Magnetohydrodynamics of Laboratory and Astrophysical Plasmas

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Notes by J.P. Goedbloed and R. Keppens
based on *PRINCIPLES OF MAGNETOHYDRODYNAMICS*
by J.P. Goedbloed & S. Poedts (Cambridge University Press, 2004)

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   waves and instabilities of stationary plasmas, shocks
7. Toroidal Plasmas [future Volume 2]
   equilibrium and stability of tokamaks and accretion disks

Literature

Introductory plasma physics:

- F.C. Chen, *Introduction to Plasma Physics and Controlled Fusion* (1984).
- J.A. Bittencourt, *Fundamentals of Plasma Physics* (1986).
- R.J. Goldston and P.H. Rutherford, *Introduction to Plasma Physics* (1995).

Magnetohydrodynamics:

- J.P. Freidberg, *Ideal Magnetohydrodynamics* (1987).
- D. Biskamp, *Nonlinear Magnetohydrodynamics* (1993).
- J.P. Goedbloed and S. Poedts, *Principles of Magnetohydrodynamics* (2004).
  http://www.cambridge.org/uk/catalogue/catalogue.asp?isbn=0521626072
  www.rijnh.nl/users/goedbloed (ErrataPrMHD.pdf)

Plasma astrophysics:

- E.R. Priest, *Solar Magnetohydrodynamics* (1984).
- A.R. Choudhuri, *The Physics of Fluids and Plasmas, intro for Astrophysicists* (1998).
- R.M. Kulsrud, *Plasma Physics for Astrophysics* (2004).

Plasma physics on www

- **Fusion energy**

  www.fusie-energie.nl *(nuclear fusion and ITER, in Dutch)*

- **Solar physics**

  dot.astro.uu.nl *(Dutch Open Telescope)*

  www.spaceweathercenter.org *(space weather)*

- **Plasmas general**

  www.plasmas.org *(basics, applications of plasmas)*

- **These notes**

  mesonpi.cat.cbpf.br/cbpfindex *(downloadable pdf files)*
  or
  www.rijnh.nl/users/goedbloed *(downloadable pdf files)*

Chapter 1: Introduction

Overview

- **Motivation:** plasma occurs nearly everywhere, magnetized plasma unifying theme for laboratory and astrophysical plasma physics; [book: Sec. 1.1]

- **Thermonuclear fusion:** fusion reactions, conditions for fusion, magnetic confinement in tokamaks; [book: Sec. 1.2]

- **Astrophysical plasmas:** the standard view of nature, why it fails, examples of astrophysical plasmas; [book: Sec. 1.3]

- **Definition of plasma:** usual microscopic definition (collective interactions), macroscopic definition (the magnetic field enters). [book: Sec. 1.4]

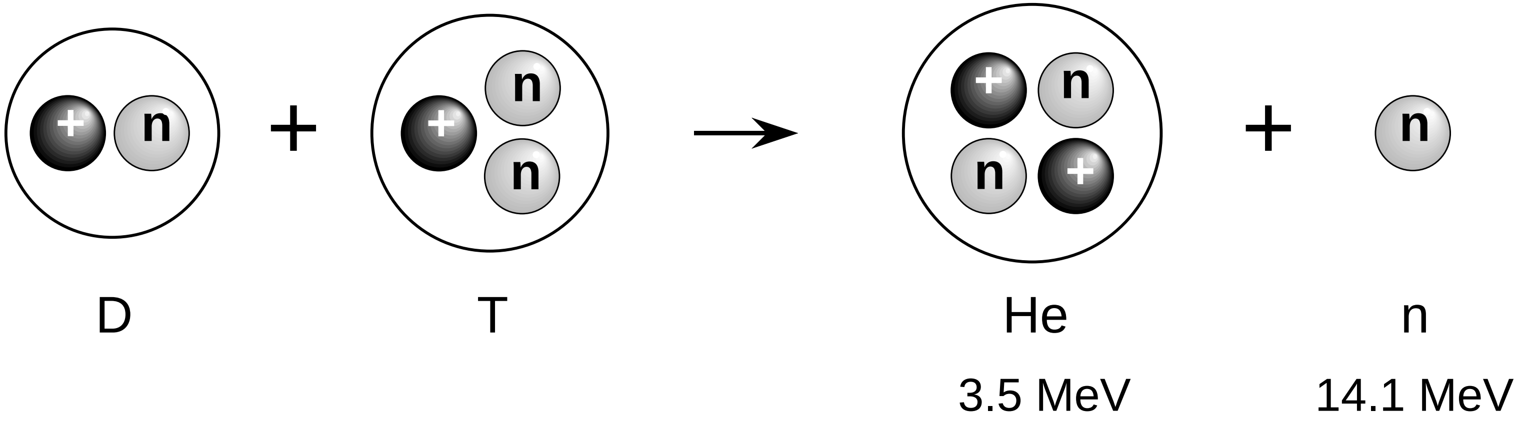
Plasma

- Most common (90%) state of matter in the universe.
- On earth exceptional, but obtained in laboratory thermonuclear fusion experiments at high temperatures ($T \sim 10^8\ \mathrm{K}$).
- Crude definition: *Plasma is a completely ionised gas, consisting of freely moving positively charged nuclei and negatively charged electrons.*

Applications

- Magnetic plasma confinement for (future) energy production by Controlled Thermonuclear Reactions.
- Dynamics of astrophysical plasmas (solar corona, planetary magnetospheres, pulsars, accretion disks, jets, etc.).
- Common ground: *Plasma interacting with a magnetic field.*

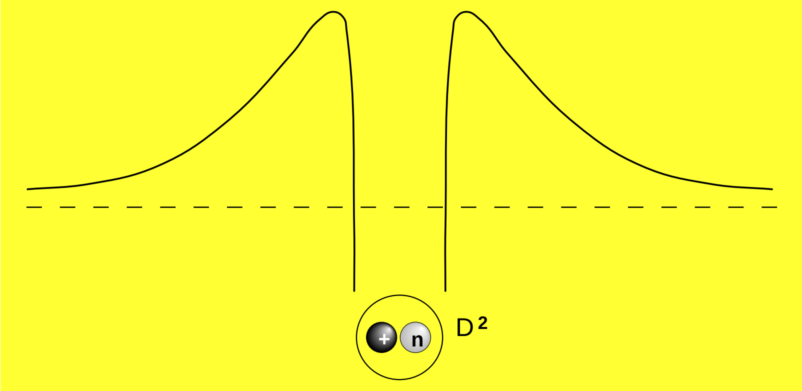
Reactions of hydrogen isotopes



Two products

- Charged α particles:
  capture in plasma magnetic field $\Rightarrow$ α particle heating
- Neutrons:
  capture in Li^6 blanket $\Rightarrow$ fusion energy $+$ T^3 breeding

Why plasma?



- To overcome electrostatic repulsion of nuclei need $10\,\mathrm{keV}$
  $\Rightarrow\ T \sim 10^8\,\mathrm{K}$ (ionisation at $14\,\mathrm{eV}$).

$\Rightarrow$ Plasma $\equiv$ completely ionised gas consisting of freely moving positively charged nuclei and negatively charged electrons.

How to confine?

- Magnetic fields:
  1. charged particles gyrate around field lines;
  2. fluid and magnetic field move together ("$\mathbf{B}$ frozen into the plasma");
  3. thermal conductivity: $\kappa_{\|} \gg \kappa_{\perp}$.

$\Rightarrow$ Need: Closed magnetic geometry.

Power balance

Power contributions ($\widetilde{T}$ in units of keV):

- thermonuclear output $P_\mathrm{T} = \frac{1}{4}\langle\sigma v\rangle n^2 E_\mathrm{T} \equiv n^2 f(\widetilde{T})\,, \quad E_\mathrm{T} \approx 22.4\,\mathrm{MeV}\,,$
- Bremsstrahlung losses $P_\mathrm{B} = \alpha n^2 \widetilde{T}^{1/2}\,, \quad \alpha \approx 3.8 \times 10^{-29}\,\mathrm{J}^{1/2}\,\mathrm{m}^3\,\mathrm{s}^{-1}\,,$
- heat transport losses $P_\mathrm{L} = 3n\widetilde{T}/\tau_\mathrm{E}\,.$

(a) Original idea (Lawson): three power contributions externally available for conversion into electricity and back again into plasma heating with efficiency $\eta \approx 0.33$,

$$P_\mathrm{B} + P_\mathrm{L} = \eta\,(P_\mathrm{T} + P_\mathrm{B} + P_\mathrm{L}) \tag{1}$$

⇒ **ignition condition:**

$$n\tau_\mathrm{E} = \frac{3\widetilde{T}}{(\eta/(1-\eta))\,f(\widetilde{T}) - \alpha\widetilde{T}^{1/2}}\,. \tag{2}$$

(b) Present approach (more restrictive): ignition when power losses are balanced by α-particle heating P_α,

$$P_\mathrm{B} + P_\mathrm{L} = P_\alpha = \frac{1}{4}\langle\sigma v\rangle n^2 E_\alpha \equiv n^2 f(\widetilde{T})\,, \qquad E_\alpha \approx 3.5\,\mathrm{MeV} \tag{3}$$

⇒ formally condition (2) still applies, but now with new f and $\eta \approx 0.135$.

Power balance (cont'd)

- Fusion power ⇔ radiation + transport losses:

  (a) Lawson criterion: lower curve,

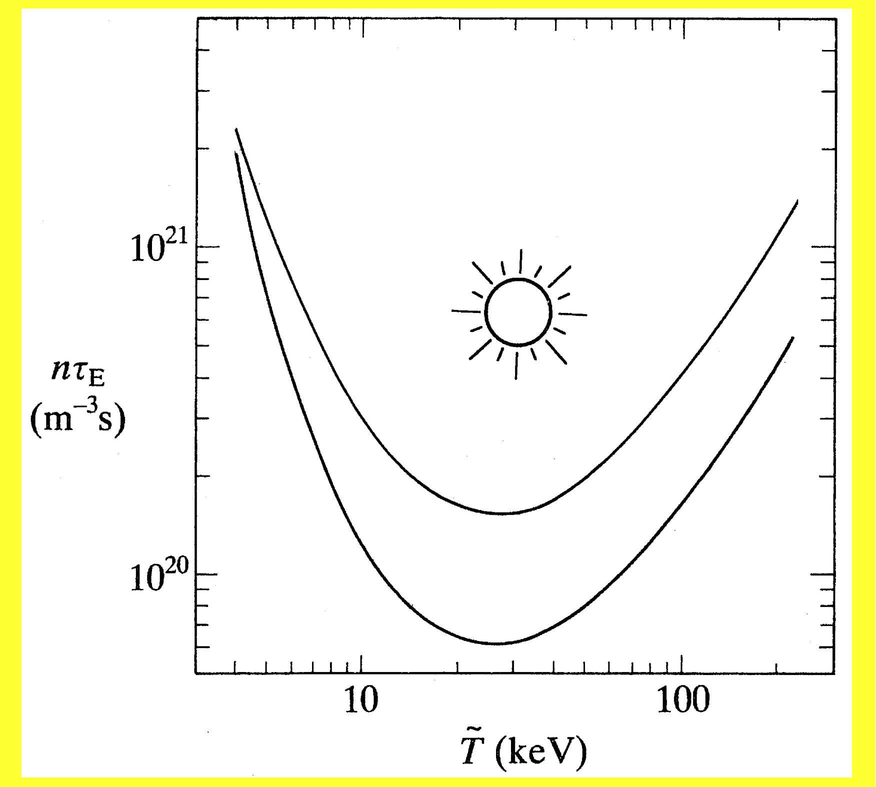
  (b) Modern approach: upper curve.

- Upper curve at minimum ($\widetilde{T} \sim 20\,\mathrm{keV}\,!$):

  $n\tau_E \sim 3 \times 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}\,;$

  typically:

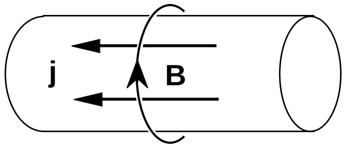
  $n \sim 10^{20}\,\mathrm{m}^{-3} \;\; \rightarrow \;\; \tau_E \sim 3\,\mathrm{s}\,!$



⇒ Magnetic fields provide the only way to confine matter of such high temperatures during such long times.

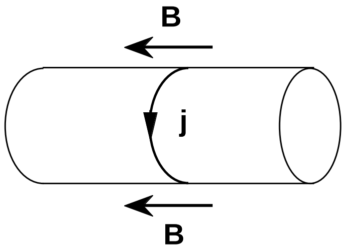
Interaction of currents and magnetic fields

- Schematic history of fusion experiments:



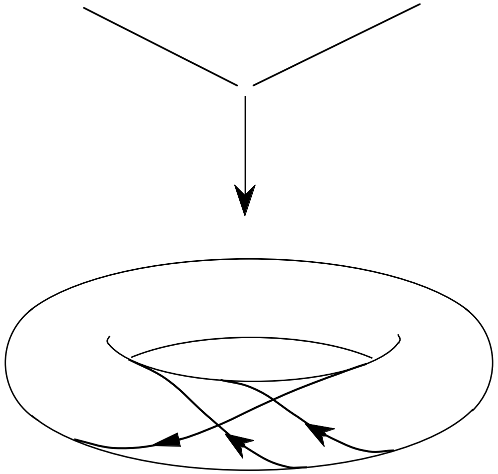
z - pinch:

very unstable
(remains so in a torus)



θ - pinch:

end-losses
(in torus: no equilibrium)

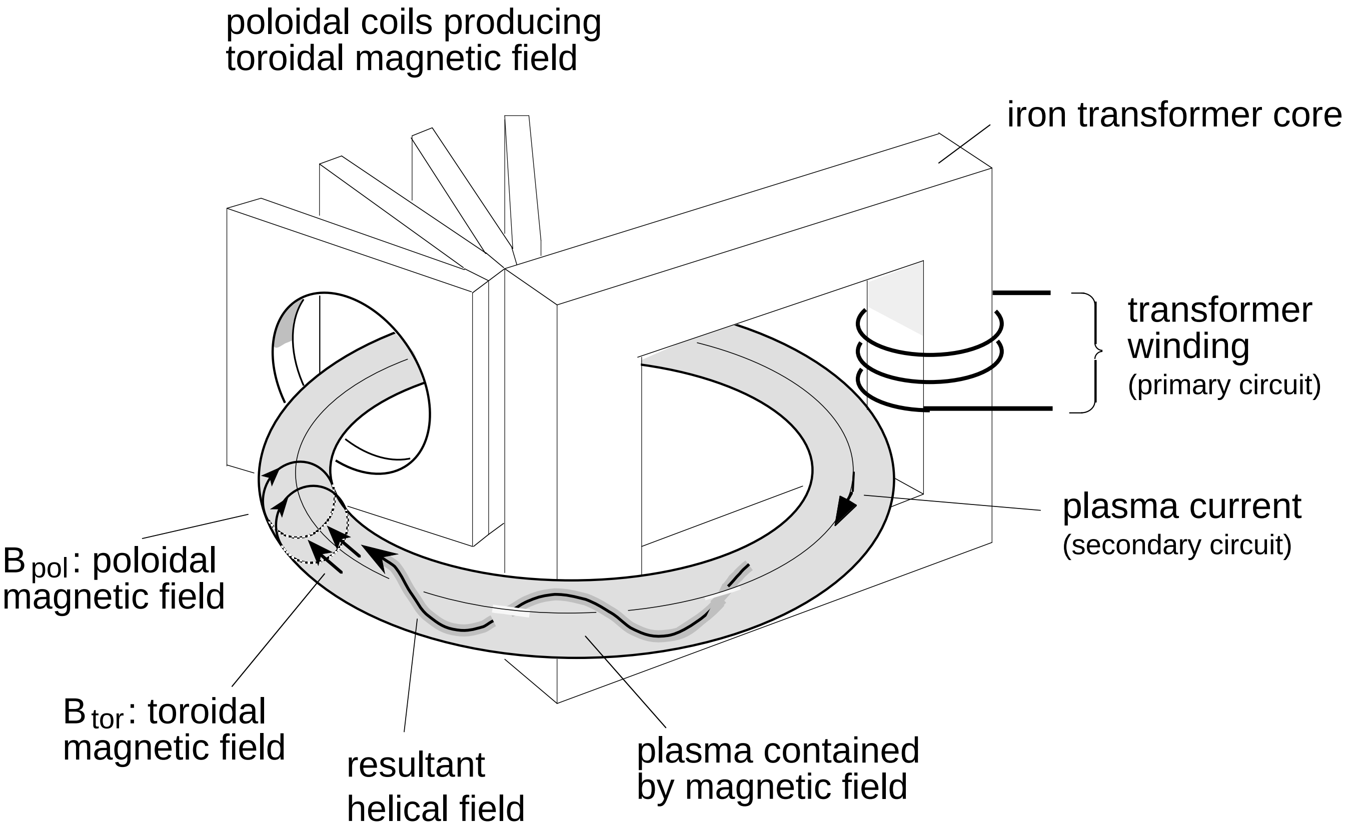


Tokamak:

delicate balance between equilibrium & stability

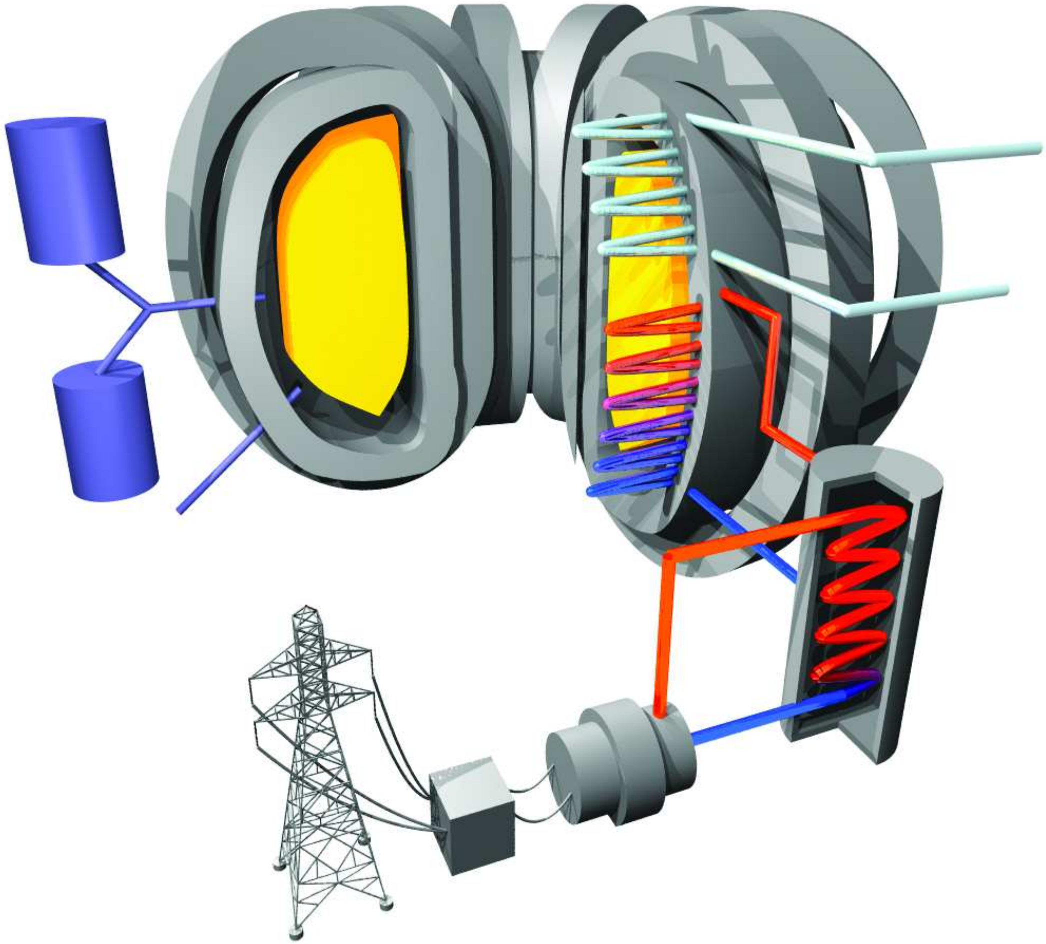
Tokamak

- Magnetic confinement:

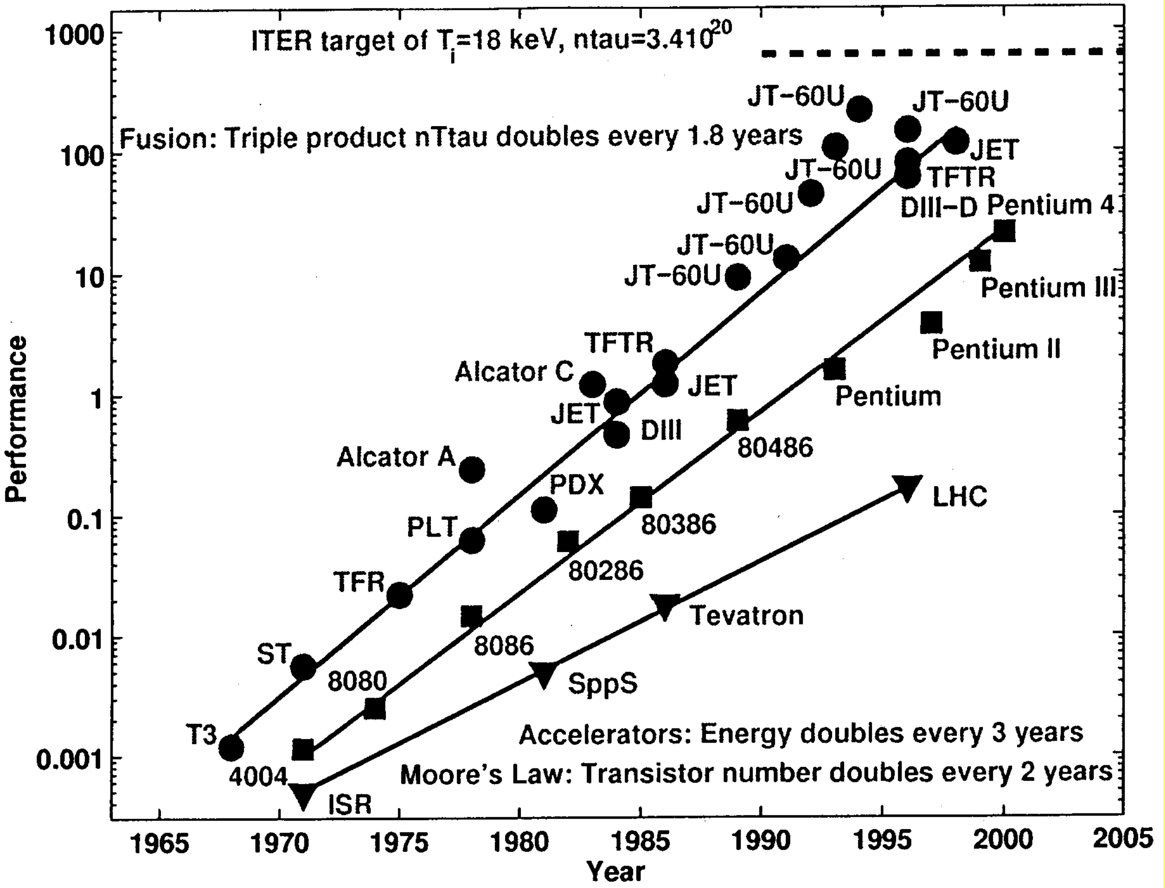


Tokamak (cont'd)

- Goal is electricity producing power plants:



Progress in fusion research



(from: CRPP Annual Report 2000)

- Progress made in controlled fusion over the years shows the same impressive advance as other fields recognized as world leaders.

The Standard View of Nature

Nuclear forces

⇓

quarks / leptons

nuclei (+) / electrons (−) 10^{-15} m

Electrostatic forces

⇓

atoms / molecules 10^{-9} m

(ordinary matter: electrically neutral)

⋮

Gravity

⇓

stars / solar system $10^{9}/10^{13}$ m

galaxies / clusters $10^{20}/10^{23}$ m

universe 10^{26} m

However, ...

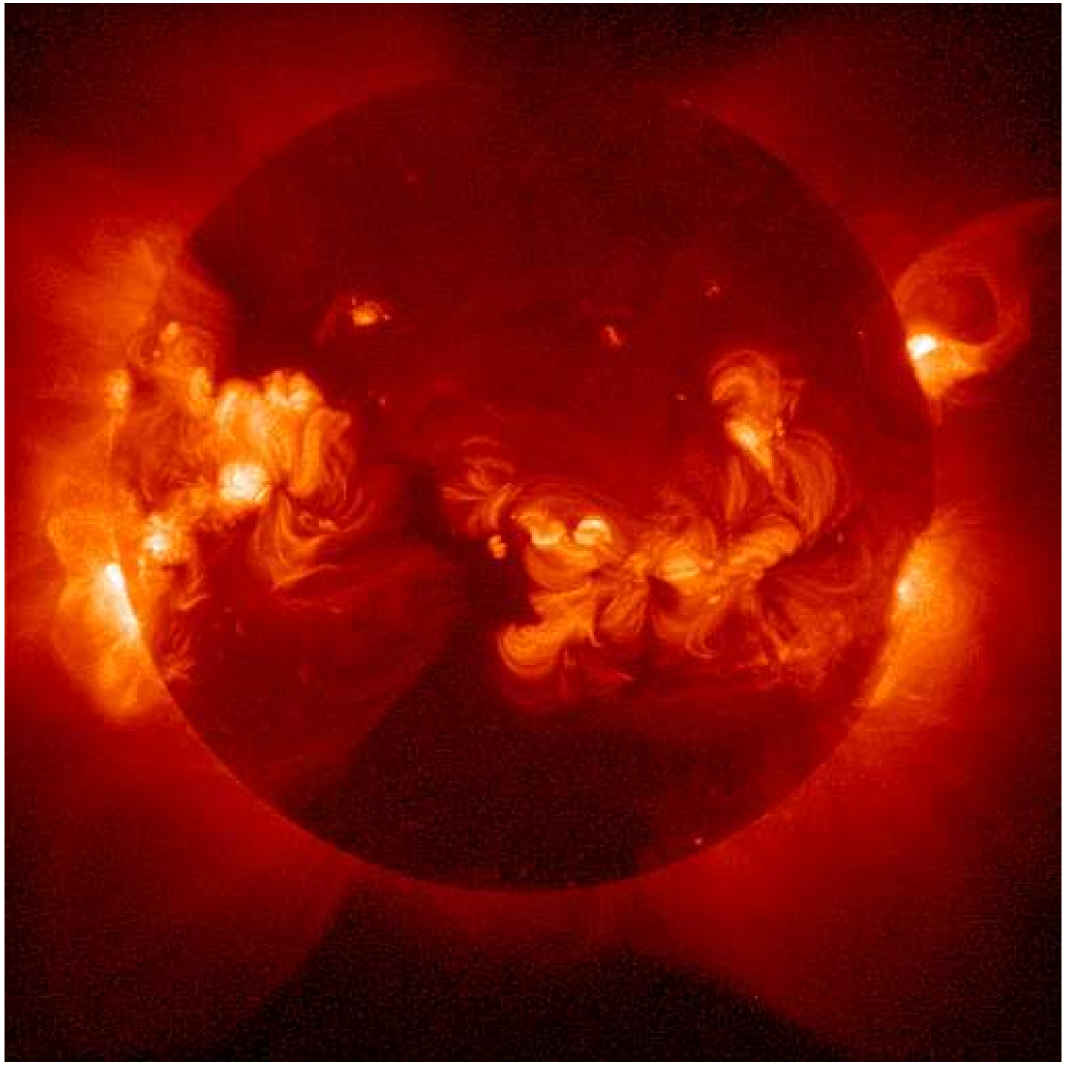
The universe does not consist of ordinary matter

- $> 90\%$ *is plasma:*
  electrically neutral, where the nuclei and electrons are not tied in atoms but
  freely move as fluids.
- The large scale result is *Magnetic fields*
  (example: interaction solar wind – magnetosphere).

Geometry

- Spherical symmetry of atomic physics and gravity (central forces) not present on the plasma scale:
  $\nabla \cdot \mathbf{B} = 0$ *is not compatible with spherical symmetry*
  (example: solar flares).

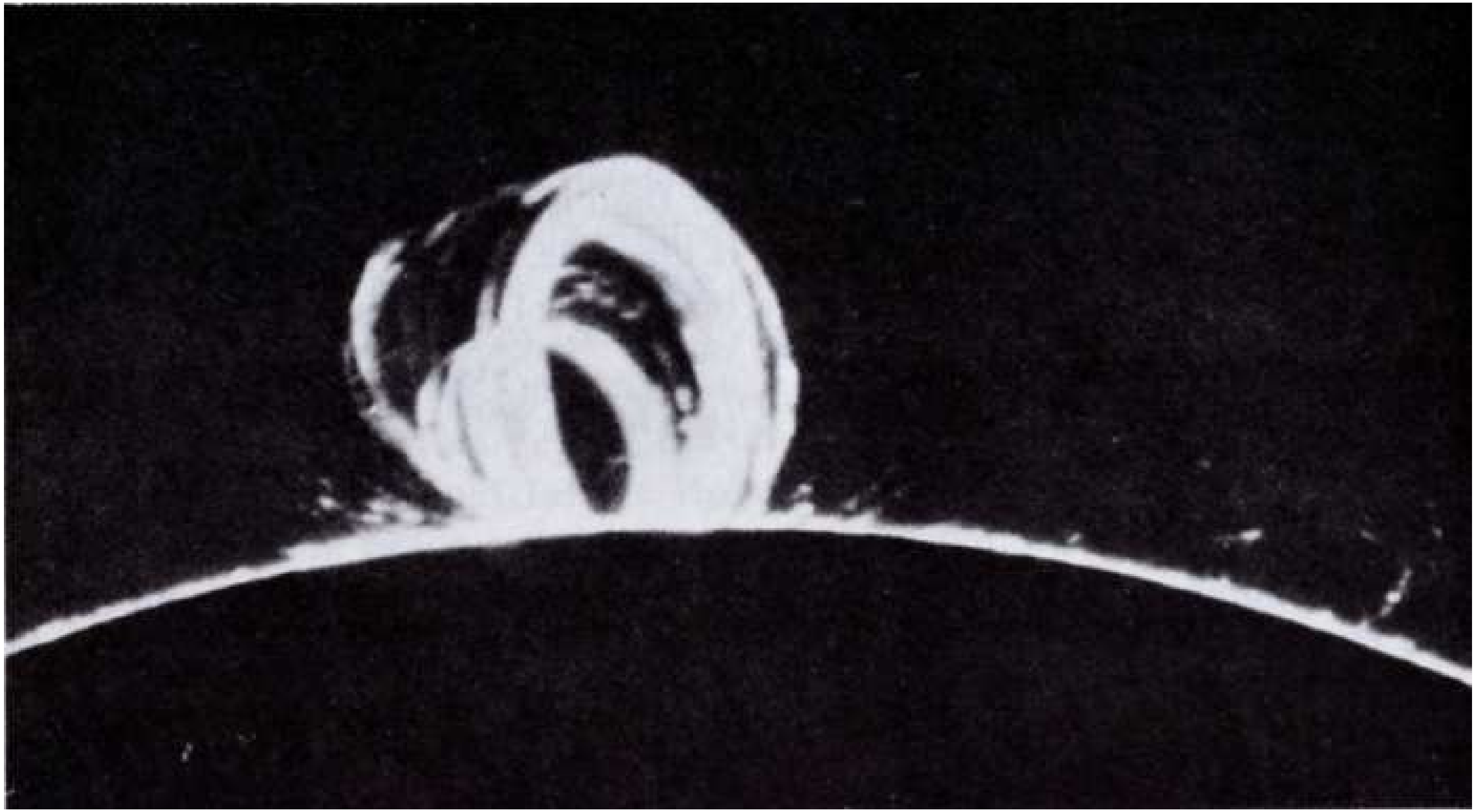
Example: The Sun



a magnetized plasma!

(sunatallwavelengths.mpeg)

Example: Coronal loops

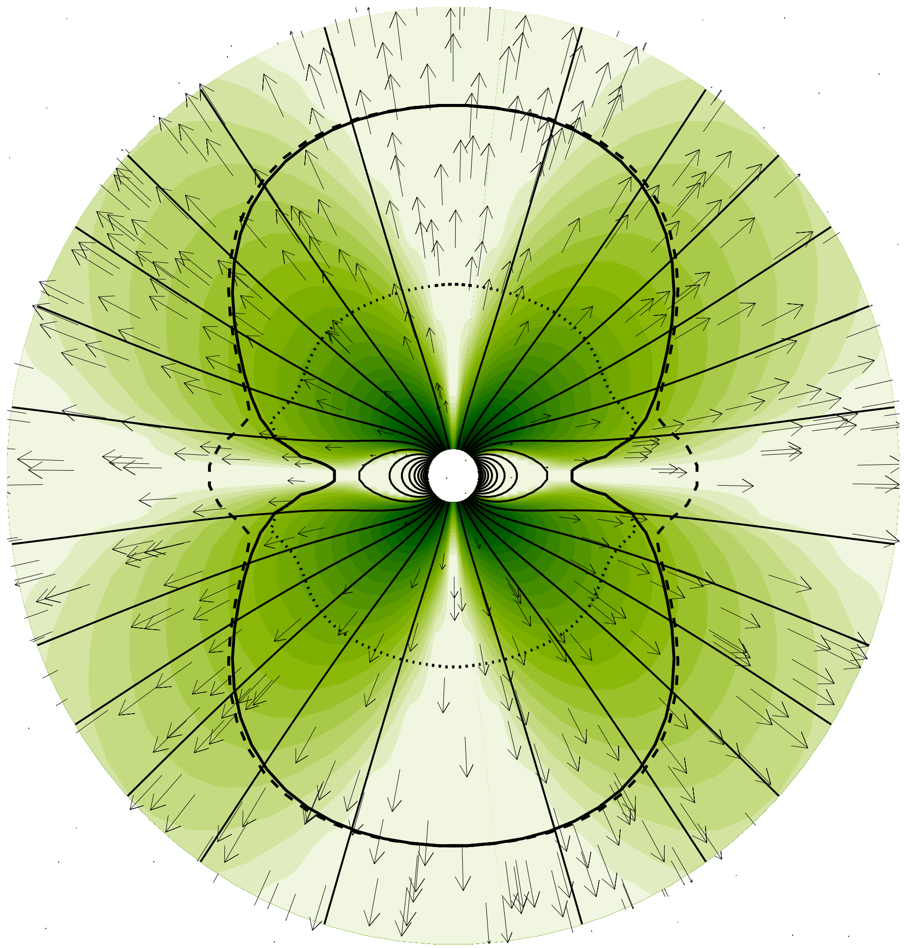


[from Priest, Solar Magnetohydrodynamics (1982)]

Example: Coronal loops (cont'd)

[from recent observations with TRACE spacecraft]

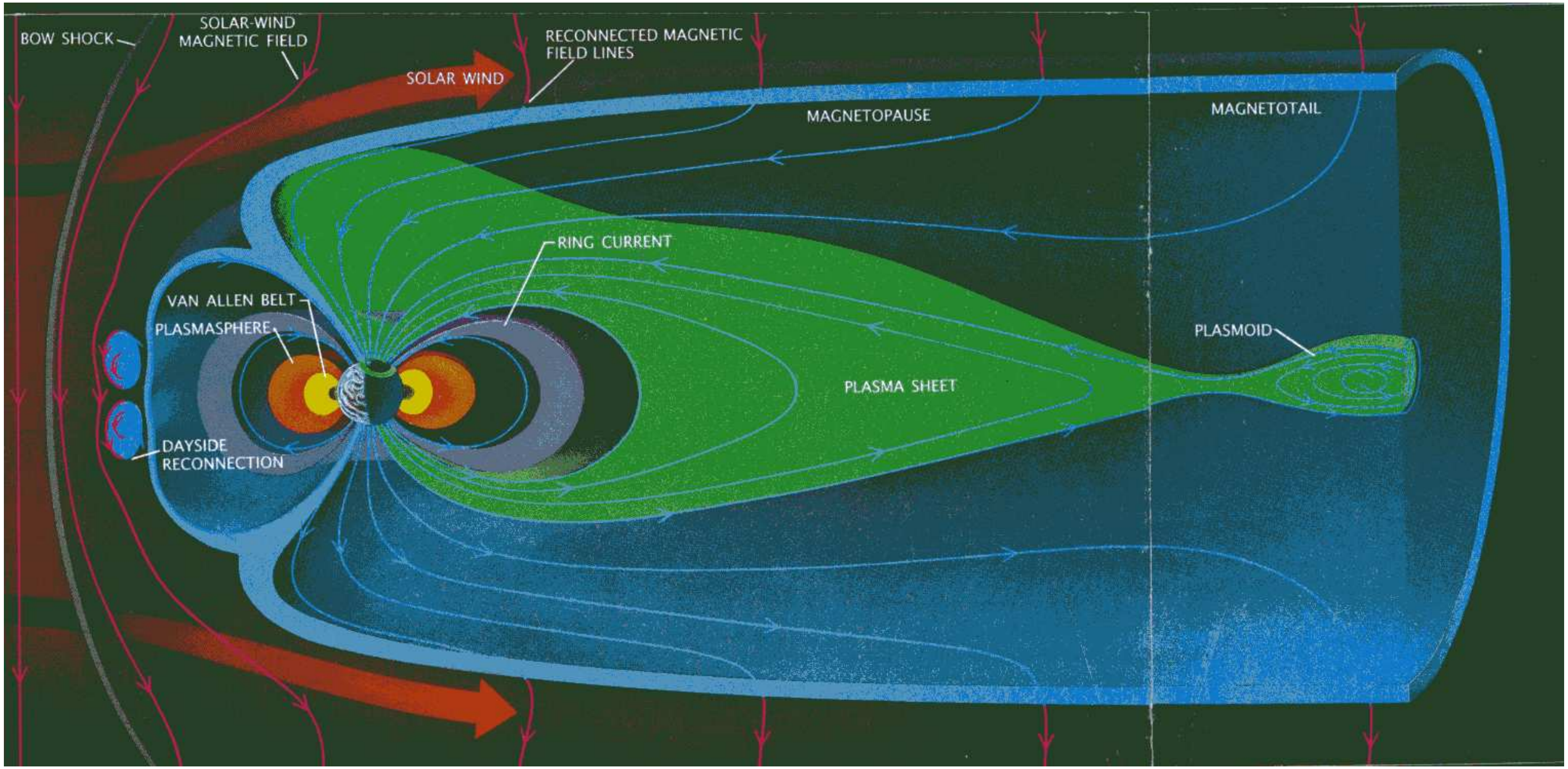
Example: Stellar wind outflow (simulation)



- Axisymmetric magnetized wind with a ‘wind’ and a ‘dead’ zone

  [Keppens & Goedbloed, Ap. J. **530**, 1036 (2000)]

Example: Magnetosphere

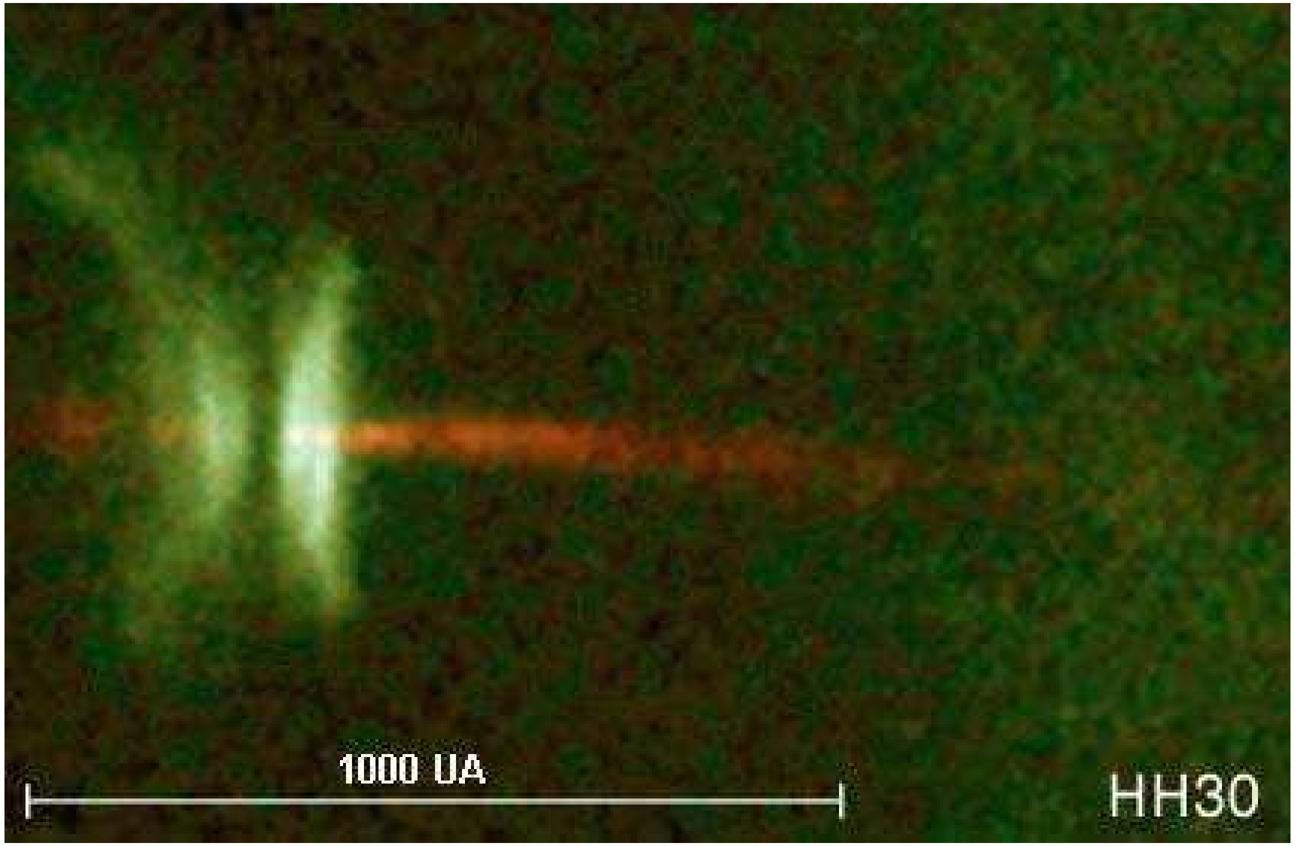


Example: Polar lights

Beauty of the polar lights (a1smallweb.mov)

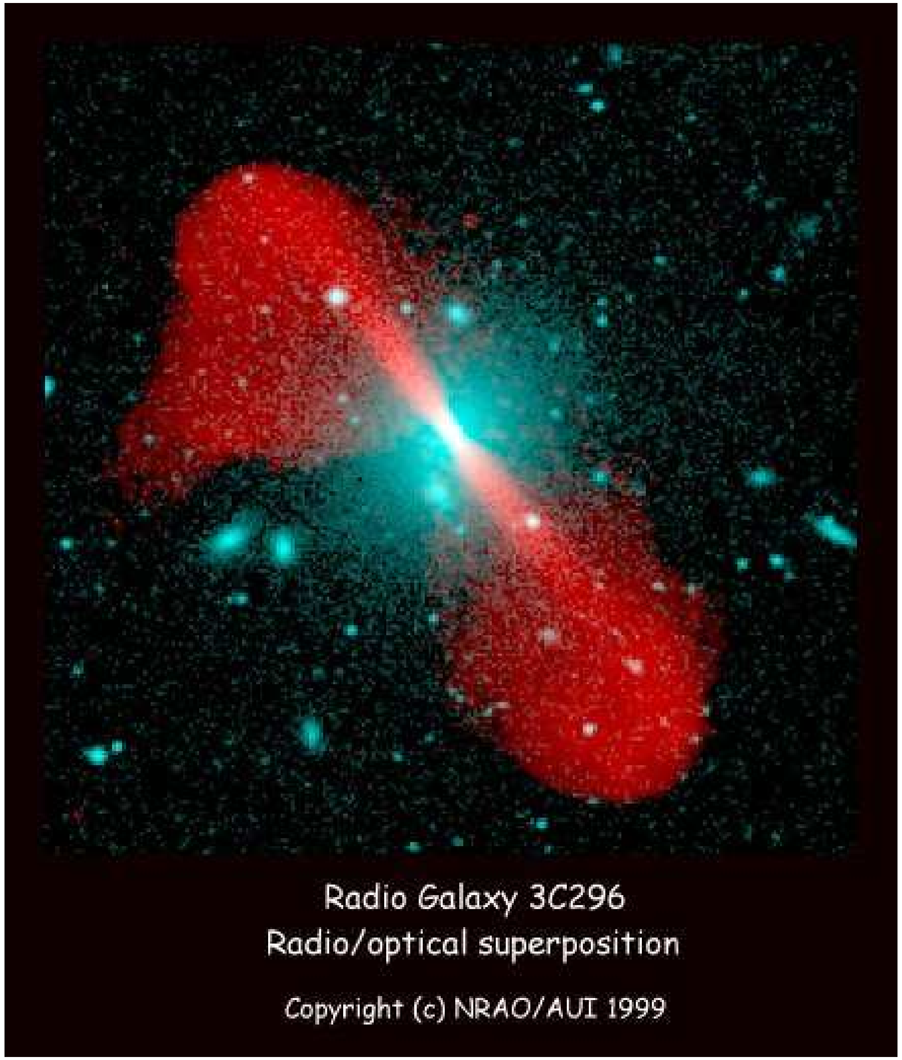
Solar wind powering auroral displays (fuvmovie.mpeg)

Example: Accretion disk and jets (YSO)



Young stellar object ($M_* \sim 1 M_\odot$): accretion disk ‘seen’ edge-on as dark strip, jets colored red.

Example: Accretion disk and jets (AGN)



Radio Galaxy 3C296
Radio/optical superposition

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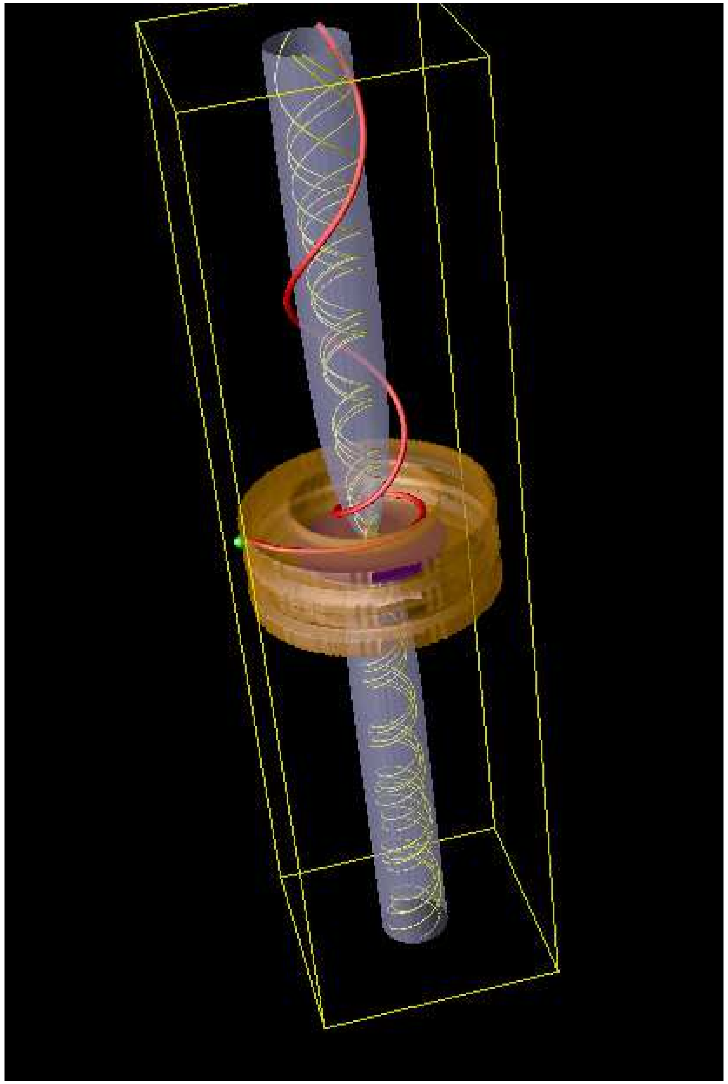
Active galactic nucleus ($M_* \sim 10^8 M_\odot$): optical emission (blue) centered on disk, radio emission (red) shows the jets.

Example: Accretion disk and jets (simulation)



Stationary end state from the simulation of a Magnetized Accretion Ejection Structure: disk density surfaces (brown), jet magnetic surface (grey), helical field lines (yellow), accretion-ejection particle trajectory (red).
[Casse & Keppens, Ap. J. **601**, 90 (2004)]

Crude definition:

Plasma is an ionized gas.

Rate of ionization: $$\frac{n_i}{n_n} = \left(\frac{2\pi m_e k}{h^2}\right)^{3/2} \frac{T^{3/2}}{n_i} e^{-U_i/kT}$$ (Saha equation)

– air: $T = 300\,\mathrm{K}$, $n_n = 3 \times 10^{25}\,\mathrm{m}^{-3}$, $U_i = 14.5\,\mathrm{eV}$ $\Rightarrow$ $n_i/n_n \approx 2 \times 10^{-122}$ (!)

– tokamak: $T = 10^8\,\mathrm{K}$, $n_i = 10^{20}\,\mathrm{m}^{-3}$, $U_i = 13.6\,\mathrm{eV}$ $\Rightarrow$ $n_i/n_n \approx 2.4 \times 10^{13}$

Microscopic definition:

Plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behaviour (Chen).

(a) Long-range collective interactions dominate over binary collisions with neutrals

(b) Length scales large enough that quasi-neutrality ($n_e \approx Z n_i$) holds

(c) Sufficiently many particles in a Debye sphere (statistics)

Collective behavior

Conditions:

(a) $\tau \ll \tau_n \equiv \frac{1}{n_n \sigma v_{\rm th}}$

tokamak: $\tau \ll 2.4 \times 10^6\,\mathrm{s}$

corona: $\tau \ll 2 \times 10^{20}\,\mathrm{s}\,;$

(b) $\lambda \gg \lambda_D \equiv \sqrt{\frac{\epsilon_0 k T}{e^2 n}}$

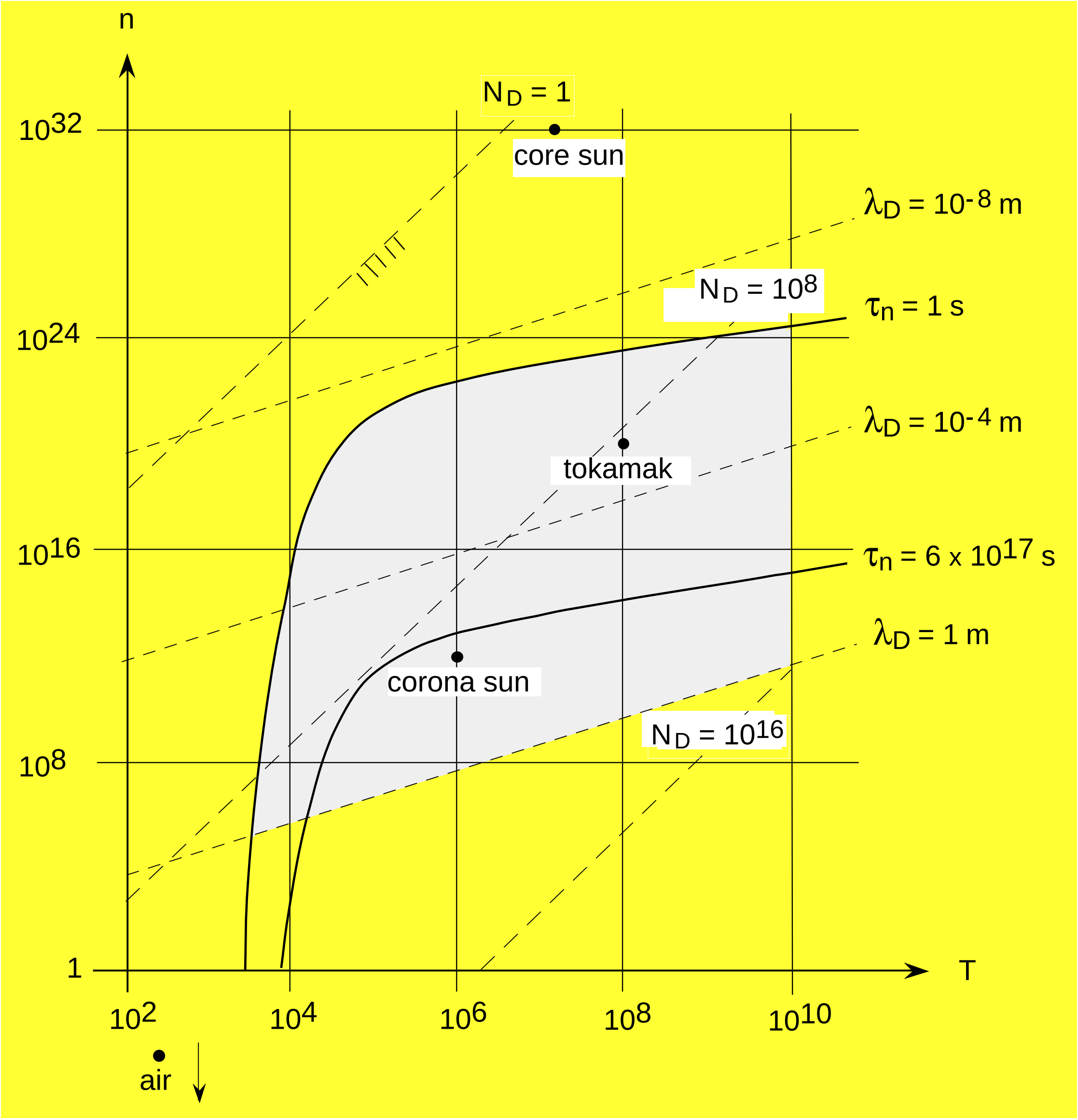
tokamak: $\lambda_D = 7 \times 10^{-5}\,\mathrm{m}$

corona: $\lambda_D = 0.07\,\mathrm{m}\,;$

(c) $N_D \equiv \frac{4}{3}\pi \lambda_D^3\, n \gg 1$

tokamak: $N_D = 1.4 \times 10^8$

corona: $N_D = 1.4 \times 10^9\,.$



So far, only the electric field appeared. (LOCAL)

Macroscopic definition:

For a valid macroscopic model of magnetized plasma dynamical configurations, size, duration, density, and magnetic field strength should be large enough to establish fluid behavior and to average out the microscopic phenomena (i.e. collective plasma oscillations and cyclotron motions of electrons and ions).

Now, the magnetic field enters: (GLOBAL !)

(a) $\tau \gg \Omega_i^{-1} \sim B^{-1}$ (time scale longer than inverse cyclotron frequency);

(b) $\lambda \gg R_i \sim B^{-1}$ (length scale larger than cyclotron radius).

$\Rightarrow$ MHD $\equiv$ magnetohydrodynamics