

Conventions spreading in open-ended systems

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New Journal of Physics **11** (2009) 023018 (11pp)

Received 25 September 2008

Published 11 February 2009

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/11/2/023018

Abstract. We introduce a simple open-ended model that describes the emergence of a shared vocabulary. The ordering transition toward consensus is generated only by an agreement mechanism. This interaction defines a finite and small number of states, despite each individual having the ability to invent an unlimited number of new words. The existence of a phase transition is studied by analyzing the convergence times, the cognitive efforts of the agents and the scaling behavior in memory and time.

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1. Introduction

The general process where domains characterized by a specific homogeneous quantity emerge out of an initial disordered state is a paradigmatic problem in statistical physics. A classic example is the study of magnetization, the Ising model being the simplest and more common representation of such a phenomenon.

Interestingly, it is possible to redirect to this kind of situation problems coming from other areas of knowledge [1]. The spreading of opinions or conventions in social systems is a good example. The analogy, in this case, can be built up through the identification of individuals with agents characterized by some quantity, and representing the interaction between them as a result of the implementation of a set of simple rules. A straightforward realization of this scheme is possible using models inspired by the kinetic Ising model. In this case, agents' opinions are characterized by the spin values and the interaction reduces to spin flips, according to some particular rule. For example, in two-state models, the opinion update (a spin flip) is implemented following the local majority [2, 3] or directly emulating another opinion [4, 5].

Starting from these straightforward models, various levels of complexity have been reached with the introduction, for example, of noise [6] or with the implementation of a q -state discrete space (Potts-type models) [7]. A particularly interesting case was obtained by coupling some Potts-type models [8]. Finally, a quite different modeling strategy, which departs from the classical Ising prototype, also appeared [9].

The specific situation where agents' opinions represent words and the social dynamics describing the evolution toward the emergence of a shared vocabulary raised some novel questions that forced us toward different modeling schemes. In fact, these dynamics are characterized by some peculiar facts [10]. Linguistic units, because of their conventional nature [11], are characterized by a strong arbitrariness, either in their changes or in their established forms. This open-ended nature is constrained by the correlation imposed by communication: the freedom of an individual's inventiveness is bounded by the interaction with the community. The effect of these social interactions is to reduce the arbitrariness of the used symbols and to convey the community toward a consensus. Ultimately, it is necessary to build up a connection between these two founding opposing factors: the arbitrary convention, which allows totally free choice, and the social interactions during the passage of time, which fix that choice [12].

These last considerations suggest we abandon the Ising-like canon. First of all, arbitrariness and variation in linguistic units force us toward a model scheme implemented in an open-ended word system. This means that agents must be characterized by a memory capable of accumulating an unlimited number of possible states, each one characterized by an infinite number of possible choices. Moreover, individuals must be able to constantly create new, different words. Secondly, the interaction between individuals should be characterized by rules that capture two essential features of this process: the presence of memory and learning/feedback effects. Unfortunately, the herding behavior described by Ising-like interaction cannot account for these effects.

This purpose has been recently achieved by the so-called Naming Game [13], where the implemented microscopic mechanisms, the negotiation dynamics, are based on these features. In this dynamical system, originally inspired by an artificial experiment based on robots [14], the negotiation dynamics are able to order the system toward consensus without the intervention of any central control and revealed itself richer and more realistic than the classical implementation based on Ising-like formalization.

In the present work, we will describe, through a modification of the Naming Game of Baronchelli *et al* [13], an open system where each agent can actually store an unlimited number of different names. This fact is possible thanks to a new dynamics that allows the introduction of different words at every MC step [15], contrasting with a dynamics that allows the introduction of different words just in the first MC steps [13].

With this setting, we investigate a minimal interaction mechanism capable of generating the convergence toward consensus. In the original implementation, two cognitive mechanisms work: the agreement and the learning (or overlapping) mechanism. The first one erases the information collected in the agent's memory if it does not take part in the last agent's successful communication. The second one, in the case of a failure, allows the memorization of a new word, directly learned from the speaker. This last mechanism causes a fast increase in the overlap between the words used in the community [13, 15].

In our work, we looked for a lower bound for the cognitive mechanisms, defining a dynamics governed just by the agreement mechanism, with no internal overlapping mechanism. In this situation, all the new words collected in individuals' memory are self-invented, picked up by an external reservoir of words. In [13], after a transient, when everybody attains at least one word, the game is characterized by a fixed number of different words that are interchanged through the overlapping mechanism and are reduced by the agreement mechanism. In our model, new words are continuously invented and the convergence does not simply emerge like an effect of the dissemination of the most common word caused by the overlapping mechanism. In this way, we test the new hypothesis that it is possible to reach consensus even for memories that can store an infinite number of different words and with a simple interaction that can only compare and not exchange symbols.

In this implementation, the nature of the words introduced in the system is totally independent of the social mechanisms of interaction. Anyway, the nature of the linguistic convention is not totally arbitrary and relates to society's communicational use of the material supplied by the current language. Continuity with the past constantly restricts freedom of choice [12]. This fact is a source of bounds in the open-ended nature of the invented signs. These considerations, along with other practical reasons, suggested we characterize the reservoir of words by a distribution that models the presence of some constraints, differentiating between more and less probable words.

To sum up, in the present work, we will try to understand some basic features of ordering processes governed by an agreement dynamics with an unlimited number of memory states. The importance of these dynamics is evident in the description of conventions spread, as, for example, in the convergence on the use of a particular word or category in real communities or among embodied software agents or tagging systems on the web. The fundamental process acting in these dynamics is a memory-based negotiation strategy, made up of a sequence of trials that shape and reshape the system memories. This kind of interaction is at the heart of numerous cognitive processes, either in a biological [16], social [17] or technological [18] context, and it is more widespread than the simple imitation mechanism that can be described by Ising-like dynamics. For these reasons, the definition of the simplest mechanism sufficient for its performance is fundamental. In the present study, we will show that the agreement mechanism can be sufficient for reaching consensus and no other mechanisms, which can help in diffusing the most common symbols, are necessary. This fact is important not only from a theoretical point of view, but also for a possible transposition into practical applications, such as, for example, the

designing of artificial communication systems, which can be represented either by robots [18] or by peer-to-peer information systems on the web [17].

2. The model

The game is played by P agents. An inventory, which can contain an arbitrary number of words, represents each agent. Population starts with empty inventories. At each time step, two agents are randomly selected; the first one assumes the role of the speaker and the second one, of the hearer. Then, the following microscopic rules, which are a modification of the ones used in the original Naming Game [13], control their actions:

- (1) The speaker retrieves a word from its inventory or, if its inventory is empty, invents a new word.
- (2) The speaker transmits the selected word to the hearer.
- (3a) If the hearer's inventory contains such a word, the communication is a success. The two agents update their inventories so as to keep only the word involved in the interaction.
- (3b) Otherwise the communication is a failure. The speaker invents a new word, different from all the other ones that are in its memory.

The players invent new words from a distribution that decays following a simple Gaussian law: $\exp(-x^2/2\sigma^2)$. We choose this continuous distribution because its implementation is easy and it allows fast simulation runs. Anyway, words are represented only by positive integer numbers. To sum up, invention of new words corresponds to picking them up from a reservoir, whose effective size is tuned by σ . The speaker chooses a new word which is different from all the other ones that are in its memory but which can be present in the memory of different agents or could have been present, at earlier time steps, in the same agent's memory. By the way, this model implementation could generate homonymy if the agents try to achieve consensus on a set of names to be used for signifying a number of different meanings (objects).

In accordance with our aims, the dynamics are governed only by the agreement mechanism and the procedure for inventing new words is independent of the social mechanisms of interaction. This implementation defines an open-ended system where an unlimited number of words can be invented. Players invent new words if their inventory is empty (that happens only in the early stages of the simulation), or if their communication is a failure. In fact, we can think that, in real life, individuals that are not able to communicate are naturally led to look for new words.

3. Results

We will describe the time evolution of our system looking at some usual global quantities [13, 15]: the total number of words (N_{tot}) present in the population, the number of different words (N_{dif}) and the success rate (S), which measures an average rate of success in communications.

A fast initial transient exists. Agents start with an empty inventory and, in each interaction, each speaker invents at least one new word and each hearer can possibly learn one. After this early stage, the system self-organizes in a state where N_{dif} and S display a long plateau. Similarly, the total number of distinct words displays a plateau, but slightly decreasing along

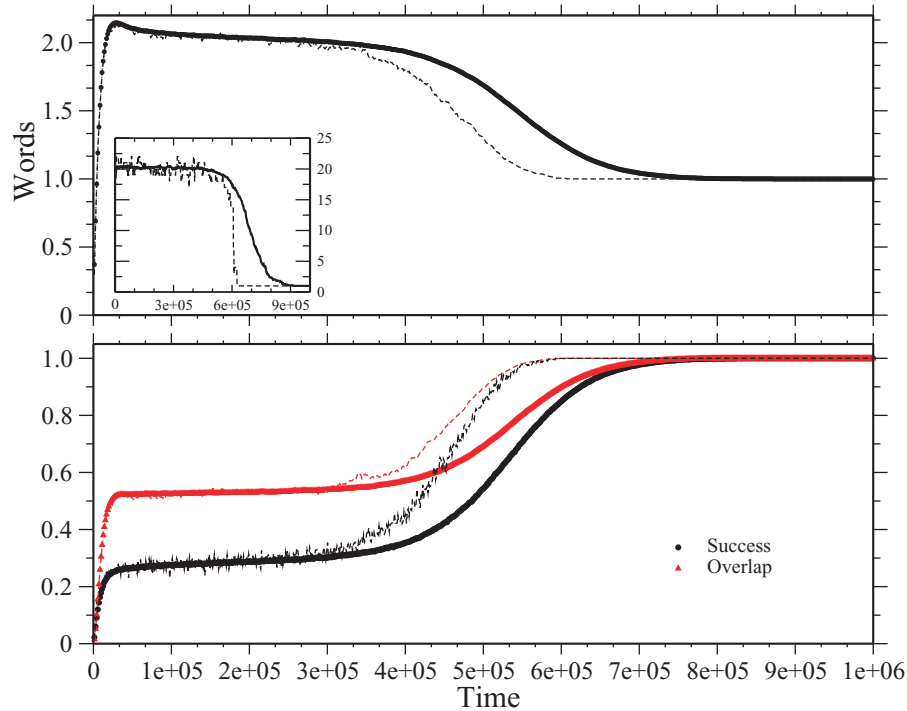


Figure 1. Top: temporal evolution for the total number of words divided by the total population. Inset: total number of different words ($N_{\text{dif}}(t)$). Bottom: the success rate $S(t)$ and the overlap function \mathcal{O} . The dashed curves represent a single realization. All other data are averaged over 100 simulations ($P = 7000$ and $\sigma = 5$).

the time evolution (see figure 1). Quite abruptly, perhaps because of a large fluctuation, the ordering transition takes over. The number of successful plays increases and N_{tot} changes its concavity and begins a decay toward the consensus state, corresponding to one common word for all the players, reached at time T_c . In contrast, the number of different words maintains a constant value until the convergence process has led the system nearer to the absorbing state. In figure 1, we report the temporal evolution for $N_{\text{tot}}(t)$, $N_{\text{dif}}(t)$ and $S(t)$. Generally, the time evolution of the single runs displays a large variation and can be quite different from the average.

With the aim of characterizing our system with more details, describing directly its state and not just the outcome of the game, we looked at an overlap function [13]. It corresponds to the total number of words common to all the possible agent pairs:

$$\mathcal{O} = \frac{2}{P(P-1)} \cdot \sum_{i \geq j} a_i \cap a_j.$$

The behavior of this quantity is similar to $S(t)$, with a well-defined plateau followed by a sudden transition toward one (see figure 1). The long plateau is characterized by a constant mean value, which rapidly decreases with increasing σ value. We will present a more comprehensive analysis of the behavior of this quantity at the end of this section.

We investigated how the macroscopic observables scale with the population size P . First, we look at the scaling behavior of the system memory size. By a numerical exploration, we could

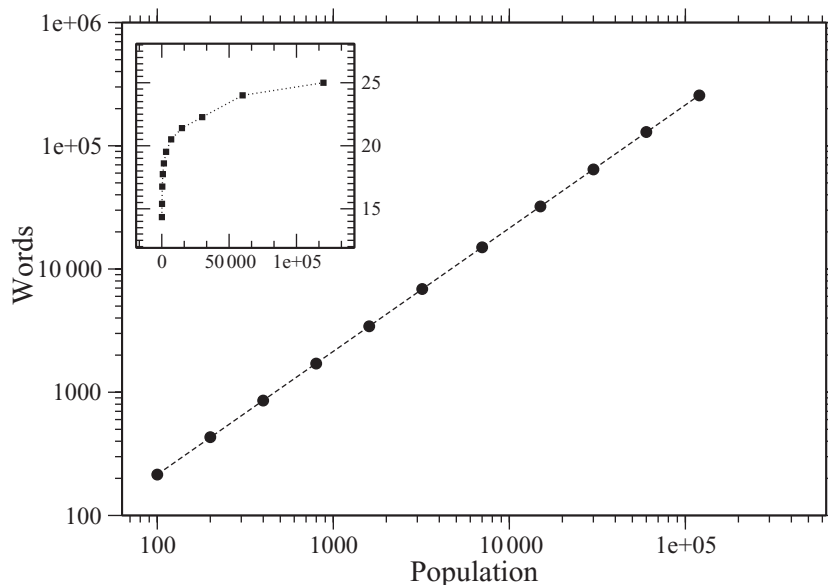


Figure 2. Maximum number of total words (the dashed line has slope 1) and, in the inset, maximum number of different words for different population sizes. In the original Naming Game [13], the maximum number of total words scales with $P^{1.5}$ and the maximum number of different words scales with P . In our model, the first one scales with P and the second one is almost not dependent on P , for sufficiently large P . Data are averaged over 100 simulations with $\sigma = 5$.

see that the maximum number of different words is almost not dependent on P for sufficiently large P (see figure 2).

The maximum number of total words linearly scales with the population number. In other words, the number of total words of each player is not dependent on the dimension of the community. We can demonstrate it with a simple analytical consideration, already used in [13]. We represent the mean total number of words for the agent, at time step t , with $n(t)$, and the mean total number of different words with $D(t)$. If we assume that $n(t)$ scales with β , unknown, then we can write

$$n(t+1) - n(t) \propto \frac{1}{n(t)^\beta} \left(1 - \frac{n(t)^\beta}{D(t)} \right) - \frac{1}{n(t)^\beta} \frac{n(t)^\beta n(t)^\beta}{D(t)}. \quad (1)$$

We consider that the probability for the speaker to communicate a specific word is $1/n(t)^\beta$ and the probability for the hearer to own that word is $n(t)^\beta/D(t)$. It follows that the first term represents the gain term for a failed communication and the second one, the loss term. We can use this equation for describing the P dependence. D , for large P , can be considered as a constant. For this reason, at the stationary state, where we should impose $n(t+1) - n(t) = 0$, our equation reduces to $(1/n^\beta) \propto n^\beta$, which forces $\beta = 0$. This fact implies that the number of total words for each player is not dependent on P . It follows that $N_{\text{tot}} \propto P$, as confirmed by the numerical results shown in figure 2.

We look at the behavior of our model varying the σ value in the distribution of the invented words. As can be seen in figure 3, N_{dif} linearly scales with σ . This result is quite intuitive if we approximate our distribution of new words with a box of dimension σ .

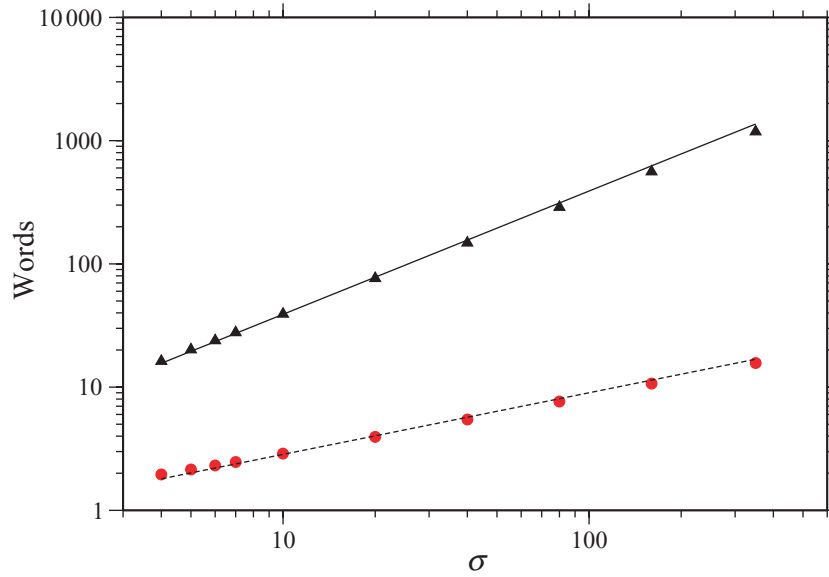


Figure 3. Maximum number of total words (circles) and maximum number of different words (triangles) for different σ values. The dashed line has slope 1/2 and the continuous line has slope 1. Data are averaged over 100 simulations with $P = 5000$.

We can use another time equation (1) for examining the dependence of N_{tot} on σ . We will look for a power-law dependence on σ ; for this reason, we consider that $n \propto \sigma^\gamma$ with γ being unknown. Using the fact that, at equilibrium, $D \propto \sigma$, we obtain

$$\frac{1}{\sigma^\gamma} \left(1 - \frac{\sigma^\gamma}{\sigma}\right) \propto \frac{1}{\sigma^\gamma} \frac{\sigma^\gamma \sigma^\gamma}{\sigma}.$$

This equation implies that, at the stationary state, $\gamma = 1/2$. It follows that $N_{\text{tot}} \propto \sigma^{1/2}$, a result confirmed by the numerical data shown in figure 3.

Finally, we explored the behavior of the convergence time T_c . As stated before, this is the time at which the system reaches the consensus state, corresponding to one shared word for all the players. When the system does not achieve convergence, the system persists in the plateau regime displayed in figure 1. There, each agent stores in its memory some words, which are continuously suppressed by successful communications and invented by failed communications.

We stated the dependence of T_c on P averaging over different simulations. The T_c value grows linearly with P (see figure 4). We have also observed that the distribution of the convergence times follows a log-normal-like shape with large variances.

The T_c dependence on σ has more intriguing behavior. We report T_c as a function of the parameter σ , for different P values (see figure 4). We display simulations up to values of σ for which convergence emerges in the limits of typical computational times (2×10^9 steps). For low σ values, T_c increases exponentially. For higher values, the divergence gets super-exponential and the linear scaling, as a function of P , breaks down. By increasing the system size, the growth of T_c becomes steeper. Anyway, it is difficult to state if there exists a σ_{crit} that corresponds to a phase transition between a stationary state characterized by consensus and one characterized

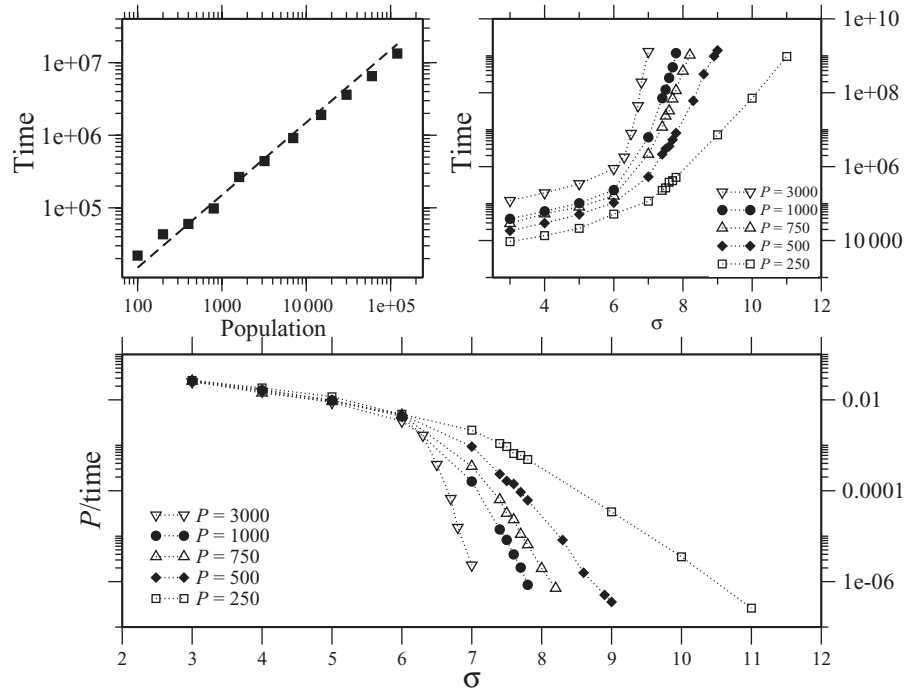


Figure 4. Top: on the left, convergence time (convergence of the mean simulation) for different population sizes ($\sigma = 5$) is shown. The dashed line has slope 1. On the right, mean convergence time for different σ values and different P is shown. Bottom: normalized inverse of the convergence time for different σ values and different P .

by disorder. The plot of $(T_c)^{-1}$ can suggest this fact (see figure 4), with $6 < \sigma_{\text{crit}} < 7$ for large P . Unfortunately, because of the open-ended nature of our model, analytical results, which can confirm this supposition, are still lacking. An interesting system where, even though with a different mechanism, an analogue transition has also been described analytically can be found in [19]. In that paper, a consensus phase transition, controlled by a parameter that represents the efficiency of the negotiation dynamics, is described. This parameter constrains the level of noise present in communications (no noise corresponds to the Naming Game). For a sufficiently low level of noise, consensus is reached. Otherwise, several opinions, continuously exchanged between agents, coexist. There is a similarity between increasing the noise level, which decreases the efficiency of the erasing mechanism, and increasing the broadness of our reservoir, which corresponds to increasing the variance of the words entering the system.

We conclude this analysis by comparing our results with those of the original Naming Game [13]. In the thermodynamic limit, the usual Naming Game stores an infinite number of different words (P dependence), and also after the temporal rescaling $t \rightarrow t/P$, the convergence time diverges ($P^{1.5}$ dependence). In our model, in the thermodynamic limit, the number of different words seems to be finite and, at least for $\sigma < \sigma_{\text{crit}}$, the system can converge toward consensus (P dependence). Now, for $\sigma \rightarrow \infty$, the number of words becomes infinite and consensus is not attained. This fact suggests that, if the number of words effectively present in the system is infinite, convergence cannot be attained.

Finally, we study the dependence of the convergence toward consensus on the initial conditions of our simulations. The overlap function \mathcal{O} is the one that can better represent

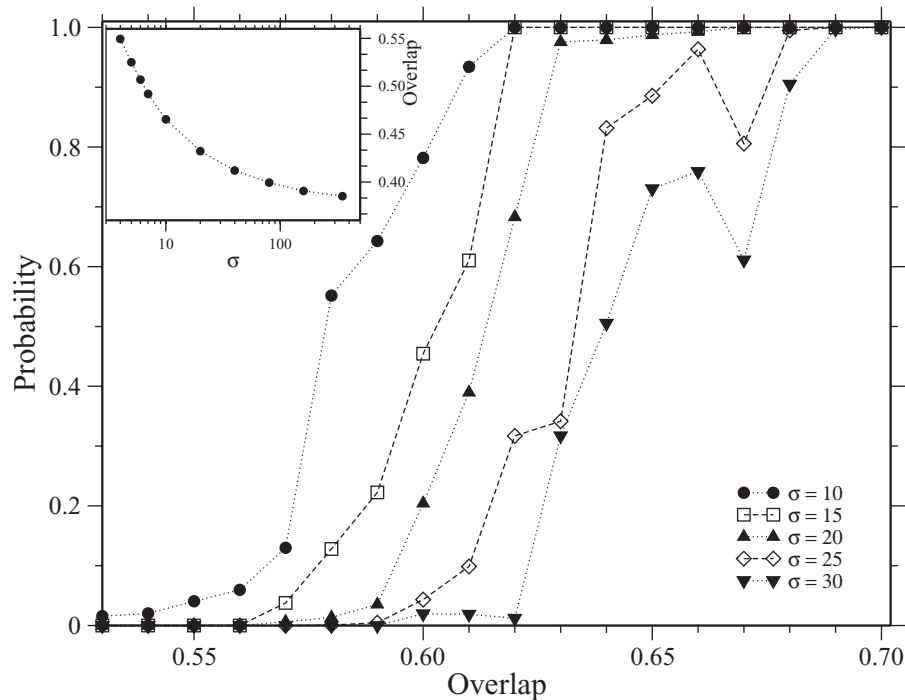


Figure 5. Probability of reaching consensus as a function of the \mathcal{O} value of the initial conditions, for different σ values ($P = 1000$). In the inset, the mean value of \mathcal{O} in the plateau region, as a function of σ , for standard initial condition is shown.

the system internal state. For this reason, we characterize the initial conditions with their \mathcal{O} value. We obtain an ensemble of different initial conditions running a simulation with some fixed parameters and picking up some particular configurations obtained at a specific time step. Then, we run new simulations, with different parameters, starting from these collected configurations.

In figure 5, we report the frequency of runs that reach consensus as a function of the \mathcal{O} value of the initial conditions, for different σ values. The probability of reaching the ordered state increases with higher \mathcal{O} values. The same probability decreases by raising the σ value of the run. In fact, higher σ values cause a slower ordering or even do not allow the transition to take over. With this approach it is easier to determine whether the system reaches consensus or not, without having to wait for a very long time. In fact, we consider that the system does not evolve toward consensus if the initial overlap value decreases until reaching the stationary state value corresponding to that σ . We consider that the stationary \mathcal{O} value (a function of σ) is the one previously found at the end of very long simulations run with standard initial conditions (see the inset of figure 5).

A simple interpretation of these results is possible. A critical overlap value exists: when the system reaches that value, the ordering transition takes place and a very fast convergence process toward the absorbing state occurs. The convergence probability corresponds to the probability that, because of a fluctuation, such a critical value is achieved starting from the \mathcal{O} value of the plateau. For this reason, different initial conditions in the \mathcal{O} value ($\mathcal{O} \neq 0$) cause the convergence for σ values that do not allow convergence for initial configuration having $\mathcal{O} = 0$. Moreover,

such a critical \mathcal{O} value grows with σ and this is the reason why the convergence is not always attained.

4. Conclusions

We propose a very simple model, a modification of the Naming Game of Baronchelli *et al* [13], which explores some questions raised by the study of semiotic dynamics. We implement an open-ended system, where only the agreement mechanism is responsible for the ordering process. This interaction is sufficient for defining a finite and small number of states, despite each individual having the ability to invent an unlimited number of new words. We found that, for some values of the parameters, the ordering process led to a consensus state, characterized by the use of a single word. With these results, we confirmed the hypothesis that it is possible to reach consensus even for memories that can store an infinite number of different words and with a simple interaction that can only compare and not exchange symbols. This is a quite unexpected result. In fact, if exchanging symbols in a closed system can quite intuitively lead to convergence, it would be hard to foresee the same behavior by simply erasing symbols in an open system. These facts are explored through an extended numerical study that analyzes the scaling behavior for memory and time, with respect to the population size. Moreover, we study the relation between convergence times and the shape of the invented words distribution and the dependence on initial conditions.

Our analysis assesses the robustness of the ordering transition for models with a very simple interaction mechanism and gives some new general insights for systems characterized by an unlimited number of memory states. It describes some characteristics of the birth of neologisms in real communities, modeled through the introduction of a reservoir of words which reflect the fact that the nature of the linguistic convention is not totally arbitrary. These results, interesting from a theoretical point of view, perhaps could be transposed into practical applications for designing artificial communication systems for robots [18] or for other semiotic dynamics [17].

Acknowledgment

We thank the Brazilian agency CNPq and the FAPERJ for financial support.

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