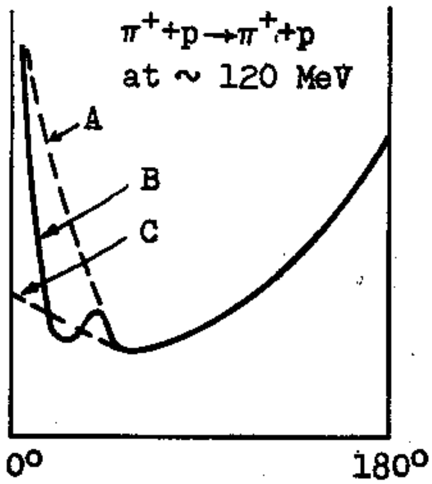


Fig. 6

Once the sign of one phase shift is fixed, one gets the signs of the rest of them. Also dispersion relations provide a way of determining directly the sign of the phase shifts.

With regard to α_3, it turns out to be negative all over the low energy region, decreasing approximately as - q. It has been accurately measured only in the very low energy region, and a behavior indicated by the dotted line is attributed to it further on.

With respect to α_{31}, it is badly known; the only thing that one can say is that it must be very small, and negative.

For the $T = \frac{1}{2}$ state, only α_1 is known; it turns out to be positive and increasing towards higher energies nearly as q.

Nothing can be said precisely about it for higher energies, eventhough some calculations give a behavior indicated by the dotted curve. About the other two phase shifts, α_{11} and α_{13} one only knows that they are very small, thus confined to the small region shadowed on the graph.

This is the situation about the behavior of the phase shifts; as one can see, it is not very brilliant but it is enough to know the phenomena. About checking, any nearly good theory gives α_{33} correctly, but a really good theory should give also the other phase shifts.

4 - Photomeson Production

When a γ-ray strikes a proton we have the reactions $\gamma+p\rightarrow p+\pi^0$ and $\gamma+p\rightarrow n+\pi^+$ where we neglect Compton scattering ($\gamma+p\rightarrow\gamma+p$) that gives only a small contribution.

In the photoproduction the cross-section is much smaller than in the π-nucleon scattering being 2 or 3 orders of magnitude smaller.

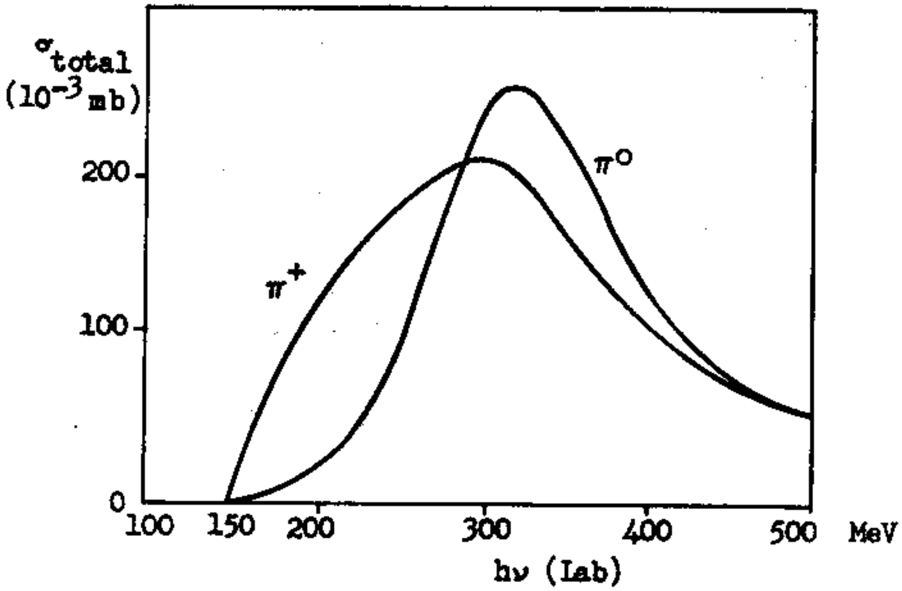
We could expect this since in the first case we have the interaction of γ with p which involves the coupling constant $\frac{e^2}{\hbar c} = \frac{1}{137}$ and in the scattering, using pseudoscalar coupling appears $G^2\sim 15$.

As to the accuracy of the experiments we have the same situation both for photoproduction and scattering except at very low energy where the first is more accurate.

The γ-rays come from bremsstrahlung thus they will have a continuous spectrum, but their incident energy can be well defined by fixing the momentum of the produced pion.

We can see the "big facts" by looking the plot of the total cross sections for the photoproduction of neutral and charged pions from hydrogen which exhibit a maximum around 350 Mev for the incident laboratory energy of the γ-ray (figure 7).

We want to compare photomeson production ~~with meson scattering~~ and such a comparison should of course be made at the same total energy in the C. M. system.



Incident photon energy in Lab (Mev)

Fig. 7

Then we can show that the laboratory energy of the γ-ray (E_γ) and the kinetic energy of the π meson (T_π) are connected by

$$E_\gamma = T_r + T_\pi$$

where $T_r = m_\pi \left(1 + \frac{m_\pi}{M_N}\right)$ is the threshold energy for photoproduction.

Now if we remember that $T_r \simeq$ 150 Mev we see that photoproduction and scattering have the same behavior, the position of the maxima being practically the same. This shows that there is a close relation between both processes.

Let us note that photoproduction of π^+ is different from π^0-photoproduction and this is due to the fact that the incident γ sees the charge and magnetic moment of the particle; in the π^+ case we have an extra term which is described by the diagram in

Fig. 8.

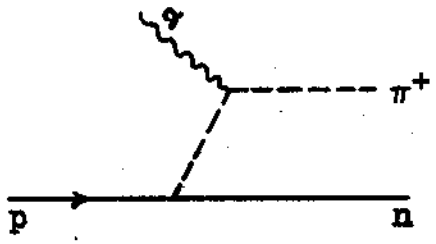


Fig. 8

The connexion between photoproduction and scattering can be checked in several ways:

1) By comparing the reaction $\gamma + p \rightarrow p + \pi^0$ with $\pi^0 + p \rightarrow p + \pi^0$

Now π^0 beams are not available but using charge independence we can get the phase shifts and the cross section for the scattering which we can compare with the first reaction, which can be measured. This was done in Illinois and comes out that both results are the same.

Now let us give a simple explanation of why this is so.

We can understand both processes as two step processes in which the intermediate "state" is the same:

$$\gamma + p \rightarrow Y_{3/2\ \ 3/2} \rightarrow p + \pi^0$$

$$\pi^0 + p \rightarrow Y_{3/2\ \ 3/2} \rightarrow p + \pi^0$$

where $Y_{3/2\ 3/2}$ is a kind of isobar excited state.

We could argue about the physical meaning of this intermediate "state" since the 3/2, 3/2 resonance is very broad, but from the point of view of the model it is convenient to say that the intermediate state exists and that it has a short lifetime.

The decay of Y only depends on g^2 and once it is formed it does not depend on the way in which it was formed.

In the present problem the wave length of the γ-ray is of the same order than the radius of interaction and so we have no way to say if the electric or magnetic absorption is dominant. The nuclear interaction of the isobar decay selects the pole of the γ-ray which is absorbed and turns out to be that the absorption is M1.

2) Another way of checking the relation between both processes is to compare their angular distributions.

We have for $\gamma + p \rightarrow p + \pi^o$ a distribution $2 + 3 \cos^2 \theta$ and for $\pi^o + p \rightarrow \pi^o + p$ we have practically the same result, $1 + 3 \cos^2 \theta$.

3) Another way of stablishing a connection between photoproduction and scattering is via the Panofsky ratio.

The capture of a π^- by proton can led to the reactions

$$\pi^- + p \rightarrow n + \pi^o \qquad (a)$$

$$\pi^- + p \rightarrow n + \gamma \qquad (b)$$

and the Panofsky ratio is defined by

$$P = \frac{\sigma(\pi^- + p \rightarrow n + \pi^o)}{\sigma(\pi^- + p \rightarrow n + \gamma)}$$

By detailed balance the reaction (b) can be connected with its inverse ($\gamma + n \rightarrow \pi^- + p$) by

$$\sigma(\pi^- + p \rightarrow n + \gamma) = 2\left(\frac{P_\gamma}{P_\pi}\right)^2 \times P \times R \times \sigma(\gamma + p \rightarrow n + \pi^+)$$

where:

$$R = \frac{\sigma(\gamma + n \rightarrow p + \pi^-)}{\sigma(\gamma + p \rightarrow n + \pi^+)}$$

With the value of $R = 1.35 \pm 0.10$ and knowing the cross sections we get $P \sim 1.5$ and actually from direct experiments $P = 1.47 \pm 0.07$.

5 - The intermediate region

Our treatment of the intermediate region will begin by seeing which is the general situation about σ_{π^+} and σ_{π^-}, the total cross section for $\pi^+ + p$ and $\pi^- + p$ scattering respectively.

The experimental information comes from the work of three groups, namely: Berkeley, MIT and Saclay.

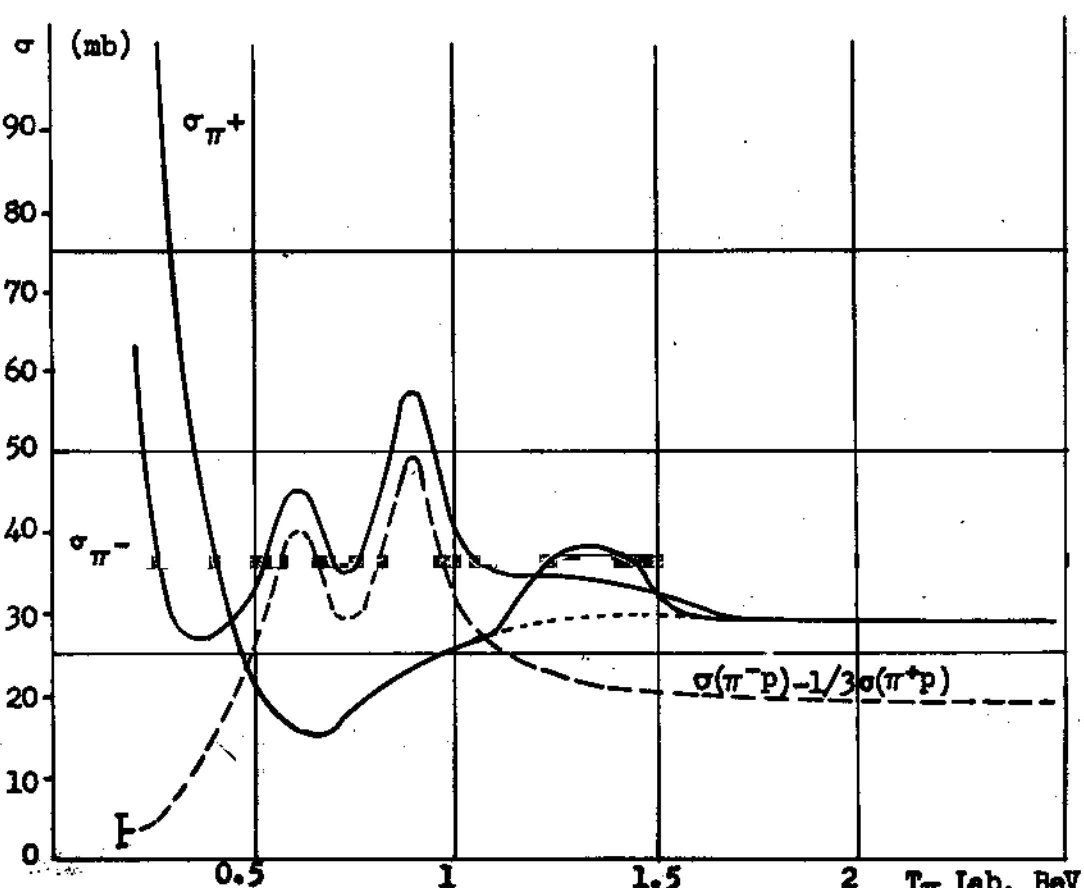


Fig. 9

As one can see from Fig. 9, there are two maxima in σ_{π^-} at the energies of (600 $\pm$ 15) Mev and (890 $\pm$ 20) Mev. In σ_{π^+} we find, appart from a slight indication of a possible maximum at about 900 Mev, a broad maximum at (1350 $\pm$ 100) Mev. In the old measurements of Piccioni et al. both maxima in σ_{π^-} at 600 and 900 Mev, appeared as a single broad maximum.

Having in mind the close conexion between scattering and photoproduction at low energies, let us see now what happens with photoproduction at these new energies. The solid curve of Fig. 10 which represents the cross-section for photoproduction of a π^0, presents appart from the already studied high peak near 350 Mev, a small waving at higher energies which is not meaningfull by itself. In the photoproduction of π^+ it appears also the maximum in the low energy region and a finite clear peak near 750 Mev.

Remembering that the energy that a photon must have in order to give the same energy in the center of mass system as a π is:

$$E_\gamma = 150 \text{ Mev} + T_\pi ,$$

then we see a close relation between the behavior of the cross section for scattering and for photoproduction also at these energies. In this way, the information given by one of the processes is used to imposed the knowledge about the other. For example, the separation between the peaks at 600 and 900 Mev was not seen in the earlier experiments on scattering, but the waving around

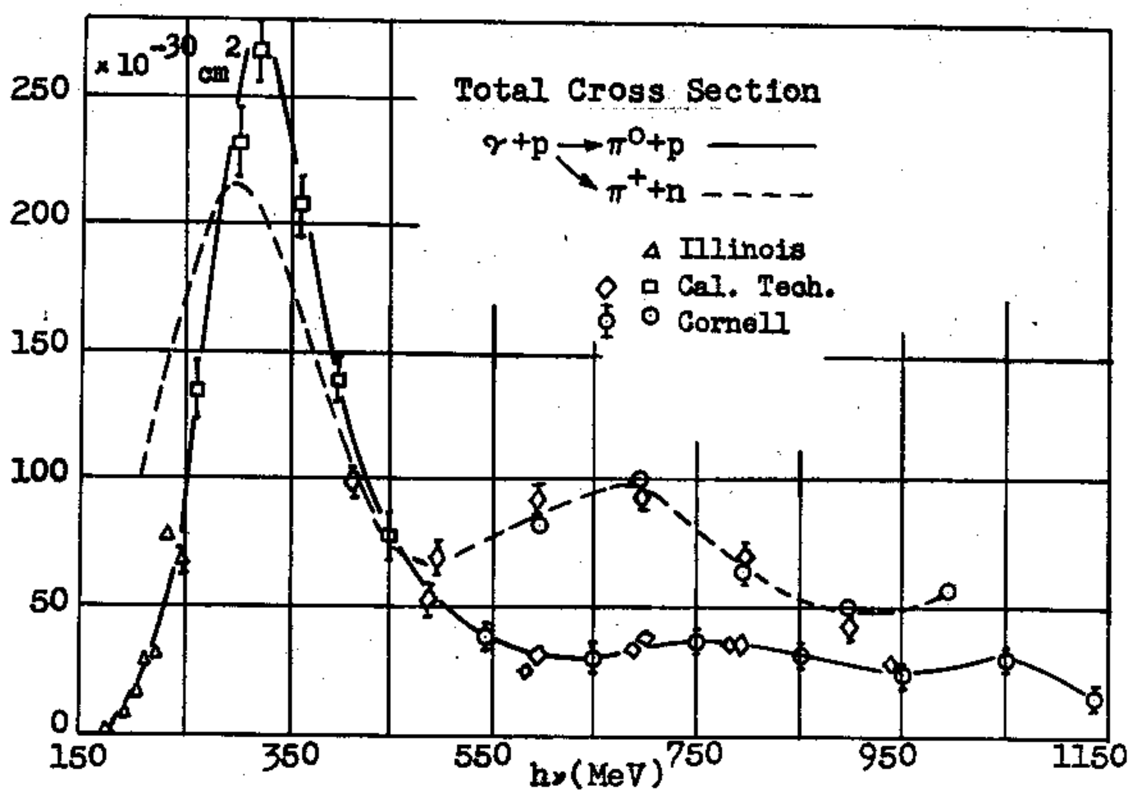


Fig. 10

the corresponding energies in the photoproduction curve was taken as an indication of some particular behavior, and then the experiments on $\pi^- + p$ scattering were improved and the two peaks clearly appeared.

Now, as we did for the low energy resonance, our first task in the intermediate region will be to assigne isotopic spin, angular momentum and parity to each maximum.

5a) <u>Assignement of isotopic spin</u>

Here the arguments are quite simple and conclusive.

Since the peak in $\pi^+ + p$ was much bigger than the corresponding peak in $\pi^- + p$ in the low energy region, and since $\pi^+ + p$ is a pure $T = 3/2$ state of the pion-nucleon system, we assumed T for that first resonance to be 3/2. The same reasoning for the maximum at 600 Mev gives $T = 1/2$ and so on. Thus we have the following assignements of I-spin for the several maxima:

1^{st}	maximum	$T = 3/2$
2^{nd}	"	$T = 1/2$
3^{rd}	"	$T = 1/2$
4^{th}	"	$T = 3/2$

The same conclusions come from photoproduction. In fact: for the two possible channel for photoproduction: $p + \pi^o$ and $n + \pi^+$, the probabilities for $T = 1/2$ and $T = 3/2$ are given by Clebsch - Gordon coefficients:

	$T = 1/2$	$T = 3/2$
$p + \pi^o$	$-\sqrt{1/3}$	$\sqrt{2/3}$
$n + \pi^+$	$\sqrt{2/3}$	$\sqrt{1/3}$

Then, if we imagine the photoproduction processes as taking place through the same intermediate state (as we did for low energies) the ratio between both processes should be given by the ratio of the corresponding Clebsch-Gordon coefficients. Thus, if the intermediate state were $T = 1/2$ it will be:

$$\frac{\sigma(\gamma + p \to p + \pi^o)}{\sigma(\gamma + p \to n + \pi^+)} = \frac{\sigma^o}{\sigma^+} = \frac{1}{2}$$

and if it were $T = 3/2$: $\frac{\sigma^o}{\sigma^+} = 2$.

From the two experimental curves of Fig. 10 one concludes that for each peak one can think of a definite dominant T namely the ones listed in (5.1). In fact, we find for instance, for the second peak at $\sim$ 750 Mev, that the ratio σ^0/σ^+ is nearly 1/2 which corresponds to T = 1/2, and a similar situation for the other peaks.

5b) <u>Assignement of angular momentum</u>.

The first argument comes from the evaluation of the total cross-sections.

If the most important contribution to the cross section for scattering comes from only one angular momentum state, we have:

$$\sigma \leq \sigma_{max} = \frac{4\pi}{k^2} (J + \tfrac{1}{2}) \qquad (5.2)$$

Then we can see which is the minimum J which is necessary to account for the experimental value. From these considerations it turns out to be $J \geqslant 3/2$ for the 2nd maximum and $J \geqslant 5/2$ for the 3rd maximum.

This is true, we insist, if only one state contributes, and must be taken only as a first indication of the value of J.

A second, and better argument is given by the analysis of the angular distributions. There are good experiments in the case of photoproduction.

The angular distribution for the photoproduced π^0 at about 750 Mev (Fig. 11) is fitted by a polynomial: $3.5 - 2.8 \cos^2 \theta$, which means that the distribution is parabolic and this corresponds to a strong $J = 3/2$. From the shape of the curve it seems that there is no appreciable contribution of other J states to the second maximum.

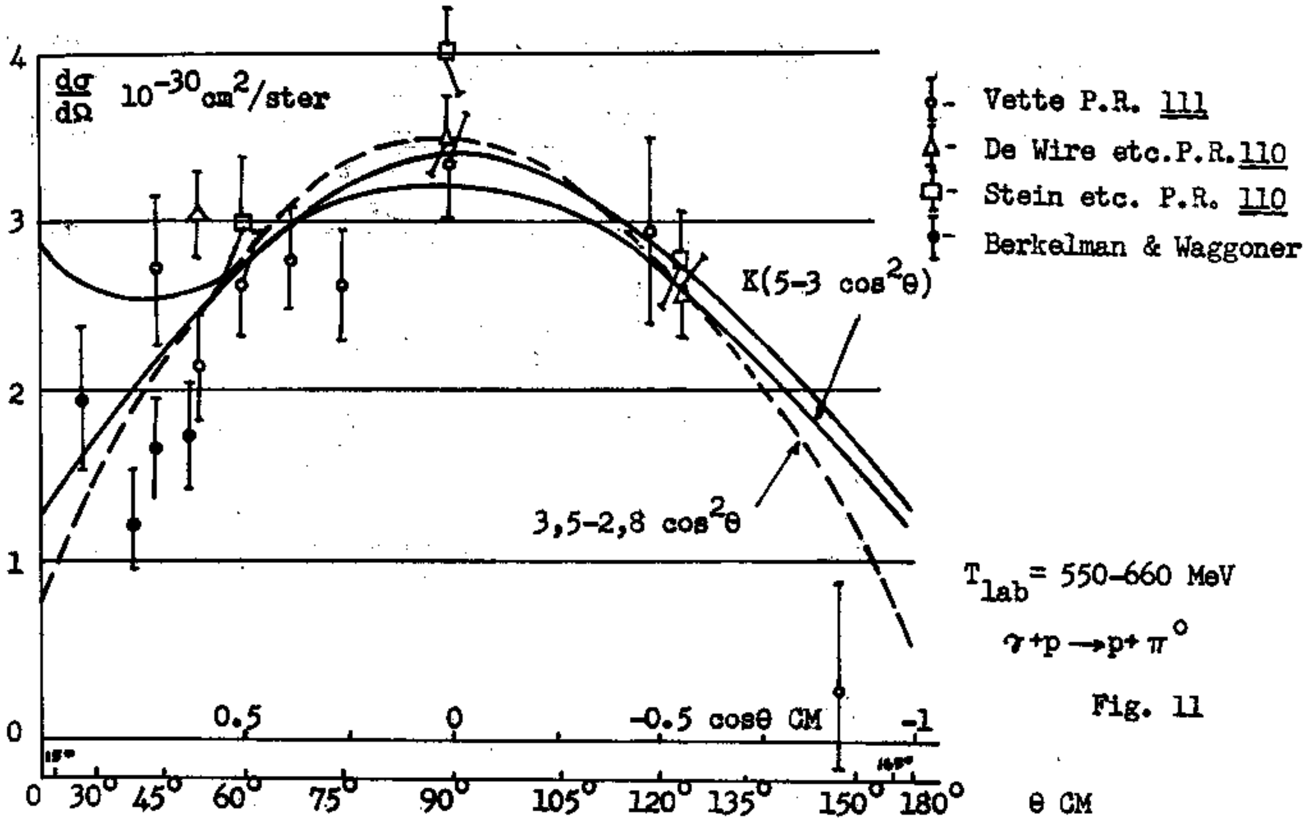


Fig. 11

Moreover, going back to the expression for the total cross-section if we put in (5.2) J = 3/2, it must be $\sigma \cong \sigma_{max}$ in order to account for the 43 mb of $\sigma_{\pi-}$ at 600 Mev. (Fig.9). That means that the phase shift passes at the peak through 90°. Summarizing, the situation at the second maximum can be described by a single resonant state of T = 1/2, J = 3/2.

Now about the third peak at ~ 900 Mev., we have quite good information about the angular distribution for elastic scattering $\pi^- + p$ at 915 Mev. (Fig. 12).

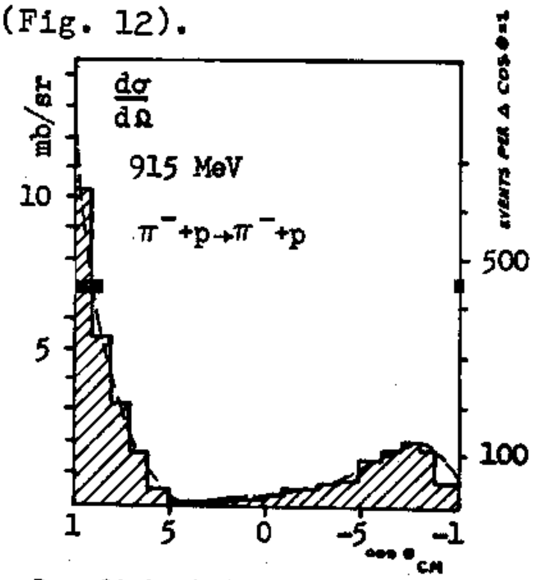


Fig. 12 - Corrected angular distribution with the sixth degree least square fit curve.

Because of the presence of the very big forward peak, the elastic scattering at this energy is assumed to be just the shadow of the inelastic process. It has also a noticeable backward peak attributable to spin flip. The curve is fitted by a polynomial of 5^{th} degree. It is not easy to decide which is the dominant J necessary to account for that behaviour, but evidently one needs the presence of an f wave, that is of a J = 5/2 state.

5c) Assignement of parity

As we have finished to see, it is rather easy to assigne a definite isotopic spin and angular momentum to the maxima in the cross-section in the intermediate region in particular to the second resonance at 600 Mev.

Let's go now to the assignement of parity which is a much more difficult problem. We concluded that a single resonant state in J = 3/2 could account for both the value of the total cross section and the angular distribution at the second resonance. But the problem to be solved next is to know whether the state is $P^{+}_{3/2}$ or $D^{-}_{3/2}$.

We will go again to the data arising from photoproduction processes. One can imagine that within 0 and 700 Mev both the photoproduction of π^{+} and π^{0} are dominated by two states: the ${}_{3/2}P^{+}{}_{3/2}$ which would account for the big first resonance, and the unknown state which we can call B, that will be responsable for the second resonance, and about which we have to decide whether it is $P^{+}_{3/2}$ or $D^{-}_{3/2}$. In that case, one can write for the cross-sections for photoproduction of π^+ and π^o:

$$\sigma^{+} \propto |\ a_1\ A^{+}_{33} + a_2\ B^{?}_{13} + a_3\ C^{-}_{s}\ |^2 \qquad (5.3)$$

$$\sigma^{o} \propto |\ b_1\ A^{+}_{33} + b_2\ B^{?}_{13}\ |^2 \qquad (5.4)$$

where A stands for $P^{+}_{3/2}$. In the case of σ^+ we had to add the term C which is responsable for the electromagnetic interaction between the charged pion and the photon through E_1 absorption, thus a term in s-wave.

By squaring (5.3) and (5.4) we get:

$$\sigma^{+} \propto \left[\frac{2}{3}\ |A|^2 + \frac{4}{3}\ |B|^2 + |C|^2 + \left\{ \begin{matrix} \alpha \,\mathrm{Re}\ A^{*}B \\ \alpha \,\mathrm{Re}\ B^{*}C \\ \alpha \,\mathrm{Re}\ A^{*}C \end{matrix} \right\} \right]$$

$$\sigma^{o} \propto \left[\frac{4}{3}\ |A|^2 + \frac{2}{3}\ |B|^2 + \left\{ \alpha \,\mathrm{Re}\ A^{*}B \right\} \right]$$

Two solutions have been proposed for B, non of which have strong theoretical support, neither agreement with experiments is clear cut.

One proposal comes from Peierls and Sakurai, who assumed B to be $D^{-}_{3/2}$. The simplest to be discussed is the expression for σ^o because of the absence of the term C, but, as in the energy region considered here A and C are small, the analysis of σ^+ turns out to be also easy. Let us remember that in this energy region the ratio between the total cross-sections is:

$$\sigma^+/\sigma^0 = 2 .$$

Thus, assuming $2\,|A|^2 = |C|^2$ one can account for the observed ratio between both cross-sections. But also if we assume A and C to be negligibly small, the former ratio is again accounted for. What onecan conclude from this is that C must be more important than A.

Next thing one can do is to neglect the interference term in (AC) which gives a small and nearly constant contribution over this region.

About the interference (BC) if B and C had opposite parities, this would mean a change in sign in the interference when passing through the maximum at 600 Mev. It has not been observed any such a change, thus indicating that B has the same parity as C_s, namely negative parity. Therefore A and B have opposite parities and the interference (AB) should be present in both angular distributions. But its effect has not been detected in the experiments performed over all the energy region we are considering. Thus one must conclude that A and B must be 90° out of phase.

Peierls tried to justify this conclusion in the following way: the cross section for π^0 photoproduction can be resolved into the two peaks shown in Fig. 13. We know that A contributes predominantly to the first maximum and that for the energy of the resonance the phase is exactly 90°. Since at this same energy, the contribution of B is negligibly small, its own phase should be $\sim 0^{\circ}$. Going towards the energy of the second resonance δ_{33} tends to 180° while the phase of B must rise reaching 90° at 750 Mev., at which energy we thus find again both phases differing in 90°.

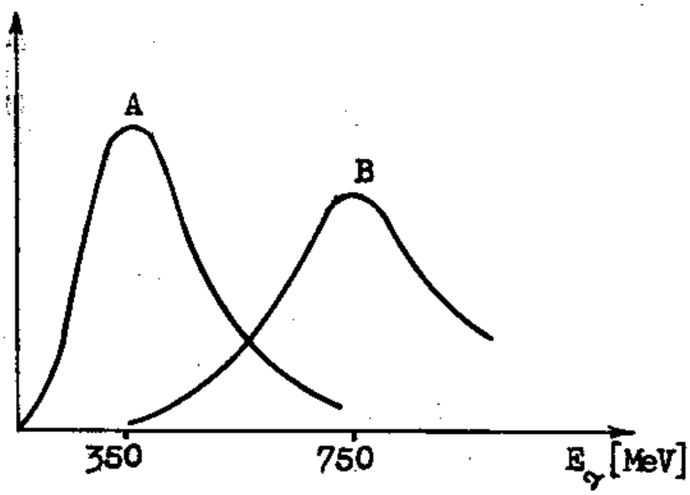


Fig. 13

If the argument is good, then an immediate consequence follows: there must be a region in which $|A|^2$ and $|B|^2$ are comparable and if in that region we have $\mathrm{Re}\,(AB^*) = 0$ then

$$\mathrm{Im}\,(AB^*) \sim |A|^2 \sim |B|^2 .$$

But the expression giving the polarization of the final proton in $\gamma + p \longrightarrow \pi^0 + p$ is:

$$P\,(\theta_{CMS}) = \frac{4\ \mathrm{Im}\,(AB^*)\sin\theta}{(|A|^2 + |B|^2)\,(1 + \frac{3}{2}\sin^2\theta + \ldots)}$$

which, in the case considered above, will give:

$$P\,(\theta_{CMS}) = \frac{4\sin\theta}{2 + 3\sin^2\theta + \ldots}$$

and this would mean an 80% of polarization over a wide region around

90°. Therefore, the analysis of the polarization of the recoil proton would permit to check the model provided of course one believes in the arguments.

Experiments have been performed to analyse polarization, in which the proton going out from the reaction $\gamma + p \rightarrow \pi^0 + p$ was made to collide again against carbon (which is a good analyser for polarization) observing left to right asymmetry of the outgoing protons (Fig. 14). The results of the experiments performed by Stern at Cornell are listed in table I.

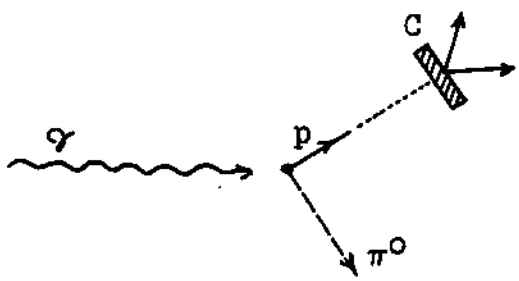


Fig. 14

We said, based on the very naive arguments given above, that one should expect about 0.8 for polarization around 750 Mev. Thus, the fact of finding a large degree of polarization is a strong indication in favour of the model. But next thing is to see whether the same results would not be accounted for by other assignements of parity to the state B.

TABLE I

E	Stern (experimental)	Peierls Sakurai (assuming $D^-3/2$ contribution)	Wilson Stoppini (assuming $P^+3/2$ contribution)
550	0.30 ± 0.12		... 0.3
700	0.59 ± 0.07	0.8	... 0.45
900	0.1 ± 0.1		0.32 ...

Wilson and Stoppini tried to see if $P^{+}_{3/2}$ would give the same result with regard to polarization. They calculated the values listed in the third column of Table I, which also give a nearly good agreement with the experimental data.

Therefore, all one can conclude from the above analysis is that the assignement of parity to the second resonance is a very difficult task and is about a matter of believing in some arguments, most of which are merely guesses, since no theory has yet been developed in this energy region.

Another thing one can try then, is to look into the possibility of extending to the intermediate region the procedures of the better known low energy theory.

This theory gives in a natural way, p and s waves (corresponding to the diagrams a) and b) of Fig. 15) through M_1 and E_1 absorption respectively. D waves must be looked for only in higher order

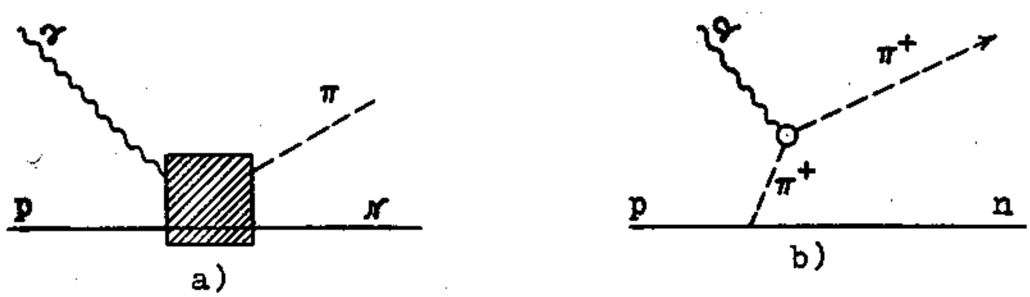


Fig. 15

terms in which the photon can interact with a recoil nucleon (in fact it interacts with the nucleonic current). But there are two difficulties: 1) in the conventional theory those contributions turn out to be small. 2) the nucleonic current is present only in the photoproduction of π^0. The experiments on the contrary show that photoproduction of π^+ is dominant at these energies. This is not a very strong argument, because of the extrapolation into this energy region of procedures which are valid only in the low energy region, but it is the only thing with which we can play. So, looking from this point of view, the assignement of $P^+_{3/2}$ will appear as more natural. Besides, there is nothing in contrary to assume that α_{13} is increasing in the high energy region.

On the contrary, a very simple origin for a strong d wave can be found if we imagine that the interaction proceeds through a $\pi\,\pi$ block like that shown in Fig. 16, because in this case the proton can interact with the π current and one can find all the poles one likes. A diagram of this kind (Fig. 17) is important for a discussion of the electromagnetic structure of the nucleons. Furthermore, a similar diagram works also for the double production (Fig. 18). Therefore, this way of attack seems very promis-

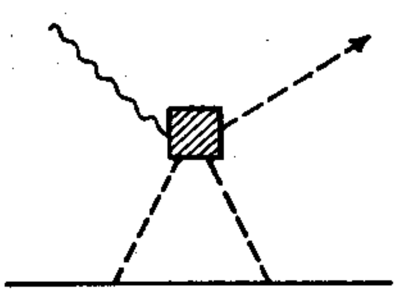


Fig. 16

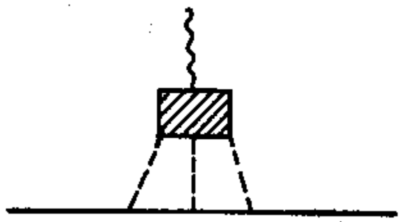


Fig. 17

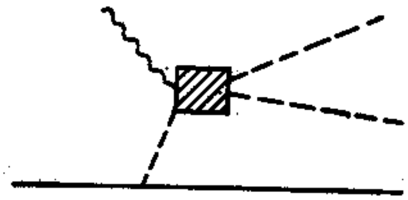


Fig. 18

ing although not yet quantitative calculations have been carried out.

Lastly we will see in the forthcomming lectures, that a $D^{-}_{3/2}$ wave comes out in a very simple way in the isobar model for 2π production. So, despite of the lack of theory it seems that $D^{-}_{3/2}$ would be a good channel.

One of the main difficulties of the intermediate region is the existence of an inelastic part in all the processes. There is a collection of experiments where they have tried to determine which is the contribution to the cross section of the inelastic part.

One could draw a curve such as the dotted curve indicated on Fig. 19, in which case the waving in the total cross section should be attributed to the behaviour of the elastic process. But of course one could also take into account the oscilations in the values of $\sigma_{inel.}$ and then the behaviour of the elastic part should be considered as more regular. Eventhough no defi-

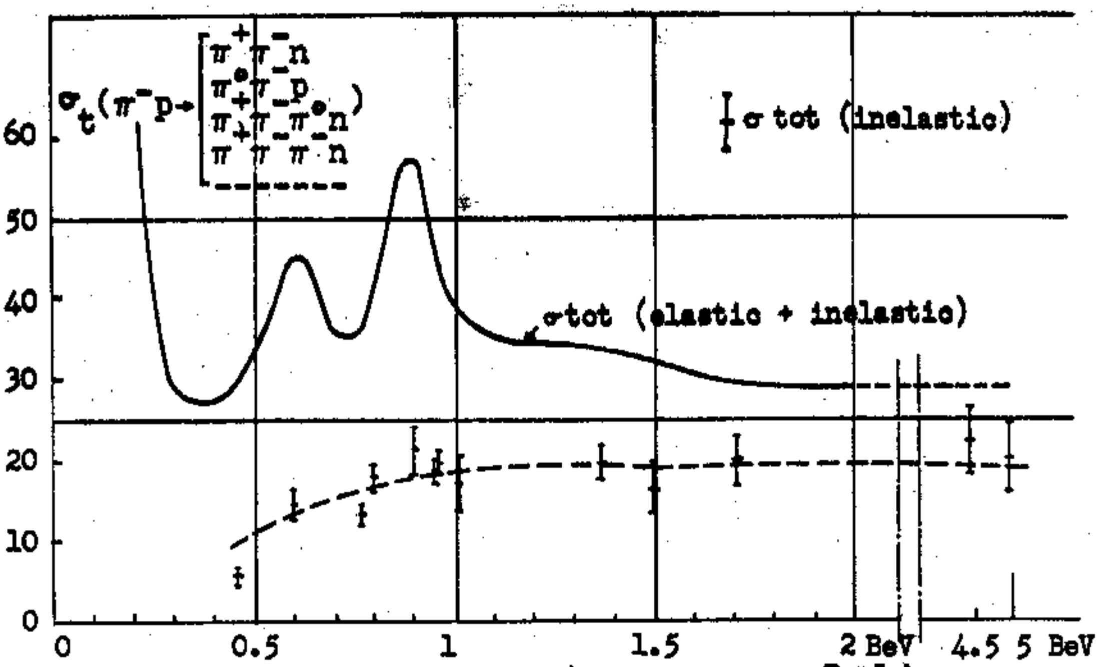


Fig. 19

nite conclusion can be extrated, the first possibility seems to be more reliable if one observes also Fig. 20.

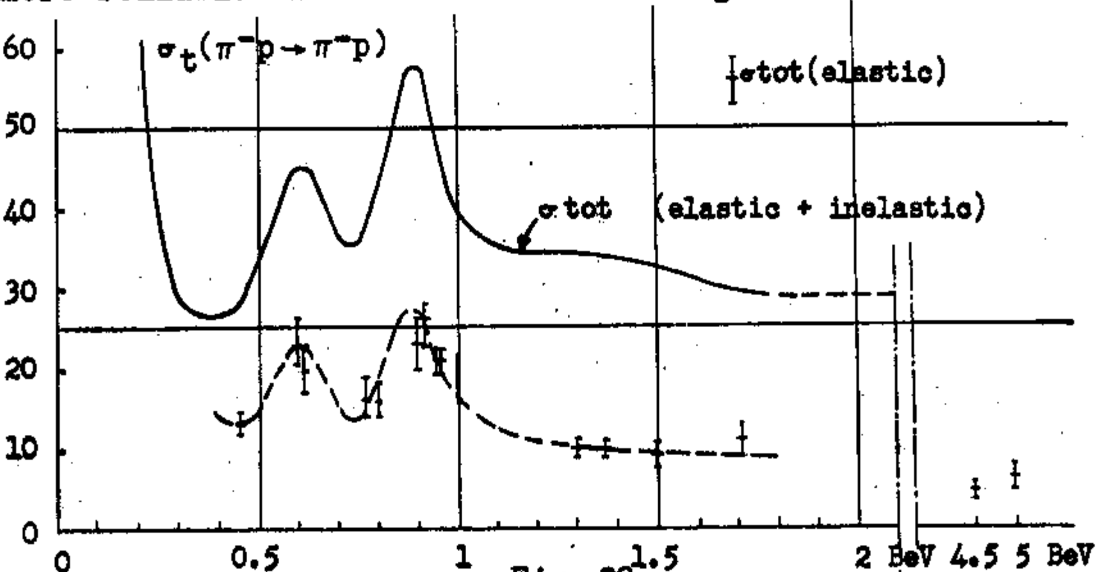


Fig. 20

Anyhow is too dangerous to affirm anything, until some more points be determined.

6 - Models for π production

As we have mentioned in the last lecture we have two kinds of isobar models: the nucleonic isobar model and the pionic isobar model.

In this lecture we shall deal with the first one and after we will touch in the second model.

The nucleonic isobar model is an attempt to describe some phenomena in the intermediate region without introducing any new concept, i.e., using only concepts of the low energy.

In intermediate region we can have the reactions

$$\pi + N \rightarrow \begin{cases} N + \pi \\ N + 2\pi \\ N + 3\pi \quad \text{etc.} \end{cases} \qquad\qquad \gamma + N \rightarrow \begin{cases} N + \pi \\ N + 2\pi \\ N + 3\pi \quad \text{etc.} \end{cases}$$

and others producing strange particles that are very much less frequent.

The double pion production by both π or γ is very rich, the reactions being of the same order of magnitude than $\pi+N \rightarrow \pi+N$ and $\gamma+N \rightarrow \pi+N$ (scattering and single photoproduction) respectively.

The abundance of double pion production is very peculiar and it is difficult to understand it in terms of say, "statistical theory" and so it needs a special explanation.

Note that the terminology "double π production" for the reactions $\pi+N \rightarrow N+2\pi$ and $\gamma+N \rightarrow N+2\pi$ is measleading since only in the second case we have the production of 2 pions; in the former case we have the production of only 1 extra pion.

Let us now see some numbers that give the relative importance of the processes involved.

Take for instance the $\pi^- + p$ case at 960 Mev.

Then we have the following table:

elastic scattering (1π)	40%
"double" production (2π)	34%
"triple" production (3π)	6%
all neutrals ($1\pi^0$, $2\pi^0$, $3\pi^0$)	17%
strange particles production	2%

We have analagous data for photomeson production and this gives us an idea of the branching ratios in the 1 Bev region.

As far as mesonproduction is concerned, $\pi^- + p$ can give the following reactions:

elastic: $$\pi^- + p \rightarrow \pi^- + p \qquad \text{(a)}$$

2π: $$\pi^- + p \rightarrow \begin{cases} n + \pi^+ + \pi^- & \text{(b)} \\ p + \pi^0 + \pi^- & \text{(c)} \end{cases}$$

3π: $$\pi^- + p \rightarrow \begin{cases} p + \pi^- + \pi^+ + \pi^- & \text{(d)} \\ p + \pi^- + \pi^0 + \pi^0 & \text{(e)} \\ n + \pi^0 + \pi^+ + \pi^- & \text{(f)} \end{cases}$$

all neutrals: $$\pi^- + p \rightarrow \begin{cases} n + \pi^0 & \text{(g)} \\ n + \pi^0 + \pi^0 & \text{(h)} \\ n + \pi^0 + \pi^0 + \pi^0 & \text{(i), etc.} \end{cases}$$

All these process can be distinguished by kinematical consi-derations, with exception of the last group.

In fact the mentioned experiment on $\pi^- + p$ interaction at 960 Mev. was performed with an hydrogen buble chamber exposed within a magnetic field.

The fact that the chamber contains only protons, permits one to know that all the interactions will be produced in elementary $\pi - p$ collisions. In addition to this, one is able to determine with a good accuracy the momenta of the charged particles, by means of the curvatures of their tracks in the magnetic field.

In the reactions listed above, with exception of the last three, what one sees in the photographs is one of the tracks of the beam stopping at one point from which secondary tracks emerge. For instance in (a), (b) and (c) one sees two secondary tracks (Fig. 21). One knows already that the negative particle, that is the one which has the curvature in the same sense as the incomming π^- is again a π^-. Analizing the above listed reactions, one sees that the positive can be either a p or a π^+. Now

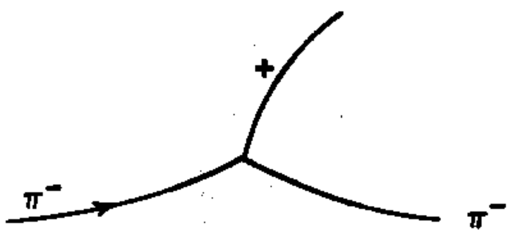


Fig. 21

the curvature of a track depends only of the momentum of the particle and then magnetic field cannot decide about the nature of the particle. But particles with the same momenta and different masses ionize differently and so the analysis of the ionization of the positive particle indicates in most cases whether it is a π^+ or a p.

Next step is to perform a balance of energy and momentum in order to get the mass of the neutral particle; this is done on the basis of the measured momenta of the charged secondaries, the known momentum of the incomming π and the knowledge of the mass of the secondaries.

If the event is elastic, what comes out from the balance is that there is no neutral particle and this situation can also be checked by means of the analysis of coplanarity and angular correlation.

When the event is one of cases b) or c) the neutral mass turns out to be, within the errors of the measured momenta, that of a neutron or of a π^0 respectively. When in trying to adjust the neutral mass to a single particle, one has to go to twice or more times the experimental errors in the momenta in order to get the mass of a π^0 or a neutron, then according to the nature of the positive one knows that it is dealing with one of cases e) or f).

Of course case d) is the most adequate to analize the feature of double production since there one is able to measure the momenta of all the secondaries.

Therefore the kinematical analysis of the registred events, permits a clear identification of the case one is dealing with.

Let us see the thresholds for meson production in the π^- + p case. We have the following table in which we take m_π = 138 Mev. and all energies are in Mev ;

	2π	3π	4π	5π	6π
$T_\pi^{lab.}$	190	370	590	810	1050
$E_\gamma^{lab.}$	340	520	740	960	1200
$W_{C.M.}$	$M+2\mu$	$M+3\mu$	$M+4\mu$	$M+5\mu$	$M+6\mu$

$W_{C.M.}$ is the total energy of the system in C.M.

Let us note that 190 Mev is just the energy for the (3/2, 3/2) resonance, 590 Mev is the energy for the (1/2, 3/2) and the third resonance happens between 810 and 1050 Mev ...

About the explanation of these pion process we can say that the "statistical theory" does not work. The 2π process is not a regular process, showing a cross section that increases from 400 to 600 Mev and then it is of the same order than scattering. This fact cannot be explained by any proposed theory.

6a) <u>The isobar model</u>

The "nucleonic isobar model" is an attempt to understand double pair production without introducing any new concept and dealing only with a well known phenomena "the big (3/2 3/2) resonance".

actually his first application was in the explanation of pion production in p - p and p - n collision but it seems that the most clear discussion can be followed in π production from

πN collision.

Let us describe the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ in terms of the "nucleon isobar model" in order to see how it works. Essentially we have Bohr's idea of intermediate excited state which after a short time decays. We describe the process by:

$$\pi^- + p \rightarrow \pi^- + Y \rightarrow \pi^- + \pi^+ + n$$

The nucleon is initialy in the proton state (ground state) described by ($T = 1/2$, $J = 1/2^+$) and then goes to the excited Y state ($T = 3/2$, $J = 3/2^+$) which has a short lifetime and then decays into $\pi^- + n$, i.e., by emission of one pion it backs to the ground (nucleon) state.

The model assumes as quantum numbers for this excited Y state the characteristic numbers of the "big (3/2, 3/2) resonance". In the model Y is a definite object and the first part of the reaction is a true two body problem; we have a real transition from the point of view of the strong interactions.

Nowadays we are not able to see the isobar because its life time is too short ($\sim 10^{-22}$s) but perhaps in future we will be able to produce it at suficient high velocity and then we could observe it.

So we are in front of a clear cut model where Y is a particle and the 1st part of the reaction is a true two body problem.

In general we would have the two step reaction

$$\pi + N \rightarrow \pi + Y \rightarrow \pi + \pi + N$$

which we represent by the diagram in Fig. 22.

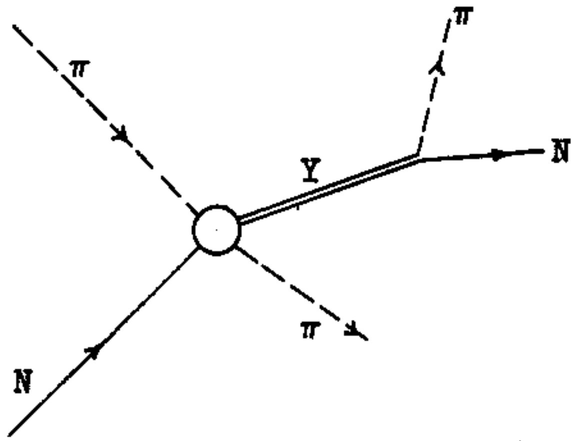


Fig. 22

Let us see the others assumptions.

a) As we said the process (1^{st} part) is a two body process and we have an statistical factor which we call $F = F\ (W_{C.M.}, m_y)$ depending on the mass of the isobar.

This statistical factor is $\propto \dfrac{p_\pi^{\ 2}\ dp_\pi}{(dE_\pi + dE_y)} = \dfrac{p_\pi^{\ 2}}{V_\pi + V_y}$

Since the lifetime of Y is so short, its mass is not well defined and so there must be a number which gives the probability that Y is formed with a certain mass. We call this number $\sigma(m_y)$ and the probability of the process depend on it. So the total cross section is proportional to

$$\int_{M+\mu}^{W_c-\mu} F(W_c, m_y)\ \ \sigma(m_y)\ dm_y$$

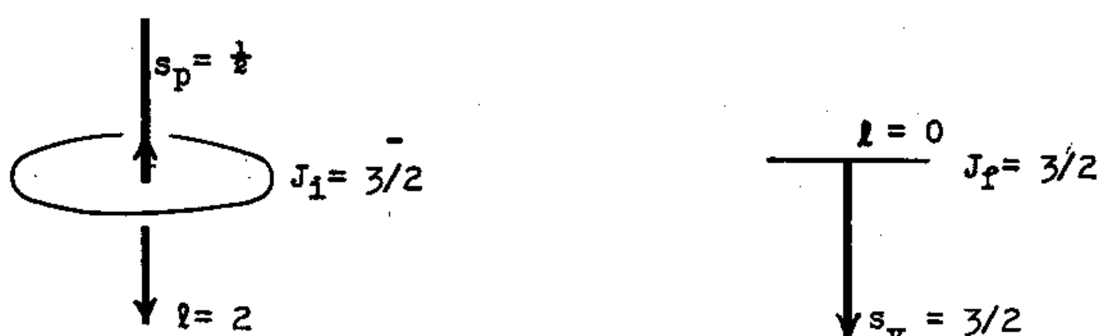
b) The model assumes that $\sigma(m_y) \propto \sigma(\pi + p)$ i.e. to the scattering cross section in the (3/2, 3/2) state.

Through the form of the curve $\sigma(\pi+p)$ we see that Y would not be formed unless we have nearly 190 Mev. So the required energy for the double pion production would have a mass term $\sim M+3\mu$ to give the mass of the π and Y ($m_y = M+\mu+190$ Mev $\sim M+2\mu$) and a kinetic energy term for things to go out. Taking this equal to μ we would have $M + 4\mu$ for the total energy of the system and this implies about 600 Mev for the pion incident energy which is just the position of the second resonance.

It is clear that owing to the fact that Y is a $P^{+}_{3/2}$ we have the following available states for the reaction:

a) If the relative angular momentum of Y and π is S^{-} the total final angular momentum reduces to the spin of the isobar and then the only possible entrance channel is $D^{-}_{3/2}$.

This is ilustrated in the following way (Fig. 23).



initial system (πp) in $D^{-}_{3/2}$. final system (Yπ) in $S^{-}_{3/2}$.

(a) (b)

Fig. 23

b) if the relative angular momentum of Y and π is P^+ then we have the entrance channels $P^+_{1/2}$, $P^+_{3/2}$, $F^+_{5/2}$.

c) if we have D^- for the final system then we would have the possibilities $S^-_{1/2}$, $D^-_{3/2}$, $D^-_{5/2}$, $G^-_{7/2}$ for the initial state and so on.

As we said in last lecture 1) $D^-_{3/2}$ would be important for the 2^{nd} resonance ($\sim$ 600 Mev) and 2) $F^+_{5/2}$ would be important in the 3^{rd} resonance (P waves may be not important in this energy range).

The isobar model makes a quite definite prediction for the energy spectrum of the produced pions. We write again

$$\pi + N \rightarrow \pi_e + Y \rightarrow \pi_e + N + \pi_d$$

where π_e means "extra-π" and π_d means "decay π" ($Y \rightarrow N + \pi_d$). Now we see that the "extra π" momentum is dominated by m_y and as we have a two body problem this momentum is well defined. The spectrum of the "decay-π" is dominated by v_y and will be symmetrical in the rest system of Y.

We give in Fig. 24 the predicted momentum distribution of both pions for 960 Mev incident of π^- in the reaction

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n$$

The "extra-π" ("decay-π") can be the π^+ or π^- and this we ilustrate in Figs. 25 and 26.

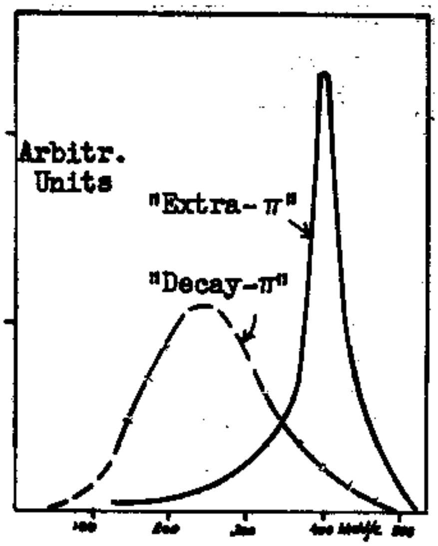


Fig. 24

In the first case the extra-π is π^+ and in the second graph is π^-.

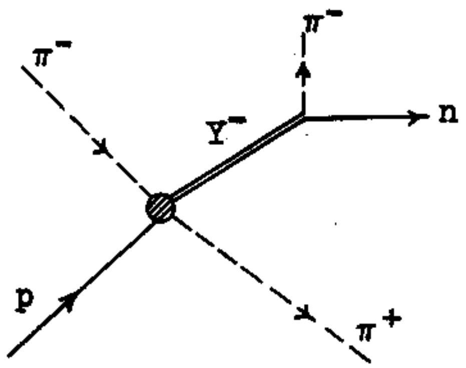


Fig. 25

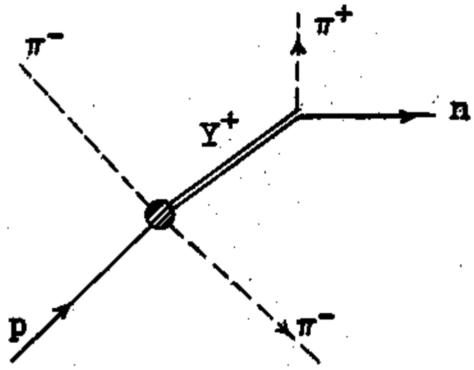


Fig. 26

Now we show (Fig. 27) the experimental result for the π^+ - momentum distribution (C.M.S.) together with the theoretical curves of the isobar and statistical theory.

This shows that the isobar model can fit the experiment, showing that from both final pions, "extra-π" is mainly π^+. Then the final π^- would be mainly the "decay-π" and this we can check

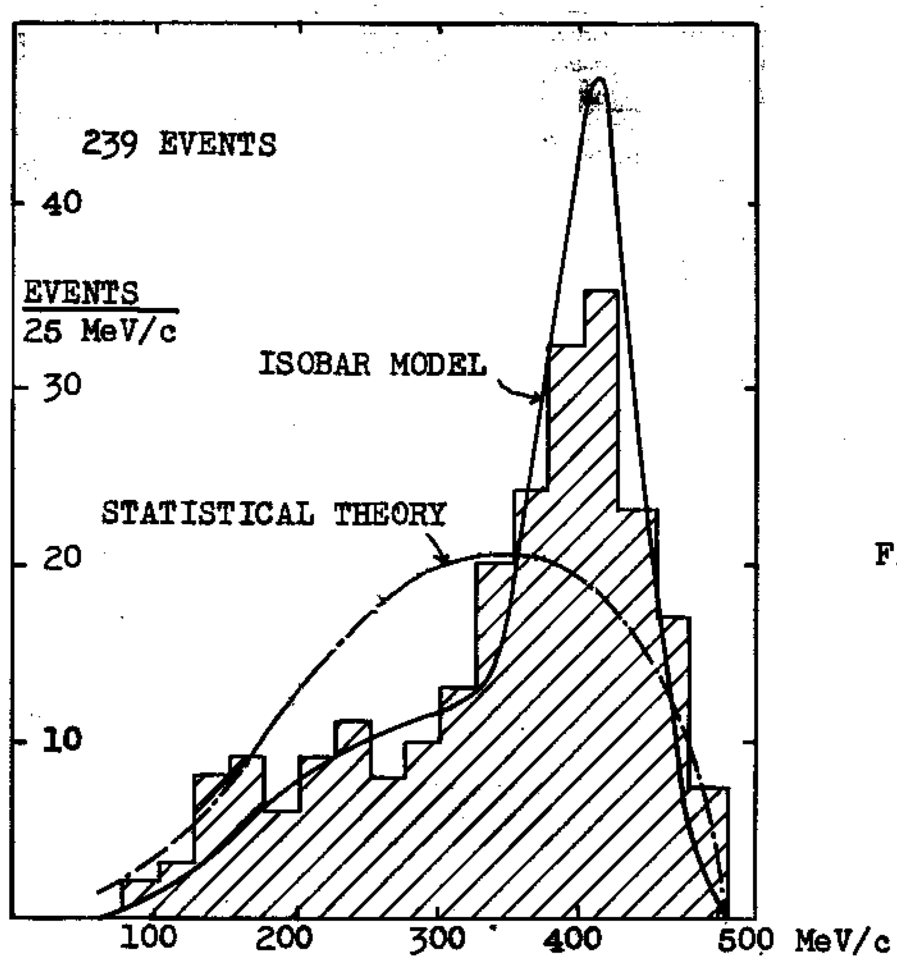


Fig. 27

looking the π^--momentum distribution (Fig. 28) below.

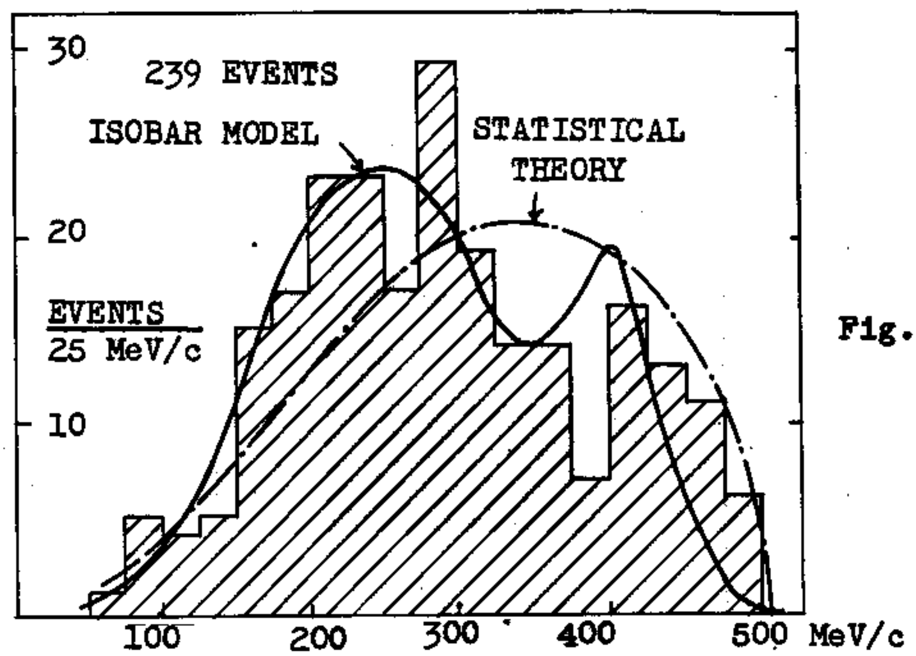


Fig. 28

We can now to point out one attempt to measure the spin of Y by using Adair argument applied for the determination of the spin of Λ . (See chapter 9). The value 3/2 for the spin of Y seems the correct one, but we cannot eliminate the possibility 1/2. (See Alles-Borelli et al. - N.Cim. 14, 211, 1959).

Let us now try to obtain the branching ratios between the different channels for production of one pion within the framework of the isobar model. Let us consider for sake of simplicity the case:

$$\pi^+ + p \longrightarrow p + \pi^+ + \pi^0 \qquad \text{a)}$$

$$\longrightarrow n + \pi^+ + \pi^+ \qquad \text{b)}$$

since we are dealing in this case with only one isotopic spin state, namely the $T = 3/2$ state.

The possible intermediate states for those reactions are:

$$\pi^+ + p \longrightarrow \pi^0 + Y^{++} \longrightarrow \pi^0 + (\pi^+ + p) \qquad \text{a)}$$

$$\pi^+ + p \longrightarrow \pi^+ + Y^+ \longrightarrow \pi^+ + (\pi^0 + p) \qquad \text{b)} \quad (6.0)$$

$$\pi^+ + p \longrightarrow \pi^+ + Y^+ \longrightarrow \pi^+ + (\pi^+ + n) \qquad \text{c)}$$

The wave function for the system can be written in terms of the third component of I-spin, in the following way:

$$\psi = c_1 \ \varphi^{3/2}_{3/2} \ \xi^0_1 + c_2 \ \varphi^{1/2}_{3/2} \ \xi^1_1 \qquad (6.1)$$

where $\varphi^{\tau_z}_{\tau}$ and $\xi^{\tau'_z}_{\tau'}$ are the wave function of the isobar and of the extra pion respectively and c_i the corresponding Clebsch Gordon coefficients. But the wave function of the isobar can be written also in terms of the wave functions of its decay products:

$$\varphi^{3/2}_{3/2} = c_3 \, \mu'_1 \, X^{1/2}_{1/2} \qquad (6.2)$$

$$\varphi^{1/2}_{3/2} = c_4 \, \mu^{o}_1 \, X^{1/2}_{3/2} + c_5 \, \mu^{1}_1 \, X^{-1/2}_{1/2} \qquad (6.3)$$

being μ_T^{Tz} and $X_{T'}^{T'z}$ the wave functions of the decay π and of the nucleon respectively and again the C's the adequate Clebsch Gordon coefficients. Therefore, replacing (6.2) and (6.3) into (6.1), and squaring the resulting expression, we can obtain the following expressions for the spectra of the differently charged pions:

$$I_a(\pi^+) = \frac{3}{5} \, I_d + \frac{4}{15} \, I_e$$

$$I_a(\pi^o) = \frac{4}{15} \, I_d + \frac{3}{5} \, I_e$$

$$I_b(\pi^+) = \frac{2}{15} \, (I_e + I_d)$$

where I_d and I_e stand for decay and extra pion spectrum respectively. Now, suming up the two channels of reaction a) and comparing with b), we get the value of the branching ratio:

$$R = \frac{(\pi^+ + p \rightarrow \pi^+ + \pi^o + p)}{(\pi^+ + p \rightarrow \pi^+ + \pi^+ + n)} = \frac{13}{2} = 6.5$$

Unfortunately, there are not enough experimental results regard ing $\pi^+ + p$ reactions, as to check this clear prediction of the model.

There are on the contrary good experiments on $\pi^- + p$ scattering which can provide the possibility of checking, only that the situation is not so clear-out as in the former case, since now one has a mixture of T = 3/2 and T = 1/2 states. Because of these two possible channels one must analyse this reaction in terms of two parameters:

$$\rho = \frac{\sigma_{3/2}^{in}}{2\sigma_{1/2}^{in}} \qquad \text{and} \quad \varphi$$

where $\sigma_{3/2}^{in}$ and $\sigma_{1/2}^{in}$ are the cross sections for production of one pion for the states T = 3/2 and T = 1/2 respectively and φ is the phase difference between the matrix elements for one pion production in the T = 3/2 and T = 1/2 states.

Of course if there were good experimental information on $\sigma_{3/2}^{in}$ and $\sigma_{1/2}^{in}$, ρ would not be any more a parameter, but this is not the case.

With an adequate choise of the values of ρ and φ, one gets consistency between the experimental results on branching ratios and the predictions of the model and as we have already seen, one can account in a quite satisfatory way for the momentum distribution of the pions. This last turns out to be the best check of the model and the clean cut predictions on this point are a consequence of the strong selection of the energies of the extra pion, due to the formation of the isobar.

6b)

Up to now we have been dealing with the cross-sections for pion production in the different states between the possible charge states of the intermediate isobar. This implies the possibility to observe the isobar so as to be able to decide which is its charge state. But this is not the actual situation due to the very short lifetime of the isobar and what one observes is a certain final state which can be produced through different intermediate states.

Nevertheless it is possible to develope an isobar model introducing the amplitudes for the different channels instead of the cross-sections, and by this way take into account the interferences between those channels.

The advantages of such a procedure would be: a) one is able to theoretically justify the amplitudes one is introducing.; b) it is possible to derive the isobar model as presented by Lindenbaum and Sternheimer, and establish quite well which are the terms one is neglecting in such particular model.

Recently Stanghelini et al* developed that idea. They considered in particular the process

$$\pi^+ + p \longrightarrow \pi^+ + \pi^0 + p$$

* Bergia, Bonsignori and Stanghelini - N. Cim. in press.

for which, in the C.M.S. one has the situations described in Fig. 29, a) and b) for the initial and final states respectively.

a) b)

Fig. 29

One does not know anything about the mechanism through which the final particles have been produced, but only that there are two pions and a proton, with momenta q', q" and p' respectively, and that there is a strong interaction between the proton and the pions in the 3/2 3/2 state, thus being likely that the proton and one of the pions emerge in a bound state characterized by the mentioned values of J and T.

Therefore, the process will be described by the diagram of Fig. 30, and the scattering amplitude can be written:

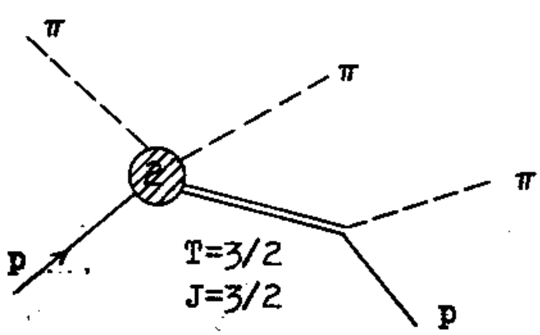


Fig. 30

$$T \sim R\,(E_o, \theta, \varphi)\,\tau(m) \qquad (6.4)$$

where $R\,(E_o, \theta, \varphi)$ represents the unknown process through which the final particles appear (which could involve for example a $\pi\,\pi$ interaction); E_o

the total energy in the C.M.S.; $\mathcal{T}(m)$ the amplitude for the production of isobar with total energy m in the C.M.S. This $\mathcal{T}(m)$ is closely connected with the amplitude for $\pi - p$ elastic scattering in the 3/2 3/2 state, thus would involve the cross section for $\pi^+ + p$ scattering at the energy m in the C.M.S. besides a phase space factor also depending on m. From an approximate solution of dispersion relations, one gets for the amplitude of the elastic channel in $\pi^+ + p$ scattering:

$$T_{33} = \frac{1}{q} e^{i\delta_{33}} \text{ sen } \delta_{33} = \frac{\Gamma(m)}{(m - m_1) - i\,\Gamma(m)\underline{q}} \qquad (6.5)$$

where $\Gamma(m)$ is the width of the resonance, m the total energy of the $(\pi^+ p)$ system in their C.M.S., m_1 a parameter and $\underline{q}$ the pion momentum. Expression (6.5) is the solution of the Feynman graph of Fig.31 corresponding to the whole scattering process, where each identical vertex contributes with a $\sqrt{\Gamma(m)}$ and the propagator with the denominator $\left[(m-m_1) - i\Gamma(m)\underline{q}\right]$. For the production process represented in Fig. 30, the contribution of the left hand side vertex is involved in the unknown factor $R(E_0, \theta\, \varphi)$. After that vertex a propagator appears, and then the second vertex, which is identical to those appearing

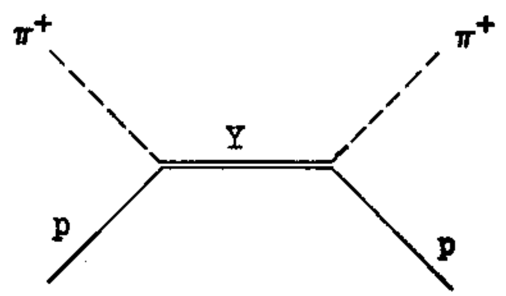


Fig. 31

in the diagram for elastic scattering of Fig. 31. Therefore, the amplitude for production can be written:

$$T_{prod.} = R\,(E_o,\theta,\varphi)\ \frac{\sqrt{\Gamma(m)}}{(m-m_1) - i\ \Gamma(m)\ q} \qquad (6.6)$$

Further more, once the two pions and the proton are created in the unknown step of the process, we do not know which is the pion which is actually interacting with the proton, since the sharpe state of the isobar is not observable. Due to the fact that the momenta of the pions will be in general different the corresponding masses of the two possible isobars will be also different and in order to consider both possibilities, we have to add a similar term in (6.6). Therefore, the total production amplitude will be given by:

$$R\,(E_o,\theta,\varphi)\ \left[\frac{\Gamma(m)}{(m-m_1) - i\Gamma(m)\ q} + \frac{\Gamma(m')}{(m'-m_1) - i\Gamma(m')\ q}\right] \qquad (6.7)$$

By squaring (6.7) we obtain the cross-section for production which will thus contain interference terms accounting for the two possible isobars which give an identical final state, namely $Y^+\ (\pi^o,\ p)$ and $Y^{++}\ (\pi^+,\ p)$ of a) in (6.0).

If one neglects the interference terms when squaring (6.7), one obtains for the cross-section the same expressions than Lindenbaum and Sternheimer.

Figs. 32 and 33 represent the predicted spectra for π^0 and π^+ respectively, from the reaction $\pi^+ + p \longrightarrow \pi^+ + \pi^0 + p$ as compared with the corresponding spectra calculated with the Lindenbaum and Sternheimer model and with the statistical theory.

In both figures one can readily see that the interference terms enphasise the separation between the peaks corresponding to the extra $-\pi$ and decay $-\pi$.

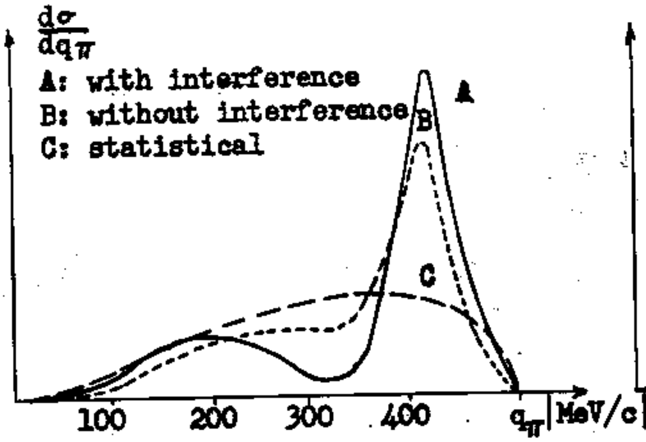


Fig. 32

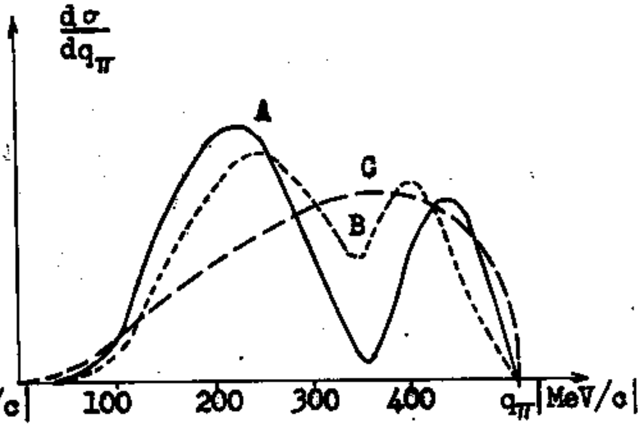


Fig. 33

For the case of $\pi^- + p$ reaction, Figs. 34 and 35 show about the same situation for the π^+ and π^- spectra from reaction $\pi^- + p \longrightarrow \pi^- + \pi^+ + n$.

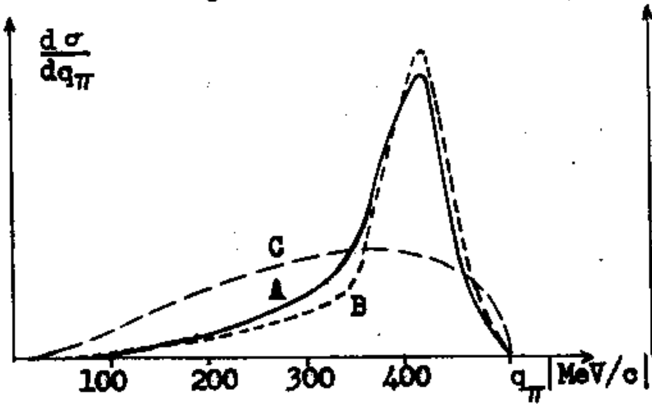


Fig. 34

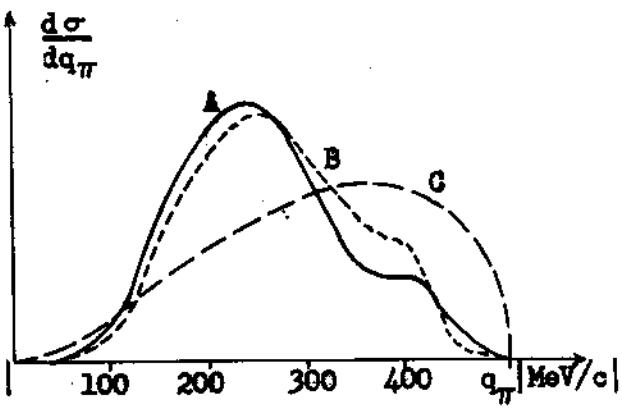


Fig. 35

The agreement with the experimental situation (See Figs. 27 and 28) is slightly improved.

We can conclude that taking into account the interference, there is a tendency to magnify the qualitative feactures of the isobar model and this is perhaps the reason for its good agreement with the experiment, despite of the fact that it probably does not represent the whole actual situation.

6c) <u>The $\pi - \pi$ interaction</u>

There have been lastly some indications about the possibility of a strong $\pi - \pi$ interaction. They are not very relevant but we believe that in a near future new experiments will provide more evidence on that subject.

The hypothesis of $\pi - \pi$ interaction based on a very naive idea, namely that all the particles taking part in strong interactions, interact strongly with each other. We know already that there is a strong interaction between $\pi - N$, $K - N$ and $N - N$. Then in Fig. 36 we can imagine than all the other boxes, will be

	π	K	N	Y
π	X		X	X
K			X	
N	X	X	X	
Y	X			

Fig. 36

also filled up, and let us begin by putting some mark in the first box.

About $\pi - \pi$ interaction, the experimental features which can give a first indication on it appear in some high energy phenomena. They are the following: 1) the levelling off (see fig. 9) towards about 30 mb of the total cross sections for $\pi^- + p$ and $\pi^+ + p$ scattering from 2 Bev upwards. If we imagine the nucleon as a black sphere its geometrical cross section will be πR^2, where, in order to account for the 30 mb, R must be of the order of the Compton wave length of the π ($\lambda_{C\pi} \sim 1$ fermi) which is much greater then the Compton wave length of the nucleon λ_{CN} (0.2 fermions). This fact indicates that the incoming pion, would collide mainly with the nucleon cloud. Within the cloud one finds π, K and Y, so that the cross section for the interaction of a pion with a nucleon, must be built up by the cross sections of the pion with pions, kaons and hyperons. 2) The angular distribution of elastically scattered pions by nucleons at energies around 1 Bev., presents the already mentioned big forward peak (Fig. 12) which is interpreted as diffraction scattering. Therefore, from that forward peak one is able to estimate the radius of the object which absorbes part of the incoming wave packet giving rise to the diffraction. By this means one finds again for $R \sim \lambda_{C\pi}$.

Now, from all the particles which are present in the nucleon cloud, the pion is the one which is able to go to the largest distances because of its lower mass, so that the probability of a collision between an incoming pion with a pion is greater than the probability to collide with the other particles for large impact parameters, i.e. for large energies. Furthermore from the knowledge of the nucleonic structure, that is, of the charge and magnetic moment distributions, an estimate can be done of the number of pions in the nucleon cloud and that number turn out to be ~ 1. Thus, in order to account for a cross section of ~ 30 mb, the $\pi\,\pi$ interaction should be very strong.

There are also other facts indicating the existance of a $\pi\,\pi$ interaction, such as for instance, the possibility of a better understanding of the low energy behaviour of the $\pi\,N$ scattering by means of the introduction of such $\pi\,\pi$ interaction. Also the production of pions in $\pi\,N$ interactions could be interpreted by diagrams taking into account a $\pi\,\pi$ interaction.

All these are merely indications, and not strong evidences. One should look then for a suitable experiment to give the cleanest information about that process. Actually few phenomena are able to give a clear answer. The ideal would be of course to analyse direct $\pi\,\pi$ scattering, performing the experiment by means of an arrangement as indicated in Fig. 37, taking the pion beams produced in two accelerators and making them to collide in the vacuum, but up to now there is not enough intensi ty in the available accelerators. More feasible than the former, yet still technically difficult to be performed, is the electromagnetic production of pions in the collision between two electrons. The experiment woul be able to discover a resonance in the $\pi\,\pi$ interaction because if that resonance would exist, the cross-section for the production of pions should show the same resonance.

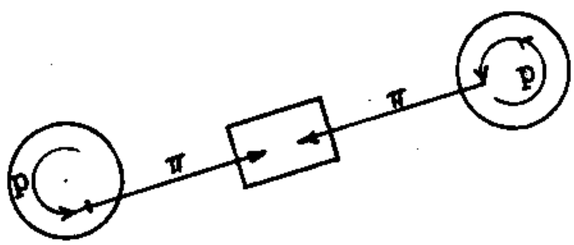


Fig. 37

Due to the practical difficulties of these experiments, one is obliged to think of less clean, but still reliable situations, such as the photoproduction of pions on nuclei, where one can select events in order to use electromagnetic field of the nucleus outside the radius of the nuclear force (Fig. 38) so that the production would not be complicated by the strong interaction between the pions and the nucleons in the final state. For that aim, one should select events with very low momentum transfer to the nucleus.

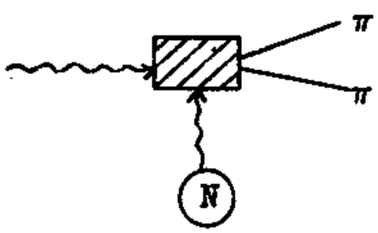


Fig. 38

Thus, if K is the energy of the incoming photon, ω the total energy of the two pions in the cms. and ϵ the energy of the emitted photon, since $\epsilon \sim \omega^2/_{2K}$ in order to go outside the radius of the nucleus, one needs:

$$K > 600 \text{ Mev. } A^{1/3}.$$

One can be sure that one is dealing with this kind of phenomena, by looking to the double dependence with respect to A and Z.

Ferretti suggested also an experiment which could be performed now at CERN, on the production of three pions in the collision of π on nuclei (Fig. 39). The pions would present a peculiar angular distribution due to the small momentum transfer by the electromagnetic field of the nucleus. None of these experiment have been performed up to now and the only practical possibility is to try to extract some information from pion production in π - N interaction, assuming the process as a result of the collision between the incoming pion with a virtual pion of the nucleon cloud. The corresponding diagrams for such a process should be that of Fig. 40 a), the box involving the π π cross-section, but there is also the possibility of a final state

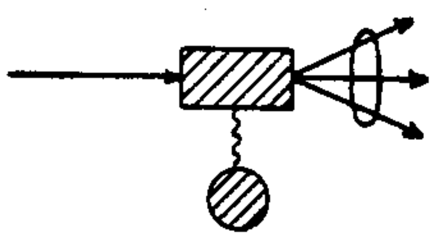
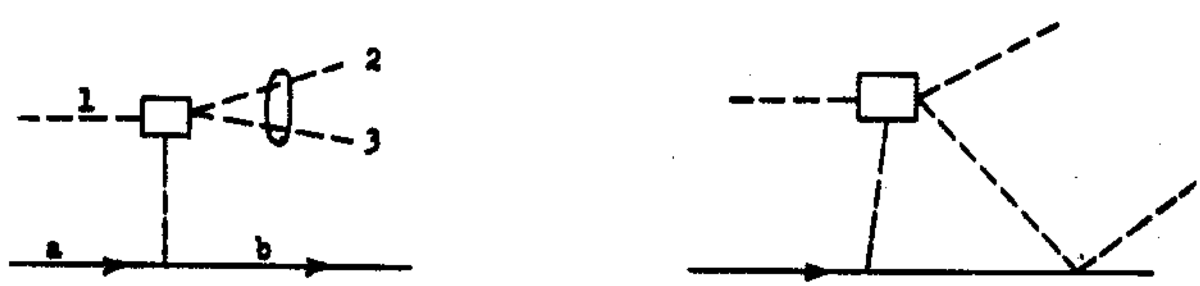


Fig. 39



a)

b)

Fig. 40

π - N interaction, as indicated in Fig. 40 b). Let us assume for the time being that the final state interaction is unimportant so we have merely the first graph.

But we already saw that the process: $\pi + N \rightarrow 2\pi + N$ is well interpreted by the isobar model, so one is tempted to say why to look for another model ? But if there is a clear indication of the presence of the nucleonic isobar Y due to the strong interaction between π and N, one is obliged not to neglect the possibility of the existence of a pionic isobar I_π if there is the suspiction of a strong $\pi\pi$ interaction. This I_π would be favoured in cases of low energy transfer to the nucleon, while Y would be more likely formed for large momentum transfer.

Let us see now how is it possible to get information on $\pi\pi$ interaction from the analysis of π N collisions. Chew and Low were the first who put in evidence the existence of a pole in the scattering amplitude in a diagram such as that of Fig. 26 a). Suppose for a moment that the two pions were produced in a nearly bound state, i.e. as an object of total mass ω and momentum p_{23} as a result of a strong interaction between those pions in a certain definite value of their total energy in its c.m.s., angular momentum J and parity P. Thus one is thinking of the two pions going as a single particle with momentum p_{23} and energy ω, and in consequence we are reducing the diagram to one for four particles (Fig. 41). The dispersion relations theory has been able to give, in a few cases, the poles appearing in graphes of four particles, and from there, one can generalize

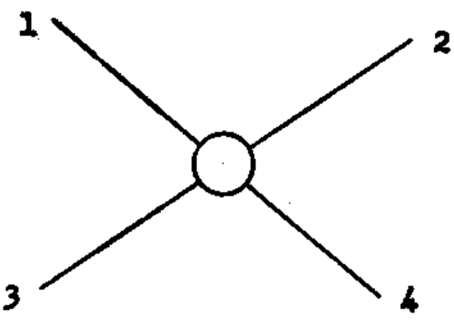


Fig. 41

the way of obtaining the poles in the scattering amplitude. Consider any pair of particles, such as for example 1 and 3; if those particles can be thought in a state for which the quantum numbers are the same than those of a physical particle, there is a pole in the scattering amplitude. Therefore if 1 and 3 are a π and a N respectively the system formed by them can be in a state of $T = 1/2$, $J = 1/2$, $P = +1$, but those quantum numbers correspond to a physical nucleon. Thus the scattering amplitude should present a pole in the energy.

In the described situation there must be a pole for the $\pi\,\pi$ scattering amplitude when one looks the diagram of Fig. 26 a) from the top, the pole being given by a pion. Now, going back to the notation of Fig. 26 a), one can define the following quantities:

$$s_1 = (p_1 - p_{23})^2 = (p_a - p_b)^2$$
$$s_2 = (p_1 - p_b)^2 = (p_{23} - p_a)^2$$
$$s_3 = (p_1 + p_a)^2 = (p_{23} + p_b)^2$$

If we look the diagram from the left, s_3 is the total energy in the c.m.s. where the momentum of 1 and a are opposite; in the same situation, s_1 represents the momentum transfer. If we now look the diagram from the top, s_1 is this time the total energy in the c.m.s. and in the variable s_1 we will have a pole when the specifications of the channel through its quantum numbers, coincides with the specification of a pion. Let us calculate s_1 in the laboratory system where, in the initial state we have an incoming pion colliding with a nucleon at rest, so that $E_a = M$ (M = mass of the nucleon) and $\vec{p}_a = 0$. In the final state, one has for the nucleon:

$$\vec{p}_b = \vec{p} \quad ; \quad E_b = \sqrt{p^2 + M^2}$$

Thus:

$$s_1 = \left(\sqrt{M^2 + p^2} - M \right)^2 - p^2 = 2M^2 - 2M\,(M + T) = -\,2M\,T_{Lab}$$

We already said that the pole will appear for a π, so that

$$s_1 = 2M\,T_{Lab} = \mu^2$$

but in a non relativistic approximation: $2M\,T_{lab} = p^2$ so that the pole will be at $p^2 = -\mu^2$. In consequence the pole will be in the unphysical region for the energy, otherwise it would not be a stable particle.

In order to extract the evidence of a $\pi\,\pi$ interaction from an experiment on production of pions in π N collisions, one should then represent the differential cross section with respect to p^2, assuming ω to be constant, and extrapolate the resulting curve from the positive values of p^2 into the unphysical region as is roughly indicated in Fig. 42, the ordinate at $p^2 = -\mu^2$ depending on the strenght of the $\pi\,\pi$ interaction. The reliability of the extrapolation depends on the goodness of the fit to the experimental data. But from the experiment what one gets directly is not $d\sigma(p^2)/dp^2$ but $\partial^2\sigma(p^2,\omega^2)/\partial\omega^2\partial p^2$. Where ω is the total energy of all the final state particles except the recoil nucleon in their c.m. system. Therefore the first step will be to analyse a two dimensional representation of the cross--section. Both variables p^2 and ω^2 have limitations, because they depend on the total energy of the system, thus the points representing each event should be contained within a certain region, as it turns out to be in Fig. 44 and 45.

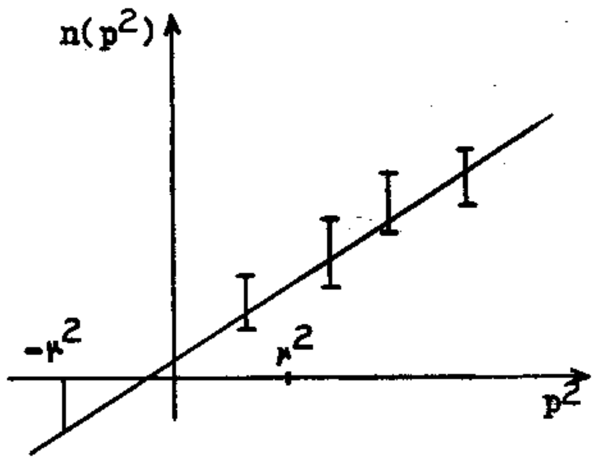


Fig. 42

Now, if the process were dominated by a strong $\pi\,\pi$ interaction, what one should see is a clustering of the poinst around a defined value of ω^2, such as indicated in Fig. 43.

In this case, the one dimensional graph $(n\,(p^2))$ could be constructed and the extrapolation into negative values of p^2 could be performed. This is not the actual experimental situation, since the graph dont present such a clustering but a rather uniform distribution of the points.

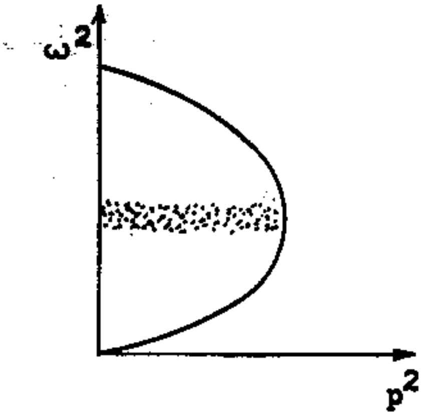


Fig. 43

Bonsignori and Selleri (N.Cim. XV, 465, 1960) analysed in this way the results of the already mentioned experiment performed at Bologna on $\pi^- + p$ interactions at 960 Mev (Figs. 44 and 45) where no indication of a strong $\pi\,\pi$ interaction can be extracted from any of the

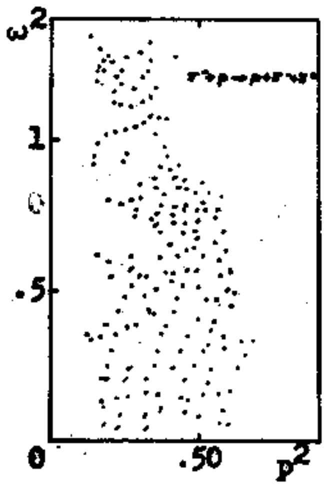


Fig. 44

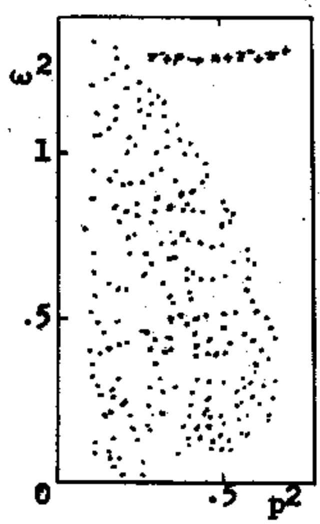


Fig. 45

two dimensional graphs. Then they tried to elaborate a little more the results, assuming that the only important diagram describing the process is that of Fig. 46. This means that the fundamental process is one of elastic $\pi\,\pi$ scattering. The matrix element for such a process have been calculated by Chew and Low, obtaining, after integration over all variables but p^2 and ω^2,

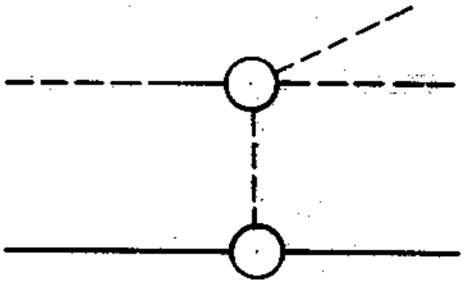


Fig.46

$$\frac{\partial^2\sigma}{\partial^* p^2\, \partial\omega^2} \longrightarrow \frac{f^2\mu}{2\pi}\ \frac{\sigma_{\pi\pi}(\omega)}{q_{\pi L}}\quad q'_\pi\,(\omega)\ \ \frac{p^2/\mu^2}{(p^2+\mu^2)^2}$$

form which is valid only when the square of the four momentum of the spectator nucleon approaches the unphysical limit $-\mu^2$.

Bonsignori and Selleri took the expression of Chew and Low as valid also for physical values of p^2, with the argument that the value $p^2 = -\mu^2$ where it holds exactly is not too far from the physical zone.

Then, as it is impossible to select the appropiate ω^2 from the two dimensional graphs, they integrated the expression with respect to ω, neglecting the terms containing any contribution coming from diagrams different from that of Fig. 40 a). This procedure has been objected because two things can happen.
1) That there is no resonance in $\sigma_{\pi\pi}$ in which case one is quite safe integrating over ω ; 2) that there is a resonance, in which case by taking an average over a large interval, one is spreading out the information.

Nevertheless, in order to check their model, Bonsignori and Selleri applied the same criteria to another kind of process, namely $N + N \longrightarrow \pi + N + N$. Here one has a diagram as indicated in Fig. 47, where now the pair (2,3) will be a π and a N in the strongly resonant 3/2, 3/2 state. Then if we treat this case in the same way as we did for the graph of Fig. 40 a) we will obtain:

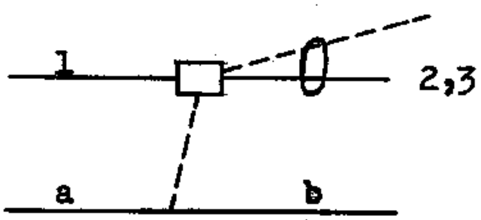


Fig. 47

$$\frac{1}{f^2} \frac{\partial\sigma}{\partial p^2} \propto \overline{\sigma_{\pi p}}$$

but now, one has the way of checking the goodness of the model, since both f^2 and $\sigma_{\pi p}$ are well known from the experiments.

Considering again the case of π production in $\pi^- + p$ collisions, one important point is the choice of the process which is more likely to give the features of the $\pi\,\pi$ interaction. We have two possibilities for the production of charged secondaries in $\pi^- + p$ interactions, namely:

$$\begin{aligned} \pi^- + p &\longrightarrow \pi^+ + \pi^- + n \qquad \text{a)} \\ &\longrightarrow \pi^- + \pi^0 + p \qquad \text{b)} \end{aligned} \qquad (6.8)$$

They gess that both reactions are dominated by a $\frac{3}{2}$, $\frac{3}{2}$ state, but in a) there is a (π N) system that is in a pure

3/2 3/2 state (namely the π^- + n pair), while in b) no such system exists. Thus, reaction a) is more likely to give rise to the formation of the nucleonic isobar than reaction b) and in consequence, if the features of the $\pi\pi$ interaction have to come out from the experiment is natural to look for then in b).

The last point is to look for the proper experimental information which is to be compared with the predictions of the model. What one calculates after the integration with respect to ω is the differential cross-section per unity interval of kinetic energy of the recoil nucleon. Therefore, the spectrum of the recoil nucleon will be the experimental information which should be more sensitive to the model. In fig. 48 and 49, we present the calculated spectra of the nucleon from reactions $N + N \longrightarrow \pi + N + N$ and $\pi + N \longrightarrow \pi + \pi + N$ respectively for about the same kinetic energy of the incoming particles in the

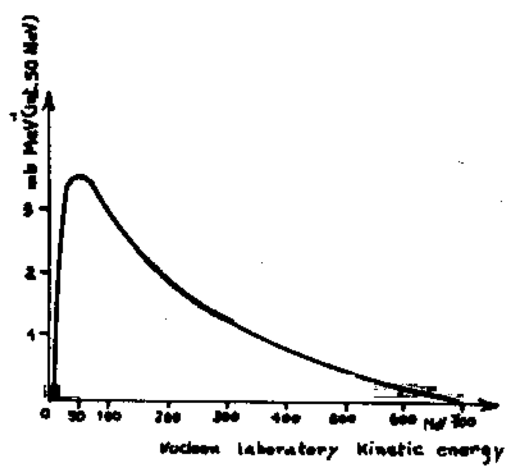


Fig. 48

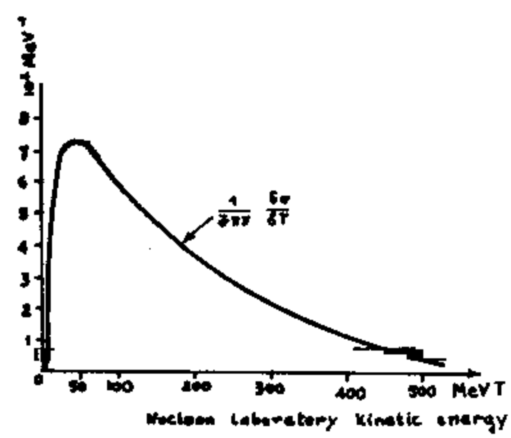


Fig. 49

laboratory system, namely ~ 960 Mev. The most striking feature is a strong maximum in both distributions, below T = 100 Mev.

In Fig. 50 we present the experimental distribution of about 270 events of the kind $p + p \rightarrow p + n + \pi^+$ with respect to the kinetic energy of the neutron in comparison with the spectrum predicted by the statistical theory and in Fig. 51, the spectrum of the neutron of reaction a) of (6.8) is compared both with predictions of the isobar model and the statistical theory.

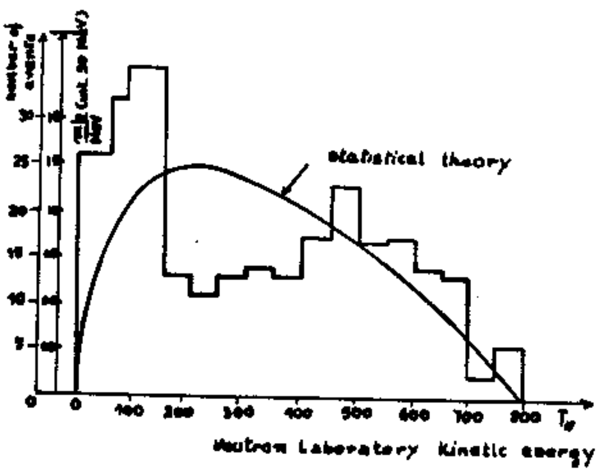


Fig. 50

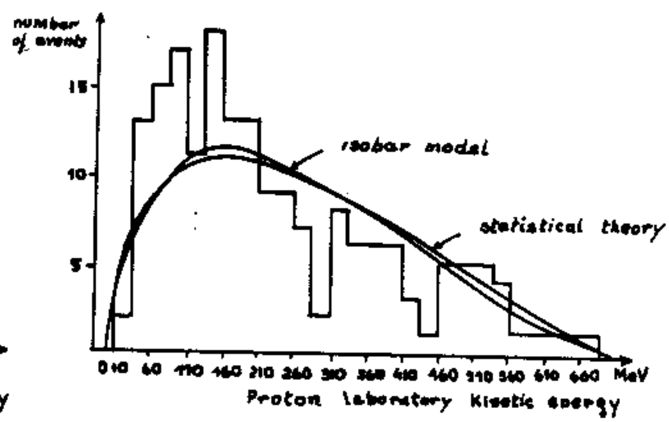


Fig. 51

There is in both figures a good evidence of the existence of a peak below 100 Mev. fact which gives some support to the assumption of an influence of the diagram of Fig. 46.

It's therefore reasonable to assume that both the Y and I_π are present in the pion production processes and that for small momentum transfers to the nucleon the second would be dominant.

7 - The problem of checking charge independence.

This problem has not been fully developed because the actual experiments are not good enough but still I would like to draw your attention on it because in next year a lot of investigation will be done.

One way of checking charge independence is to look in scattering where the possibility of analysis in terms of isospin the various channels is an evidence for the validity of charge independence.

We can do the analysis in two ways and the good would be to go through phase shift analysis. Unfortunately this cannot be done because the experimental situation is not sufficiently good to draw any conclusion from the known behaviour of the phase shifts and so we must appeal to the overall analysis in which we try to make the checking by considering relations (inequalities) between the cross sections.

As we have already said there are 3 process

$$\pi^+ + p \longrightarrow \pi^+ + p \qquad (a)$$

$$\pi^- + p \longrightarrow \pi^- + p \qquad (b)$$

$$\pi^- + p \longrightarrow \pi^0 + n \qquad (c)$$

From the point of view of isospin we have two open channels, 1/2 and 3/2 states, and the amplitudes T (W,0) as we have seen can be written:

$$\text{for (a)} \qquad T^{+} \equiv T_{3/2}$$

$$\text{for (b)} \qquad T^{-} \equiv \frac{1}{3}\,(T_{3/2} + 2\,T_{1/2})$$

$$\text{for (c)} \qquad T^{0} \equiv \frac{\sqrt{2}}{3}\,(T_{3/2} - T_{1/2})$$

From this follows the relation

$$\sqrt{2}\;T^{0} = T_{+} - T_{-}$$

Those quantities are complex and so we can represent them in the complex plane (Fig. 52).

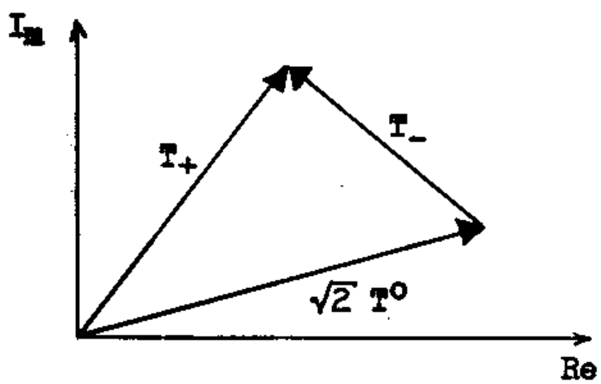


Fig. 52

As a result the modulus of the amplitudes must obey the usual triangle inequalities

$$\Big|\,|T_{+}| - |T_{-}|\,\Big| \leqslant |\sqrt{2}\;T_{0}| \leqslant |T_{+}| + |T_{-}|$$

and all the ciclic permutations.

As $\sigma = |T|^{2}$ we have $|\sqrt{\sigma_{+}} - \sqrt{\sigma_{-}}| \leqslant \sqrt{2\sigma_{0}} \leqslant \sqrt{\sigma_{+}} + \sqrt{\sigma_{-}}$

Now those inequalities must hold for each angle and energy and of course they are true also for the corresponding total cross sections. Some of those inequalities have been used for checking the charge independence. For instance we have

$$\sqrt{\sigma_+} - \sqrt{\sigma_-} - \sqrt{2\sigma_o} \leqslant 0$$

which was used by Salzman at three energies 100, 150 and 190 Mev.

The result is not exactly true but it is compatible. At Chicago with charge exchange experiments the agreement is becoming better. They are doing experiments at various angles and energies in order to prevent some fortuitus agreement.

Other kind of relations used by Stanghelini is

$$3 (\sigma^- + \sigma^o) = \sigma^+ + 2 \sigma_{1/2}$$

where $\sigma_{1/2}$ means the cross-section for 1/2 state. This relation is useful because the influence of 1/2 state is usually very small (for instance, at resonance $\sigma_{1/2}$ is equal to a few percent of σ_t) except at very low energy.

Then we can write

$$3 (\sigma^- + \sigma^o) - \sigma^+ \gtrsim 0$$

which provides a stringent checking for charge independence.

Other more sophisticated relation is the following:

$$\begin{aligned} 2\sigma_o &= |T_+ - T_-|^2 \\ &= |\text{Re } T_+ - \text{Re } T_-|^2 + |\text{Im } T_+ - \text{Im } T_-|^2 \end{aligned}$$

This seems complicated but becomes simpler in forward direction ($\theta = 0$) where we can apply the optical theorem for the total cross-section:

$$\sigma_{total} \equiv \Sigma = \frac{4\pi}{k} \text{ Im } T(0)$$

Then
$$2\sigma_o(0) = |\text{Re } T_+(0) - \text{Re } T_-(0)|^2 + \left(\frac{k}{4\pi}\right)^2 (\Sigma_+ - \Sigma_-)^2$$

The real parts can be eliminated by using

$$\sigma_{\pm} = |\text{Re } T_{\pm}|^2 + |\text{Im } T_{\pm}|^2$$

and so

$$|\text{Re } T_{\pm}(0)|^2 = \sigma_{\pm}(0) - \left(\frac{k}{4\pi} \Sigma_{\pm}\right)^2$$

where the sign of Re $T_{\pm}(0)$ we take from phase shift analysis.

Using this we get $\sigma_o(0)$ in function of the total and forward differential charged cross-section. We can now compare with experiment. From all those experiments available we can say that charge independence does hold within an accuracy of 10%.

Another kind of experiment was carried out by Harting et al Phys. Rev. Letters (3 52, 1959) which measure

$$R = \frac{p + d \rightarrow H^3 + \pi^+}{p + d \rightarrow He^3 + \pi^o}$$

This is a very good experiment for checking charge independence because the answer comes only from one number, the value of R that we calculate now:

As the I-spin for deuteron is zero the total I-spin of the entrance channel is $T = 1/2$ ($T_z = +1/2$); the final state could be in both cases $T = 1/2$ or $3/2$ as the I-spin of H^3 and He^3 is 1/2.

If now we believe in charge independence, the total isospin T is conserved and then the only value is $T = 1/2$ ($T_z = +1/2$) also for the final state.

Using Clebsch-Gordon coefficients the wave function for the final state can be written

$$\sqrt{\frac{2}{3}}\ Y_1^{\,1}\ X_{\frac{1}{2}}^{-\frac{1}{2}} + \sqrt{\frac{1}{3}}\ Y_1^{\,o}\ X_{\frac{1}{2}}^{+\frac{1}{2}}$$ where

$Y_T^{T_z}$, $X_{T'}^{T_z'}$ are the wave functions for the π and the H^3 or He^3.

So the probability for finding $H^3 + \pi^+$ in the final state is $\frac{2}{3}$ and for finding $He^3 + \pi^o$ is $\frac{1}{3}$.

Then

$$R = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$$

In the experiment a magnetic field was used in order to separe H^3 from He^3 and time of flight measurements was used for prevent confusion of H^3 with protons.

The value obtained for the ratio was $R = 2.26 \pm 0.11$. There are few corrections to take into account; the difference in kinematics because $m_{\pi^0} \neq m_{\pi^+}$ and Coulomb distortion for the wave describing the outgoing π^+. This gives an effect of about 4% in the right direction. We can say that the confidence in the check is of $\sim$ 10%.

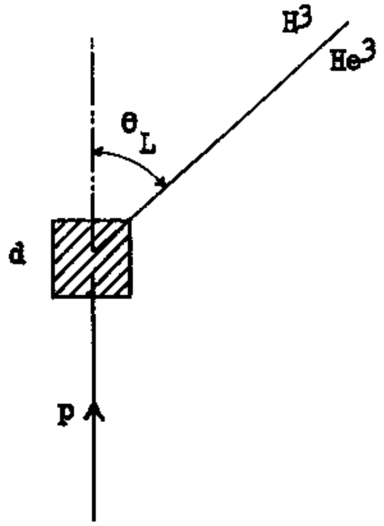


Fig. 53

Another experiment has been performed by Greue et al. (Phys. Rev. Letters 2 269, 1959) where they measured the ratio R at four different angles in C.M. system. It turns out to be $R = 1.91 \pm 0.25$. Again the confidence in the checking of charge independence is of about 10%.

Till now we have no experiment that could down the accuracy to let us say 1%.

8 - Production of strange particles

We will treat now the production of strange particles and as we have already done for pion production our first step will to discover the important quantum numbers that describes the phenomenon.

The production of strange particles is possible in various ways as for instance from the following reactions

$$N + N \longrightarrow N + Y + K$$

$$\pi + N \longrightarrow Y + K$$

$$\gamma + N \longrightarrow Y + K$$

where the corresponding thresholds in L.S. are $T_N \simeq 1600$ Mev, $T_\pi \simeq 770$ Mev and $T_\gamma \simeq 920$ Mev.

All those reactions are possible under control with the big machines that are nowdays available.

Another source for production of strange particles is for instance the capture of K^- by nuclei, but then we need a beam of K^- and so we can say that the three reactions mentioned in the begining are representative for strange particle production.

As usual I will go to discuss production by π beams because the number of experiments is the greatest and also because I believe that most important information come from there.

Looking in the 1 Bev region we can distinguish 4 kinds of reactions with π on hydrogen:

$$\pi^- + p \longrightarrow \Sigma^- + k^+ \qquad (a)$$
$$\pi^- + p \longrightarrow \Sigma^0 + k^0 \qquad (b)$$
$$\pi^+ + p \longrightarrow \Sigma^+ + k^+ \qquad (c)$$
$$\pi^- + p \longrightarrow \Lambda^0 + k^0 \qquad (d)$$

The thresholds are respectively aproximately igual to 910 Mev for (a), 890 Mev for (b) and (c) and 770 Mev for (d) and the different values are connected with the different masses of the produced hyperons.

We discuss the phenomena only near threshold because of the following reasons:

a) this situation is the one available.

b) we hope few partial waves to take part in the process.

Near thresholds the order of magnitude of the cross section for production of strange particles all together is ~ 1 mb i.e. $\sim 1.5\%$ of the total cross section for π N collision. The same holds for incident γ instead of π.

At first we could say that this value is small but this is not so because as we are at threshold only few partial waves are of interest and in fact only one is important.

Looking π-N collision at channel $J = 1/2$ the total inelastic cross section is ~ 3 mb and so we see that the value of ~ 1 mb for the cross section of strange particle production is small only when compared with π-N collisions corresponding to the overall channels available.

Let us now have a glance to the experimental situation where we have a lot of experiments accumulated in the last 3 years.

Most of them were planned for production of strange particles but this production was used only to determine spin of the particles involved, non conservation of parity etc.

No sistematic study of the production in itself (cross sections and angular distributions) has yet been done. This will be a program for the next two years where a lot of work will be done in order to understand the dynamical behaviour of the phenomena involved.

As I said no good experiments concerning the production cross section is available and so I will try only to sketch the experimental situation. We have a graph of the type sketched in Fig. 54.

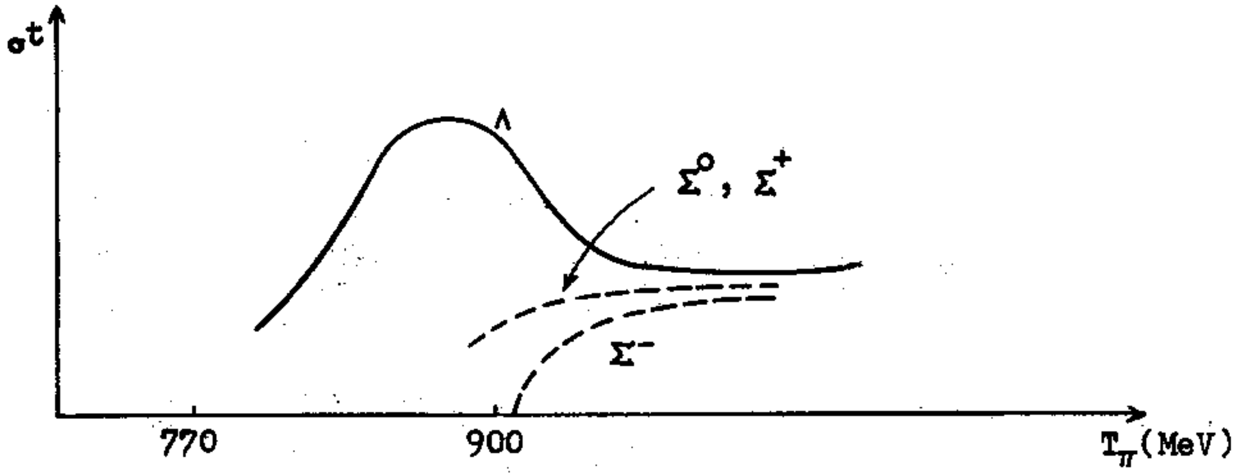


Fig. 54

Actually the Λ curve starts from zero because we are dealing with a threshold but we have no data in this region. We see that σ_Λ has a maximum but we do not know if it represents a resonance for some state involved in Λ production or if it is due to the starting of Σ^0, Σ^+ production in which case we would have a cusp in the cross section. Concerning Σ^- I believe that there exists a fast rise just after threshold and then a leveling of the cross section and so we have not a so exciting situation as for Λ. For Σ^0, Σ^+ we have an intermediate behaviour.

Resuming, after 1 Bev we have a cross section of $\sim$ 1 mb and in lower energy we see a maximum for production of Λ near the threshold for Σ^0, Σ^+.

Now comes the indications from angular distribution about the partial waves that are important in the production.

The experimental information is that the Σ^- angular distribution is isotopic and then the production would start in S wave.

For Λ no experimental information at threshold are available but there are evidence for a large contribution of S wave in the energy region explored; so we have some indication that the channel for the production is J = 1/2.

Now the angular distribution of Λ in the maximum region shows a big interference term explained by S and P waves, in which both have the same relative importance. Then it seems that the production would start in S wave in which case the cross section would follow a $\sqrt{E}$ law just above threshold.

Actually this seems directly established only for Σ^- because only 10 Mev above threshold we find a somewhat large cross section.

So very probably all production start in channel J = 1/2.

Now let us see about isospin assignement (if there is meaning for it in this situation). For this we will extend the method used in π physics.

Let us look for the 3Σ's reactions and see if i-spin is a useful tool. We have as entrance channel a mixture of 1/2 and 3/2 states for (a) and (b) and a pure 3/2 state for (c). Because we make the assignement i-spin = 1, for the Σ's and 1/2 for the K's we have the same situation as in π physics; there is only a change of names for the final state from the point of view of isotopic spin.

So following the same manipulation as before we have for the amplitudes for the Σ^-, Σ^o and Σ^+ case the expressions:

$$f_- = 1/3\,(f_{3/2} + 2\,f_{1/2})$$
$$f_o = \sqrt{2/3}\,(f_{3/2} - f_{1/2})$$
$$f_+ = f_{3/2}$$

Now the first thing is to check if the hypothesis works and for this we will play with the inequalities that comes from i-spin assignment.

The experimental analysis where made at two energies 960 Mev and 1,10 Bev. For the 1,10 Bev case we have for the production total cross sections the values

$$\Sigma_- = 0.27 \pm 0.028 \text{ mb}$$
$$\Sigma_o = 0.39 \pm 0.037 \text{ mb}$$
$$\Sigma_+ = 0.15 \pm 0.050 \text{ mb}$$

The first two are Berkeley data and the last is from Michigan group (the bigger error in the last case come because π^+ experiment is more difficult to perform due to proton contamination in the π^+ beam).

The useful inequality in this case is

$$\sqrt{2\Sigma_o} \leqslant \sqrt{\Sigma_+} + \sqrt{\Sigma_-} \tag{8.1}$$

because of the great value of Σ_o. With the values mentioned above we have

$$\sqrt{2\Sigma_o} \not\leqslant \sqrt{\Sigma_+} + \sqrt{\Sigma_-}$$ and

so the triangle for the amplitudes colapses almost in a straight line but still we have compatibility with the established inequality.

We have also the angular distributions for all 3 cases. Below we have the results provided by the old Michigan work (Brown at al Phys. Rev. 107 906, 1957) and the new Berkeley data. (F.S. Crawford at al Phys. Rev. Letters 3 394, 1959).

First we have the experimental results for Σ^0 and Σ^- case. (Figs. 55 and 56).

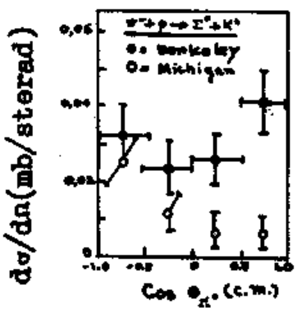


Fig. 55

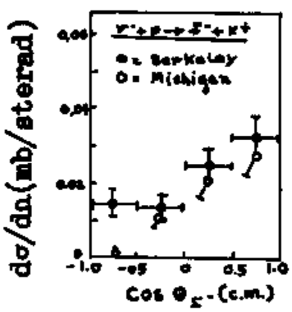


Fig. 56

In fig. 57 we present for the Σ^+ case the Michigan experimental results; the solid circles represent the lower limit allowed by combining the Σ^0 K^0 and Σ^- K^+ production results of the Berkeley experiment with the triangle inequality for the differential cross section, similar to (8.1), implied by charge independence.

The calculated values are all compatible with Michigan measurements except for the backward point. But because the same discrepancy appears between both measurements in the charge symmetric reactions $\pi^- + p \rightarrow \Sigma^- + k^+$ it may be considered as due to the same error in the old results. Using only the Berkeley data for σ_0 and σ_-, subjected to the assumption of a triangle of zero area, we have for the 1/2 and 3/2 amplitudes

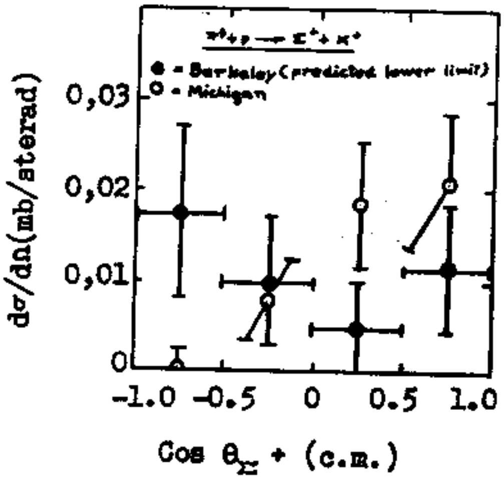


Fig. 57

$$f_{1/2} = + (3.05 \pm 0.11) \times 10^{-14} \text{cm}$$

$$f_{3/2} = - (1.14 \pm 0.16) \times 10^{-14} \text{cm}$$

So $f_{1/2} \simeq 3\, f_{3/2}$ which means that in the production the 1/2 state contributes with $\sim 90\%$, appearing then as the most important I-state.

Summarizing the situation, we have at threshold as the most important quantum numbers $J = 1/2$ and $T = 1/2$.

9 - Determination of the spin of strange particles.

A quite simple and general method is due to Adair (Adair, Phys. Rev. 100, 1540, 1955). No detailed analysis such as phase shifts analysis is used but only conservation laws.

Let us analyse the reactions:

$$\pi^- + p \longrightarrow \begin{cases} \Lambda^0 + k^0 \\ \Sigma^- + k^+ \end{cases}$$

and take the direction of the incoming π as the axis of quantization of angular momenta. This is only in order to make the analysis more easy but does not mean any loose of generality. In such a case, m_ℓ for the initial system will be zero, remaining as the only component of the total angular momentum, the spin of the proton. Thus, the situation in the c.m. system for the initial state can be represented as in Fig. 58.

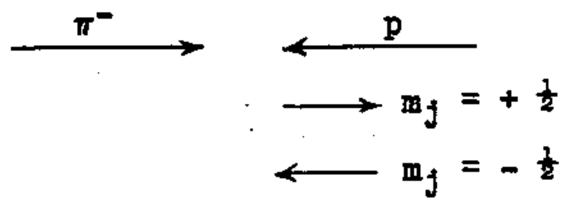


Fig. 58

The final state will be in general like in Fig. 59.

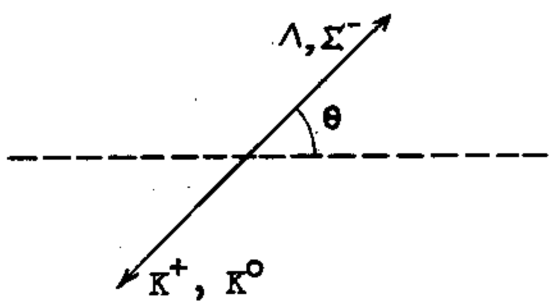


Fig. 59

while $m_{\ell'}$ can be either 0 or 1 in order to conserve m_j. One has then, some probability a for $m_{\ell'} = 0$ and certain probability b for $m_{\ell'} = 1$. If one developes the scattering amplitude for each of those possible values of $m_{\ell'}$ one gets, for small angles:

$$\frac{b}{a} \sim \text{sen}^2\,\theta$$

Therefore, if we take only the particles produced at $\theta = 0^o$ in the c.m. system, $m_{\ell'}$ will be zero and m_j would be determined only by the spins of the final particles. Furthermore, if we accept that the spin of K is zero, m_j will be determined by the spin of the $\Lambda\,(\Sigma^-)$.

All the argument can be performed, starting with m_j (initial)= $= +\frac{1}{2}$ or m_j (initial) $= -\frac{1}{2}$. These two states are orthogonal, therefore the analysis can be applied to $m_j = +\frac{1}{2}$ and afterwards the similar contribution of $m_j = -\frac{1}{2}$ can be added incoherently.

The hyperon, be it Λ or Σ^-, will decay into a π and a N (Fig. 60).

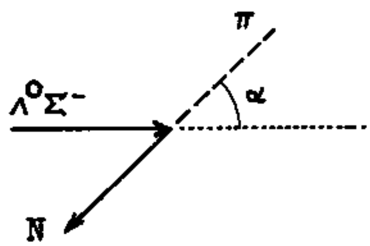


Fig. 60

Since the decay products go away in any direction, their relative angular momentum could have $m_\ell \neq 0$. Let us see now which are the possible final state which conserve m_j.

Total angular momentum	Parity of the final (πN) system	Orbital angular momentum in the final state	Partial waves in the final state
$J = 1/2$	$P = +1$ $P = -1$	$\ell = 1$ $\ell = 0$	$P^{+}_{1/2}$ $S^{-}_{1/2}$
$J = 3/2$	$P = +1$ $P = -1$	$\ell = 1$ $\ell = 2$	$P^{+}_{3/2}$ $D^{-}_{3/2}$
$J = 5/2$	- - - - - - - -	- - - - - - - - - -	- - - - - - -
- - - -	- - - - - - - -	- - - - - - - - - -	- - - - - - -

Let us now study the angular distribution for the decay process. In order to do that, we must expand the angular part of the wave function in spherical harmonics, taking always as axis of reference for the angles, the line of flight of the initial system. A typical term of such expansion will be of the form:

$$C\ (\ell,\ S,\ J,\ m_\ell\ ,\ m_s)\ Y_\ell^{m_\ell}\ X_s^{m_s}$$

where: $C\ (\ell,\ S,\ J,\ m_\ell\ ,\ m_s)$ is the corresponding Clebsch-Gordon coefficient, $Y_\ell^{m_\ell}$ are the normalized spherical harmonics and $X_s^{m_s}$ the spin function of the nucleon.

Therefore, for the simplest case $S^{-}_{1/2}$, the angular part of the wave function turns out to be:

$$f(\theta,\varphi) = C\,(0,\tfrac{1}{2},\tfrac{1}{2},0,\tfrac{1}{2})\; Y_0^{\,0}\, X_{\frac{1}{2}}^{+\frac{1}{2}} + C\,(0,\tfrac{1}{2},\tfrac{1}{2},0,-\tfrac{1}{2})\; Y_0^{\,0}\, X_{\frac{1}{2}}^{-\frac{1}{2}}$$

where the second term correspond to the contribution of the $m_j=-\frac{1}{2}$ state.

The angular distribution will thus be given by:

$$\sigma_{S_{\frac{1}{2}}^{-}}(\alpha) = |f(\theta,\varphi)|^2 = \left|C(0,\tfrac{1}{2},\tfrac{1}{2},0,\tfrac{1}{2})\, Y_0^{\,0} X_{\frac{1}{2}}^{+\frac{1}{2}}\right|^2 + \left|C(0,\tfrac{1}{2},\tfrac{1}{2},0,-\tfrac{1}{2}) Y_0^{\,0} X_{\frac{1}{2}}^{-\frac{1}{2}}\right|^2$$

$$= 2\left|\sqrt{\tfrac{1}{2}}\right|^2 = 1$$

For the $P_{\frac{1}{2}}^{+}$ state one has:

$$\sigma_{P_{\frac{1}{2}}^{+}}(\alpha) = \left|C(1,\tfrac{1}{2},\tfrac{1}{2},1,\tfrac{1}{2}) Y_1^{\,0} X_{\frac{1}{2}}^{\frac{1}{2}}\right|^2 + \left|C(1,\tfrac{1}{2},\tfrac{1}{2},1,-\tfrac{1}{2}) Y_1^{\,1} X_{\frac{1}{2}}^{-\frac{1}{2}}\right|^2 +$$

$$+ \left|C(1,\tfrac{1}{2},\tfrac{1}{2},0,-\tfrac{1}{2}) Y_1^{\,0} X_{\frac{1}{2}}^{-\frac{1}{2}}\right|^2 + \left|C(1,\tfrac{1}{2},\tfrac{1}{2},-1,\tfrac{1}{2})\; Y_1^{\,-1}\; X_{\frac{1}{2}}^{\frac{1}{2}}\right|^2 =$$

$$= 2\left\{ \left|-\sqrt{1/3}\;\sqrt{3/2}\;\cos\alpha\right|^2 + \left|\sqrt{2/3}\sqrt{3/4}\;\sin\alpha\right| \right\} = 1$$

As expected we see from the preceding results that the angular distribution does not depend on the parity but only on the total angular momentum of the state. In both case, the angular distribution of decay turns out to be isotropic.

If we write down in the same way the expression for the $J = 3/2$ states, we get:

$$\sigma_{3/2}(\alpha) = \frac{1 + 3\cos^2\alpha}{2}$$

and for $J = 5/2$:

$$\sigma_{5/2}(\alpha) = \frac{3}{4}\left[1 - 2\cos^2\alpha + 5\cos^4\alpha\right]$$

One sees immediately that the angular distribution takes a peculiar form for each value of J and this is due to the selection of the direction of production (0^o or 180^o) in order to get a single value for m_j.

Now, these angular distributions must be compared with the experimental ones. But the preceding argument was developed for production exactly at 0^o or 180^o. Since the probability to find a particle produced exactly at those two angles is zero, one is obliged to take a certain cone, with axes at 0^o or 180^o. The greater the angle, the larger will be the number of events at our disposal. But we must put a limit because as we increase the angle, the contribution of $m_\ell \neq 0$ will also increase, as we have already seen, as $\sin^2 \theta$. Only in the particular case of having $\ell = 0$ one could take the Λ produced in any direction. Therefore, one has to arrive to a compromise between the contamination of $m_\ell \neq 0$ and the number of events required to give statistical significance to the analysis.

As an example, in Fig. 61 is represented by the full curve the fitting of some previous experimental data on the angular distribution of Λ with s and p waves and by the dotted curve the maximum contribution of $m_\ell \neq 0$. (see reference below).

From the backward part one can take up to $\cos \theta = - 0.5$ without having a contamination of $m_\ell \neq 0$ greater than 10%.

In figs. 62 and 63 (from Eisler et al. N. Cim. VII, 222, 1958) appear the experimental data on the angular distribution for

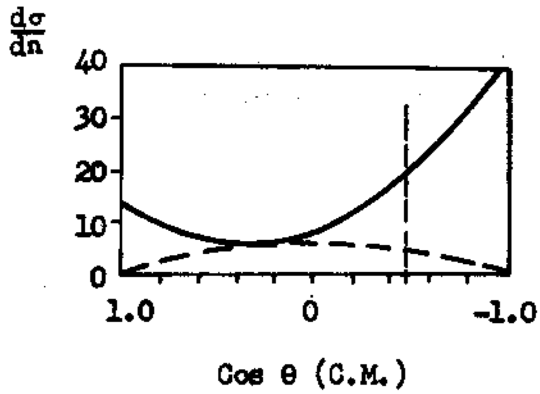


Fig. 61

decay of Λ and Σ^- respectively produced by π^- of $\sim$ 1 Bev, within a cone such that the contamination with $m_\ell \neq 0$ is expected to be smaller than 10%. Also the calculated curves corresponding to spin 1/2, 3/2 and 5/2 are there represented.

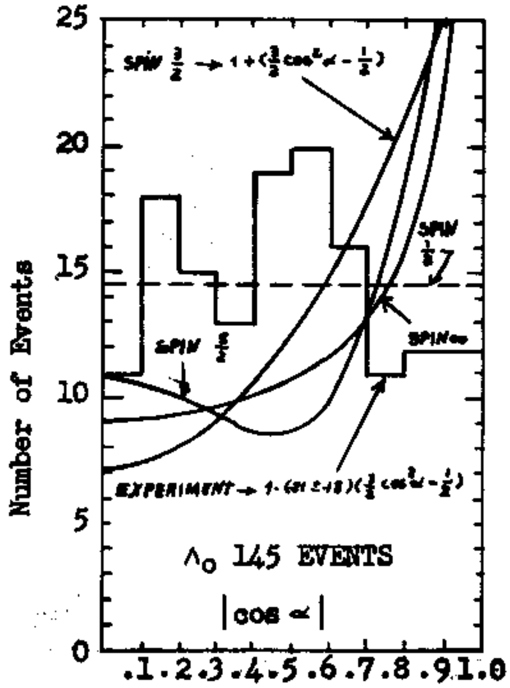


Fig. 62

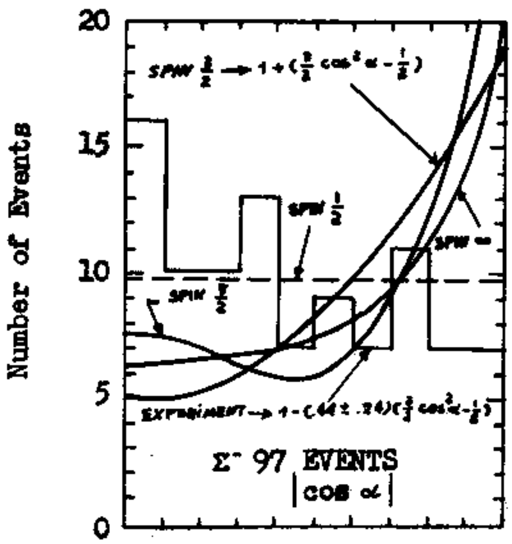


Fig. 63

One can readily see that the best fit is got in both cases with $s = 1/2$. This is the accepted value for both Λ and Σ^-.

A similar analysis can be performed on the absorption of K^- by hydrogen: $K^- + p \rightarrow \pi^+ + \Sigma^-$. Also in this case it is necessary to assume spin zero for the K.

Adair's argument provides a quite general method which can be applied for the determination of the total J of a system.

Lee and Yang developed another argument for the determination of the spin of Λ and Σ^- which does not need any assumption on the spin of the K. It is based in the nonconservation of parity in the decay of those particles. Experimentally, an analysis must be acomplished on the up and down asymmetry in the decay but a much larger number of events than in the Adair's analysis is required in order to be able to put in evidence that effect.

For the Λ its spin determination has been also attempted by this method and again the value 1/2 was indicated. Up to now a similar analysis has not been possible to be performed for the case of Σ^-.

10 - The problem of the relative parity between Λ and K.

In Chapter 8 we could conclude that the available experimental information agrees with the assumption that the most important channel for the production of Λ and Σ is, at threshold, the $T = 1/2$, $J = 1/2$ channel.

But there is still an open question which is the determination of the parity of that channel and this problem is connected with the question of the relative parity between Λ and K.

In order to be able to arrive to some conclusion about this matter we must analyse reactions initiated by particles whose relative parity is well established and in which the parity is conserved. In this way we will be able to say something about the relative parity in the final state where the Λ or the Σ appear.

But, because of the fact of the associated production of strange particles, we only can speak of the relative parity between two of those particles.

Starting with the reaction which requires a lowest threshold namely:

$$\pi^- + p \longrightarrow \Lambda + K^0 \qquad (10.1)$$

our first conclusion will then be about the relative parity between Λ and K.

We would have a different situation for the determination of the parity of the Ξ by the analysis of the reaction:

$$\pi + N \longrightarrow \Xi + K + K \qquad (10.2)$$

Since in the final state we have two identical particles, the relative parity between those two is P = + 1, then we could compare directly the parity of Ξ with that of the system (πN).

But in any case of the type of reaction (10.1) where one has only two strange particles, the only thing one can establish is the relative parity between both strange particles, be them Λ and K or Σ and K.

There is a lot of speculation about this subject, most of them insufficient by themselves because in all the cases the conclusions are based in supplementary assumptions, and it is difficult to establish which is the level of confidence on the different arguments. Nevertheless there are several pieces of information using different supplementary hypothesis, all of them favouring the same conclusion about the relative parity between Λ and K.

This is a good indication of the correctness of the conclusion and we can nowadays believe with some support that the parities of Λ and K must be opposite.

Let us see now which are the possible sources of information.

a) The analysis of some peculiarities in the production processes at threshold.

There is an argument originally given by Wigner for the case of nuclear reactions, which says that if a new process occurs through a certain channel that can also be observed at elastic scattering, there is a perturbation in the cross-section for elastic scattering just at the threshold for the new process. In a more general way it can be affirmed that at any time that the derivative of the cross section for a certain process has a singularity, it perturbs the cross-sections for all the other processes which proceed through the same channel.

If this is true, in the case of $\pi^- + p$ one has as the main processes the elastic and inelastic scattering. As we have already said, there is some indication on the starting of the cross-section for Λ production is s-wave, and in this case, there is a $\sqrt{E}$ dependence of the cross-section which means a strong singularity in its derivative. Then, one should observe a perturbation in the elastic scattering just in the same channel of J, T and P which is involved in the production of Λ. Therefore, if we knew very well which are the states of J and P involved in the scattering process we could analyse the behaviour of the phase shifts and discover the expected peculiarities.

But this analysis is hopeless in this case, since for the threshold energy for Λ production one has about 28 phase shifts involved in the scattering process.

The conditions are better if we tried to analyse the behaviour of the cross-section for Λ production at threshold for Σ production, since as we saw, it seems that at this energy, only s and p waves are involved in the Λ cross-section. Still we have very few experimental data both on Λ and Σ as to get a definite answer. We said for instance that it is still impossible to decide whether the maximum in σ_Λ around the threshold for production of Σ is due to a resonance in the S or P states involved, or to a perturbation of one of the phase shifts due to the Σ production process proceeding through the same channel, or to a more complicated situation.

b) Analysis of the reaction $p + p \rightarrow K^+ + \Lambda^0 + p$ (G. Costa and B. T. Feld Phys. Rev. 109 607, 1958).

Also here, a carefull analysis of the angular distribution and of the momentum dependence of the cross-section at threshold, could give some information about the relative parity between Λ and K if we supplement that analysis with some assumption on the most important final state interaction.

If the bombarding energies are sufficiently close to threshold, one can assure that most K's should be produced in S and P states and the resulting $(p\Lambda)$ system is most likely to be in a relative S state.

In this way, one can analyse which are the initial state corresponding to each possible final state, thus, making it possible to predict which must be the angular distribution and the momentum dependence of the final particles.

In constructing Table II, we have, for simplicity taken the intrinsic parity of the hyperons to be identical with that of the nucleon, but, since the parity is a quantity which must be conserved in the product, the result would be the same if we changed our choice of the intrinsic parity of the hyperon. Also in preparing Table II one assumes a spin-0 K and spin - 1/2 Λ.

In the two last column of Table II, the energy and angular dependence of the cross-section are presented.

From this Table one can readily see that with only the assumptions up to now proposed, there are many ambiguities and nothing can be said about the relative parity of Λ and K. But there are

Table II

(ΛK) parity	Final (pΛ) state	ℓ_K	Initial (pp) state	Angular distribution	Momentum dependence
+	$^1S_0$	0	$^1S_0(a_0)$	$\|a_0\|^2$	k
		1	$^3P_1(a_1)$	$9/2\|a_1\|^2\sin^2\theta$	k^3
	$^3S_1$	0	Forbidden	-	
		1	$^3P_{0,1,2}(a'_{0,1,2})$	$\left\{\frac{9}{4}\|a'_2-a'_1\|^2+\|a'_2-a'_0\|^2\right\}\mathrm{sen}^2\theta+\left\{\frac{9}{2}\|a'_2+a'_1\|^2+\|2a'_2+a'_0\|^2\right\}\cos^2\theta$	k^3
-	$^1S_0$	0	$^3P_0(b_0)$	$\|b_0\|^2$	k
		1	Forbidden	-	
	$^3S_1$	0	$^3P_1(b'_1)$	$\|b'_1\|^2$	k
		1	$^1S_0(b'_0)$, $^1D_2(b'_2)$	$\frac{1}{3}\|b'_0-(\frac{5}{2})^{1/2}b'_2\|^2+\left\{\|b'_0+(\frac{5}{2})^2b'_2\|^2-\|b'_0\|^2\right\}\cos^2\theta$	k^3

indications based in the analysis of hyperfragments and on theoretical considerations, that the Λ-p force is strongest in the $^1S_0$ state. If this is true, we can expect $C_1 + C_2 \,\mathrm{sen}^2\,\theta$ for even P and const. for odd P, and some definite conclusion could be worked out. This would be the necessary supplementary assumption we were referring to at the begining. Therefore, if the supplementary assumption is reasonable, this analysis would provide a good indication on P (ΛK).

But the fact is that nobody has yet performed a complete experiment on production of Λ at threshold, so that the number of events is too small to give any significance to an analysis of angular distribution or to a test of momentum dependence.

c) analysis of the reactions

$$\pi^+ + He^4 \longrightarrow {}_{\Lambda}He^4 + K^+$$
$$\pi^- + He^4 \longrightarrow {}_{\Lambda}H^4 + K^o$$

In this case with only one assumption, one can determine the relative parity between Λ and K and this assumption refers to the state in which the hyperfragments are produced.

Let us assume that the only bound state for the hyperfragment is $J = 0$. If this is true all the particles have spin zero and for any angular momentum it will be $\ell_i = \ell_f$, so there will be no influence of the angular momentum on the parity of the final state relative to the initial state.

Then if we believe in conservation of parity in strong interactions, and since the relative parity of the π and the nucleon is $P = -1$ also K and Λ must have apposite parity.

Therefore if some support can be found to the spin zero for the hyperfragments, the existence of these reactions would be a proof of the opposite parity of Λ and K. Unfortunately no information on $\pi^- + He^4$ experiment is available for the moment.

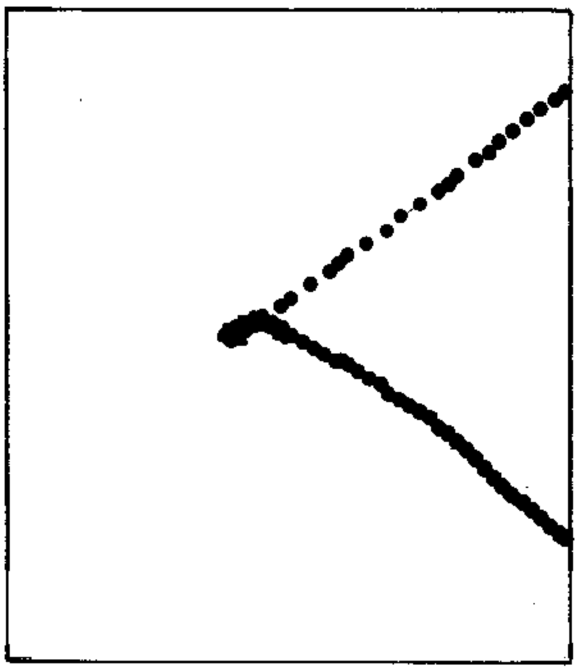
Nevertheless a carefull experiment is being carried out at present (He bubble chamber collaboration) on reactions $K^- + He^4$. Here we have with regard to the occurring of hyperfragments the same situation as that just described for $\pi^- + He^4$. The following reactions can occur:

$$K^- + He^4 \longrightarrow {}_{\Lambda}He^4 + \pi^- \qquad (a)$$

$$K^- + He^4 \longrightarrow {}_{\Lambda}H^4 + \pi^0 \qquad (b)$$

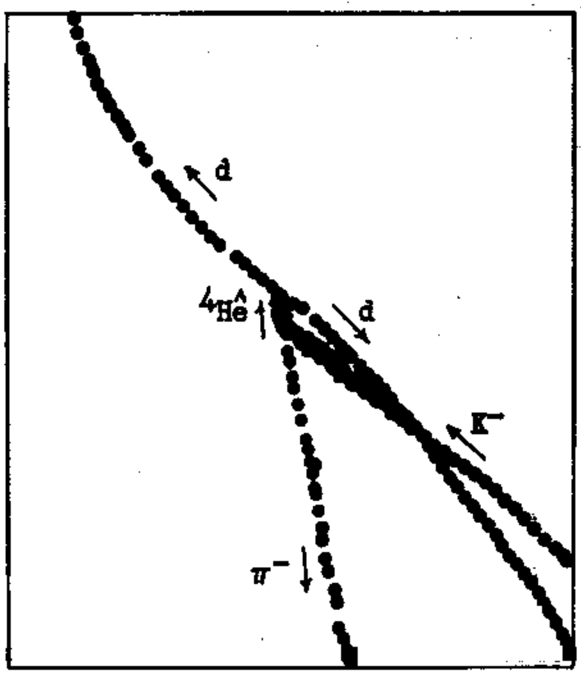
where again with the assumption of spin zero for the hyperfragment, the existence of these reactions indicates P (ΛK) to be odd.

In figs. 64 and 65 two typical cases for reaction (a), founded in the mentioned experiment are presented.



$K^- + He^4 \rightarrow \pi^- + {}^4\hat{He}$
$\hookrightarrow \pi^0 + He^4$

Fig. 64



$K^- + He^4 \rightarrow \pi^- + {}^4\hat{He}$
$\hookrightarrow d + d$

Fig. 65

The results up to now are summarized in the following table

Reaction	n° of cases
$K^- + He^4 \rightarrow {}_\Lambda He^4 + \pi^-$	15
$K^- + He^4 \rightarrow {}_\Lambda H^4 + \pi^o$	6
$K^- + He^4 \rightarrow {}_\Lambda H^3 + \pi^- + n$	2

The frequency of reactions involving hyperfragments in comparison with the total K^- absorption turns out to be of a 2%, but compared with the direct production of Λ one finds that the frequency is of a 17%. This is a quite big number which tells definetely that those reactions are allowed, and we insist, if the reactions are allowed and $J = 0$ for the hyperfragments, $P\ (\Lambda K)$ must be odd.

Furthermore the expected ratio between the first two cases assuming charge independence, is obtained taking into account that for both reactions $T = 1/2$ and $T_Z = - 1/2$.

Then the Clebsch Gordon coefficients are $-\sqrt{2/3}$ for the first case and $\sqrt{1/3}$ for the second. Thus the expected ratio is $R = 2$ which is in good agreement with the result $R_{exp} = \frac{15}{6} = 2.5$.

But this of course constitutes an interval test. One can conclude that charge independence holds and if one accepts beforhand that the efficiency in scanning is good.

Still there cannot be any cut conclusion about the relative parity between Λ and K until the argument in favour of $J = 0$ for the hyperfragment will not be improved.

One of the arguments in this sense is due to Dalitz who tried to explain the binding energy of the hyperfragments by means of a phenomenological approach in terms of two body forces. In first approximation, this approach gives that J must be zero, but also for approach J = 1 state some very low still finite binding energy is obtained.

The approach based in two body forces can not be satisfactory and then Dalitz introduced also three body forces which improve the calculations and the result is that the J = 1 state must be unbounded.

More recently Fujii and Sudarshan trated again the problem and saw that even without the introduction of 3 body forces, both $_{\Lambda}He^4$ and $_{\Lambda}H^4$ turn out to be unbounded for J = 1. The results are E_o = - 2.3 Mev and E_1 = + 0.20 Mev.

There are also others indications concerning the ratio between mesonic and non mesonic modes of decay of light hyperfragments, about the state of J involved and also this arguments favour the J = 0 state.

All the questions can be sumarized as follows: the experimental information, which unfortunately exists only in the region $E_K \sim$ 1 Bev gives agreement with the assumption that J = 0 is the most important channel for the production of hyperfragments and indicates that the reactions are allowed.

Still the arguments on which a definite conclusion on the scalarity or pseudoscalarity of K is based must be somehow forced.

d) Photoproduction of K^+ in hydrogen

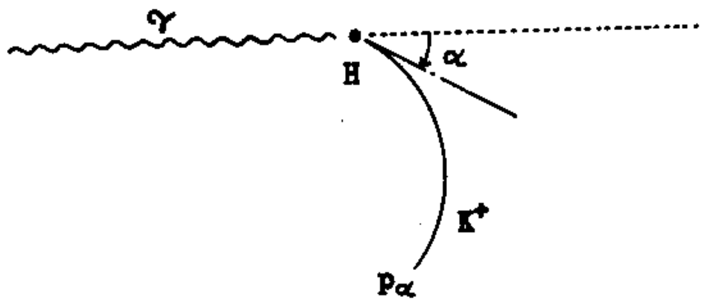


Fig. 66

The reaction $\gamma + p \rightarrow \Lambda^0 + K^+$ is also able to supply some information about $P(\Lambda K)$. The experimental set-up is given by a beam of γ-rays (bremsstrahlung) incident upon an hydrogen target and producing a charged K^+ meson. In order to fix the energy of the incident γ-ray, we must use a device to measure the momentum of the K^+ and the angle of production α refered to the line of flight of the γ-ray. This can be performed by using a magnetic field. (Fig. 66).

The experiments are, in principle, easy to do. But, up to now, the available machine supplies only photons of 1 Bev, and as this energy is near the threshold the cross-section is low and the statistics is poor. We expect that the situation will become better at 2 or 3 Bev, where we expect a bigger number of events because of a larger number of usefull photons with $E_\gamma > E_{\gamma\ \text{threshold}}$.

There are different experiments made at differents places and the statistical accuracy is poor and the situation not very good. The best set of experiments are those from Cornell and Caltech at the energy E_γ = 1000 ÷ 1010) Mev. (Fig. 67).

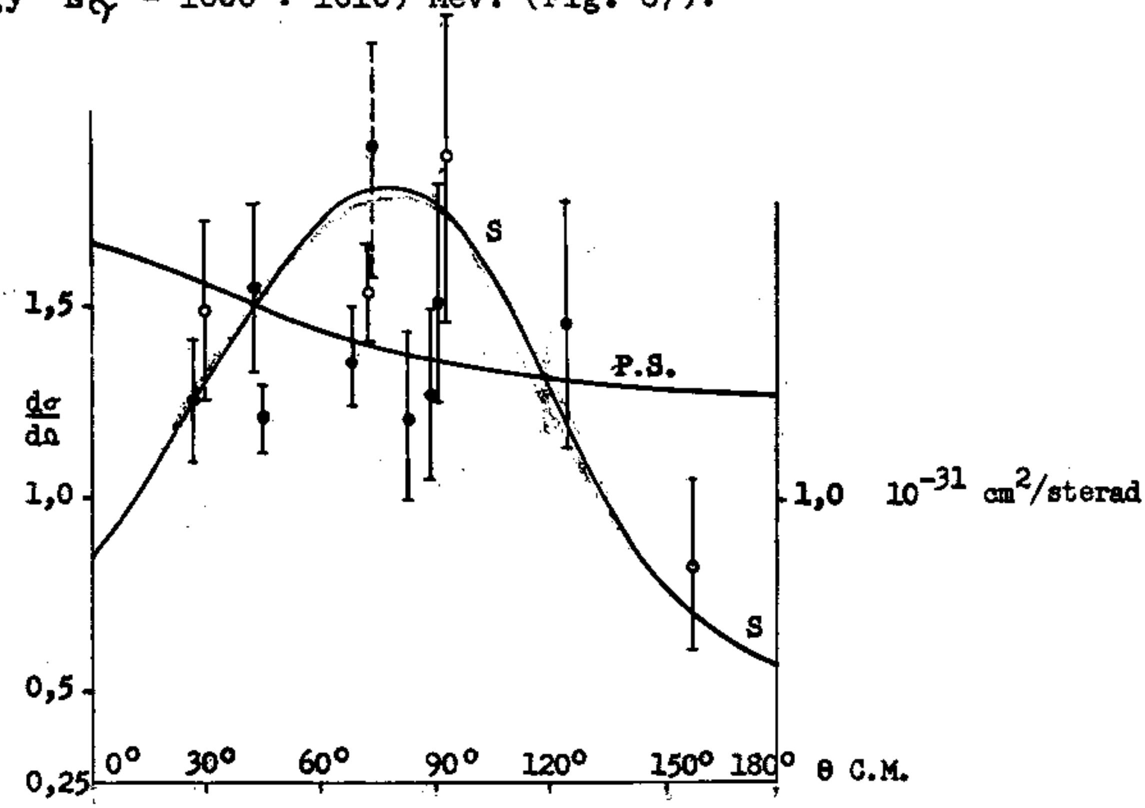


Fig. 67

The angular distribution of K^+ does not show any outstanding feature, the errors are still large and the points are somewhat scattered. Together with the experimental points we plot also two curves due to Fujii and Sudarshan, which are standard calculations for each scalar and pseudo-scalar case.

As we can see it is difficult to discriminate between those two curves.

Then we should try another way in order to obtain information about the parity P (ΛK) basing on photoproduction. The process can be represented by a graph of the type of Fig. 68.

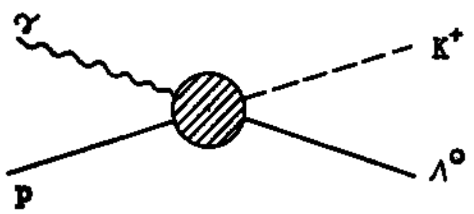


Fig. 68

As far as our understanding of this interaction becomes improved, the black box becomes greater and greater. This means that our analysis should become too much envolved, with too many parameters.

We should try a more limited program, directed on the information about P (ΛK) only.

For this objective, we must remember the possibility of the graph of Fig. 69.

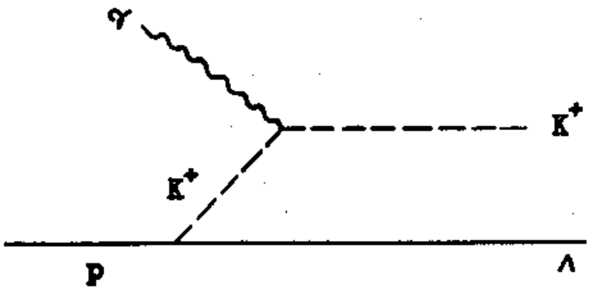


Fig. 69

A graph analogous to this one appeared at the photoproduction of π^+ and it was just the photoelectric term, which was shown experimentaly to exist. So we can believe this graph to be present also here.

Now, as in this graph there is an intermediate K^+ particle, the amplitude will have a pole at a certain point. The transition amplitude $T = T(W, \Delta^2)$ for $\gamma + p \rightarrow \Lambda + K$ is a function of W and Δ^2 i.e., the total energy and momentum transfer in the c. m. system.

The photoelectric term in the transition amplitude must have a pole in the variable Δ^2, the momentum transfer, but not in W, as we saw in similar reasoning. So we expect that the pole will be on the variable cos θ, at the angular distribution. The pole can be shown to be outside the physical region, i.e., $|\cos\theta| > 1$.

So, we can show that:

$$T = T\,(W, \Delta^2) = \frac{M_1}{1 - v_K \cos\theta} + T_2$$

where the term is M_1 is the photoelectric term and T_2 comes from other sources. The quantities M_1 and T_2 are regular near $\cos\theta = 1/_{v_K}$. This pole is expected to be the most near pole to the physical region, as compared to other poles. We can show by kinematical considerations that the pole is located at $|\cos\theta| = 2.7$ for $E_\gamma = 1$ Bev ($v_K \cong 0.3$).

Now, in order to calculate $d\sigma/_{d\Omega}$ we must take the square $|T|^2$.

Then:

$$\frac{d\sigma}{d\Omega} = \frac{|M_1|^2}{(1 - v_K \cos\theta)^2} + \frac{A}{(1 - v_K \cos\theta)} + B$$

where A comes from the interference between M_1 and T_2; $|M_1|^2$, A and B are regular finite quantities at the pole given by

$1 - v_K \cos\theta = 0.$

So, in order to isolate M_1 we apply the standard way:

$$(\alpha_o - \cos\theta)^2 \frac{d\sigma}{d\Omega} = f(W, \cos\theta) = |M_1|^2 + A(\alpha_o - \cos\theta) + B(\alpha_o - \cos\theta)^2 \tag{10.2}$$

That is, we multiply the experimental angular distribution $d\sigma/d\Omega$ by $(\alpha_o - \cos\theta)^2$ and plot the result against $\cos\theta$. (in c.m. system). So using the eq. (10.2), we see that as $\cos\theta \to \alpha_o$, $(\alpha_o - \cos\theta)^2 \frac{d\sigma}{d\Omega} \to |M_1|^2$.

So if we are able to extrapolate the experimental points up to the unphysical region, we can get information about the residue whose magnitude depends on the coupling constant and whose sign depends on the parity P (ΛK).

The analysis has been performed by Moravczik (fig. 70).(M. Moravczik Phys. Rev. Letters 2 352 1959).

In order to extrapolate we can try polinomials of different order n = 1, 2 or 3. All those extrapolation look to be indistinguishable in the physical region, inside the statistical inaccuracy. [We can also try a X^2 method]. The result can be plotted against the order of polinomial used for the fit. (Fig. 71). Now, there is no criteria for the choice of n. The general impression one has is that a small negative value for the residue is present for all n. The solution is near zero.

If we try to calculate what we should expect for the magnitude of the residue, we can use a Born approximation, for which the

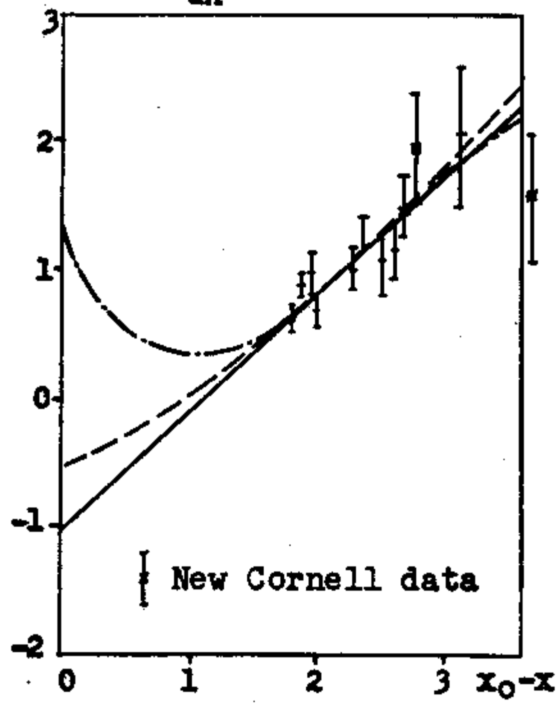


Fig. 70

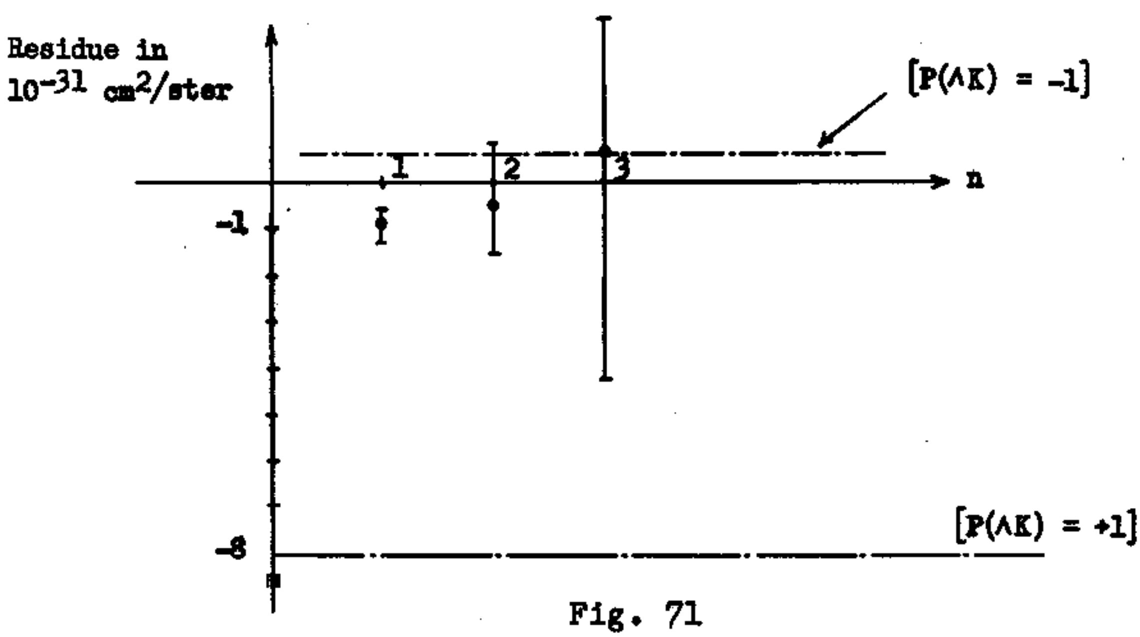


Fig. 71

photoelectric term, besides a kinematic factor, is proportional to $g^2_{p\Lambda K} \cdot e^2/\hbar c$.

Now the general belief is that $g^2_{p\Lambda K}$ is slightly smaller than $g^2_{\pi N}$, but still strong. Moravczik takes $g^2_{NK\Lambda} \sim 1$.

So, by performing the calculations, we find that the residue is slightely positive for pseudoscalar coupling and negative and near $-8 \times (10^{-31}\ cm^2/ster)$ for scalar coupling. This should give evidence for P $(\Lambda K) = -1$. It looks that the choice of g^2 is not so bad.

In conclusion, we can say that each peace of information is not quite sound, but all different experiments points in the direction of a value P $(\Lambda K) = -1$.

11 - <u>Dispersion Relations approach to relative parity of Λ-K.</u>

We do not believe that the conclusions of this approach can give an unambiguos answer to the problem but they can be improved by a better knowledge of the cross sections. So it is interesting to establish which are the arguments.

Consider the process:

$$K^+ + p \longrightarrow K^+ + p \qquad \text{(a)}$$

$$K^- + p \longrightarrow K^- + p \qquad \text{(b)}$$

which are described by the scattering amplitudes $f_+(\omega,\theta)$ and $f_-(\omega,\theta)$ respectively, where ω is the total energy of the system and θ is the scattering angle.

We do not know how to calculate those amplitudes because there is no theory available but if we consider only forward scattering we will see that we can stablished some properties of these functions (now only depending on ω) in order to have information about the relative parity of Λ and K and about the strength of the coupling constant.

From dispersion relation theory we have the following expression connecting the real and imaginary part of a forward scattering amplytude

$$\mathrm{Re}\ f(\omega) = \frac{1}{\pi} \fint_{-\infty}^{\infty} \frac{\mathrm{Im}\ f(\omega')}{\omega'-\omega}\, d\omega'$$

($\fint$ means principal value).

Actually this expression is just Cauchy's theorem for analitic functions and so is very general. For the scattering amplitude the analicity condition comes essentially from the requirement of microcausality.

Now we will try to see how to work with this expression.

The difficulty is to express the integral and the first thing to do it is to look the phenomenon in all energy range.

As ω is the total energy of the system we must have $\omega \geqslant m_N + m_K$ which defines the physical region.

Now we must discover if there is some pole in the variable ω in the scattering amplitude and for this we considerer first reaction (a). Remembering that the strangeness of K^+ is +1 and that we must have conservation of strangeness and barionic number at each vertex, it is easy to see that we cannot connect the initial and final state with a particle; then we have no pole in the corresponding amplitude $f_+(\omega)$.

The situation is different for reaction (b) because now we can connect the initial and final state by a Σ^0 or Λ (Fig. 72). Then we have for the corresponding amplitude $f_-(\omega)$, two poles at $\omega = m_\Lambda$, m_Σ. The problem now is that there are more accident in $f_-(\omega)$ because we can add a π to the intermediate state (Fig. 73) and we can do this in an infinity number of ways which means that we have not just a pole but a continum.

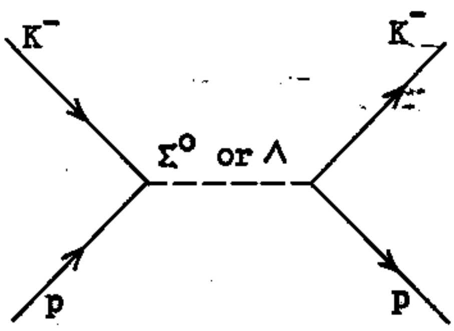


Fig. 72

Simbolicaly we can represent the situation for f_- in the following way: (Fig. 74).

After this analysis we split the integral into various pieces but before doing this let us see how to deal with the integration range from $-\infty$ to 0. For this we use a trick known as "crossing symmetry" which states that

$$f_+(\omega) = f_-^*(-\omega) \qquad (1)$$

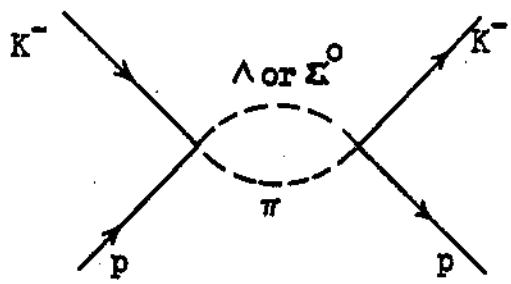


Fig. 73

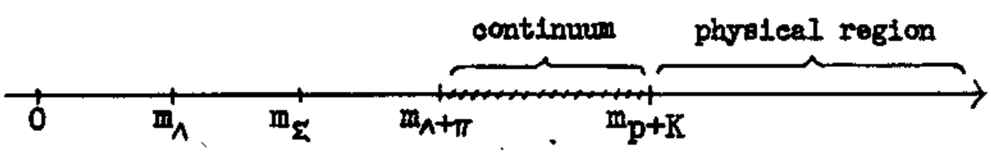


Fig. 74

with which we can pass from $\int_{-\infty}^{0}$ to $\int_{0}^{\infty}$.

Now, as we saw there is no pole for f_+ and so we can write

$$\mathrm{Re}\ f_+(\omega) = \frac{1}{\pi} \int_{m_K}^{\infty} \frac{\mathrm{Im}\ f_+(\omega')}{\omega' - \omega}\, d\omega' + \frac{1}{\pi} \int_{-\infty}^{0} \frac{\mathrm{Im}\ f_+(\omega')}{\omega' - \omega}\, d\omega'$$

Using (1) we can write

$$\mathrm{Re}\ f_+(\omega) = \frac{1}{\pi} \int_{m_K}^{\infty} \frac{\mathrm{Im}\ f_+(\omega')}{\omega' - \omega} + \frac{1}{\pi} \int_{0}^{\infty} \frac{\mathrm{Im}\ f_-(\omega')}{\omega' + \omega}\, d\omega'$$

Since we have singularities only for $f_-(\omega)$ we can write

$$\mathrm{Re}\ f_+(\omega) = U_{cont} + \frac{X(\Lambda)}{\omega_\Lambda + \omega} + \frac{X(\Sigma)}{\omega_\Sigma + \omega} +$$

$$+ \frac{1}{\pi} \int_{m_K}^{\infty} \left[\frac{\mathrm{Im}\ f_-(\omega')}{\omega' + \omega} + \frac{\mathrm{Im}\ f_+(\omega')}{\omega' - \omega} \right] d\omega'$$

where U_{cont} is the contribution of the unphysical region and the 2^{nd} and 3^{rd} terms are the contributions of the poles of $f_-(\omega)$.

Using the optical theorem

$$\mathrm{Im}\ f(0) = \frac{k}{4\pi}\ \sigma^t ,$$

we can write

$$\mathrm{Re}\ f_+ = U_{cont} + \frac{X(\Lambda)}{\omega_\Lambda + \omega} + \frac{X(\Sigma)}{\omega_\Sigma + \omega} +$$

$$+ \frac{1}{4\pi^2} \int_{m_K}^{\infty} k' \left[\frac{\sigma_-^t(\omega')}{\omega'+\omega} + \frac{\sigma_+^t(\omega')}{\omega'-\omega} \right] d\omega'$$

and analagously for f_-:

$$\mathrm{Re}\, f_- = U_{cont} + \frac{X(\Lambda)}{\omega_\Lambda - \omega} + \frac{X(\Sigma)}{\omega_\Sigma - \omega} +$$

$$+ \frac{1}{4\pi^2} \int_{m_K}^{\infty} k' \left[\frac{\sigma_-^t(\omega')}{\omega'-\omega} + \frac{\sigma_+^t(\omega')}{\omega'+\omega} \right] d\omega'$$

At this point we have introduced all the measurable quantities that we have from experiment or that we can calculate like the residues of poles, except the contribution from the continuum in the unphysical region.

Of this continuum we can say very little. The term $\mathrm{Im}\, f_-$ must vanish at the threshold of this region i.e. at $m_{\Lambda+\pi}$. (Fig. 75).

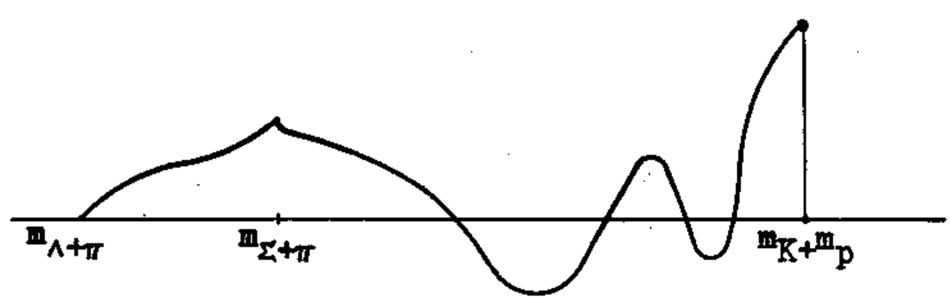


Fig. 75

At the threshold of the physical region $\mathrm{Im}\, f_-$ has a finite value as can be seen immediatly in this way: σ_-^t at zero kinetic energy behaves like an S wave of an exotermic reaction and then $\sigma_-^t \propto \frac{1}{k}$. Using the optical theorem we have $\frac{\text{Im } f_-}{k \to 0}$ = constant.

But in the interior of the region it is impossible to know how it behaves; it can have oscillations and cusps. Because of this situation the only hope is that the contribution of U_{cont} is small in comparison with other terms and that it is possible to neglect it.

For what concerns the contribution of the residues of poles, we proceed with the usual method which consist in calculating X on the basis of Born approximation of the scattering process. This term can be calculated assuming that the K is scalar or pseudo-scalar with respect to Λ .

Now, neglecting the contribution from the continuum in the unphysical region, the only terms which contains explicity informa tion about relative K parity and strength of the coupling constant are the residues of the poles. So from the calculations of the other terms by means of the experimental data it is possible to decide about relative parity. Unfortunately the dispersion relations written just now are still insatisfactory for another reason which is the poor convergence, if any, of the integral over the total cross sections when $\omega \to \infty$. For this reason people prefer to use dispersion relation derived from the first ones by subtraction.

The result is

$$m_K \left\{ \text{Re } f_+(m_K) - \text{Re } f_-(m_K) \right\} + \frac{m_K^2}{4\pi^2} \int \frac{d\omega'}{k'} \left[\sigma_-^t(\omega') - \sigma_+^t(\omega') \right] =$$

$$= \begin{cases} 2(G_\Lambda^2 + G_\Sigma^2)/4\pi & \text{if } P_{KH} = +1 \\ -2\left(\frac{m_K}{2m_N}\right)^2 (G_\Lambda^2 + G_\Sigma^2)/4\pi & \text{if } P_{KH} = -1 \end{cases}$$

At least in the energy interval that we have experimental information the integral is positive since $\sigma_-^t > \sigma_+^t$ and so all things relies in the $\{\quad\}$ term, in order to decide between the two possibilities presented in the right side of the equation.

The experiments are not good enough to settle the problem; we have incertitude in the sign of the phase shifts and so all depends on the guess on the atractive or repulsive nature of K^-- p interaction.

For K^+- p we have repulsion and then $\mathrm{Re}\, f_+ < 0$.

Coulomb interference in elastic K^-- p scattering speaks in favor of atraction but there is not a so definite answer because constructive interference is difficult to detect.

But people believe in the atraction and then $\mathrm{Re}\, f_- > 0$ which means that $P_{KH} = -1$.

We cannot draw any definite conclusion but we have learned the way in which we can work out the problem.

12 - Parity conservation in strong interactions

The speculations about the possibility that parity would not be conserved to some extend in strong interactions, started soon after the discovery that parity conservation and charge conjugation were violated in weak interactions.

Probably the most satisfactory way to look into the problem on theoretical grounds is in terms of the so called conservation of combined parity, first considered by Landau (Nucl. Phys. 3, 127, 1957) and Lee - Yang (Phys. Rev. 105, 1671, 1957). This hypothesis states that in all kind of interactions (weak,electromagnetic and strong), only the combined P C parity is conserved in local and non local theories. The operator P.C means simultaneous space reflexion and change of particles into antiparticles.

In the case of weak interactions, due to P C T. conservation and to the fact that T violation has not been observed, the conservation of P C has good evidence. In this case, both P and C are in general violated, but the combined P C is not.

For electromagnetic interactions, auxiliary invariance requirements (gauge invariance) assure the conservation of both P and C if P C is conserved.

For strong interactions the problem is still open and clearly of great interest. In particular it will be interesting to investigate theoretically if there is in this case any auxiliary requirement which could guarantee conservation of P like in the electromagnetic case.

Let us consider some examples of the mentioned cases where P is conserved only as a consequence of P C conservation and some auxiliary requirement.

We have first the case of a spinor in the electromagnetic field. The requirement of P C invariance gives a Lagrangian of the form:

$$\mathcal{L} = \bar{\psi}(x)\left[-i\gamma_\mu \left(\frac{\partial}{\partial x_\mu} - ie\, A_\mu\right) + m - e'\gamma_5\gamma_\mu A_\mu\right]\psi(x) + \text{H.C.}$$

where the term in e' is responsible for non conservation of parity. However as this term is not gauge invariant must be discarded. Thus in quantum electrodynamics, as a consequence of the gauge invariance condition, the requirement of invariance with respect to the P C operation automatically leads to invariance under the space reflection operation.

A similar situation arises when one considers the case of (π, N) interaction, that is, a spinor in a pseudoscalar field. The most general Lagrangian invariant under P C is:

$$\mathcal{L} = g(\bar{\psi}_p\gamma_5\psi_n\,\phi + \psi_n\gamma_5\,\psi_p\,\phi^*) + g_o'\,\bar{\psi}_p\gamma_5\,\psi_p\,\phi^o +$$

$$+ g_o''\,(\bar{\psi}_n\gamma_5\,\psi_n\,\phi^o) + ig'\,(\bar{\psi}_p\,\psi_n\,\phi - \bar{\psi}_n\,\psi_p\,\phi^*)$$

ϕ representing the meson field, and ϕ^* being the charge conjugate of ϕ, for instance $\phi \to \pi^+$ and $\phi^* \to \pi^-$ and finally g, g_o', g_o'', g' are real quantities. The last term of that expression is not invariant under space reflections. But it is well known that π - N interaction does conserve isotopic spin, and so one has also charge symmetry. Thus, the two terms inside the parenthesis in g', are charge symmetric and are cancelled out, remaining the usual Lagrangian which is invariant under P and C. Therefore in the case of (π, N) interactions, the conservation of parity is a consequence of conservation of combined parity and of I-spin.

Summarizing, a consequence of the conditions of gauge invariance in quantum electrodynamics and of isotopic spin conservation in pseudoscalar meson theory is that the interaction Lagrangian invariant under the operation of combined inversion P C also become invariant under space reflection P.

The situation is quite different when one introduces interactions involving K mesons. In fact, let us see how the different field operators transform under P C.

a) Spinor fields (nucleons, Hyperons)

$$\psi'(x) = S^{-1}\,\psi^{+}(-\vec{X}, x_4), \qquad \psi^{+\prime}(x) = \psi(-\vec{X}, x_4)\,S$$

where:

$$S\,\gamma_\mu\,S^{-1} = -\gamma_\mu^T \qquad \gamma_4 = \beta;\ \gamma_k = \beta\alpha_k\ (\mu = 1,2,3,4)$$

b) Pseudoscalar field (π mesons): $\pi^+ = \phi,\ \pi^- = \phi^*,\ \pi^0 = \phi_0 = \phi_0^*$

$$\Phi_0' = -\,\Phi_0\,(-\vec{X}, x_4) \qquad \Phi' = -\,\Phi^*\,(-\vec{X}, x_4)$$

c) Scalar field:

$$\varphi_0' = \varphi_0\,(-\vec{X}, x_4) \qquad \varphi' = \varphi^*\,(-\vec{X}, x_4)$$

d) K meson field (if pseudoscalar):

$$K^+ = K;\quad K^- = K^*;\quad K^0 = K_0;\quad \bar{K}^0 = K_0^*$$

$$K_0' = -K_0^*\,(-\vec{X}, x_4) \qquad K' = -K^*\,(-\vec{X}, x_4)$$

One sees from all this the difference between π and K. Let us write down for example the Lagrangian for $KN\Lambda^0$ interactions:

$$\mathcal{L} = g\,\bar{\psi}_\Lambda\left[\psi_N\,\phi^-_{K_o} + \psi_p\,\phi_{K^-}\right] + g^*\left[\bar{\psi}_N\,\psi_\Lambda\,\phi_{K_o} + \bar{\psi}_p\,\psi_\Lambda\,\phi_{K+}\right]$$

which, although invariant under rotations in I-spin space, is not charge symmetric in the usual sense, because one cannot obtain charge symmetry by rotation in I-spin space. Thus, there is no relation between the terms in g and in g^*.

Therefore, an interaction involving strange particles can be invariant under P C but not under P and C operations separately.

This is true also if strange particles appear only in a virtual state in an intermediate step of the interaction.

$$\pi + N \longrightarrow Y + K \longrightarrow \pi + N$$

and could produce a certain effect of P nonconservation also in pion or nuclear physics. Therefore it is reasonable to try to see experimentally where and how such a parity violation can be detected. There are in this sense three possibilities: experiments directly involving K's; experiments at high energy where K's can play a role in an intermediate state; and experiments in low energy physics where still K's could influence to some extent.

The experiments in low energy physics, suitable for detecting parity violation deal with the search for reactions which are forbidden for parity conservation.

The most general way to observe a violation of parity conservation is to detect the presence of a pseudoscalar quantity like

$$\vec{\sigma} \cdot \vec{p} \quad \text{or} \quad \vec{p}_1 \wedge \vec{p}_2 \cdot \vec{p}_3$$

Such pseudoscalar term should appear in cases of nucleon-nucleon scattering if in the initial state we had the two unpolarized nucleons, and in the final state some degree of longitudinal polarization of the outgoing nucleons were detected. Another case would be the detection of some dependence of the angular distribution of π's produced by a polarized particle shotted against an unpolarized target, on a term $\vec{\sigma} \cdot \vec{k}$ where $\vec{k}$ is the momentum of the outgoing π.

Some experiments have been already performed namely:

1) The Rochester group, working with π^+ produced by polarized protons tried to detect a term $\vec{\sigma}_p \cdot \vec{k}_\pi$. They got a precision of about 1%, and the answer was negative.

2) At CERN, they analized the production of π^0 by polarized neutrons of energy up to 540 Mev on a C target. A fore-aft asymmetry in the produced π^0, relative to the polarization vector of the neutron beam was sougt, because it would be indicative of the parity violating term $\vec{\sigma}_n \cdot \vec{k}_\pi$. The experiment was sensible to asymmetries of 1 in 10^3 and no effect was detected, thus the value of the parity violating term should be of the order of 1%.

3) Also an experiment has been performed by Jones et al. looking for the detection of polarized neutrons at 0° produced by unpolarized 385 Mev protons. Again no indication of violation of parity has been found.

There have been some rumors some years ago about the violation of parity conservation in interactions producing Λ. It has been already stablished that the degree of violation of parity in the decay of the Λ is about $\alpha \geqslant 0.7$. Now, if one measures the angular distribution of the decay products of the Λ with respect to the line of flight of Λ, one should find a distribution $1 + P\beta \cos \theta$, where P is the polarization of the Λ, thus related to the violation of parity conservation in the production process.

The first measurements by cosmic rays people in cloud chambers seemed to reveal some degree of violation of parity in Λ production, since they found in all their experiments some forward-backward asymmetry in the Λ production. The statistics on which they based their conclusions was too low, and as soon as more information was available from bubble chambers experiments, people tried again and one can say that as the goodness of the statistics increases, the indications of the parity violation tend to desappear. The experiments in bubble chambers on production of Λ by processes initiated by pions are particularly adequate because one has a quite well defined energy of the incoming π and the momenta of the Λ are low enough as to make their measurement easy. These experiments agree in giving un upper limit of about 10% to the parity violation in strong interactions.

The second rumor about some degree of parity violation in strong interactions, came from very preliminary results on capture of K^- in deuterium, from the Alvarez group. With regard to Λ production, they found three possible sources:

1) $K^- + D \rightarrow \pi^- + \Lambda + p$ (direct production)

2) $K^- + D \rightarrow \pi^- + \Sigma^0 + p$

$\Sigma^0 \rightarrow \Lambda + \gamma$ (indirect production through decay of the Σ^0)

3) $K^- + D \rightarrow \pi + N + \Sigma^{\pm}$

$\Sigma^{\pm} + N \rightarrow \Lambda^0 + N$ (indirect production through conversion of the $\Sigma^{\pm}$)

There is a way for distinguishing between these cases, and this is done through the analysis of the momenta distribution of the accompanying pion. This distribution, turns out to be of the type shown in Fig. 76 with two marked peaks of comparable size at about 180 and 250 Mev/c.

On the other hand, if one represents the momenta distribution of the accompanying π in reactions where only charged Σ are produced, a graph as that of Fig. 77 results with a peak at the same energy as the lower peak of Fig. 76.

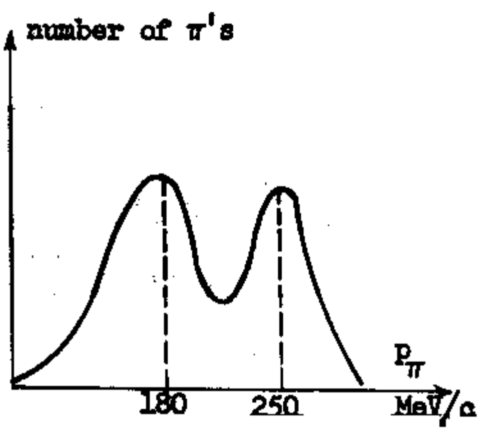


Fig. 76

Then it is natural to attribute this lower peak to the cases 2) and 3) where Σ's appear.

The consequence was that selecting those Λ produced by conversion of Σ's an effect β P of the order of 0,38 $\pm$ 0,16 was found in the direction of flight of the Λ, effect which turned out to be of the same order of magnitude as the observed by cosmic rays people.

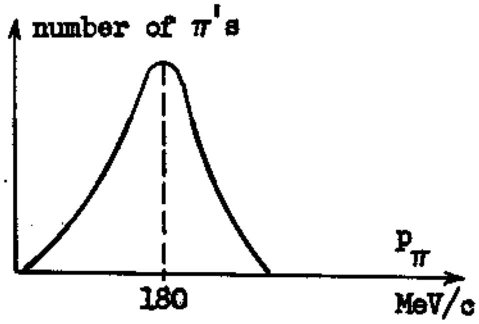


Fig. 77

Nevertheless, it was very difficult to understand how, existing such a high degree of violation of parity conservation in that case, no effect could be detected in the low energy physics.

The situation changed since Alvarez and coworkers improved the statistics, the detected effect going rapidily down.

The same analysis has been performed looking to Λ production in a He bubble chamber. There are about 500 events already analysed and no effect has been detected. In fact, on the basis of 482 events from a He bubble chamber experiment, a forward-backward asymmetry of 0.04 $\pm$ 0.08 has been measured.

The general conclusion is that if parity conservation is violated in strong interactions it cannot be in a degree greater than 10^{-8} in the low energy region and than about a 10% in high energy phenomena.

13 - Interactions of K's with nucleons

Most of our discussion on this subject will be concentrated on K^--N interactions, since the K^++N scattering cross-section does not present any outstanding feature, being nearly constant all over the investigated energy range.

With regard to K^-, there is already considerable experimental information on the following reactions:

$$\begin{array}{ll} K^- + p & (a \\ K^- + D & (b \\ K^- + He^4 & (c \\ K^- + \text{nuclei} & (d \end{array} \qquad (12.1)$$

both at rest and in flight.

The simplest to be analyzed is of course the first case. Let us then start with $K^-+ p$ and once one has understood something about this process one can try to analyze the more complicated reactions in terms of the elementary interaction with protons. This is a general way of working, since every method applied for two body cases, becomes more difficult to be applied to the many body problems.

For interactions $K^-+ p$ in flight, one has three-main channels:

$$K^- + p \rightarrow \begin{cases} K^- + p & \text{elastic scattering} \\ K^0 + n & \text{charge exchange} \\ \text{inelastic} & \text{production processes} \end{cases}$$

In the low energy region, up to 200 Mev, the angular distribution for elastic scattering turned out to be in all the experiments already performed, compatible with isotropy, indicating that S waves dominate the process. This is a first simplification in the proposed analysis.

A second simplification of the picture would consist in extending I-spin concepts to those reactions. Since both K and p have $T = \frac{1}{2}$, in the incoming channel one has $T = 0$ or $T = 1$, and two amplitudes M_0 and M_1 must be introduced. In terms of those amplitudes, the cross-sections for elastic scattering and charge exchange will be expressed:

$$\sigma_{scatt} = |M_0 + M_1|^2$$
$$\sigma_{ch.ex.} = |M_0 - M_1|^2 \qquad (12.2)$$

respectively.

Assuming only S waves to be present, the expansion in spherical harmonics takes only one term. Each amplitude will be written down us usually:

$$M_i = \frac{1 - e^{2i\delta_i}}{2i\,q} \qquad (12.3)$$

where the only important point is that δ_i is a complex quantity, because of the existence of inelastic processes. It should then be expressed: $\delta_i = \alpha_i + i\,\beta_i$ and with these elements we should be able to represent the experimental data. If the analysis would not work, it would be an indication that more channels should be considered or that charge independence does not hold.

If one expresses the cross-sections for S wave one gets the following relations:

$$\sigma_{el} = \pi\lambda^2 \left| 2 - e^{-2\beta_0} e^{2i\alpha_0} - e^{-2\beta_1} e^{2i\alpha_1} \right|^2 \qquad (12.4)$$

$$\sigma_{ch.ex} = \pi\lambda^2 \left| e^{-2\beta_0} e^{2i\alpha_0} - e^{-2\beta_1} e^{2i\alpha_1} \right|^2 \qquad (12.5)$$

$$\sigma_{absorption} = \pi\lambda^2 \left[(1 - e^{-4\beta_0}) + (1 - e^{-4\beta_1}) \right] \qquad (12.6)$$

where the indeces 0 and 1 refer to the T = 0 or T = 1 state respectively. In consequence, we have the four quantities α_0, β_0, α_1, and β_1 to be determined through only these three relations. One can try to obtain more information from the inelastic processes, namely the following:

$$\begin{aligned} K^- + p &\longrightarrow \pi^+ + \Sigma^- && (a \\ &\longrightarrow \pi^- + \Sigma^+ && (b \\ &\longrightarrow \pi^0 + \Sigma^0 && (c \\ &\longrightarrow \pi^0 + \Lambda && (d \end{aligned} \qquad (12.7)$$

The cross-sections for these reactions must be represented by the two terms of the expression (12.6). They can be analyzed again in terms of the possible states of isotopic spin. Since in the final state for reactions a) b) and c) one has two objects of T = 1, the possible states are 0, 1 and 2, the last being eliminated because in the initial state one has only 0 and 1. The cross-sections for reactions a), b), c) and d) of (12.7) will be:

a) $$\sigma \propto \left|\frac{1}{\sqrt{6}} A_o + \frac{1}{2} A_1\right|^2$$

b) $$\sigma \propto \left|\frac{1}{\sqrt{6}} A_o - \frac{1}{2} A_1\right|^2$$

c) $$\sigma \propto \left|\frac{1}{\sqrt{6}} A_o\right|^2$$

d) $$\sigma \propto \left|\frac{1}{\sqrt{2}} B\right|^2$$

The ratio $\Sigma^-/_{\Sigma^+}$ will depend therefore on the interference terms arising from these expressions. From each one of the listed reactions (12.7) one gets a number: n_1, n_2, n_3 and n_4 with which one is able to calculate A_0, A_1 and B and with those values one can go back to (12.6) from where β_0 and β_1 can be obtained. The values for β_0 and β_1 calculated by this method are the following:

$$\beta_0 \simeq 0.445$$

$$\beta_1 \simeq 0.04$$

which indicate that for inelastic processes, T = 0 is the most important channel.

Taking the values of β_0 and β_1 into the expressions (12.4) and (12.5), α_0 and α_1 can be calculated. There is an ambiguity in the solution of the system, with regard to the sign of the pair α_0, α_1.

In order to resolve the ambiguity one can try to decide if the real phase shift α_1 is positive or negative, or, what is the same, if the potential is attractive or repulsive. We have already mentioned, when studying the phase shifts for π - N scattering, that a method to determine the sign of a potential is by means of its interference with the Coulomb potential. There are some indications in the present case, in the sense that the interference with the Coulomb potential is constructive implying attractive potential and therefore, $\alpha_1 > 0$.

We must remark that the interference effects with the Coulomb potential are quite difficult to be detected if the interference is constructive, being much easier to be detected a destructive interference. In the present case, since no destructive interference has been detected, one concludes that it is constructive. But one can easily understand that this fact cannot be taken as a definitive argument, but only as an indication.

Nevertheless, even remaining this ambiguity, our analysis has been useful for a better understanding of the phenomena. We have been able to conclude that only S waves and T = 0 are important states.

Through a more refined analysis, for instance through the study of the ratio Σ^-/Σ^+ , one could get the relative sign of the phase shifts. For K^- at rest, these ratio varies quite rapidily when passing from $K^-+ p$ to $K^-+ D$ or $K^-+ He^4$. Plotting Σ^-/Σ^+ versus E_K (energy of the K) one gets Fig. 78.

The ratio is practically equal unity for every value of E_K in the case of $K^-+ p$, with the exception of $T_K= 0$. For deuterium and He, as the interaction takes place with bound nucleons, $E_K < 0$,

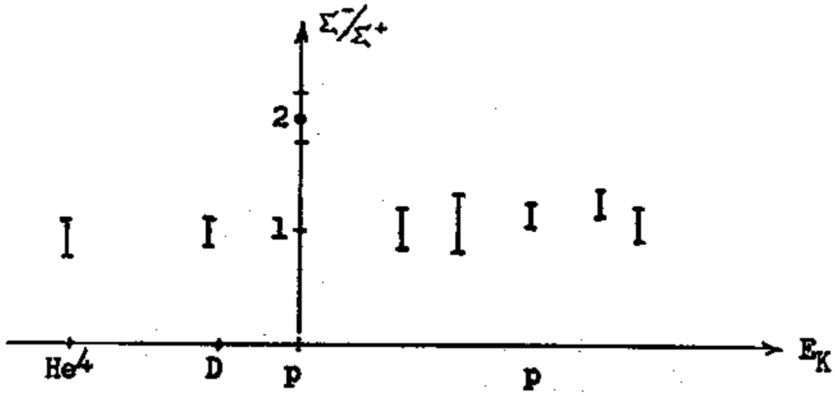


Fig. 78

and here again the ratio is about 1. The exception in $T_K = 0$ must be due to the presence at this energy of higher order waves.

Another exciting feature when passing to bound system, is the high rate of conversion of the Σ with the nucleon ($\Sigma + N \rightarrow \Lambda + N$). About one half of the Λ produced in the capture of K^- by deuterium, are of indirect origine. The situation is more drastic in He, where about 2/3 of the Λ are originated by conversion of the Σ's, while only 1/3 are directly produced.

This fact is not easy to be understood. One must assume a very large $\Sigma - N$ cross-section in order to justify the results but the validity of impulse approximation (which considers the second nucleon as a mere spectator) and the mentioned high rate of conversion, seem to be in strong contradiction.

14 - Results on the He bubble chamber collaboration

The following possibilities arise for K^- capture in He:

$$
\begin{aligned}
K^- + He^4 &\longrightarrow \Sigma^- + \pi^+ + (pnn) && \text{(a} \\
&\longrightarrow \Sigma^+ + \pi^- + (pnn) && \text{(b} \\
&\longrightarrow \Sigma^0 + \pi^0 + (pnn) && \text{(c} \\
\\
&\longrightarrow \Sigma^- + \pi^0 + (ppn) && \text{(d} \\
&\longrightarrow \Sigma^0 + \pi^- + (ppn) && \text{(e} \\
\\
&\longrightarrow \Lambda + \pi^0 + (pnn) && \text{(f} \\
&\longrightarrow \Lambda + \pi^- + (ppn) && \text{(g}
\end{aligned}
\qquad (12.8)
$$

The π can be absorbed by the group of nucleons, so that also reactions with a final state consisting only on Σ and nucleons or Λ and nucleons should be found.

The first thing in order to analyze these reactions will be to compare the results with what happens in more elementary reactions, i.e. in $K^- + D$ or $K^- + p$. Immediatly something interesting will appear, and this is about the kind of hyperons. In He as well as in deuterium or protons, only Σ and Λ hyperons will be produced. The situation with regard to the number of hyperons of each type produced in the three mentioned elements is summarized in Fig. 79 where the percentages refer to the total number of captures.

How can this change in yield be justified ? A reason can be found immediately in the fact that in interactions with protons, the initial state can be either in $T = 0$ or $T = 1$ while in

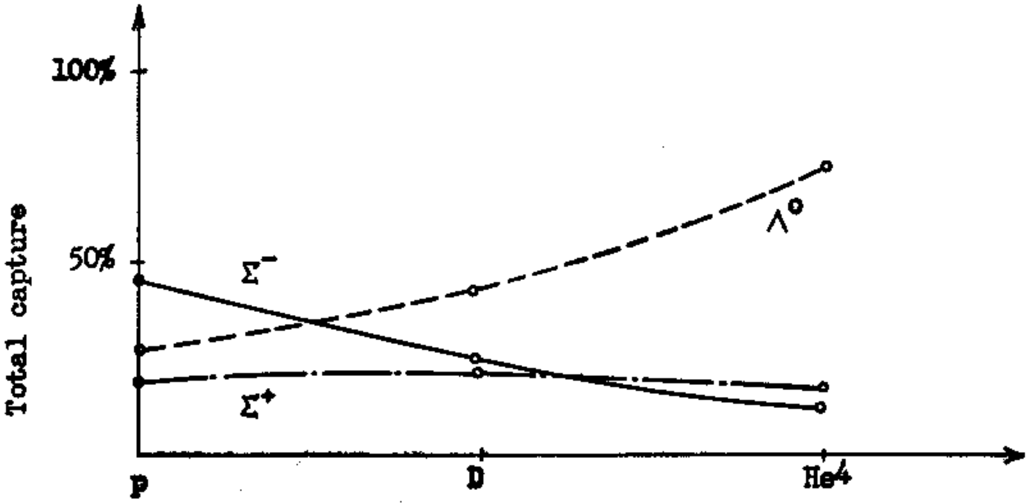


Fig. 79

deuterium and He one has only one possibility, which is $T = \frac{1}{2}$. Therefore there is a difference in what concerns the incoming channel.

A second point is the good evidence that the processes listed in (12.8) proceed via three body reactions, where the (pnn) and (ppn) systems are bound as tritium and He^3 respectively. The evidence for the existence of these bound system comes from the analysis of events of the two types represented in Figs. 80 and 81.

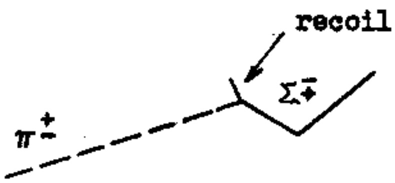


Fig. 80

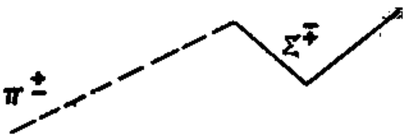


Fig. 81

The analysis of the events in which the recoil is visible, can be fully accomplished. In fact, one measures the momenta of the Σ and π, and the angle between them, and with these elements, the direction of the recoil can be determined and according to the assumption for the recoil to be a tritium or a He^3, one can also estimate its range which is to be compared with the one observed. In particular for tritium, the range can be well measured in bubble chambers and the kinematics can be checked.

It turns out to be

$$\frac{N^{\underline{b}} \text{ of 3 prongs } (\Sigma + \pi + T)}{\text{All 3 prongs } (\pi \Sigma)} \simeq \frac{1}{2} \qquad (12.9)$$

This ratio is given by the events having a visible recoil, whose kinematics is compatible with a 3 body process. But, as the number of these is small, the criterium can be extended to the cases not having visible recoil. Of course we lose here the possibility of checking both the coplanarity and the range, but the criterium is that if the unbalance in momenta gives a mass approximately equal to that of a tritium, the event can be considered as a 3 body case. Now the ratio turns out to be:

$$\frac{N^{\underline{b}} \text{ of 2 prongs compatible with } (\Sigma + \pi + T)}{\text{All 2 prongs } (\Sigma \pi)} \simeq \frac{2}{3}$$

and it depends on the criteria of acceptance of the events. One can take as a good information (12.9) so that about 50% of the events in which one sees a Σ and a π can be considered as 3 body processes. This fact could seem astonishing because of the total energy available for the bound system (about 84 Mev taking into account the 494 Mev of the mass of the K) in front of the 8 Mev of the binding energy of tritium. Nevertheless, one should not forget that as the π is a light particle, it should take most of the available energy, and then the energy left to the (PNN) system must be comparable with the 8 Mev of binding energy.

Some attempts have been done in order to understand the main features of these 3 body reactions. Since no theory has yet been developed, one must try only to compare the experimental facts with the predictions of some phenomenological models.

1) The first possibility is given by the statistical model. In our case if the model works, it would mean that the energy transfered to the particles in the final state, would not depend on the momentum dependence of the matrix element for that transition, either on the structure of the absorbing system. One can see even intuitively that this model cannot describe the experimental situation since it ignores these two important facts which should influence the mechanism of production. There is a way to prove this, namely the fact that the statistical model predicts large momentum transfers to the tritium, thus long range for the recoil tracks. Therefore, events supporting the predictions of the statistical model should be the most easy to be measured and despite this fact, no long range recoil has been measured.

2) A second model which takes into account the structure of the

absorbing system, but still neglects the momentum dependence of the matrix element is the one called the impulse approximation by the theoreticians and the spectator model by the experimentalists.

The structure of the He^4, known from electron-He scattering experiments, is used in order to write the wave function of the proton (the actor) whose Fourier transform to the momentum space will give its distribution of momenta. The rest of the nucleons will just support the recoil momentum of the actor. The predictions of this model with regard to the momentum distribution of the tritium, are quite different from those of the statistical model, since the large momentum transfer are cut down. Fig. 82 is a rough representa tion of the predictions of both models, compared with the experimental results. These are much better described by the impulse approximation, at least with regard to the position of the maximum, eventhough its hight is not well accounted for, thus meaning that some improvement of the model is necessary.

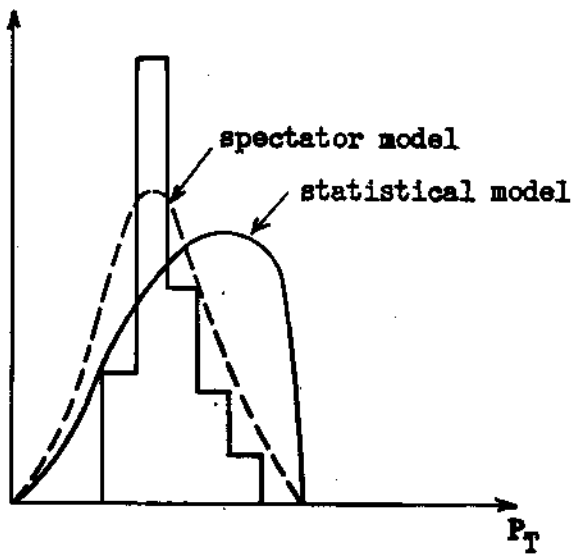


Fig. 82

3) A next step would be to take into account that the absorption of K's by the actor, with production of the π and the Σ :

$$K^- + p \longrightarrow \Sigma + \pi$$

will depend on the total energy available for the reaction.

The marked peak in the momentum distribution of the tritium is an indication, eventhough not yet a proof, of a $\pi - \Sigma$ interaction in the unphysical region for (K p). In fact, a diagram like Fig. 83 could be imagined, where one sees that there is a pole in the momentum transfer which could be affecting the energy spectrum. As the lower vertex is well known, what is left is the cross-section for K-p in the unphysical region. The analysis of K^- capture in He^4 would then provide a way of extrapolating our knowledge into the unphysical region for (Kp).

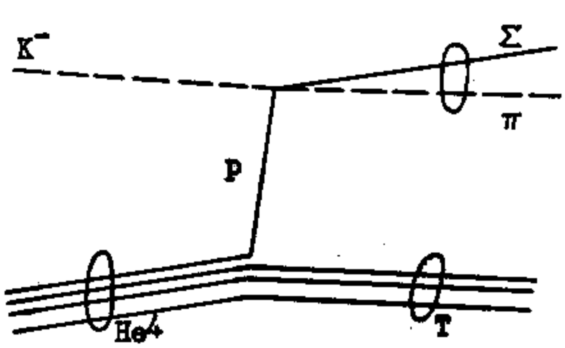


Fig. 83

It could be also that a graph like in Fig. 84 is important, which could account for the high rate of conversion Σ- N.

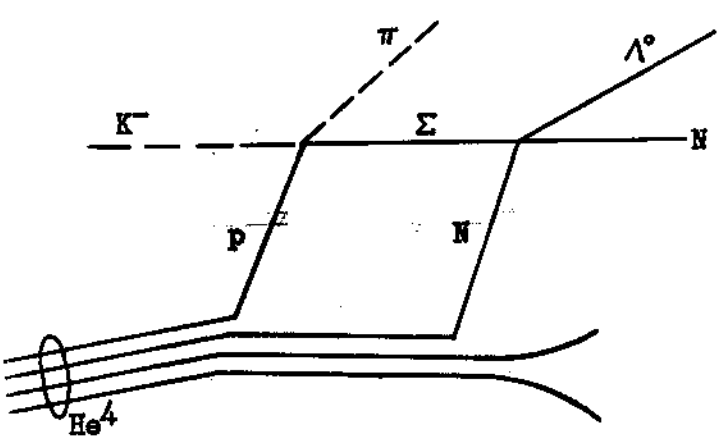


Fig. 84

With respect to the strong rate of conversion of the Σ's it can be worked out by looking into the events with Λ and π^- in the final state, i.e. reactions b), e) and g) of (12.8). It was already shown by Alvarez et al. that the peak at higher energy in the spectrum of the pions (Fig. 76) corresponds to pions going with the Λ directly produced (reaction g) of (12.8)) while the peak at lower energy comes from π accompanying Λ's produced through decay of Σ^0 or through conversion of charged Σ's. In deuterium the ratio between the two peaks is 1:1 while in He is 2:1.

It is easy to prove that no Λ comes from conversion of Σ^-. In fact, for Σ^+ and Σ^0 one has the following possibilities:

$$\Sigma^+ + n \longrightarrow \Lambda + p$$
$$\Sigma^0 + \begin{Bmatrix} p \\ n \end{Bmatrix} \longrightarrow \Lambda + \begin{Bmatrix} p \\ n \end{Bmatrix}$$

In the case of Σ^- one has only the possibility of conversion with a proton, but then three neutrons would be left together which is an extremely unlikely situation. Therefore one should not find any case of Λ associated with a π^+, and this is in fact the experimental situation.

Furthermore, there is another criterium which enables one to discover the cases of Λ coming from conversion of the Σ^+. In fact, due to the difference in mass between Σ^+ and Λ (about 80 Mev), there will be a fast proton in the final state, and this is easy to be experimentally observed. A further analysis in terms of I-spin would permit to extend the conclusion about Σ^+ conversion, to the case of Σ^0.

Finally it is also interesting to point out which is the proportion of events without π's in the final state. These constitute about a 10% of the total, and can be due to conversion of the π's.

All this constitutes a very wide picture of results from the He^4 bubble chamber collaboration, which have not yet been fully elaborated.

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