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BOUNCE AND WORMHOLES

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We investigate if theories yielding bouncing cosmological models also generate wormhole solutions. We show that two of them present sensible traversable static wormhole solutions, while for the third possibility such solutions are absent.

Keywords: Bouncing cosmological models; wormholes; energy conditions violation; exact solutions.

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1. Introduction

With the observation of the present acceleration of the Universe,[1,2] it seems that fluids that violate the strong energy condition (SEC) must indeed exist in nature,[3] and antigravitate. It is also speculated that such acceleration might also be driven by yet more exotic fluids, called phantom fields, which violate the weak energy condition (WEC), and may cause a future Big-Rip singularity.[4] These fluids can also be effectively obtained through nonminimal couplings between gravity and other fields,[5,6] quantum corrections,[7–9] or nonstandard interactions among ordinary fields satisfying all the energy conditions.[10]

Such kind of fluids may also play a fundamental role in the early universe, either yielding an inflationary period,[11–15] and avoiding the initial singularity as in the bouncing models.[16–20] Regarding the later possibility, it is also known that the violation of SEC may not be sufficient to produce a bounce in the past: some period of WEC violation is also required if the spatial sections have no positive curvature.[21]

Fluids violating WEC are also necessary to obtain wormhole solutions,[22] hence, in view of all those scenarios in which exotic sources are seriously taken into account (the usual quantum instabilities which sometimes plague these sources are circumvented by assuming that, in these cases, they should be considered as effective fluids, not fundamental fields), we can investigate in which of them traversable wormhole solutions can be obtained.

In the present paper we investigate the existence of wormhole solutions in theoretical frameworks which produce bouncing cosmological models. The existence and properties of these cosmological bounces have been studied in many papers (see Refs. 23–26), but it remains to be investigated if such theoretical frameworks can produce wormhole solutions, which is a natural expectation to have due to the enormous parallel between bounces and wormholes (which are nothing but bounces in space).

Our work will be based in three models of bouncing universes. The first one considers a nonminimal coupling between vector and gravitational fields. Such coupling leads to nonlinearities in the theory and, consequently, violations of the energy conditions that enabled not only a bounce,[23] but also a very nice traversable wormhole solution, as we will see in Sec. 2. One may associate this vector field with the vector potential of electromagnetism. In this case, nonminimal couplings can be caused by quantum electrodynamics corrections to general relativity,[23,27] which could be the origin of the presence of magnetic fields observed in galaxies and intergalactic medium.[28–30]

The next one considers a model of universe in a Weyl integrable space–time[24] (WIST), yielding an effective negative energy scalar field, which evolves from a Minkowski configuration in the infinite past, goes through a bounce and finish in another asymptotic flat universe. This pure geometric model can also give rise to a perfectly reasonable traversable wormhole.

The last case treated involves nonlinear corrections to the electromagnetic field. Such corrections are needed when very intense fields are considered, where the particle creation phenomenon occurs. These nonlinearities lead to the violation of energy conditions and allow a bounce,[26] but not a wormhole. We will show that whenever one imposes the boundary conditions that characterizes the wormhole throat, the asymptotic behavior of the spacetime solution becomes unacceptable. Therefore, we have an example of a theoretically acceptable source that violates WEC and do not lead to a wormhole, showing that this condition is necessary but not sufficient to obtain such type of geometries.

Finally, we conclude that, although we have explicitly shown examples of wormholes solutions obtained using sources already present in other frameworks, this is not enough to consider such type of configuration as real as long as they are static. This still depends on dynamical treatments, and investigations about their classical and quantum instabilities, which are beyond the scope of this paper.

2. Nonminimal Coupling between Vector and Gravitational Fields

Nonminimal coupling between electromagnetic and gravitational fields appears when one takes into account the vacuum polarization influence on the propagation of photons in a gravitational background using the one-loop approximation.[27]

If gauge invariance is not imposed, one may obtain a mass term for the photon proportional to the Ricci scalar, which should be important only in the presence of large gravitational fields (see Refs. 31–33 for theoretical and observational discussions concerning the possibility of a massive photon). These nonminimal couplings were used to study the production of primordial magnetic fields in the universe.[28–30] One can also work with a vector field not necessarily identified with elecromagnetic fields, where nonminimal couplings appear already at tree level. These kind of fields were used to obtain nonsingular cosmological models,[23] to yield acceleration in the past[34,35] (inflation) and in the present[35–37] (dark energy), and as models for the aether.[38]

In the present Section we will consider wormholes in the framework of an arbitrary vector field nonminimally coupled to gravity as described by the Lagrangian

$$L = \sqrt{-g}\left[\frac{1}{2k}(1+\lambda A_\mu A^\mu)R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{1}$$

where $k = 8\pi G$, G is the Newton's constant, λ is the nonminimal coupling constant and $F^{\mu\nu}$ is the usual Maxwell tensor constructed from A^μ. The variation of this Lagrangian with respect to $g_{\mu\nu}$ and A^μ yields the following equations:

$$(1+\lambda A^2)G_{\mu\nu} + \lambda\Box A^2 g_{\mu\nu} - \lambda\nabla_\nu\nabla_\mu A^2 + \lambda R A_\mu A_\nu = kT_{\mu\nu}\,, \tag{2}$$

and

$$F^{\mu\nu}{}_{||\nu} = -\frac{\lambda}{k}RA^\mu\,, \tag{3}$$

where $G_{\mu\nu}$ is the usual Einstein tensor, $A^2 \equiv A^\mu A_\mu$, and $T_{\mu\nu}$ is the Maxwell stress–energy tensor,

$$T_{\mu\nu} = F_{\mu\alpha}F^\alpha_\nu + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\,. \tag{4}$$

We will look for solutions with the same source used in the cosmological solution obtained in Ref. 23, which presents a bounce, where $A_\mu = w_{|\mu}$. With this choice for the vector potential, it follows from (3) that $R = 0$, and the set of equations reduces to

$$R_{\mu\nu} = \frac{\Upsilon_{|\mu||\nu}}{\Upsilon}\,, \tag{5}$$

$$g^{\mu\nu}\Upsilon_{|\mu||\nu} = 0\,, \tag{6}$$

in which the last one comes from the trace of (2) and we have introduced the new variable $\Upsilon \equiv 1 + \lambda A^2$.

We are interested in a static spherical geometry, with metric given by

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{\left(1-\frac{b(r)}{r}\right)} + r^2 d\Omega^2\,. \tag{7}$$

The field equations (5) and (6), in this case, reduce to

$$-\left(1-\frac{b}{r}\right)\frac{\phi'\Upsilon'}{\Upsilon}=\frac{b'}{r}\,, \tag{8}$$

$$\frac{1}{\Upsilon}\sqrt{1-\frac{b}{r}}\left(\sqrt{1-\frac{b}{r}}\Upsilon'\right)'=-\frac{b}{r^3}+2\left(1-\frac{b}{r}\right)\frac{\phi'}{r}\,, \tag{9}$$

$$\left(1-\frac{b}{r}\right)\frac{\Upsilon'}{r\Upsilon}=\left(1-\frac{b}{r}\right)\left[\phi''+\phi'\left(\phi'+\frac{1}{r}\right)\right] - \frac{(b'r-b)}{2r^2}\left(\phi'+\frac{1}{r}\right), \tag{10}$$

$$\frac{\sqrt{1-\frac{b}{r}}}{r^2e^{\phi}}\left(r^2e^{\phi}\sqrt{1-\frac{b}{r}}\Upsilon'\right)'=0\,. \tag{11}$$

This last one implies that

$$r^2e^{\phi}\sqrt{1-\frac{b}{r}}\Upsilon'=D\,, \tag{12}$$

a constant, so that (9) can be rewritten as

$$\frac{D}{\Upsilon}\sqrt{1-\frac{b}{r}}\left(\frac{1}{r^2e^{\phi}}\right)'=-\frac{b}{r^3}+2\left(1-\frac{b}{r}\right)\frac{\phi'}{r}\,. \tag{13}$$

Note that if one turns off the nonminimal coupling, $\lambda = 0 \Rightarrow \Upsilon = 1$, and as we are taking $A_\mu = w_{|\mu}$ (a pure gauge), the unique solution to the system above is the Schwarzschild geometry (spherically symmetric solution in vaccum).

A simple exact solution to this nonlinear system of differential equations reads

$$b(r)=r_0\,, \tag{14}$$

$$\phi(r)=\phi_0\,, \tag{15}$$

$$\Upsilon(r)=\frac{2D}{r_0e^{\phi_0}}\sqrt{1-\frac{r_0}{r}}\,. \tag{16}$$

Let us now examine the properties and traversability of this wormhole, following the lines of Ref. 22. The embedding function $z(r)$ reads

$$z(r)=2r_0\left(\frac{r}{r_0}-1\right)^{1/2}, \tag{17}$$

showing that it is a parabolic wormhole. The proper distance is given by

$$l(r)=\sqrt{r}\sqrt{r-r_0}-\frac{r_0}{2}\ln\left(\frac{\sqrt{r}-\sqrt{r-r_0}}{\sqrt{r}+\sqrt{r-r_0}}\right), \tag{18}$$

which tends towards r when r is big.

As long as ϕ is constant, one can traverse the wormhole with constant speed, and the tidal radial acceleration is null. For the tidal lateral acceleration to be less than one Earth gravity, one has the condition on the speed v:

$$\frac{r_0\gamma^2}{2r^3}\frac{v^2}{c^2} \lesssim \frac{1}{(10^{10}\ \text{cm})^2}\,, \tag{19}$$

where γ is the usual relativistic γ-factor, which we are assuming to be approximately equal to one, since we are considering nonrelativistic velocities. Its maximum value is at the minimum of the throat at r_0, which implies that

$$v \lesssim 42\ \text{m/s}\frac{r_0}{10\ \text{m}}\,. \tag{20}$$

If the trip through the wormhole begins and ends in stations where the curvature of the wormhole is negligible, with deviations, say, of 0.01% from flatness, then they must be located at coordinate distances of order $r \approx 10^4 r_0$, yielding a total proper distance of $\Delta l \approx 2 \times 10^4 r_0$. This gives a total time travel of

$$\Delta t = \frac{\Delta l}{v} \gtrsim 1\ \text{h}\ 19\ \text{min}\,. \tag{21}$$

For a one year travel one needs

$$v \approx 6.34 r_0 10^{-4}/\text{s}\,. \tag{22}$$

Note that the standard vector field energy required to construct this wormhole contained in $T_{\mu\nu}$ is null because $F_{\mu\nu}$ is null. Only the nonminimal interacting energy between the vector field and gravity is present in this scenario.

3. Effective Negative Energy Scalar Field

As mentioned in the Introduction, cosmological bounces can also be induced by negative energy fluids.[25,39,40] We will consider in this Section a negative energy stiff matter fluid represented by a massless free scalar field, which can appear due to interactions among positive energy fluids,[10] or through a pure geometrical model where space–time is represented by a manifold with a geometric structure determined by the Weyl geometry.[24] In this later case, the model is empty, it starts as a flat Minkowski space–time which, through instabilities, begin to collapse into a homogeneous and isotropic Weyl geometry.

The dynamics of the model is obtained from the action

$$S = \int \sqrt{-g}\big(\hat{R} + \xi\hat{\nabla}_\alpha W^\alpha\big)\,, \tag{23}$$

where $\hat{R}$ is the Ricci tensor in the Weyl geometry, W^α is the Weyl vector satisfying $\hat{\nabla}_\alpha g_{\mu\nu} = W_\alpha g_{\mu\nu}$, and $\hat{\nabla}_\alpha$ is the covariant derivative with the Weyl affinity.

Taking a Weyl integrable space–time, where the Weyl vector is a gradient given by $W_\lambda = \varphi_{|\lambda}$, and performing the variation of the action S with respect to the pair

$(g_{\mu\nu}, \varphi)$ of independent variables yields the following equations of motion in terms of Riemannian geometrical quantities (see Ref. 24 for details):

$$G_{\mu\nu} = -\lambda^2 \left(\varphi_{|\mu}\varphi_{|\nu} - \frac{1}{2}\varphi_{|\alpha}\varphi^{|\alpha} g_{\mu\nu} \right) \equiv -\lambda^2 T^{\varphi}_{\mu\nu} \,, \tag{24}$$

$$\Box\varphi = 0 \,, \tag{25}$$

in which $\lambda^2 = (4\xi - 3)/2$. Note that if this constant is positive, one has an effective negative energy fluid, which gives rise to cosmological bounces.[24,25,39,40]

We are looking for the possibility of a static wormhole solution for the above set of equations. Using the spherical geometry given by the metric (7), Eqs. (24) and (25), after a simple manipulation, read

$$b' = -\frac{\lambda^2}{2}\left(1 - \frac{b}{r}\right)(\varphi')^2 r^2 \,, \tag{26}$$

$$\phi' = \frac{b - \frac{r^3\lambda^2}{2}\left(1 - \frac{b}{r}\right)(\varphi')^2}{2r(r-b)} \,, \tag{27}$$

$$\left[\frac{1}{2}\left(1 - \frac{b}{r}\right)(\varphi')^2\right]' = -\left(1 - \frac{b}{r}\right)(\varphi')^2\phi' - \frac{2}{r}\left(1 - \frac{b}{r}\right)(\varphi')^2 \,. \tag{28}$$

This last one can be rewritten as

$$\left[(r^2 e^{\phi})^2 \left(1 - \frac{b}{r}\right)(\varphi')^2\right]' = 0 \,. \tag{29}$$

Now it is easy to integrate the resulting system of equations. The solution is

$$\varphi(r) = -\frac{\sqrt{2}}{\lambda}\arctan\left(\frac{r_0}{\sqrt{r^2 - r_0^2}}\right) + \frac{\pi\sqrt{2}}{2\lambda} + \varphi(r_0) \,, \tag{30}$$

$$b(r) = \frac{r_0^2}{r} \,, \tag{31}$$

$$\phi(r) = \phi_0 \,. \tag{32}$$

The space–time generated by this solution is represented by the geometry described by the following metric:

$$ds^2 = -e^{2\phi_0}\, dt^2 + \frac{dr^2}{\left(1 - \frac{r_0^2}{r^2}\right)} + r^2 d\Omega^2 \,. \tag{33}$$

This solution was already studied in Refs. 41–43.

As it was done in the previous Section, let us examine the properties and traversability of this wormhole, again following the lines of Ref. 22. The embedding function $z(r)$ now reads

$$z(r) = r_0 \,\mathrm{arccosh}\left(\frac{r}{r_0}\right) , \tag{34}$$

showing that it is a hyperbolic wormhole. The proper distance is given by

$$l(r) = r\sqrt{1 - \left(\frac{r_0}{r}\right)^2}\,, \tag{35}$$

which, again, tends towards r when r is big. Also, as long as ϕ is constant, one can traverse the wormhole with constant speed, and the tidal radial acceleration is null. For the tidal lateral acceleration to be less than one Earth gravity, one has the condition on the maximum speed v at $r = r_0$ inside the throat,

$$v \lesssim 30\ \mathrm{m/s}\frac{r_0}{10\ \mathrm{m}}\,. \tag{36}$$

For a 0.01% departure from flatness at the stations, they must be located at coordinate distances of order $r \approx 10^2 r_0$, yielding a total proper distance of $\Delta l \approx 2 \times 10^2 r_0$. This gives a total time travel of

$$\Delta t = \frac{\Delta l}{v} \gtrsim 67\ \mathrm{s}\,. \tag{37}$$

For a one-year travel one needs

$$v \approx 6.34 r_0 10^{-6}/\mathrm{s}\,. \tag{38}$$

In this case, some (negative) energy for the scalar field is required to construct this wormhole as long as its energy momentum tensor $T^{\varphi}_{\mu\nu}$ is not null. The energy density is $\rho = -r_0^2/r^4$, which in the throat at $r = r_0$ reaches its maximum value $\rho = -1/r_0^2$, and the total (negative) mass of the scalar field reads $M = -2\pi^2 r_0$. Hence, smaller values of r_0 requires less mass but bigger densities inside the throat. Note that this wormhole has zero ADM mass.

4. Nonlinear Electrodynamics

In Ref. 26 bouncing solutions where obtained considering a generalization of Maxwell electrodynamics, given by nonlinear local covariant and gauge-invariant terms which depend on field invariants up to second order, as the source of classical Einstein's equations. This modification is expected to be relevant when the fields reach large values, as occurs in the early universe. One expects that such modifications may also be important inside the hypothetical throat of a wormhole, generated by this source.

The generalization of Maxwell electrodynamics, up to second order, is determined by the following Lagrangian:[26]

$$L = -\frac{1}{4}F + \alpha F^2 + \beta G^2\,, \tag{39}$$

where

$$F := F_{\mu\nu}F^{\mu\nu}\,,$$

and

$$G := \frac{1}{2}\eta_{\alpha\beta\mu\nu}F^{\alpha\beta}F^{\mu\nu}\,,$$

where α and β are arbitrary constants and $\eta_{\alpha\beta\mu\nu}$ is a totally antisymmetric tensor. According to that same reference, in the early universe, the source should be identified with a hot primordial plasma. The high temperature of the plasma suggest we model the electromagnetic field as a statistically isotropic random field. In order to do that, a spatial average should be taken into account. This average procedure was introduced in Ref. 44 and used in many situations in cosmology.[26,45–48] It can be done by associating a reference frame adapted to the coordinates and defined by the velocity vector field $V^\mu = (g_{00})^{-1/2}\delta_0^\mu$. This reference frame defines the space where the averaged procedure is done given the following mean values to the electric ($\boldsymbol{E}$) and magnetic ($\boldsymbol{H}$) fields:[44]

$$\overline{E_\mu} = 0\,, \quad \overline{H_\mu} = 0\,, \quad \overline{E_\mu H_\nu} = 0\,, \tag{40}$$

$$\overline{E_\mu E_\nu} = -\frac{1}{3}E^2 h_{\mu\nu}\,, \tag{41}$$

$$\overline{H_{i\mu} H_\nu} = -\frac{1}{3}H^2 h_{\mu\nu}\,, \tag{42}$$

The tensor $h_{\mu\nu} = g_{\mu\nu} - V_\mu V_\nu$ is the metric on the hypersurface perpendicular to the vector field V_μ. The average is done according to the definition

$$\bar{C} \equiv \lim_{V\to V_0} \frac{1}{V}\int C\sqrt{-h}d^3x^i\,, \tag{43}$$

to an arbitrary quantity C, in which $V = \int \sqrt{-h}d^3x^i$ and V_0 represents a sufficiently large three-volume in order to allow the average procedure but small compared to the other relevant length in the model.

The high temperature of the plasma led us to consider the situation where only the squared magnetic field H^2 survives and the electric field is put to $E^2 = 0$. The resulting averaged energy–momentum tensor is identified as a perfect fluid with modified expressions for the energy density ρ and pressure p given by

$$\rho = \frac{H^2}{2}(1 - 8\alpha H^2)\,, \tag{44}$$

$$p = \frac{H^2}{6}(1 - 40\alpha H^2)\,. \tag{45}$$

This source violates the energy condition $\rho + 3p \geq 0$, as in the models presented before. Nonetheless, it is worth to call attention that, in this case, the pressure is isotropic. We will look for a static solution in a spherical geometry given by the metric (7). The field equations become

$$b' = \rho r^2\,, \tag{46}$$

$$\phi' = \frac{b + pr^3}{2r^2\left(1 - \frac{b}{r}\right)}\,, \tag{47}$$

$$p' = -(\rho + p)\phi'\,. \tag{48}$$

Eliminating ϕ' from these equations we obtain

$$b' = \rho r^2\,, \tag{49}$$

$$p' = -(\rho + p)\frac{(b + pr^3)}{2r^2\left(1 - \frac{b}{r}\right)}\,. \tag{50}$$

The boundary conditions at the wormhole's throat, r_0, are

$$b_0 \equiv b(r_0) = r_0\,, \qquad b'_0 \equiv b'(r_0) \le 1\,, \tag{51}$$

$$p_0 \equiv p(r_0) = -\frac{1}{r_0^2}\,, \qquad p'_0 \equiv p'(r_0) = \frac{b'_0 - 3}{r_0^3} < 0\,, \tag{52}$$

which imply for $X(r) \equiv H^2(r)$:

$$X_0 \equiv X(r_0) = \frac{1 + \sqrt{1 + \gamma}}{80\alpha} \ge \frac{1}{16\alpha}\,, \qquad X'_0 \equiv X'(r_0) > 0\,, \tag{53}$$

where $\gamma = 960\alpha/r_0^2$.

Equations (49) and (50) become

$$b' = \frac{X}{2}\,(1 - 8\alpha X)r^2\,, \tag{54}$$

$$X' = \frac{X(16\alpha X - 1)\left(40\alpha X^2 r - Xr - \frac{6b}{r^2}\right)}{\left(1 - \frac{b}{r}\right)(80\alpha X - 1)}\,, \tag{55}$$

and any solution to it with the above boundary conditions has the property that the radial coordinate possesses a minimum value at r_0. However, for this case, these boundary conditions turn the solution completely unacceptable as we move away from the throat. We can see this by examining the behavior of $X(r)$. This function begins with a positive value and increases, so it must reach a maximum value, $X_m \equiv X(r_m) > X_0$, at $r_m > r_0$ if we want the functions $p(r)$, $\rho(r)$, $b(r)$ and $\phi(r)$ to remain finite. Otherwise, the resulting space–time would not be acceptable. Imposing that $b(r)/r$ be finite (requirement of asymptotic flatness), we can see by (55) that $X(r)$ will only stop increasing if one of these equations holds true:

$$X_m = 0\,, \tag{56}$$

$$16\alpha X_m - 1 = 0\,, \tag{57}$$

$$40\alpha X_m^2 r_m - X_m r_m - 6\frac{b_m}{r_m^2} = 0\,, \tag{58}$$

with $b_m \equiv b(r_m)$.

It is clear that the first one cannot be true, for it would imply that $X_m < X_0$. This same argument discards the second one, which states that $X_m = 1/16\alpha \le X_0$ and not greater. The last one gives

$$X_m = \frac{1 + \sqrt{1 + 960\alpha\frac{b_m}{r_m^3}}}{80\alpha} = \frac{1 + \sqrt{1 + \gamma\frac{r_0^2 b_m}{r_m^3}}}{80\alpha}\,, \tag{59}$$

which is also smaller than X_0, since it is given by (53), and $b_m \leq r_m$ and $r_0 < r_m$ implies that $r_0^2 b_m / r_m^3 < 1$.

Therefore, $X(r)$ diverges positively, leading the quantities $p(r)$, $\rho(r)$, $b(r)$ and $\phi(r)$ to diverge negatively, which would imply in a infinitely negative mass and a horizon for large values of r. The only case in which $X(r)$ (and the other functions) can stop increasing is if $b(r)$ increases in such a way that $b(r)/r$ do also diverges and makes $X'(r) = 0$, which is not compatible with the requirement of asymptotic flatness. Hence we can conclude that any attempt to impose a minimum value for the radial coordinate in this case results in a too much problematic space–time for it to be considered a satisfactory solution. This case does not allow a reasonable wormhole solution.

5. Conclusion

In this paper we have investigated the existence of wormhole solutions in theories which produce cosmological bounces. We have shown that in one of them, concerning nonlinear electrodynamics, no wormhole solution is possible, showing that it is not mandatory that violation of WEC, or theories which produce cosmological bounces, can also result in wormholes.

In the other two theories we investigated, we were able to obtain simple, traversable (in the sense of Ref. 22) wormhole solutions with very nice properties. Both depend on a parameter r_0, which cannot be very big in order to not impose unattainable speeds or a very long time to traverse the wormhole (see Eqs. (22) and (38)).

In the case of Sec. 2, nonminimal coupling between gravity and a vector field, one arrives at the nice situation that no vector field free energy (the one contained in Eq. (4)) is required to produce the wormhole. The parameter r_0 appears only in the denominator of the amplitude of the vector potential (see (16) and the definition of Υ). However, as there is another arbitrary constant present in (16), one can adjust it to satisfy physical requirements while preserving values of r_0 compatible with traversability with reasonable speeds.

In the case of Sec. 3, the parameter r_0 appears in the energy density of the field (in the denominator), and in its total mass (in the numerator). Hence, a small r_0 would imply a small total quantity of scalar field, which is good, but a high concentration of the field in the throat. However, if this field does not interact with matter and other fields, the traveler could pass through the throat without any harm.

Finally, we would like to point out that the solutions obtained here are all static, as if those wormholes always existed. In order to see if they can really arise, it is necessary to consider a dynamical geometry. The resulting equations and solutions should then show if some initial configuration could evolve into a wormhole, and if it will be classically and quantum mechanically stable. Situations where stability is lost in different scenarios are presented in Ref. 49. It should also be possible to investigate if during the bouncing period, where WEC is only preserved when spacial sections have positive curvature, wormholes would have time to arise or not.

We have shown that the existence of static wormholes is indeed a perfectly plausible theoretical possibility, in reasonable theoretical frameworks. Their production in the history of the universe or even in the laboratory depends on dynamical developments of the present investigation, which will be the subject of our future investigations.

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