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Magnetorotational instability in the Hall regime in a hot-electron plasma

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The magnetorotational instability (MRI) in the Hall regime in a hot-electron plasma is analyzed. The dispersion relation is derived for a rotating plasma with both finite electron pressure and pressure anisotropy. It is pointed out that the former effect has to be taken into account in the Hall regime, for $\beta \gtrsim 1$, where β is the ratio of electron pressure to the magnetic field pressure. As a whole, the effects of order β weaken or suppress the MRI. It is shown that in the presence of electron pressure anisotropy, a hybrid of MRI and anisotropic instability appears, and that anisotropy of type $T_\perp > T_\parallel$ is destabilizing, while $T_\parallel > T_\perp$ is stabilizing, where $T_\perp$ and $T_\parallel$ are the perpendicular and parallel electron temperatures, respectively. © *2007 American Institute of Physics*.
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I. INTRODUCTION

The magnetorotational instability[1–3] (MRI) is one of the most actively investigated phenomena of recent plasma physics and astrophysics. As is known, the initial astrophysical applications of MRI were focused on the problem of anomalous viscosity in accretion disks.[4,5] An important step in development of the MRI theory has been taken in Ref. 6, with the derivation and analysis of the kinetic dispersion relations for a collisionless plasma. One of relevant results of this reference was to elicit that, for $\beta \gg 1$ (β is the ratio of the plasma pressure to the magnetic field pressure), the growth rate of MRI in the collisionless regime is greater than that produced by the magnetohydrodynamic (MHD) theory. The approach of Ref. 6 has been generalized in Ref. 7 to the case in which the mean free path of particles is arbitrary compared to the wavelength of the perturbations. One more step in the kinetic theory of MRI was to allow for the equilibrium plasma pressure anisotropy.[8] As was noted in Ref. 7, one motivation for studying MRI in the collisionless regime is understanding the mechanism of the radio and x-ray source Sagittarius A* in our galaxy.

Let us note also the development of the theory of the helical MRI presupposing the existence of both the longitudinal and azimuthal components of the equilibrium magnetic field.[9,10] This instability was observed in the laboratory.[11]

One of the important trends in astrophysical applications of MRIs is their investigation in the so-called Hall regime.[12–29] This class of works (the Hall trend) discusses, in particular, a possible role of MRI in the problem of evolution of protostellar disks[12,13,22,26] and the quiescent phase of dwarf nova disks.[14,15]

Following Ref. 12, the Hall trend treats both the collisionless and collisional effects beyond the standard MHD (magnetohydrodynamic) regime.[1–3] For a physical understanding of the essence of the Hall regime, it is convenient to separate collisionless effects from the collisional ones, as performed in Ref. 13. It was then elucidated that the collisionless version of the Hall regime is nothing but the whistler regime modified by plasma rotation effects. In this context, it is well known from plasma theory (see, e.g., Refs. 30–33) that the whistler regime is pure electronic; i.e., that ions do not play a role in it. This made it necessary the development of a theoretical scheme to study the collisionless variety of the Hall regime by means of pure electronic MHD equations.

The main physical difference between collisionless varieties of all regimes is the degree of plasma magnetization. The standard MHD regime is a regime of strong magnetization, implying that the ion cyclotron frequency ω_{Bi} exceeds substantially the plasma rotation frequency; i.e., $\omega_{Bi} \gg \Omega$. In the opposite case, when $\omega_{Bi} \ll \Omega$, the standard MHD regime gives place to the whistler regime; i.e., to the regime of intermediate magnetization. For much smaller magnetization, when the plasma rotation frequency Ω becomes of the order of the electron cyclotron frequency ω_{Be}, i.e., $\Omega \simeq \omega_{Be}$, the MRI theory has to properly allow for electron inertia.[34] Such a region of magnetization can be called the weak magnetization regime. An important region in this regime is the subregime of unmagnetized electrons treated in Ref. 34.

The above-listed papers concerning the Hall trend have all been oriented to applications for a scenario of a rather cold plasma, so that the effects of finite electron temperature have been neglected; this is a somewhat forced approximation. The fact is that the electron dynamics in the Hall regime is described by the magnetic diffusion equation, derived from a generalized statement of Ohm's law, i.e., from the electron equation of motion, by taking the curl of it. As a rule, the temperature effects are allowed for in terms of the pressure gradient of corresponding particle species. However, if one complements the electron equation of motion with the term including the electron pressure, taking the curl of this gradient leads to a vanishing result. In this context, the situation differs radically from the standard MHD regime, when one deals with the ion equation of motion, which does not undergo the curl operation.

At a first sight, it seems that the situation could be improved by augmenting the terms with the pressure anisotropy in the electron equation of motion. However, there is a tool to elucidate whether such a procedure is adequate. The fact is that, if one neglects the term with the gradient of the rotation frequency responsible for the MRI, one should arrive at the dispersion relation for a homogeneous plasma. Such a dispersion relation can be obtained by means of the electrodynamic approach, i.e., using the permittivity tensor; thereby the electrodynamic dispersion relation can be used as a benchmark for dispersion relations derived through different schemes. The result is that the dispersion relation derived with allowance for the pressure anisotropy does not coincides with the benchmark. Therefore, using the anisotropic electron hydrodynamics in the Hall regime is invalid.

At the same time, the electrodynamic approach gives a recipe for adequate generalization of the electron equation of motion, so that, instead of the anisotropic hydrodynamics, one should use that equation allowing for only the perpendicular electron pressure and taking the parallel pressure to vanish. This is the approach of the present paper.

Taking into account of the perpendicular electron pressure allows us to study the Hall regime in both the cases of Maxwellian and non-Maxwellian (anisotropic) equilibrium electron distribution function. Thereby, it is possible to consider not only the effects inherent to electrons with isotropic equilibrium pressure, but also the effects of equilibrium electron pressure anisotropy, similar to the standard MHD regime.[8,35]

In Sec. II we present our basic equations. Section III is addressed to derivation of the dispersion relation for MRI in the collisionless Hall regime allowing for the effects of perpendicular electron pressure. In Sec. IV we analyze the resulting dispersion relation. Discussion of results is given in Sec. V.

II. BASIC EQUATIONS

For a description of the perturbed dynamics of electrons, we use their perturbed perpendicular and parallel equations of motion,

$$\delta\left[\mathbf{E} + \frac{1}{c}(\mathbf{V}_e \times \mathbf{B})\right]_\perp = -\frac{1}{en_0}\delta\nabla_\perp p_\perp, \tag{1}$$

$$\delta(\mathbf{E}\cdot\mathbf{B}) = 0. \tag{2}$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively, $\mathbf{V}_e$ is the electron velocity, $p_\perp$ is the perpendicular electron pressure, n_0 is the equilibrium plasma number density, e is the elementary charge, $\delta(\cdots)$ means the perturbed part of $(\cdots)$, and $\nabla_\perp \equiv \nabla - \mathbf{B}^{-2}\mathbf{B}(\mathbf{B}\cdot\nabla)$ is the perpendicular gradient.

Acting by the operator $\nabla\times$ on Eq. (1) and using Eq. (2) and the Maxwell equation

$$\nabla \times \mathbf{E} = (-1/c)\partial\mathbf{B}/\partial t, \tag{3}$$

we arrive at the freezing condition in the form

$$\frac{\partial\mathbf{B}}{\partial t} - [\nabla \times (\mathbf{V}_e \times \mathbf{B})] + \frac{c}{en_0}(\nabla \times \mathbf{G}) = 0. \tag{4}$$

Here, the vector $\mathbf{G}$ describes the effect of perpendicular pressure

$$\mathbf{G} = \frac{1}{\mathbf{B}^2}\mathbf{B}(\mathbf{B}\cdot\nabla)p_\perp. \tag{5}$$

We consider an axisymmetric plasma cylinder placed in a magnetic field $\mathbf{B}_0$ directed along its axis; i.e., $\mathbf{B}_0 = (0,0,B_0)$. We use the cylindrical coordinates (R,ϕ,z), with ϕ as the azimuthal coordinate. For simplicity, the field B_0 is assumed to be uniform; i.e., $dB_0/dR=0$. We suppose that plasma rotates in the azimuthal direction, so that its equilibrium velocity $\mathbf{V}_0$ is given by $\mathbf{V}_0=(0,V_0,0)$, where $V_0=R\Omega$, Ω is the rotation frequency, dependent on the radial coordinate R, i.e., $\Omega=\Omega(R)$.

We take the time and spatial dependence of each perturbed function $\delta F(\mathbf{r})$ in the form

$$\delta F(\mathbf{r}) = \delta F(R)\exp(-i\omega t + ik_R R + ik_z z), \tag{6}$$

where ω is the oscillation frequency, and k_R and k_z are, respectively, the perpendicular and parallel projections of the wave vector. The radial dependence of the function $F(R)$ is assumed to be negligibly weak.

Taking the (R,ϕ)th projections of Eq. (4), we obtain

$$-i\omega\delta B_R - ik_z B_0 \delta V_{eR} = 0, \tag{7}$$

$$-i\omega\delta B_\phi - \frac{d\Omega}{d\ln R}\delta B_R - ik_zB_0\delta V_{e\phi} + \frac{ck_Rk_z}{en_0}\delta p_\perp = 0. \quad (8)$$

Here, δB_R and δB_ϕ are the (R,ϕ)th projections of the perturbed magnetic field, δV_{eR} and $\delta V_{e\phi}$ are the projections of the perturbed electron velocity, and $\delta p_\perp$ is the perturbed part of the perpendicular electron pressure.

The perturbed perpendicular pressure $\delta p_\perp$ is expressed in terms of the perturbed distribution function δf by[30]

$$\delta p_\perp = M\int \frac{v_\perp^2}{2}\delta f d\mathbf{v}. \quad (9)$$

Here, $v_\perp$ is the perpendicular particle velocity, $\mathbf{v}$ is the velocity space volume, and M is the electron mass.

According to Ref. 30, in the case of strong magnetized particles, the function δf is equal to

$$\delta f = \frac{Mv_\perp^2}{2T_\perp}\left(1 - \frac{T_\perp}{T_\parallel} + \frac{\omega}{\omega - k_zv_\parallel}\frac{T_\perp}{T_\parallel}\right)f_0\frac{\delta B_z}{B_0}. \quad (10)$$

Here, $T_\perp$ and $T_\parallel$ are, respectively, the equilibrium perpendicular and parallel temperatures of electrons, and f_0 is their equilibrium distribution function taking to be bi-Maxwellian.

III. DERIVATION OF DISPERSION RELATION

It follows from Eqs. (9) and (10) that

$$\delta p_\perp = p_{\perp 0}\left\{1 - \frac{T_\perp}{T_\parallel}\left[1 + \frac{i\sqrt{\pi}\omega}{|k_z|v_{T\parallel}}W\left(\frac{\omega}{|k_z|v_{T\parallel}}\right)\right]\right\}\frac{\delta B_z}{B_0}. \quad (11)$$

Here, W is the plasma dispersion function[36] defined as

$$W(x) = \exp(-x^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^x \exp(t^2)dt\right]. \quad (12)$$

δB_z is the zth projection of the perturbed magnetic field related to δB_R through

$$\delta B_z = (-k_R/k_z)\delta B_R, \quad (13)$$

which is a consequence of the Maxwell equation $\nabla\cdot\delta\mathbf{B}=0$. As a result, Eq. (8) yields

$$\delta V_{e\phi} = -\frac{\omega}{k_zB_0}\left(\delta B_\phi - \frac{i}{\omega}\frac{d\Omega}{d\ln R}\delta B_R\right) + \frac{ic}{en_0B_0^2k_z} \times k_R^2p_{\perp 0}\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + i\sqrt{\pi}\frac{\omega}{|k_z|v_{T\parallel}}W\right)\right]\delta B_R. \quad (14)$$

In the case of pure electronic perturbations, the perturbed electron velocities are related to the perturbed magnetic field δB_R, δB_ϕ by

$$\delta V_{eR} = i\frac{ck_z}{4\pi en_0}\delta B_\phi, \quad (15)$$

$$\delta V_{e\phi} = -\frac{ick^2}{4\pi ek_zn_0}\delta B_R, \quad (16)$$

where $k^2=k_R^2+k_z^2$. Substituting Eqs. (15) and (16), into Eqs. (7) and (4), we arrive at the equation system for δB_R, δB_ϕ:

$$\frac{c^2k_z^2}{\omega^2}\delta B_\phi - i\frac{\omega_e}{\omega}\delta B_R = 0, \quad (17)$$

$$i\frac{\omega_e}{\omega}\delta B_\phi + \frac{c^2k^2}{\omega^2}\left\{1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln R} + \frac{k_R^2}{k^2}\frac{\beta_\perp}{2}\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + i\sqrt{\pi}\frac{\omega}{|k_z|v_{T\parallel}}W\right)\right]\right\}\delta B_R = 0, \quad (18)$$

where

$$\beta_\perp = 8\pi p_{\perp 0}/B_0^2, \quad (19)$$

$$\omega_e = 4\pi en_0c/B_0. \quad (20)$$

Hence, we find the dispersion relation

$$\omega^2 = \frac{c^4k_z^2k^2}{\omega_e^2}\left\{1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln R} + \frac{\beta_\perp}{2}\frac{k_R^2}{k^2}\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + i\sqrt{\pi}\frac{\omega}{|k_z|v_{T\parallel}}W\right)\right]\right\}. \quad (21)$$

Note also that the function $W(x)$ has the asymptotics[36]

$$W(x) = \begin{cases} \dfrac{i}{\sqrt{\pi}x}, & x \gg 1, \\ 1, & x \ll 1. \end{cases} \quad (22)$$

IV. ANALYSIS OF DISPERSION RELATION

A. MRI in cold-electron plasma

Taking $\beta_\perp \to 0$, Eq. (21) reduces to

$$\omega^2 = \frac{c^4k_z^2k^2}{\omega_e^2}\left(1 + \frac{\omega_e}{c^2k^2}\frac{d\Omega}{d\ln R}\right). \quad (23)$$

This is the dispersion relation for MRI in the Hall regime given in Ref. 13. The condition of this instability is[12,13]

$$\omega_e d\Omega/d\ln R < -c^2k^2. \quad (24)$$

B. Anisotropic instability in nonrotating plasma

Neglecting the term with $d\Omega/d\ln R$ in Eq. (21), we arrive at

$$\omega^2 = \frac{c^4k_z^2k^2}{\omega_e^2}\left\{1 + \frac{\beta_\perp}{2}\frac{k_R^2}{k^2}\left[1 - \frac{T_\perp}{T_\parallel}\left(1 + i\sqrt{\pi}\frac{\omega}{|k_z|v_{T\parallel}}W\right)\right]\right\}. \quad (25)$$

For $T_\perp \neq T_\parallel$, this dispersion relation describes anisotropic instability in nonrotating plasma.[30] The simplest form of this instability is revealed in the approximation $\omega \ll |k_z|v_{T\parallel}$. Equation (25) is then transformed to

$$1 - \frac{T_\perp}{T_\parallel}\left(1 + i\sqrt{\pi}\frac{\omega}{|k_z|v_{T\parallel}}\right) + \frac{2}{\beta_\perp}\frac{k^2}{k_R^2} = 0. \quad (26)$$

The instability condition is given by

$$\frac{T_\perp}{T_\parallel} > 1 + \frac{2}{\beta_\perp}\frac{k^2}{k_R^2}. \tag{27}$$

The growth rate $\gamma \equiv \mathrm{Im}\,\omega$ near the instability boundary is the following:

$$\gamma = \frac{|k_z| v_{T\parallel}}{\sqrt{\pi}}\left[1 - \left(1 + \frac{2}{\beta_\perp}\frac{k^2}{k_R^2}\right)\frac{T_\parallel}{T_\perp}\right]. \tag{28}$$

C. MRI for hot Maxwellian electrons

For $T_\perp = T_\parallel \equiv T$, Eq. (21) reduces to

$$\omega^2 = \frac{c^4 k_z^2 k^2}{\omega_e^2}\left[1 + \frac{\omega_e}{c^2 k^2}\frac{d\Omega}{d\ln R} - \frac{\beta}{2}\frac{k_R^2}{k^2} i\sqrt{\pi}\frac{\omega}{|k_z| v_T} W\left(\frac{\omega}{|k_z| v_T}\right)\right], \tag{29}$$

where, in accordance with Eq. (19), $\beta = 8\pi n_0 T / B_0^2$.

1. Perturbations with $\omega \gg |k_z| v_T$

Using Eq. (22), for $\omega \gg |k_z| v_T$, Eq. (29) yields

$$\omega^2 = \frac{c^4 k_z^2 k^2}{\omega_e^2}\left(1 + \frac{\beta}{2}\frac{k_R^2}{k^2} + \frac{\omega_e}{c^2 k^2}\frac{d\Omega}{d\ln R}\right). \tag{30}$$

The MRI condition (24) is then modified as follows:

$$\omega_e \frac{d\Omega}{d\ln R} < -c^2\left(k^2 + \frac{\beta}{2}k_R^2\right). \tag{31}$$

It hence follows that the free energy for driving the MRI has to be larger for finite electron pressure.

2. Perturbations with $\omega \ll |k_z| v_T$

For $\omega \ll |k_z| v_T$, using Eq. (22), we obtain from Eq. (29)

$$1 + \frac{\omega_e}{c^2 k^2}\frac{d\Omega}{d\ln R} - \frac{\beta}{2}\frac{k_R^2}{k^2} i\sqrt{\pi}\frac{\omega}{|k_z| v_T} = 0. \tag{32}$$

Evidently, this dispersion relation describes the perturbations near the instability boundary following from Eq. (24). According to Eq. (32), the growth rate of such perturbations is given by

$$\gamma = -\frac{2}{\sqrt{\pi}\beta k_R^2 c^2}\left(\omega_e \frac{d\Omega}{d\ln R} + c^2 k^2\right). \tag{33}$$

Thus, growth rate of MRI decreases with increasing the electron pressure.

D. MRI in the presence of electron pressure anisotropy

Taking $T_\perp \neq T_\parallel$ and $\omega \ll |k_z| v_{T\parallel}$, Eq. (21) reduces to

$$1 + \frac{\omega_e}{c^2 k^2}\frac{d\Omega}{d\ln R} + \frac{\beta_\perp}{2}\frac{k_R^2}{k^2}\left(1 - \frac{T_\perp}{T_\parallel}\right) - \frac{i\sqrt{\pi}}{2}\frac{\omega \beta_\perp T_\perp}{|k_z| v_{T\parallel} T_\parallel}\frac{k_R^2}{k^2} = 0. \tag{34}$$

This dispersion relation describes a hybrid of the MRI and anisotropic instability (see subsections A and B). The instability condition of such a hybrid is [cf. Eqs. (24) and (27)]

$$\omega_e \frac{d\Omega}{d\ln R} - \frac{\beta_\perp}{2}\left(\frac{T_\perp}{T_\parallel} - 1\right)c^2 k_R^2 < c^2 k^2. \tag{35}$$

The MRI is enhanced by the anisotropy effect for $T_\perp > T_\parallel$. In the opposite case ($T_\parallel > T_\perp$), the MRI is suppressed. According to Eq. (34), the growth rate of unstable perturbations near the instability boundary is given by [cf. Eqs. (28) and (33)]

$$\gamma = -\frac{2|k_z| v_{T\parallel}}{\sqrt{\pi}\beta k_R^2 c^2}\left\{\omega_e \frac{d\Omega}{d\ln R} + c^2\left[k^2 - \frac{\beta_\perp}{2}\left(\frac{T_\perp}{T_\parallel} - 1\right)k_R^2\right]\right\}. \tag{36}$$

V. DISCUSSION

We have analyzed the MRI in the Hall regime in a hot-electron plasma and derived the dispersion relation (21) allowing simultaneously for the effects of rotation, finite electron pressure, and electron pressure anisotropy.

We have pointed out that the finite electron pressure should be taken into account in the Hall regime for finite parameter β, $\beta \gtrsim 1$. For such β and $\omega > |k_z| v_T$, the MRI is weakened and can be suppressed, according to the condition (31). In the case $\omega < |k_z| v_T$, the MRI proves to be kinetic, being described by the dispersion relation (33). In the presence of electron pressure anisotropy, a hybrid of MRI and anisotropic instability described by the dispersion relation (34) appears. The anisotropy of the type $T_\perp > T_\parallel$ is destabilizing, while $T_\parallel > T_\perp$ is stabilizing. The condition of the hybrid instability is given by Eq. (35). It looks like a kinetic instability with the growth rate equal to Eq. (36).

The results of the present paper can be utilized by astrophysics dealing with the MRI in the Hall regime. In accordance with Ref. 7, it is possible that they can be useful for studying the mechanisms of the radio and X-ray source Sagittarius A*.

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