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SIMPLIFIED CHIRAL SUPERFIELD PROPAGATORS FOR CHIRAL CONSTANT MASS SUPERFIELDS

by

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Abstract

Unconstrained superfield potentials are introduced to derive Feynman rules for chiral superfields following conventional procedure which is easy and instructive. We also derive propagators for the case when the mass parameters are constant chiral superfields. The propagators reported here are very simple compared to those available in literature and allow a manageable calculation of higher loops.

I. Introduction

Soon after the supersymmetry discussed by Wess and Zumino[1] the notion of superfields was introduced by Salam and Strathdee[2] who also formulated superfield Feynman rules[2]. Superfield perturbation theory was applied in various papers[3]. Grisaru, Roček and Siegal[4] managed to simplify the original rather cumbersome rules for the case of constant mass parameters $m, m^\dagger$ so that the higher loop calculations became manageable. In their derivation they used the functional integral formulation for chiral superfields which obey differential constraints.

We introduce instead uncostrained superfield "potentials" for the chiral superfields like in electrodynamics and derive the corresponding propagators which are much simpler. The calculations is done for the case when $m(\theta)$ and $m^\dagger(\theta)$ are chiral constants (Sec. III). This case is important for the discussion of spontaneous supersymmetry breaking and for the calculation of effective potential. We may use the functional integral method but the conventional procedure is easy and very instructive. The vertex rules may be just read of from the interaction terms.

II. Unconstrained Superfields. Propagator for $m, m^\dagger$ constants:

The chiral superfields Φ, $\Phi^\dagger$ satisfy† differential constraints, $\bar{D}\phi=D\phi^\dagger=0$. It is then suggested that analogous to the case of electrodynamics we introduce unconstrained superfields "potentials" S, $S^\dagger$ such that $\Phi=-\frac{1}{4}\bar{D}^2S$, $\Phi^\dagger=-\frac{1}{4}D^2S^\dagger$. They are seen to be solutions of $D^2\Phi=-4\Box S$, $\bar{D}^2\Phi^\dagger=-4\Box S^\dagger$. The quadratic action for chiral superfields may be written as

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† We follow the notation of Ref. 5.

$$I_0 = \int d^8z\,\Phi^\dagger\Phi + \frac{m}{2}\int d^6s\,\Phi\Phi + \frac{m^*}{2}\int d^6\bar{s}\,\Phi^\dagger\Phi^\dagger$$

$$= \frac{1}{16}\int d^8z\,\left[S^\dagger D^2\bar{D}^2 S - 2m\,S\,\bar{D}^2 S - 2m^* S^\dagger D^2 S^\dagger\right] \qquad (1)$$

It is invariant under $S \to S + \bar{D}\Omega$, $S^\dagger \to S^\dagger + D\Omega^\dagger$. Varying w.r.t the unconstrained fields gives

$$A\begin{bmatrix} S \\ S^\dagger \end{bmatrix} = \frac{1}{16}\begin{bmatrix} \bar{D}^2 & 0 \\ 0 & D^2 \end{bmatrix} M \begin{bmatrix} S \\ S^\dagger \end{bmatrix} = -\begin{bmatrix} J \\ J^\dagger \end{bmatrix} \qquad (2)$$

where the right hand side originates from a classical external source given by

$$\int d^8z\,(JS + J^\dagger S^\dagger) \qquad (3)$$

and

$$M = \left[\begin{array}{c|c} -4m & D^2 \\ \hline \bar{D}^2 & -4m^* \end{array}\right] \qquad (4)$$

We note that $\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} A = 0$ and $\begin{bmatrix} P_2 & 0 \\ 0 & P_1 \end{bmatrix} A = A$. The free field Green's function for superfields $S, S^\dagger$ is thus defined by

$$A\,\Delta(z,z') = \begin{bmatrix} P_2 & 0 \\ 0 & P_1 \end{bmatrix} \delta^8(z-z') \qquad (5)$$

where $P_1 = D^2\bar{D}^2/16\Box$ and $P_2 = \bar{D}^2 D^2/16\Box$ are chiral projection operators. We have

$$\begin{bmatrix} S(z) \\ S^{\dagger}(z) \end{bmatrix} = -\int d^8z' \; \Delta(z,z') \begin{bmatrix} P_2'J(z') \\ P_1'J^{\dagger}(z') \end{bmatrix} \tag{6}$$

and find from Eq. (5)

$$\Delta(z,z') = M^{-1} \begin{bmatrix} D^2 & 0 \\ 0 & \bar{D}^2 \end{bmatrix} \frac{1}{\Box} \; \delta^8(z-z') \tag{7}$$

The operator M can be inverted following the procedure described in Ref. 5 to obtain $(m,m^* \neq 0)$

$$M^{-1} = -\frac{1}{4} \begin{bmatrix} \frac{1}{m} + \frac{\Box}{m(|m|^2-\Box)} P_1 & \frac{D^2}{4(|m|^2-\Box)} \\ \frac{\bar{D}^2}{4(|m|^2-\Box)} & \frac{1}{m^*} + \frac{\Box}{m^*(|m|^2-\Box)} P_2 \end{bmatrix} \tag{8}$$

which leads to the following free propagator‡ for unconstrained superfields

$$\Delta(z,z') = \frac{1}{(\Box - |m|^2)} \begin{bmatrix} \frac{m^*}{4} \frac{D^2}{\Box} & P_1 \\ P_2 & \frac{m}{4} \frac{\bar{D}^2}{\Box} \end{bmatrix} \delta^8(z-z') \tag{9}$$

For massless case we may set m=m*=0 in Eq. (9). The Eq. (5) is easily verified for this case. Reminding ourselves that for chiral sources $P_1J^{\dagger} = J^{\dagger}$, $P_2J = J$, $\bar{D}^2P_1 = \bar{D}^2$, $D^2P_2 = D^2$ etc we may effectively replace in pratice P_1 and P_2 in Eq. (9) by identity

‡The boundry conditions of causal Green's function are understood. For an alternative derivation see Sec.III.

operator. The propagator then is identical to that given in Ref. 4.

Towards an illustration consider the cubic interaction term

$$\frac{\lambda}{3}\left[\int d^6 s\, \Phi^3 + \int d^6 \bar{s}\, \Phi^{\dagger 3}\right]$$

$$= \frac{\lambda}{3}\int d^8 z\left[\Phi S(-\frac{1}{4}\bar{D}^2 S) + \Phi^\dagger S^\dagger(-\frac{1}{4}D^2)S^\dagger\right] \qquad (10)$$

The rule, for example, that at each Φ^3 vertex with two (three) internal lines requires a factor $(-\frac{1}{4}\bar{D}^2)$ acting on one (two) of the internal propagators and the introduction of the free propagators of the unconstrained superfields S, $S^\dagger$ rather than those of chiral superfields come out easily on applying Wick's theorem.

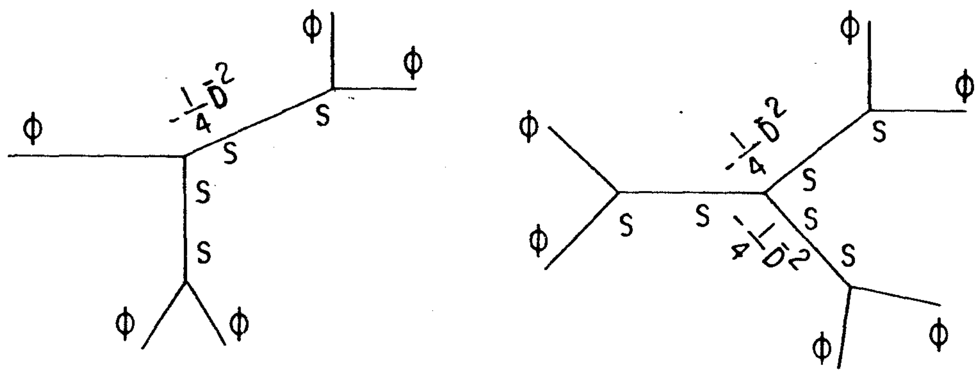

Fig. Graphs illustrating vertex rules.

III. Propagators for m, $m^\dagger$ Constant Chiral Superfields:

We now derive the propagators for the case when $m, m^\dagger$ obey the constraints $\partial_\ell m=\partial_\ell m^\dagger=0$, $\bar{D}m=0$ and $Dm^\dagger=0$. These propagators are useful when we want to calculate effective potential using superfields or discuss the spontaneous supersymmetry breaking when we are required to shift the superfields. The shift produces the effectively θ, $\bar{\theta}$ dependence of the "masses" in the free Lagrangian.

From Eq. (5) the equations determining Δ_{12} and Δ_{22} are[†]

$$-4m(\theta)\,\bar{D}^2\Delta_{12} + \bar{D}^2 D^2\Delta_{22} = 0 \tag{11a}$$

$$-4m^{\dagger}(\bar{\theta})\,D^2\Delta_{22} + D^2\bar{D}^2\Delta_{12} = 16\,P_1\,\delta^8(z-z') \tag{11b}$$

If we choose Δ_{22} to be chiral superfield we find

$$\Delta_{22} = \frac{m(\theta)}{4\Box}\,\bar{D}^2\Delta_{12} \tag{12}$$

where the right hand side is clearly chiral. Substituting in Eq. (11b) we get

$$[-m^{\dagger}(\bar{\theta})P_1 m(\theta) + \Box P_1]\Delta_{12} = P_1\,\delta^8(z-z') \tag{13}$$

Observing that both the sides are antichiral we may multiply Eq. (13) by P_1 to obtain

$$P_1[-m^{\dagger}(\bar{\theta})\,P_1 m(\theta) + \Box]\,\Delta_{12} = P_1\,\delta^8(z-z') \tag{14}$$

A solution of this equation is the superfield

$$\Delta_{12} = [\Box - m^{\dagger}P_1 m]^{-1}\,\delta^8(z-z') \tag{15}$$

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† We note that the procedure of using projection operators given in Ref. 5 is not suited to the case under discussion.

We may verify that the expression for Δ_{12} and Δ_{22} obtained above do satisfy Eq. (11). We observe also that in view of the structure of Eq. (11) we may if we wish, multiply the right hand side of Eq. (15) by P_1 and make Δ_{12} antichiral. The other two propagators are obtained to be

$$\Delta_{11} = \frac{m^{\dagger}(\bar{\theta})}{4\Box} D^2 \Delta_{21}$$

$$\Delta_{21} = [\Box - m P_2 m^{\dagger}]^{-1} \delta^8 (z-z') \qquad (16)$$

When m, $m^{\dagger}$ are independent of θ, $\bar{\theta}$ the above expressions reduce to Eq. (9) if we make (Δ_{12}) Δ_{21} (anti-) chiral†. For the case $m(\theta)=f\theta^2$, $m^{\dagger}(\bar{\theta})=f^*\bar{\theta}^2$ we find

$$\Delta_{22} = - \frac{f\,\bar{\theta}^2\,\theta'^2}{(\Box^2-|f|^2)} \delta^4 (x-x') \qquad (17)$$

We note that the propagators of unconstrained superfields given above are much simpler to work with than those of constrained superfields given in the earlier works[6]. The calculation of effective action may now be performed in a rather simple fashion and will be reported elsewhere.

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†If we go back to Eq. (18) it is easy to see that we may in fact substitute unity in the antidiagonal elements of Eq. (9).

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