# Dust-induced instability in a rotating plasma

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The effect of immobile dust on stability of a magnetized rotating plasma is analyzed. In the presence of dust, a term containing an electric field appears in the one-fluid equation of plasma motion. This electric field leads to an instability of the magnetized rotating plasma called the dust-induced rotational instability (DRI). The DRI is related to the charge imbalance between plasma ions and electrons introduced by the presence of charged dust. In contrast to the well-known magnetorotational instability requiring the decreasing radial profile of the plasma rotation frequency, the DRI can appear for an increasing rotation frequency profile. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2813188]

The investigation of linear and nonlinear collective phenomena in a dusty plasma is a currently wide tendency in plasma physics studies.[1–10] Dusty plasmas are also of long interest for space physics and astrophysics,[1,2,11] in particular, for the physics of accretion and protoplanetary disks.[12–16] According to the recent studies, collective phenomena are important for such disks in connection with plasma turbulence in them. As one possible candidate for generating such a turbulence, the magnetorotational instability (MRI) was considered.[17–19] In fact, the importance of the MRI has been demonstrated not only for astrophysical problems (see, e.g., Ref. 20), but also for various problems of applied physics (see bibliography in Ref. 21). Actually, recent plasmaphysical investigations of MRI (Refs. 22 and 23) have opened additional areas of application of this instability. Meanwhile, collective phenomena in a rotating dusty plasma have not yet been analyzed. The goal of the present brief communication is to make a first step in such an analysis. We consider here the linear problem of the instability in a rotating dusty plasma assuming the dust grains are heavy enough to be immobile.

We consider a plasma consisting of three components of charged particles: ions, electrons, and dust grains. The dust component is characterized by its number density $n_d$; each grain carries an electric charge $-eZ_d$, where $e$ is the ion charge and $Z_d$ is the number of excessive ($Z_d>0$) or deficient ($Z_d<0$) electrons on the grain. We note that standard mechanisms of dust charging by plasma currents lead to the net negative dust charge $Z_d>0$; some other mechanism such as photoionization leads to the net positive charge $Z_d<0$.[1–3] For simplicity, we assume that the dust charge is fixed; furthermore, all grains have the same size and charge, and the grain mass is assumed to be very large (so that the velocity of the dust component is negligible). This corresponds to the approximation of immobile dust.

In the presence of dust, the condition of plasma quasineutrality is written as

$$e(n_i - n_e) = eZ_d n_d, \tag{1}$$

where $n_i$ and $n_e$ are the ion and electron number densities, respectively. Thereby, in addition to the case of a standard dust-free plasma, we deal with the effect of inequality of the ion and electron number densities ($n_i \neq n_e$). This effect is the key one in our problem. Starting with the three-fluid plasma dynamical equations,[3,5,6,9,10] one can show that in the presence of immobile dust the equation of plasma motion is modified as follows:

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$$\rho \frac{d\mathbf{V}}{dt} = en_d Z_d \mathbf{E} - \frac{1}{4\pi}\left\{ \nabla \frac{\mathbf{B}^2}{2} - (\mathbf{B}\cdot\nabla)\mathbf{B} \right\} + \rho \mathbf{g}. \quad (2)$$

Here, $\mathbf{V}$ is the ion velocity component, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively, $\rho = Mn_i$ is the ion mass density, $M$ is the ion mass, $d/dt = \partial/\partial t + \mathbf{V}\cdot\nabla$, and $\mathbf{g}$ is the gravitation acceleration. For simplicity, we assume that the plasma is cold, so that the term with the plasma pressure gradient in Eq. (2) is neglected. This corresponds to the case of a small plasma kinetic pressure ($\sim n_i T$, where $T$ is the characteristic plasma temperature) as compared with the magnetic pressure ($\sim \mathbf{B}^2/8\pi$).

Thus, the consequence of the presence of immobile dust is the appearance of the electric field in the equation of plasma motion. Equation (2) is complemented by the flux freezing condition in the standard form

$$\frac{\partial \mathbf{B}}{\partial t} - [\nabla \times \mathbf{V} \times \mathbf{B}] = 0. \quad (3)$$

As in Ref. 21, we assume a cylindrically symmetric magnetic confinement equilibrium configuration. The plasma geometry is described in terms of the cylindrical coordinates $(R,\phi,z)$, where $R$, $\phi$, and $z$ are the radial, azimuthal, and longitudinal coordinates, respectively. The plasma is placed in an equilibrium magnetic field $\mathbf{B}_0$, directed along the cylinder axis, $\mathbf{B}_0 = (0,0,B_0)$, and in the presence of a radial gravitation field, $\mathbf{g} = (g,0,0)$. It rotates in the azimuthal direction, so that the equilibrium velocity of the ions and electrons $\mathbf{V}_0$ is given by $\mathbf{V}_0 = (0,V_0,0)$, where $V_0 = R\Omega$ and $\Omega$ is the radially dependent rotation frequency: $\Omega = \Omega(R)$. Taking the equilibrium velocities of plasma ions and electrons to be the same, we neglect the equilibrium electric current. As a result, the field $B_0$ proves to be uniform: $dB_0/dR = 0$.

Each perturbed quantity $\delta F(\mathbf{r},t)$ is represented in the form $\delta F(\mathbf{r},t) = \delta F(R)\exp(-i\omega t + ik_R R + ik_z z)$. Here, $\omega$ is the oscillation frequency and $k_R$ and $k_z$ are the radial and longitudinal projections of the wave vector, respectively. Radial dependence of the function $\delta F(R)$ is assumed to be negligibly weak, which corresponds to the local approximation.[19,21] Independence of the perturbed quantities on the azimuthal coordinate $\phi$ means that we restrict ourselves to an analysis of axisymmetric perturbations.

The perturbations are characterized by the perturbed magnetic ($\delta B_R$, $\delta B_\phi$), and electric ($\delta E_R$, $\delta E_\phi$) fields. In addition, we deal with the perturbed magnetic field $\delta B_z$, which is expressed in terms of $\delta B_R$ by $\delta B_z = -(k_R/k_z)\delta B_R$; this follows from the Maxwell equation $\nabla\cdot\mathbf{B} = 0$.

From Eq. (3), we find the standard relations between the perturbed magnetic field and ion velocities[21]

$$-\omega \delta B_R - ik_z B_0 \delta V_R = 0, \quad (4)$$

$$-i\omega \delta B_\phi - \frac{d\Omega}{d\ln R}\delta B_R - ik_z B_0 \delta V_\phi = 0. \quad (5)$$

From Ohm's law, $\mathbf{E} + [\mathbf{V}\times\mathbf{B}/c] = 0$, we find

$$\delta E_R = -(R\Omega\,\delta B_z + B_0 \delta V_\phi)/c, \quad (6)$$

$$\delta E_\phi = B_0 \delta V_R / c. \quad (7)$$

In turn, Eq. (2) yields

$$-i\omega \delta V_R - 2\Omega \delta V_\phi - \frac{iv_A^2}{B_0}\frac{k^2}{k_z}\delta B_R + \Omega_d\left(\delta V_\phi + R\Omega \frac{\delta B_z}{B_0}\right) - \frac{\delta\rho}{\rho_0}R\Omega\Omega_d = 0, \quad (8)$$

$$-i\omega \delta V_\phi + \frac{\kappa^2}{2\Omega}\delta V_R - \frac{iv_A^2 k_z}{B_0}\delta B_\phi + \Omega_d \delta V_R = 0, \quad (9)$$

$$-i\omega \delta V_z = R\Omega\Omega_d \delta B_R / B_0. \quad (10)$$

Here, $\kappa^2 = (2\Omega/R)d(R^2\Omega)/dR$, $v_A^2 = B_0^2/(4\pi\rho_0)$ is the squared Alfvén velocity, $k^2 = k_R^2 + k_z^2$, $\rho_0 = Mn_0$ is the equilibrium mass density of ions, and $n_0$ is the ion equilibrium number density. The value $\delta\rho$ is the perturbed mass density satisfying the continuity equation

$$\partial\delta\rho/\partial t + \rho_0 \nabla \cdot \delta\mathbf{V} = 0. \quad (11)$$

Furthermore, $\Omega_d$ is the dust-induced effective rotation frequency defined by[5]

$$\Omega_d = \omega_{Bi} Z_d n_d / n_0 \quad (12)$$

and $\omega_{Bi} = eB_0/(Mc)$ is the ion cyclotron frequency. Note that $\Omega_d$, related to the charge disbalance between plasma electrons and ions due to the presence of dust, is also the cutoff frequency for plasma Alfvén waves modified by dust.[3,5]

By eliminating the variables $\delta V_R$, $\delta V_\phi$, $\delta B_z$, $\delta V_z$, and $\delta\rho$ from Eqs. (4)–(11), we arrive at the following set of equations for $\delta B_R$ and $\delta B_\phi$:

$$\left(\omega^2 - k^2 v_A^2 - \frac{d\Omega^2}{d\ln R} + \Omega_d \frac{d\Omega}{d\ln R} - \frac{k_z^2 R^2}{\omega^2}\Omega^2\Omega_d^2\right)\delta B_R - i\omega(2\Omega - \Omega_d)\delta B_\phi = 0, \quad (13)$$

$$i\omega(2\Omega - \Omega_d)\delta B_R + (\omega^2 - k_z^2 v_A^2)\delta B_\phi = 0. \quad (14)$$

Equations (13) and (14) lead to the following dispersion equation:

$$(\omega^2 - k_z^2 v_A^2)\left(\omega^2 - k^2 v_A^2 - \frac{d\Omega^2}{d\ln R} + \Omega_d \frac{d\Omega}{d\ln R} - \frac{k_z^2 R^2}{\omega^2}\Omega^2\Omega_d^2\right) - \omega^2(2\Omega - \Omega_d)^2 = 0. \quad (15)$$

This equation can be called the rotating dusty plasma dispersion relation.

We see that the presence of immobile dust leads to two effects. First, it contributes into the magnetoacoustic part of the dispersion relation. This contribution is defined by the dust density and the plasma rotation frequency. Second, in addition to the plasma rotation (which mixes the magnetoacoustic and Alfvén waves), dust also takes part in the above wave mixing. The mixing effect of the dust depends only on the dust density in the approximation considered (immobile dust grains of the same size and the same fixed charge).

One of the most important effects of the presence of immobile dust is related to the appearance of the term with

$k_z^2R^2\Omega^2\Omega_d^2/\omega^2$ in Eq. (15). This term is a consequence of the modification of the parallel equation of motion (10) which, in turn, is a result of the appearance of the perturbed parallel electric field $\delta E_z = R\Omega\Omega_d\delta B_R/c$ in this equation. Thereby, the dust leads to a nonvanishing perturbed parallel plasma velocity $\delta V_z$. In accordance with the continuity equation (11), the velocity enters the expression for the perturbed density $\delta\rho$. On the other hand, the value $\delta\rho$ contributes into the perturbed radial motion equation (8) with the coefficient proportional to the product $\Omega\Omega_d$. This link yields the term considered with $\Omega^2\Omega_d^2$ in Eq. (15).

Let us consider $\Omega_d \ll \Omega$ for simplicity. Then Eq. (15) describes the standard MRI taking place if the condition

$$d\Omega^2/d\ln R + k^2v_A^2 < 0 \qquad (16)$$

is satisfied. Meanwhile, in addition to this instability, Eq. (15) predicts another instability described by the dispersion relation

$$\omega^2 = -\frac{k_z^2R^2\Omega^2\Omega_d^2}{k^2v_A^2 + d\Omega^2/d\ln R}. \qquad (17)$$

This instability can be called the dust-induced rotational instability (DRI). To drive the DRI, a condition opposite to that given by Eq. (16), namely,

$$d\Omega^2/d\ln R + k^2v_A^2 > 0 \qquad (18)$$

should be satisfied. Thus the domain of existence of DRI complements that of MRI. In accordance with Eq. (15), it is not obligatory for DRI that the rotation should be differential, i.e., this instability is possible even for $d\Omega/dr=0$. Note also that our analysis presupposes that, in accordance with Eq. (2), the rotation frequency is determined by $R(\Omega-\Omega_d) = g$. Using this relation, we conclude that DRI is possible both in the astrophysical conditions, when $g\neq 0$, and in the laboratory ones, when $g=0$.

We analyzed the effect of immobile dust on the stability of a magnetized rotating plasma and found a new instability called the rotational dust-induced instability. Physically, this instability is related to the charge imbalance between electrons and ions in a dusty plasma, which, being quasineutral as a whole, contains an additional charged dust component and behaves as a nonquasineutral plasma if it is treated as composed of ions and electrons only.[3,5,9,10]

As a result of such an effective "nonquasineutrality," the term with the electric field appears in the one-fluid equation of plasma motion [Eq. (2)]. This leads to a new effect dependent on the dust number density as well as the plasma rotation frequency [see Eqs. (13) and (14)]. This effect contributes into the magnetoacoustic part of the dispersion relation (15), and is the physical reason for the DRI.

It is shown that, in addition to the well-known MRI which is driven for $d\Omega^2/d\ln R<0$ and for sufficiently low wave numbers, the DRI can appear for the opposite condition $d\Omega^2/d\ln R>0$.

The described effect is obtained under assumptions that the dust grains are of the same size and (constant) charge, and their masses are high enough to ignore dust motions. The assumption of immobile dust breaks down when the frequencies of interest, in particular, $\Omega$ and $\Omega_d$ as well as the instability rate $\gamma$ exceed the larger of either the dust cyclotron frequency $\Omega_{Bd}=Z_de/M_dc$ or the dust plasma frequency $\Omega_{pd} = (4\pi n_dZ_d^2e^2/M_d)^{1/2}$. These inequalities, together with knowledge of the dust composition (dust material) define the corresponding size of grains.

In this study, we considered axisymmetric perturbations only. This is the most studied case for the MRI as well.[21] As recent study[24] shows, nonaxisymmetric perturbations can lead to a more effective MRI. Influence of nonaxisymmetric perturbations on the DRI is the subject for future research.

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