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Constrained information minority game: How was the night at El Farol?

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ABSTRACT

We introduce a variation of the El Farol Game in which the only players who surely know the outcome of the last turn of the game are those who actually attended the bar. Other players may receive this information with reduced probability. This information can be transmitted by another player who actually attended the bar in the last turn of the game or from the media. We show that since this game is not organized around the socially optimal point, arbitrage opportunities may arise. Therefore, we study how these opportunities can be exploited by an agent. An interesting application of this model is the market of goods being auctioned, such as cars being repossessed. The results obtained here seem to closely reflect the dynamics of this market in Brazil.

1. Introduction

In recent years, a promising area of research has been the dynamics of populations of agents and their collective behavior when competing for limited resources [1,2]. One specific example of this class of model is the so-called Minority Game (MG) introduced in Ref. [3] as a simplification of Arthur's El Farol Bar [4] attendance problem, which is one of the simplest complex systems that belong to this class. The Standard Minority Game (SMG) has been very well-studied—a review of these studies can be found in Refs. [2,5,6].

Since in real life agents are usually connected in social networks [7,8], several works [9–20] have considered a variation of the SMG where the networked agents who play the game access different bits of information from their neighbors. These attempts have successfully modeled the influence of local information on the dynamics of the minority game.

Another variation of the SMG has considered that the histories of the individual agents who play the game are different [21–23]. This approach has been used to analyze the effect of the amount of information available to agents on their performance.

In this paper we consider the scenario where the only players who surely know the outcome of the last round of the El Farol Game [4] are those who actually attended the bar. Other players may receive this information with reduced probability. In fact, they only access this information directly from players who actually attended the bar in the last turn of the game or from the media. As observed in real life, especially in the financial markets, outcomes of previous rounds of a game are likely disseminated after extremes, when people either celebrate good deals or complain about bad results. The media also

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emphasize these extreme good or bad performances, which increases the odds of people having access to the results. Since information about the outcome of the game is constrained to a part of the agents, we call this game "Constrained Information Minority Game" (CIMG).

In the CIMG, one can observe the two elements presented above, namely agents having access to different bits of information [9–20] and agents that make decisions based on different histories [21–23].

One interesting application of this game is as a model for the auctioning of goods, such as cars, which have been repossessed. According to the Brazilian law, personal belongings acquired with bank financing must be auctioned when people do not pay their loans.

There are some interesting characteristics of this market:

(1) Cars can be bought at auctions, but they can also be purchased at other markets.
(2) Cars auctions are conducted quite frequently.
(3) The results of a given auction are only precariously announced.
(4) The interested population only gets information about the outcome (price and quality of the objects to be auctioned) of previous auctions by means of reports from those who actually went to the auction. This information is usually shared only when people are complaining from the bad deals obtained at a previous auction or reporting good ones. The strength of this information is usually correlated to deviations in the outcome of that auction from the outcome of a typical auction.
(5) Auctions of repossessed cars closely resemble the minority game, specially to the El Farol attendance problem. If fewer people attend an auction, people who attend are likely to get better deals. On the other hand, if too many people attend the auction, the prices of the goods auctioned are high. Therefore, it is better to stay at home.

In order to consider the scenario of the auction of repossessed cars, we have modified the standard minority game to take the characteristics of this market into consideration.

This paper proceeds as follows. In Section 2 we precisely define the setup of the CIMG. In Section 3 we show the results. Finally, in Section 4 we present some final remarks.

2. Setup of the CIMG

The framework for this kind of game is quite similar to the SMG [3]. Agents compete with each other in an attempt to be on the side of the minority. In each turn t, each agent i based on his highest scored strategy chooses between two opposite actions $a = \pm 1$. A strategy in a given time is considered successful if it correctly predicts the minority side. The strategies with highest scores are those which were the most successful in the previous turns of the game. In this paper, $a = 1$ can be interpreted as the decision to go to the bar or to participate in the car auction and $a = -1$ can be interpreted as the opposite. Each specific strategy s_i defines a specific action $a^{\mu}_{s_i,i}$ for each state μ observed by the agent. Since M represents the history of the game, it is clear that the number of possible states is $P = 2^M$. Once all agents have defined their actions for round t, the sum of these actions defines the outcome of that round $A(t) = \sum_i a^{\mu(t)}_{s_{i(t)},i}$.

The rule used to update the scores of the strategies is presented below. The score $U_{s,t}$ for each strategy is initiated with 0. When there is a tie among possible strategies, the agent chooses randomly between them, using the same probability for each strategy. As usual in the SMG, strategies are updated based on the following function

$$U_{s,t}(t+1) = U_{s,t}(t) - a^{\mu(t)}_{s,i} A(t). \tag{1}$$

The action $a^{\mu}_{s_i,i}$ of each player i, linked to each strategy s_i for each state μ is initially chosen from the possible values of -1 and $+1$ with equal probability.

The difference between the CIMG and the SMG is that in the former, for each turn of the game there are two types of agents: Type 1 agents who attended the bar in the previous turn of the game and have access to the outcome of the game. Type 2 agents who did not attend the bar in the previous round of the game and access the outcome of the game with probability p, endogenously defined as a function of the attendance of the auction in the previous turn of the game.

While type 1 agents update their strategies as agents in the SMG, type 2 agents who did not receive information about the outcome of the game are not able to update their strategies nor to update their own history. Therefore, they have a different view from that of the ordinary global history.

2.1. The dynamics of the probability p

As mentioned above, some of the N agents do not know the outcome of the last turn of the game. If an agent plays $a(t) = 1$ in the last round of the game, he knows the outcome of the game with $p = 1$. Otherwise, he knows the outcome of the game with a smaller probability $p = \max\left\{\frac{|A(t)|}{N}, p_{\min}\right\}$. $p_{\min}$ refers to the minimal probability of a type 2 agent accessing the outcomes of the game. Fig. 1 presents p as a function of $A(t)$.

It is easy to understand this idea if one considers the problem of the El Farol Bar. Consider an agent who went to the bar in the last turn of the game. This agent knows with probability 1 how good the bar was that night. Those agents who did

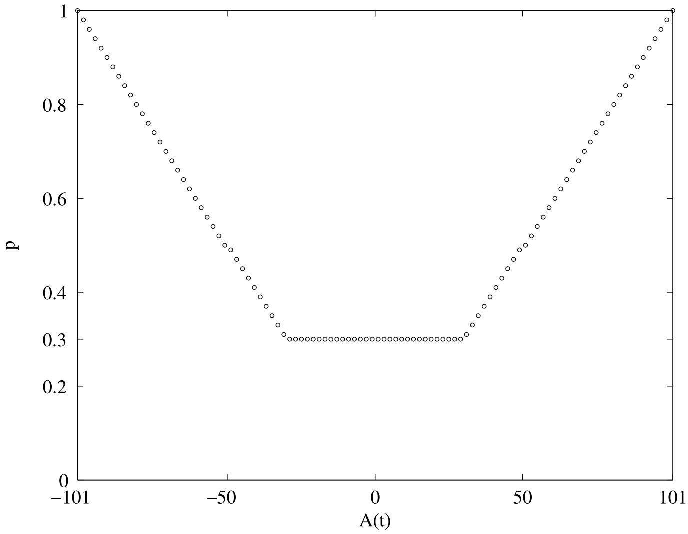

Fig. 1. Probability p of knowing the last round outcome for an agent who played $a = -1$ with $p_{\min} = 0.3$.

Table 1
Dynamics of the individual history.

Round of the game	t-8	t-7	t-6	t-5	t-4	t-3	t-2	t-1	t
Global information	0	0	1	1	1	1	0	1	0
Agent 1	S	S	N	I	N	I	S	S	S
Agent 2	I	I	N	N	N	N	I	N	S
Agent 3	N	N	I	S	N	I	N	I	N

not attend the bar may find out how the night was from other sources (radio, newspapers, friends, friends of friends etc). However, it is realistic to assume that these latter agents are less likely to know the excitement of the bar that night. Extreme results, such as a very pleasant night at the bar or a very bad night at the bar, are more likely to be disseminated to people that were not present. The same perspective happens regarding auctions. When people participate in a well-attended car auction, the outcome is widely communicated, since many participants complain about the high prices. On the other hand, when very few people participate, those who do are more likely to tell a lot of friends about their good deals. When auctions are particularly neither good nor bad, the information is not very well disseminated. As stated above, a truncated relation has been established between the absolute value of the results and the probability of access to the information about them.

Two consequences are direct effects of the proposed scenario. First, type 2 agents who did not access the results of the game, have no basis for updating their strategies scores. For these agents, $U_{s,t}(t+1) = U_{s,t}$, i.e., in this round, there is no updating. Second, due to the fact that some type 2 agents cannot update the score of their strategies, the agents of the CIMG start accumulating individual histories to base their decisions. This kind of phenomenon does not happen in the SMG where all the agents make decisions based on a global history.

2.2. The dynamics of the individual histories

In the CIMG, all agents are initiated with a random global history, just as in the SMG. However, in some round t, after the beginning of the game, the agents have their own view of reality, given by μ_i.

Table 1 explores the pieces of information available to three different agents in order to understand the dynamics of the individual histories in CIMG. In this table we consider the three possible states of an agent in the CIMG:

- State S (Standard)—The agent was present in the last round. This agent knows the outcome $A(t)$;
- State I (Informed)—The agent was not present in the last round, but got acquainted with the outcome $A(t)$;
- State N (Non-informed)—The agent was not present in the last round and did not get acquainted with the outcome $A(t)$.

Tables 1 and 2 assume that “0” is a “bad night at the bar”, “1” is a “nice night at the bar” and the game has history size $M = 3$. According to Table 1, since agent 1 has been present at the bar for the last three weeks, he knows the past outcomes. Therefore, he plays with a history equivalent to the history of an agent in the SMG. Based also on this table, while agent 2 received information about the situation at the bar at times when the outcomes were “0”, agent 3 received information about the bar at times when the outcomes were “1”. These individual histories are shown in Table 2. One may note that these three agents have different views of the same global information. The flow of information in the CIMG reflects the fact that people have different views of reality. While there are some people who are only acquainted with the good news, other

Table 2
Individual states.

Agents	t-2	t-1	t
Agent 1	0	1	0
Agent 2	0	0	0
Agent 3	1	1	1

people have only information about the bad ones. It is important to notice that the instant of time that the agent accessed the information is irrelevant. The past pieces of information are frozen as if they were generated in the last turns of the game.

3. Results

This section presents several simulations that can be used to understand the dynamics of the CIMG. First, the demand $A(t)$ that takes place in the CIMG is compared to demand $A(t)$ that arises in the SMG. We shall then consider variations in outcomes due to the proposed scenario. Finally, artificial agents and an additional strategy (constant action $a(t) = 1$) are included in an attempt to find an equilibrium for the CIMG.

3.1. Oscillation in demand

Fig. 2 shows the evolution of $A(t)$ for different values of $p_{\min}$. In particular, Fig. 2(a) and (b) clearly show the consequences of the lack of information in the CIMG. Since a sizable proportion of the agents play $a = -1$, many of them do not know the results of a round. These agents update neither their strategies nor their individual history. Therefore, they play $a = -1$ for a while. In the example of the bar or the auction of repossessed cars, it is a situation where the person stopped going and no longer received information about what was happening there. The chance of an agent remaining in this "frozen state" is inversely proportional to the absolute value of $A(t)$. This makes the negative peaks be rapidly corrected, since the agents who played $a = -1$ have a high probability of being informed and update the score of their strategies. Once negative peaks are corrected, this is followed by stagnation around a less negative value for $A(t)$. During these periods of stagnation, there are also many agents in a "frozen state" who always play $a = -1$ without updating strategies and history, since the flow of information is low. On the other hand, during these periods, there are also many agents playing $a = 1$ and being successful, since the state μ that they see does not change. The situation changes when some agents are informed about the situation and their strategies scores suggest that they should play $a = 1$. This makes $A(t)$ positive again and changes the state of the agents who were playing $a = 1$, leading to a change in their strategies choice and starting a new cycle.

The oscillation in demand for the games with a low $p_{\min}$ as shown in Fig. 2(a), (b) and (c) are quite similar. The only difference is that the cycles are reduced. This happens because, with an increase of $p_{\min}$, it is less likely that the agents get stuck in the "frozen state", since the possibility of getting acquainted with the outcomes increases.

Fig. 2(d) and (e), respectively, with $p_{\min} = 0.6$ and $p_{\min} = 0.8$ show that traces of cyclic behavior are still observed. However, the shapes of the curves are not as well defined as the ones in Fig. 2(a) and (b). Since there are always some agents in a frozen state, the average demand always tends to be less than 0. Naturally, since there are agents who stop going to the bar for some periods, the bar is more likely to be empty. Finally, with $p_{\min} = 1$, Fig. 2(f) recovers the demand function that arises in the SMG.

One interesting finding of this work is that empirically-observed cyclic patterns can be detected in the simulations. Fig. 2(a)–(e) show that the more limited the access to information is, clearer the cyclic behavior becomes.

Fig. 3 reinforces the fact that for values of $p_{\min} < 1$, the average $M_A = \frac{1}{5000}\sum_{t=1}^{5000} A(t)$ for $A(t)$ is negative. Therefore, the CIMG is not organized around the social optimal point such as in the El Farol Bar problem [24]. As $p_{\min}$ increases, however, more agents are able to organize themselves around the optimal point due to the spread of information.

This fact gives rise to the following issue. If the average gain is not null, arbitrage opportunities may arise. Furthermore, if the players could choose among more strategies or could possibly consider a larger history size, they might choose to play $a = 1$ more frequently, reducing the bias of the game. Several simulations were run. These simulations vary the number of strategies, history sizes and the minimal probability of accessing the outcomes of the games.[2] The non-null average of the demand was always observed and no asymptotically tendency to zero was found.

If the possibility of arbitrage remains for all variations tested, the problem of how to win remains. If we simply assume that most of the time people have a good time at the bar, what strategy should we adopt? Obviously, we should simply go to the bar. Therefore, the proportion of victories should be conditioned by the number of times that each agent plays $a = 1$. This becomes clear in Fig. 4. For small values of $p_{\min}$, we observe high correlation between the proportion of presences at the bar/auction and the proportion of victories. The correlation coefficients decrease according to the flow of information, until it reaches zero, when $p_{\min} = 1$. Therefore, for small values of $p_{\min}$, it is clear that the CIMG is naturally asymmetric in the sense that there is a positive correlation between success in the game and presence at the bar. Other asymmetric minority games may be found for instance in Refs. [11,25–27].

[2] With the exception of $p_{\min} = 1$ that would generate the standard minority games exactly.

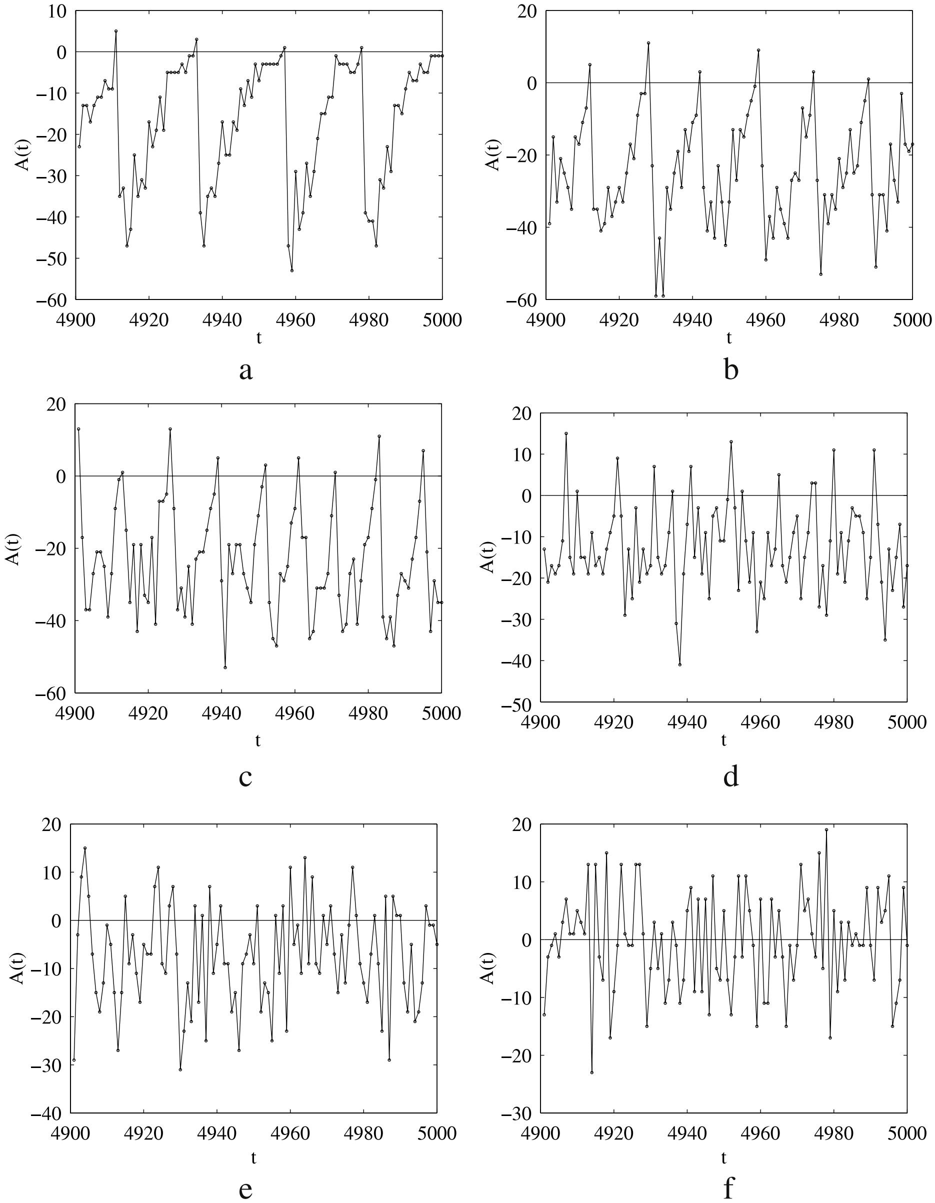

Fig. 2. (a) Oscillation in demand with $M = 8$ and $p_{\min} = 0$. (b) Oscillation in demand with $M = 8$ and $p_{\min} = 0.2$. (c) Oscillation in demand with $M = 8$ and $p_{\min} = 0.4$. (d) Oscillation in demand with $M = 8$ and $p_{\min} = 0.6$. (e) Oscillation in demand with $M = 8$ and $p_{\min} = 0.8$. (f) Oscillation in demand with $M = 8$ and $p_{\min} = 1$.

3.2. *Artificial agents*

Following the discussion of how to win in the CIMG, we introduce here some agents who constantly play $a = 1$. These artificial agents are included together with the standard agents of the CIMG in an attempt to exploit the arbitrage opportunities detected in previous games. In this context, it is interesting to mention that several others have considered games with different types of agents [22,28–32,21,33–41].

The exercise presented below evaluates the CIMG with a number n_1 of artificial agents who always play $a = 1$ for all rounds. The simulations were run with $p_{\min} = 0.4$ and with $n_1 = \{1, 5, 10, 15, 20, 25, 30, 35\}$. The number of usual CIMG agents can always be obtained by $101 - n_1$. From these simulations, the following statistics were extracted:

- proportion of victories of artificial agents (P_{a_1});
- proportion of victories of standard CIMG agents (P_a);
- average of $A(t)$, entitled M_A;
- log of the variance of $A(t)$, entitled SS_A;

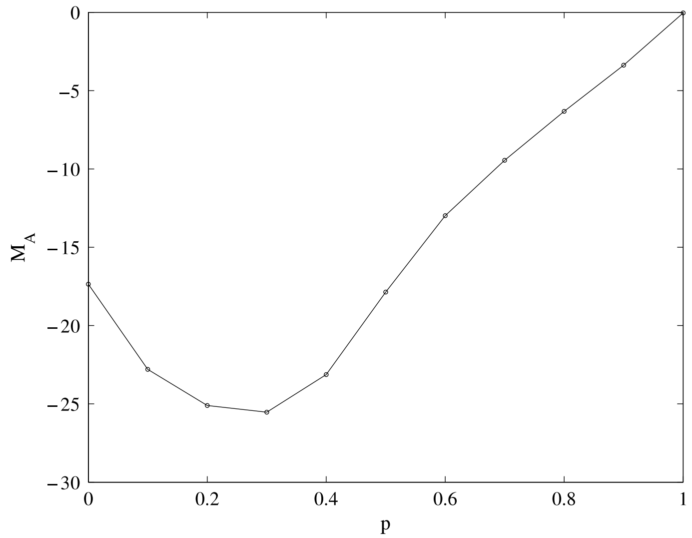

Fig. 3. Average M_A for different values of $p_{\min}$, with $M = 8$. For all simulations we observed 32 trials of 5000 rounds for each value of $p_{\min}$.

Table 3
Results: Constant action agents.

n_1	1	5	10	15	20	25	30	35
P_{a_1} (%)	91.90	91.23	90.43	86.76	82.45	72.60	38.62	10.56
P_a (%)	38.14	37.64	36.67	36.41	35.94	37.64	51.11	65.70
M_A	−21.92	−18.58	−15.14	−11.01	−8.10	−4.76	0.56	6.03
SS_A	0.62	0.42	0.17	−0.07	−0.30	−0.48	−1.24	−3.36

The first important conclusion to be drawn from this exercise involves arbitrage. When there is a single agent arbitrarily playing $a = 1$, he wins a very large percentage of times. However, when more agents adopt this strategy, the arbitrage opportunities gradually decrease, i.e., the bar gradually fills up. Naturally, if 51 agents take this same stance, they always lose because they are the majority. In fact, there is a qualitative change in this game at some point when the quantity of agents ranges from 25 to 30 agents. When 30 agents play $a = 1$, a winning average of above 50% is no longer observed. The opportunities for arbitrage have ended and the other adaptive players have prospered (Table 3).

An interesting observation is that, as the number of constant action players increase from 1 to 25, their performance gets worse, as expected. However, the performance of the regular adaptive agents is roughly the same. This means that the artificial players are not playing against the adaptive agents, but rather taking advantage of an opportunity which exists due to the inefficiency of the game. In some cases, the presence of artificial players make the game become more efficient than it would be in a game without these agents.

Another question concerning the car auctions has been answered. In an CIMG environment in which adaptive agents exist, one way to succeed is to always be present, as observed empirically. Indeed, the exercise has shown that if the same people are always present at an auction, good deals may not be so frequent.

3.3. Insertion of an extra constant action option for all agents

In this section, differently from Section 3.2, an extra strategy s_0 is made available to all the agents. The strategy s_0 plays $a = 1$ for every state μ. Since it is an additional strategy, agents can chose naturally between it and the regular strategies. In fact, this extra strategy is only used when it is the most scored strategy.

This approach is more realistic when intelligent agents are the players of the game. If one knows that the average demand of the bar is always negative, it is natural to consider a strategy that always plays $a = 1$, i.e., if one notes that the bar is frequently empty (and pleasant), it is natural that one of one's strategies is always to be present at the bar. Furthermore, when we introduce the extra strategy s_0 to all agents, we are forcing the evolution of the agents' behavior. If we did not have a clue to argue that this strategy would improve the behavior of the agents, we could proceed similarly to Refs. [42–45]. In these works, the evolution of the agents is endogenously driven by adaptive strategies.

Fig. 5 describes the oscillation in demand in a game with $p_{\min} = 0.4$, $S = 3$ (one constant strategy and two usual strategies) and $M = 8$. In contrast to what is seen in Fig. 2(c), we can no longer discern a cyclic behavior and the results $A(t)$ are much more centralized around the zero. The behavior of the oscillation of the demand looks roughly like the SMG demand. In order to reinforce these results, we have run five samples of 1000 turns each for each $M \in \{2, \ldots, 15\}$ and $p_{\min} = \{0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.95, 0.975, 0.99, 1\}$ (not shown). In none of them the average of $A(t)$ is less than -1 or

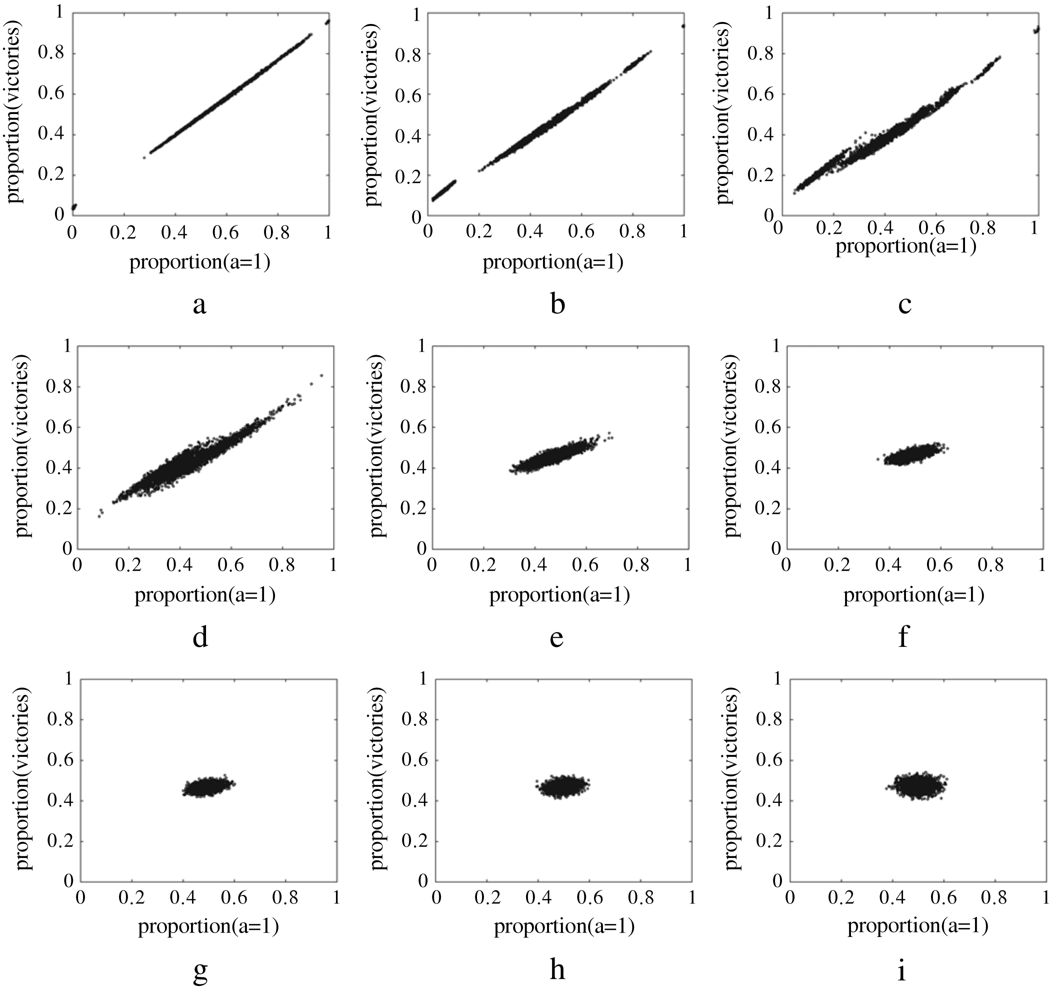

Fig. 4. Proportion of times that each agent plays $a = 1$ versus proportion of victories with $S = 2$, $M = 8$, $N = 101$ and (a) $p_{\min} = 0$, correlation coefficient (cc) $= 0.9998$ (b) $p_{\min} = 0.2$, cc $= 0.9982$ (c) $p_{\min} = 0.4$, cc $= 0.9908$ (d) $p_{\min} = 0.6$, cc $= 0.9575$ (e) $p_{\min} = 0.8$, cc $= 0.8792$ (f) $p_{\min} = 0.9$, cc $= 0.6310$ (g) $p_{\min} = 0.95$, cc $= 0.35171$ (h) $p_{\min} = 0.975$, cc $= 0.1803$ (i) $p_{\min} = 1$, cc $= -0.0005$. For all simulations we observed 32 trials of 5000 rounds for each value of $p_{\min}$. The simulations considered $N = 101$, $M = 8$ and $S = 2$.

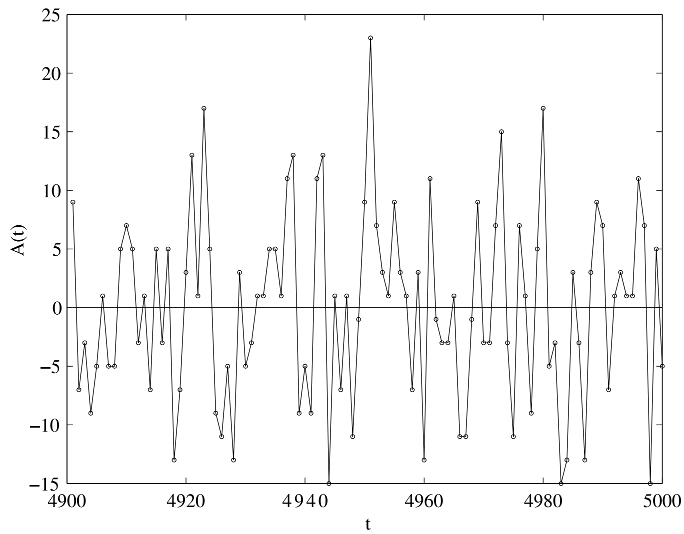

Fig. 5. Oscillation in demand with the use of additional strategy ($M = 8$ and $p_{\min} = 0.4$).

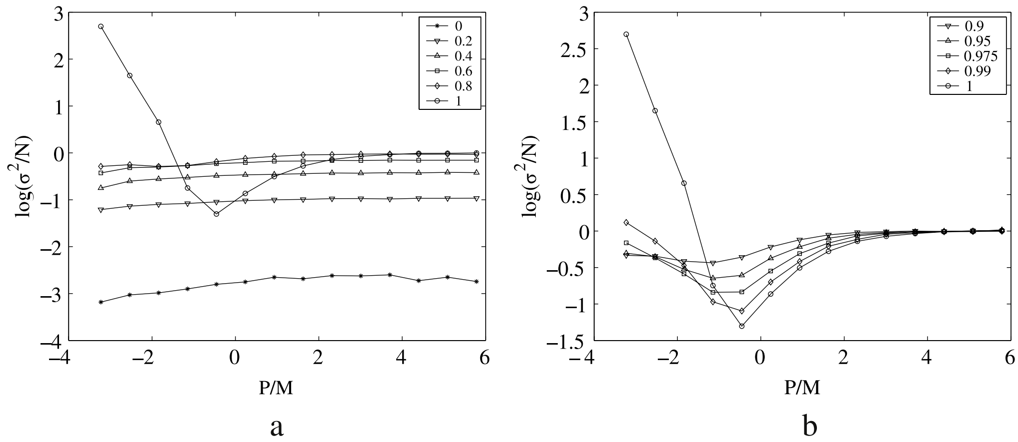

Fig. 6. Volatility of $A(t)$ for each value of M using various values of $p_{\min}$. In both figures, each point represents the average of 32 trials of 5000 rounds. The number of agents is 101 and $S = 3$ (one constant strategy and two usual strategies).

Table 4
Information × arbitrage.

$p_{\min}$	Strategy	Agents	Plays $a = 1$ (%)	Victories (%)	Avg. score
0	s_0	49.2	95.8	66.3	1078.7
	Others	51.8	6.1	32.7	−102.8
0.2	s_0	39.6	83.4	50.3	−263.3
	Others	61.4	28.3	45.9	−499.6
0.4	s_0	29.8	69.9	47.9	−937.2
	Others	71.2	41.7	46.4	−936.0
0.6	s_0	19.8	61.1	46.5	−1573.4
	Others	81.2	47.4	46.3	−1439.9
0.8	s_0	11.1	59.7	45.8	−2225.2
	Others	89.9	49.0	46.2	−1877.9
1	s_0	1.1	67.9	43.3	−4058.6
	Others	99.9	50.2	47.0	−1619.1

greater than 1. This means that when the agents choose to be arbitrarily present, equilibrium is returned to the game, which is no longer biased to zero.

Fig. 6(a) shows the plot efficiency versus memory for the CIMG games with the extra strategy s_0. It is interesting to note that the efficiency of the games with a value of $p_{\min}$ close to zero is greater than in the SMG, independent on the history size. Once the flow of information is increased, the game becomes less efficient. Fig. 6(b) shows that the well-known phase transition [46] emerges for values of $p_{\min}$ higher than 0.9 and becomes more visible as $p_{\min}$ approaches 1.

The results of the simulations for an individual history size of $M = 8$, with values of $p_{\min}$ varying in $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$, after 5000 rounds of the game, are shown in Table 4. For all agents, it is observed which of the strategies was the most highly scored at the end of the five-thousandth round. In the last column the overall accumulated score for each strategy can be observed. Again, the results represent the average of 32 trials of 5000 rounds each. As shown in this table, when access to information is more limited ($p_{\min} = 0$), almost half of the agents end up using the constant strategy as their favorite strategy.

As $p_{\min}$ increases and the access to information becomes easier, the number of times the agents use s_0 as their highest scored strategy decreases and the players who keep playing s_0 no longer have a higher rate of victories than do those who prefer other strategies. Furthermore, despite the possible advantage of s_0 after 5000 rounds, this advantage may not have happened. These agents have already adopted other strategies or they are simply using this strategy temporarily because it is the highest scored strategy at the moment. Therefore, despite being fundamental for the equilibrium of the game, to play s_0 is no longer advantageous when access to information is facilitated.

4. Final remarks

In this paper, we have introduced the so-called CIMG which is a version of the SMG where the only players who surely knows the outcome of the game in the last round are the players who actually attended the bar. Other players have a lower probability of access to this information. They only receive this information from other players who actually went to the bar in the last turn of the game or from the media.

We have shown that if $p_{\min} < 1$, this game is not organized around the socially optimal point, as in Ref. [24] and arbitrage opportunities may arise. However, if enough agents are arbitrarily present at the bar, the arbitrage opportunities disappear. As a general rule, we have found that when the level of information about the market is low, it is always beneficial to be at the bar. On the other hand, when the level of information increases, this strategy is no longer optimal.

Finally, one interesting application of this model is the auctioning of repossessed goods, such as cars. The results of this paper seem to reflect closely the dynamics of this market in Brazil.

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