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BLACK HOLE BREMSSTRAHLUNG: CAN IT BE AN EFFICIENT SOURCE OF GRAVITATIONAL WAVES?*

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We propose a new source of gravitational waves in the context of full nonlinear General Relativity: the *black hole bremsstrahlung*. A black hole initially in uniform motion is decelerated along its direction of motion; as a consequence of the deceleration, a radiative transfer process is set up through which the black hole loses its kinetic energy along with part of its rest mass by the emission of gravitational waves. Depending on the initial velocity and the strength of the deceleration, this process can correspond to a high power output in the initial pulse of gravitational radiation emitted.

Keywords: Gravitational waves; bremsstrahlung; black holes.

Nowadays, we are witnessing an enormous effort toward a direct detection of gravitational waves, which were predicted by the General Theory of Relativity. Although a strong indirect evidence on the existence of gravitational waves has been established from the measures of the Hulse–Taylor binary neutron star system PSR

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1913+16,[1] a direct detection would probably be the ultimate and definitive test of General Relativity. In addition, a new revolution in our perception of the universe through gravitational wave astronomy would be accomplished.

There are potentially good candidates for efficient production of gravitational radiation that are divided into the burst, periodic and stochastic sources. With respect to the burst sources, we mention the collapse of a star to form a black hole, the inspiral and coalescence of compact binaries (neutron stars and black holes), and the collision and merging of black holes, where the latter figures to be the most promising source of gravitational waves. However, despite the theoretical difficulties in modeling this source, it is uncertain to estimate the event rate as well as the mass spectrum of black holes themselves. Also, it is not difficult to imagine that such a catastrophic event seems to be not very common. In this way, as the subject of the present essay, we propose a new and interesting source of gravitational radiation, which we have named as *black hole bremsstrahlung* due to the analogy with the case of a charged particle or even with the usual gravitational bremsstrahlung of massive particles. The idea is quite simple. Suppose a black hole initially moving with constant velocity in a given direction relative to an asymptotic observer at rest. Owing to the interactions between the black hole and the environment, for instance a star, a cluster of stars and even other black holes, its velocity changes resulting in a net deceleration or acceleration. In both cases, it is expected by an analogy with a accelerated/decelerated charged particle that the black hole starts to emit gravitational radiation. Let us further assume that the black hole indeed decelerates. Then, according to General Relativity and contrary to the electromagnetic analogy, a fraction of the total mass of the system is radiated away along with its kinetic energy through gravitational waves. In this situation, if the black hole is initially moving with a relativistic velocity and the deceleration is such that it reduces considerably the initial velocity in a short interval of time, a formidable amount of energy would be carried out by gravitational waves.

In order to describe quantitatively the black hole bremsstrahlung, let us consider the Robinson–Trautman (RT) metrics that account for the simplest known solutions of vacuum Einstein's equations which may be interpreted as representing an isolated gravitational radiating system.[2,3] The line element is given as

$$ds^2 = \left(\lambda(u,\theta) - \frac{2m_0}{r} + 2r\frac{\dot{K}}{K}\right)du^2 + 2dudr - r^2K^2(u,\theta)(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1)$$

where u is a retarded time-type variable and a dot means u-derivative; m_0 is a constant to be interpreted as proportional to the mass of the initial stationary configuration. From Einstein's vacuum equations, it follows that $\lambda(u,\theta) = 1/K^2 - K_{\theta\theta}/K^3 + K_\theta^2/K^4 - (K_\theta/K^3)\cot\theta$, where the subscript θ stands for θ-derivative. Further, the evolution equation – denoted Robinson–Tautmann (RT) equation – is

$$-6m_0\frac{\dot{K}}{K} + \frac{(\lambda_\theta \sin\theta)_\theta}{2K^2\sin\theta} = 0. \qquad (2)$$

The above equation governs the dynamics of the gravitational field, which is totally contained in the metric function $K(u,\theta)$. In other words, Eq. (2) allows us to evolve initial data $K(u_0,\theta)$ from a given surface $u = u_0$. The Schwarzschild configuration characterized by $K(u,\theta) = K_0 = \text{const}$, $\lambda = 1/K_0^2$ and mass $M = m_0 K_0^3$ is the simplest solution of the RT equation. Another static solution for which $\lambda = 1$ is given by

$$K(\theta) = \frac{1}{\cosh\gamma + \cos\theta\sinh\gamma}, \tag{3}$$

that will be of crucial importance here. According to Bondi and Sachs[4,5] this solution can be interpreted as a *boosted black hole along the negative z-axis, with velocity parameter* $v = \tanh\gamma$. For this geometry, Bondi defines the mass function (mass aspect) as $m(\theta) = m_0 K^3(\theta)$, yielding for the mass of the gravitational configuration

$$M = \frac{1}{2} m_0 \int_0^\pi K^3(\theta)\sin\theta d\theta = m_0\cosh\gamma, \tag{4}$$

or $M = m_0/\sqrt{1-v^2}$, as should be expected for a moving particle with rest mass m_0 and velocity v. The parameter γ is associated with the rapidity parameter of a Lorentz boost given by the K-transformations of the BMS group of symmetries of asymptotically flat space–times (cf. Refs. 4 and 5). Finally, it is worth to remark that the interpretation of (3) as a boosted black hole is relative to the asymptotic Lorentz frame, which is the asymptotic rest frame of the black hole when $\gamma = 0$ (in other words, the asymptotic rest frame of the initial or final Schwarzschild static configuration).

Another appealing way of supporting this interpretation arises when thermodynamical considerations are taken into account. It is well established that a black hole can be understood as a thermodynamical system characterized by quantities such as temperature and entropy. The first is related to the surface gravity κ whereas the second, to the area of the event horizon of the black hole. Now if the solution (3) is in fact a boosted black hole, there must be relations involving the above quantities evaluated by an asymptotic observer, as was obtained for the total mass. We start with the area of the event horizon taking into account expression (3), where after a direct calculation we obtain $A_{\text{mov}} = A_{\text{rest}}$, for respectively the boosted and the black hole at rest. For the surface gravity, the following expression arises $\kappa_{\text{mov}} = \kappa_{\text{rest}}\cosh\gamma = \kappa_{\text{rest}}/\sqrt{1-v^2}$, rendering $T_{\text{mov}} = T_{\text{rest}}/\sqrt{1-v^2}$. Parenthetically, these expressions display exactly the special relativistic relations for the entropy and temperature.

The next step is to consider a nonlinear perturbation of the above solution as being the representation of the interaction between the boosted black hole with the environment. As a matter of fact, since the configuration is no longer stationary with its dynamics governed by the RT equation (2), it starts emitting gravitational waves until the equilibrium configuration is attained. At this point, it is important to mention that according to relevant analytical studies on the existence and asymptotic behavior of RT space–times given for Chrusiel and Singleton,[6] for sufficiently smooth initial data, RT space–times evolve asymptotically to the Schwarzschild black hole. Therefore, asymptotic observers would perceive that the perturbed boosted black hole decelerates until achieving the state of a Schwarzschild black hole in uniform motion, with smaller rest mass and smaller boost parameter, as a consequence of the interactions. A model for producing such a deceleration can be envisaged as a mechanism analogous to the one discussed in Refs. 7 and 8, where the black hole in relativistic motion would be decelerated as it passes in the neighborhood of a larger massive celestial object.

The invariant characterization of the presence of gravitational waves is done using the Peeling theorem.[9–11] At the radiation zone, which is sufficiently far from the source, the curvature tensor is dominated by the following term

$$R_{ABCD} \sim \frac{N_{ABCD}(u,\theta)}{r}, \tag{5}$$

as evaluated at large r and in a suitable Lorentz frame.[12] Considering the RT metrics, it can be shown that the numerator is proportional to $D(u,\theta) = \frac{1}{2}K^{-2}\partial_u(K_{\theta\theta}/K - K_\theta/K\cot\theta - 2(K_\theta/K)^2)$. The analogy between the curvature tensor and the electromagnetic tensor at the radiation zone allows one to interpret $D(u,\theta)$ as the amplitude of the emitted gravitational waves, and constitutes our crucial ingredient to quantify the wave pattern emitted. We now briefly illustrate two important aspects of the black hole bremsstrahlung. The first is the angular distribution pattern of the emitted gravitational waves evaluated in two distinct instants as shown in Fig. 1, for which the boosted black hole has initial relativistic velocity $v = 0.7$ ($\gamma = 0.88$). This clearly characterizes a gravitational bremsstrahlung process in the nonlinear regime of General Relativity, and remarkably agrees with the pattern obtained through post-Newtonian treatments[7,8] describing gravitational bremsstrahlung of point particles. The second aspect relates to the radiative transfer process that is set up when the black hole loses its kinetic energy and a fraction of its rest mass by the emission of gravitational waves. Our numerical calculations confirmed this fact, in other words, if the black hole is decelerated to rest its final mass M_∞ will be smaller than the initial rest mass m_0. In order to show more precisely the amount of extracted mass, we have depicted in Fig. 2 the dependence of the final mass M_∞ on the initial deceleration ΔV_0 imparted to the unperturbed black hole. This initial deceleration is evaluated as proportional to the square root of the change in the kinetic energy in the initial instants of the perturbation. We note that the larger the initial deceleration, the larger is the amount of rest mass extracted during the process. Finally, in our closing remarks it will be interesting to estimate the amount of energy released in the process of radiative transfer. For this, let us consider a black hole with $m_0 = 2M_\odot$ (two solar masses) and initial velocity $v \approx 0.38$. Adjusting a convenient perturbation such that a short pulse of D with width $\Delta u \sim 40$ is produced, it can be shown that about 90% of the amount of rest mass extracted in the process is extracted in the interval of the pulse of gravitational wave. Therefore, the rest mass extraction implies in a high power output

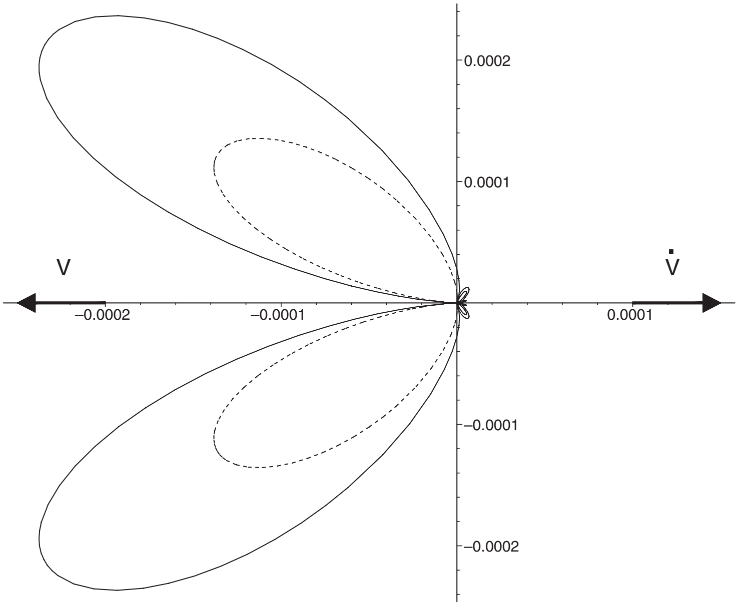

Fig. 1. Bremsstrahlung angular pattern: polar plot of the amplitude of the emitted gravitational waves, $|D(u,\theta)|$, for times $u = 90$ (solid line) and $u = 120$ (dashed line). The radiation is predominantly emitted in a sharp forward cone in the direction of motion at early times. The cone becomes sharper as the extreme relativistic limit $v \sim 1$ of the initial velocity is approached.

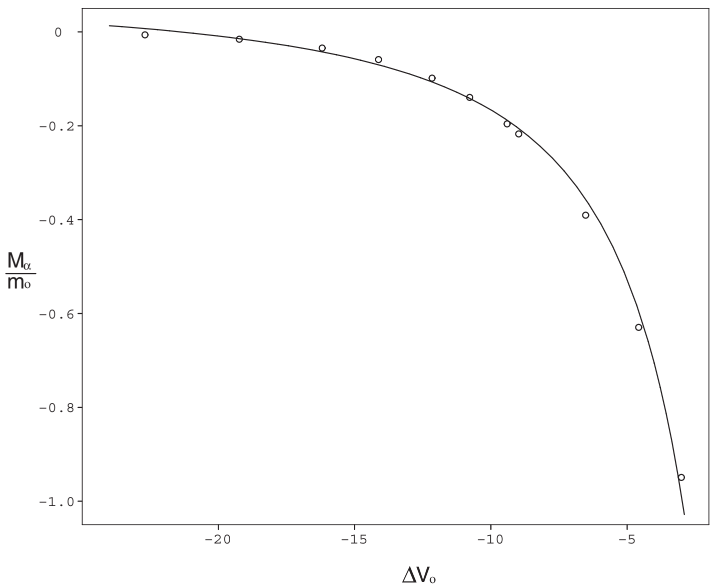

Fig. 2. Mass loss function due to bremstrahlung: log–log plot of the ratio of the final rest mass to the initial rest mass of the black hole versus the initial deceleration. The continuous curve is the best fit of the points by the law $\frac{M_\infty}{m_0} = (1 + \frac{7}{-\ln \Delta V_0})^{1.5}$. The cutoff corresponds to $v \to 1$.

mechanism. For the black hole under consideration the initial pulse has a power output of about 10^{40} GeV^2. This power output process could indeed constitute a very efficient mechanism for the generation of Gamma Ray Bursts.

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