The transverse-momenta distributions in high-energy pp collisions — A statistical-mechanical approach

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We analyze LHC available data measuring the distribution probability of transverse momenta p_T in proton-proton collisions at $\sqrt{s} = 0.9\,\mathrm{TeV}$ (CMS, ALICE, ATLAS) and $\sqrt{s} = 7\,\mathrm{TeV}$ (CMS, ATLAS). A remarkably good fitting can be obtained, along fourteen decades in magnitude, by phenomenologically using q-statistics for a *single* particle of a two-dimensional relativistic ideal gas. The parameters that have been obtained by assuming $\mathrm{d}N/p_T\mathrm{d}p_T\mathrm{d}y \propto e_q^{-E_T/T}$ at mid-rapidity are, in all cases, $q \simeq 1.1$ and $T \simeq 0.13\,\mathrm{GeV}$ (which satisfactorily compares with the pion mass). This fact suggests the approximate validity of a "no-hair" statistical-mechanical description of the hard-scattering hadron-production process in which the detailed mechanisms of parton scattering, parton cascades, parton fragmentation, running coupling and other information can be subsumed under the stochastic dynamics in the lowest-order description. In addition to that basic structure, a finer analysis of the data suggests a small oscillatory structure on top of the leading q-exponential. The physical origin of such intriguing oscillatory behavior remains elusive, though it could be related to some sort of fractality or scale-invariance within the system.

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The transverse momentum distributions of hadrons produced in collisions involving protons and also heavy ions at RHIC and LHC energies [1] have recently been successfully investigated through nonextensive statistical mechanics [2]. A statistical-mechanical approach to analytically describe these quantities was successfully used phenomenologically by Hagedorn four decades ago [3]. By adopting the idea of some kind of thermal quasi-equilibrium, he introduced the Boltzmann-Gibbs (BG) factor within his theoretical model. The Hagedorn proposal works very well at low beam-energies. However, the experiments at higher energies, which have been achieved with the modern experiments at LHC and other colliders, have produced data that neatly depart more and more from a standard thermal behavior, giving rise to power-law distributions in the asymptotic high-p_T regime.

In order to explain the discrepancy between the BG distribution and experiments at high p_T, a number of works have been done through changing the BG thermostatistical start-point by its nonextensive generalization. Broadly speaking, this change corresponds to replacing the BG exponential weight by its nonextensive q-exponential deformation. Hagedorn himself made heuristically a somewhat similar approach. With the aim of extending his analysis to higher p_T values, power-law functions conforming to pQCD [4] were used instead of exponentials, without resorting to thermostatistical justifications [3, 5][1]. Theories proposed within the nonextensive thermostatistics scenario include, for example, the generalized version of the Hagedorn asymptotic bootstrap principle [7–9], reservoir fluctuations [10], micro-canonical jet-fragmentation [11], consequences of thermodynamic consistency [12], and a relativistic hard-scattering model in perturbative QCD [13, 14]. This approach gets also its impact in description of nuclear matter under extreme conditions (as those encountered in present heavy ion collisions), cf., for example, [15] and references therein.

[1] Actually, such approach was firstly proposed in the analysis of the uncorrelated jet model, see [6].

Recently, the experimental data reveal surprisingly that the transverse momentum spectrum from the very low energy regime of many tenths of a GeV to the very high energy regime of hundreds of GeV, in high-energy pp collisions at central rapidities, is characterized by a small number of degrees of freedom [13]. However, one expects on physical grounds at least three different physical mechanisms contributing to the hadron production process. In the low p_T region at central rapidities, the mechanism of non-perturbative *string-fragmentation* is expected to play an important role [16, 17]. In this mechanism, the hadrons are produced from the string stretched between the colliding nucleons with a plateau structure in rapidity. In the low p_T region near the beam and target rapidities, the mechanism of *direct-fragmentation* is expected to play a dominant role [18]. In this mechanism, the produced hadron fragments directly from the nucleon without making a collision with the partons of the other colliding nucleon. The transverse spectrum then depends on the probability of the hadron fragmenting out of the nucleon. In the high p_T region, the *relativistic hard-scattering process* becomes important. In this mechanism, a parton from the projectile nucleon scatters elastically with a parton from the target nucleon, and the parton subsequently cascades and fragments into the produced hadrons. The transverse spectrum therefore depends on many factors, including the parton distribution in the nucleons, the parton-parton hard-scattering amplitude, the cascade and fragmentation of the scattered parton to the hadrons, as well as the running of the coupling constant, as described in [14].

The three mechanisms mentioned above (string- and direct-fragmentations, and hard-scattering) are expected to give rise to different shapes of the transverse distributions as they depend on the transverse momentum in different ways. Indeed, the string fragmentation is associated with a flux tube and the transverse momentum distribution is limited by the flux tube dimension. The direct fragmentation is governed by the parton intrinsic p_T motion inside a nucleon and the gluon radiations, whereas the transverse momentum distribution in hard-scattering is governed by the law of parton-parton scattering, parton distribution, parton cascade, parton fragmentation, and the running of the coupling constant. Therefore, one would normally expect that the three different production mechanisms will lead to different behaviors as functions of p_T, and there would consistently be a breakdown of a single description at some point of the complete spectrum.

What interestingly emerges from the analysis of the data in high-energy pp collisions is that the good agreement of the present phenomenological fit extends to the *whole* p_T region (or at least for p_T greater than $0.5\,\mathrm{GeV}/c$, where reliable experimental data are available; in fact, the ALICE $0.9\,\mathrm{TeV}$ data reliably extend even to lower values, namely down to $p_T \simeq 0.1\,\mathrm{GeV}/c$) [13]. This being achieved with very few free parameters implies the simplicity of the underlying structure and the dominance of one of the three mechanisms over virtually the whole p_T range in the central rapidity region. It is reasonable to consider the dominant mechanism to be the hard-scattering process because the other two mechanisms are unlikely to produce hadrons with high p_T. Moreover, the dominance of hard-scattering also for the production of low-p_T hadron in the central rapidity region is supported by two-particle correlation data where the two-body correlations in minimum p_T-biased data reveal that a produced hadron is correlated with a "ridge" of particles along a wide range of $\Delta\eta$ on the azimuthally away side centering around $\Delta\phi \sim \pi$ [19–21]. The $\Delta\phi \sim \pi$ (back-to-back) correlation indicates that the correlated pair is related by a collision, and the $\Delta\eta$ correlation in the shape of a ridge indicates that the two particles are partons from the two nucleons and they carry fractions of the longitudinal momenta of their parents, leading to the ridge of $\Delta\eta$ at $\Delta\phi \sim \pi$.

While the basic hard-scattering process is relatively simple in the jet production level, there are many layers of stochastic processes in the production of hadrons from jets which mask this simplicity. The hard-scattering process exhibits the power-law behavior of the transverse momentum in its basic framework for jet production [13]. However, many other stochastic elements are involved in this process which definitively increase its complexity.

In spite of this fact, it turns out, remarkably enough, to be reasonable to assume, in the lowest-order description, a "no-hair" statistical-mechanical hypothesis for the transverse-momentum distribution in the p_T region above $0.5\,\mathrm{GeV}/c$. The hadron production process appears to be well characterized by a single-particle statistical-mechanical description of QCD quanta in an relativistic $d = 2$ system. All other information about the produced hadron matter, such as the parton distribution, the hard-scattering mechanism of jet production, jet-parton cascade process followed by parton fragmentation, and the intermediary possible quark-gluon plasma, "disappear" behind the stochastic process.

Towards such a goal of description, we can postulate that for the whole range of p_T from about $0.5\,\mathrm{GeV}/c$ to very high values, the transverse distribution of the produced hadrons obeys the "no-hair" statistical-mechanical description with very few degrees of freedom of the hard-scattering process. Motivated by the good agreement of previous works, we use here the nonextensive thermostatistical background to examine the experimental high-energy pp data and we describe the distribution of hadronic transverse momenta at central rapidity $y \simeq 0$ with the following nonextensive ansatz[2]

$$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}\boldsymbol{p}_T} = \frac{1}{2\pi p_T}\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}p_T} = A e_q^{-E_T/T}\,, \qquad (1)$$

where the q-exponential function e_q^z is defined by

$$e_q^z \equiv \left[1 + (1-q)\,z\right]^{1/(1-q)} \quad (e_1^z = e^z). \qquad (2)$$

Here, E_T is the relativistic energy associated with the transverse momentum of a single particle of the beam, namely, $E_T = \sqrt{\boldsymbol{p}_T^2 + m^2}$. As most of the produced particles are pions, m was taken equal to the meson mass m_π. Notice that, if the distributions are properly normalized, the constant prefactor A is *not* an independent parameter but a straightforward function of (q, T).

The experimental results for two energies (0.9 and $7\,\mathrm{TeV}$) and various detectors (CMS, ALICE, ATLAS) [13, 14] together with the proposed distribution (1) are indicated in Fig. 1. We verify that q increases slightly with the beam energy, but, for the present energies, remains always $q \simeq 1.1$. Some indications exist that, in the limit of extremely high energies, q approaches a limiting value (possibly close to 1.2 [9, 22]). The effective

[2] We are adopting unity for the Boltzmann constant k_B.

temperature is, in all cases, $T \simeq 0.13\,\mathrm{GeV}$, neatly compatible with the meson mass $m_\pi = 0.135\,\mathrm{GeV}/c^2$. The dashed line (an ordinary exponential of E_T) illustrates how important can be the discrepancy with the analogous Boltzmann-Gibbs approach as p_T increases.

What we may extract from the behavior of the experimental data is that the Hagedorn scenario appears to be essentially correct excepting for the fact that we are *not* facing thermal equilibrium but a different type of stationary state, typical of violation of ergodicity (for a discussion of the kinetic and effective temperatures see [23, 24]). These results reinforce the pioneering connection between quantum chromodynamics and nonextensive statistics first suggested by Walton and Rafelski [25]. The successful fitting of Eq. (1) along impressive 14 decades strongly points at the present "no-hair" assumption (namely a two-dimensional relativistic single-particle system) as the simplest one on which further sophisticated models can be built. We emphasize also that, *in all cases*, the temperature turns out to be one and the same, namely $T = 0.13\,\mathrm{GeV}$.

As a concluding remark, we note that the data/fit plot in the bottom part of Fig. 1 exhibits intriguing (rough) log-periodic oscillations, which suggest some hierarchical fine-structure in the quark-gluon system where hadrons are generated. This behavior is possibly an indication of some kind of (multi)fractality in the system. Indeed, the concept of *self-similarity*, one of the landmarks of fractal structures, has been used by Hagedorn in his very definition of fireball, as was previously pointed out by Beck [9] and has been found in the analysis of jets produced in *pp* collisions at LHC [26]. These small oscillations have already been preliminary discussed in [27, 28], where the authors were able to mathematically accommodate them by essentially allowing the index q in Eq. (1) to be a complex number[3] (see also Refs. [29, 30]).

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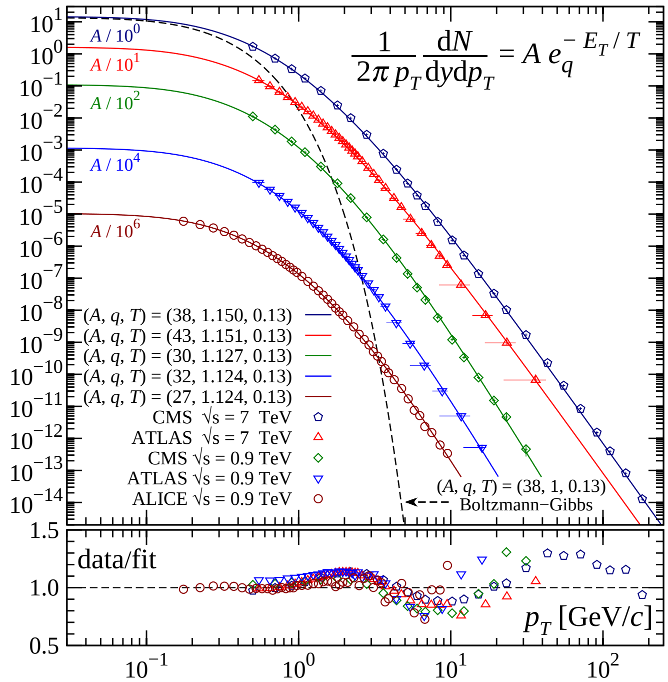

FIG. 1. Comparison of Eq. (1) with the experimental transverse momentum distribution of hadrons in *pp* collisions at central rapidity y. Herein the temperature is being measured in $[T]$ = GeV and the normalization constant in $[A] = \mathrm{GeV}^{-2}/c^3$. For a better visualization both the data and the analytical curves have been divided by a constant factor as indicated. The ratios data/fit are shown at the bottom, where a nearly log-periodic behavior is observed on top of the q-exponential one. Data from [13, 14].

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