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Nonextensive distributions of rotation periods and diameters of asteroids

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ABSTRACT

Context. To investigate the distribution of periods of rotation of asteroids of different regions of the Solar System and distribution of diameters of near-Earth asteroids (NEAs).

Aims. Verify if nonextensive statistics satisfactorily describes the data.

Methods. Data was taken from Planetary Database System (PDS) with Rel ≥ 2 (total of 3567 asteroids). Taxonomic class and region of the Solar System was also considered. Data for NEA was taken from Minor Planet Center (total of 7078 NEAs).

Results. The periods of rotation of asteroids follow a q-gaussian with $q = 2.6$ regardless of taxonomy, diameter or region of the Solar System of the object. The diameters of NEAs are described by a q-exponential with $q = 1.3$. According to this distribution, there are expected to be 1240 ± 180 NEAs with diameters greater than 1 km.

Key words. asteroids – rotation periods – diameters – nonextensivity

1. Introduction

Asteroids and comets are primordial bodies of the Solar System (SS). The study of the physical properties of these objects may lead to a better understanding of the processes of formation of the SS, and, by inference, of the hundreds of exo-Solar systems already known. Distribution of rotation periods and diameters of asteroids are two parameters that may give peaces of information concerning the collisional evolution of the SS. Pravec & Harris (2000) have analyzed a sample with 984 objects and have argued that the distribution of rotation periods of asteroids with diameters $D \geq 40$ km is maxwellian, with 99% of confidence. They suggest that objects within this diameter range are primordial, or originated from collisions of primordial bodies. It is known that for small and very small asteroids ($D < 10$ km), the distribution of rotation periods is not maxwellian. The analysis of the data suggests the existence of a spin-barrier for the asteroids with diameters between hundreds of meters and 10 km and with less than 11 rotations per day (period of about 2.2 h) (Pravec, Harris & Warner 2007). The absence of a substantial quantity of asteroids with period less than 2.2 h may be due to the low degree of internal cohesion of these objects. The majority of the sample may contain rubble pile asteroids (Davis et al. 1979, Harris 1996) that are composed by fragments of rocks kept together by self-gravitation. For objects below 0.2 km it is observed rotation periods smaller than the spin-barrier, suggesting that these objects have a high internal cohesion, implying that they may be monolithic bodies. The difficulty in the modelling of periods of rotation of asteroids as a whole may be associated to the combined action of many mechanisms such as collisions (Paolicchi, Burns & Weidenschilling 2002), gravitational interactions with planets (Scheeres, Marzari & Rossi 2004), angular momentum exchange in binary or multiple asteroid systems (Bottke et al. 2002), or torques induced by solar radiation, known as YORP effect (from Yarkovsky–O'Keefe–Radzievskii–Paddack) (Rubincam 2000). Particularly, YORP effect is strongly dependent on the shape and size of the object and its distance to the Sun.

Near-Earth Asteroids (NEAs) is a subgroup of SS asteroids whose heliocentric orbits lead them close to the Earth's orbit. More than 7000 NEAs are known up to 2011. The study of these objects is relevant once it may bring information regarding the birth and dynamic evolution of the SS. Also a special interest in these objects is related to the possibility of collision with the Earth with obvious catastrophic consequences (Alvarez et al. 1980). They also may be potential sources of raw material for future space projects.

The evaluation of the number of asteroids per year that may reach the Earth as a function of their diameters is essential for the determination of the potential risk of a collision. One of the first attempts to estimate this flux was done by Shoemaker et al. (1979).

The impact flux may be taken from the accumulated distribution of diameters of the NEAs. This distribution is indirectly obtained by the current asteroid surveys, according to the absolute magnitude H. The distribution of absolute magnitude H is described by Michelson (2004),

$$\log N = \alpha H + \beta, \tag{1}$$

where N is the number of objects and α and β are constants. The diameters of asteroids D may be given in function of their absolute magnitudes and their albedos p_v according to Bowell et al. (1989)

$$D = 1329\frac{10^{-H/5}}{\sqrt{p_v}}. \tag{2}$$

The albedo is the rate of superficial reflection and its value is essential for the estimation of the diameters of the asteroids. The values of the albedos of asteroids varies according to the superficial mineralogical composition (taxonomic complex) and object's shape. Typical values range from 0.06 ± 0.02 for low albedo objects of C taxonomic complex up to 0.46 ± 0.06 for high albedo objects of V type (Warner, Harris & Pravec 2009).

Combination of equations (1) and (2) leads to a power law behavior:

$$N(> D) = kD^{-b}. \tag{3}$$

The parameters are estimated by Stuart (2003), $b = 1.95$, $k = 1090$, and D is given in km. According to this expression and taking into account the uncertainties of the measures, Stuart & Binzel (2004) estimated that there may exist 1090 ± 180 objects with diameters equal to or greater than 1 km.

2. Nonextensive statistics

In order to model the accumulated distribution of periods and diameters of asteroids, we have applied results from Tsallis nonextensive statistics. This choice comes from observational evidences that astrophysical systems are somehow related to nonextensive behavior. It is known that system with long-range interactions (typically gravitational systems) are not properly described by Boltzmann-Gibbs statistical mechanics (Landsberg 1990). Along the last two decades it has been continuously developed the nonextensive statistical mechanics that is a generalization of Boltzmann-Gibbs statistical mechanics. Tsallis proposed in 1988 (Tsallis 1988) a generalization of the entropy,

$$S_q = k\frac{1 - \sum_i^W p_i^q}{q - 1}, \tag{4}$$

where p_i is the probability of the i-th microscopical state, W is the number of states, k is a constant (Boltzmann's constant) and q is the entropic index. As $q \to 1$ S_q is reduced to Boltzmann-Gibbs entropy $S_1 = -k\sum_i^W p_i \ln p_i$. It was soon realized that the nonextensive statistical mechanics could be successfully applied to self-gravitating systems: Plastino & Plastino (1993) found a possible solution to the problem of the existence of a self-gravitating system with total mass, total energy and total entropy simultaneously finite, within a nonextensive framework. Many examples of nonextensivity in astrophysical systems may be found. We list some instances. Nonextensivity was observed in the analysis of magnetic field at distant heliosphere associated to the solar wind observed by Voyager 1 and 2 (Burlaga & Vinhas 2005, Burlaga & Ness 2009 Burlaga & Ness 2010). The distribution of stellar rotational velocities in the Pleiades open cluster was found to be satisfactorily modelled by a q-maxwellian distribution (Soares et al. 2006). The problem of Jeans gravitational instability was considered according to nonextensive kinetic theory (Lima, Silva & Santos 2002). Nonextensive statistical mechanics was also used to describe galaxy clustering processes (Wuensche et al. 2004) and temperature fluctuation of the cosmic background radiation (Bernui, Tsallis & Villela 2006, Bernui, Tsallis & Villela 2007). Fluxes of cosmic rays can be accurately described by distributions that emerge from nonextensive statistical mechanics (Tsallis, Anjos & Borges 2003). A list of more instances of applications of nonextensive statistical mechanics in astrophysical systems may be found in Tsallis (2009).

Maximization of S_q under proper constraints leads to a distributions that are generalizations of those that appear within Boltzmann-Gibbs context. For instance, if it is required that the (generalized) energy of the system is constant (Curado & Tsallis 1991) then the probability distribution that emerges is a q-exponential,

$$p(x) \propto \exp_q(-\beta_q x), \tag{5}$$

β_q is the Lagrange multiplier. The q-exponential function is defined as (Tsallis 1994)

$$\exp_q x = [1 + (1 - q)x]_+^{\frac{1}{1-q}}. \tag{6}$$

The symbol $[a]_+$ means that $[a] = a$ if $a > 0$ and $[a] = 0$ if $a \leq 0$. The q-exponential is a generalization of the exponential function, that is recovered if $q \to 1$. If the constraint imposes that the (generalized) variance of the distribution is constant, then the distribution that maximizes S_q is the q-gaussian (Tsallis et al. 1995, Prato & Tsallis 1999),

$$p(x) \propto \exp_q(-\beta_q x^2). \tag{7}$$

The q-gaussian recovers the usual gaussian at $q = 1$, and particular values of the entropic index q turn $p(x)$ into various known distributions, as Lorentz distribution, uniform distribution, Dirac's delta (see Tsallis et al. 1995, Prato & Tsallis 1999, for details).

We have found that the distribution of diameters of NEAs follows a q-exponential and the observed rotation periods of all asteroids, regardless of their diameters, mineralogical composition or region of the SS, are well approximated by a q-gaussian.

3. Observational data

In order to obtain a distribution of asteroids rotation periods we used the compilation of 4127 periods available at the Planetary Database System (PDS) (Harris, Warner & Pravec 2010). In this sample the periods are classified according to a quality code of the reliability of the estimated period, defined by Harris & Young (1983). We have used periods with Rel ≥ 2 (Rel from reliability) that means they are accurate to ≈ 20% which resulted in 3567 asteroids taken into account. Cross-checking the samples with a compilation of taxonomic classifications, also available at PDS, has revealed that about 40% of these asteroids have approximately known mineralogical composition. The 1487 asteroids with taxonomic classification have been separated into three main classes: C , S and X complexes following the SMASS II system of Bus & Binzel (2002), with respectively 503, 663 and 321 objects. The diameters of these sub-samples have been calculated with Eq. (2) with the absolute magnitude H available from MPCORB – Minor Planet Center Orbit Database (MPC 2010).

Absolute magnitudes of 7078 NEAs were obtained from MPCORB. We have adopted $p_v = 0.14 \pm 0.02$ for the NEAs population albedo. This value was found by Stuart & Binzel (2004) and it takes into account the great variety of taxonomic groups that are found in the NEAs (Binzel et al. 2004). In order to estimate the validity of the diameters estimated by Eq. (2), we have considered the diameters of 101 asteroids obtained from Spitzer Space Telescope data (Trilling et al. 2010). This resulted in about 20% of error. We considered that this value, though not small, is reasonable for the purposes of our study.

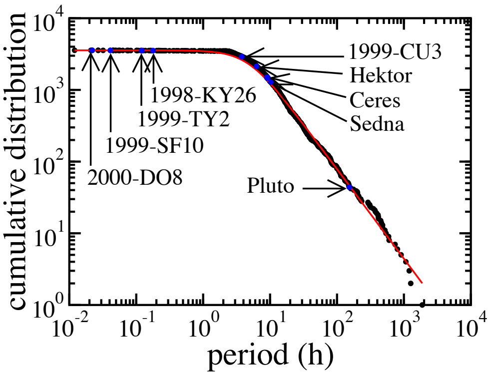

Fig. 1. Log-log plot of the decreasing cumulative distribution of periods of 3567 asteroids (dots) with Rel ≥ 2 taken from the PDS (NASA) and a q-gaussian distribution (solid line), Eq. (7), with $q = 2.6, \beta = 0.028$ h^{-2} and the normalizing constant $M = 3567$. Particular asteroids are indicated with blue dots (see Tables 1 and 2).

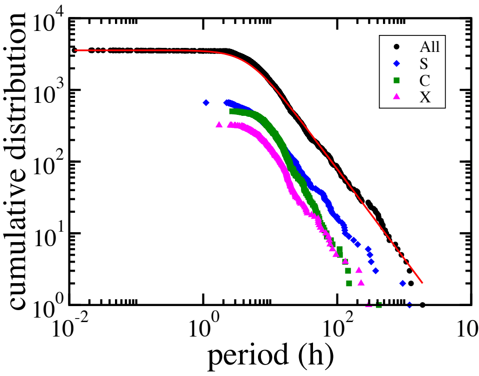

Fig. 2. Log-log plot of the cumulative distribution of periods of 3567 asteroids (circles) with Rel ≥ 2, and 663 S-complex asteroids (diamonds, blue on-line), 503 C-complex asteroids (squares, green on-line), 321 X-complex asteroids (triangles, magenta on-line). q-Gaussian of the fitting for all asteroids is also displayed (red curve).

Table 1. Asteroids separated by the region in the Solar System. Data from PDS (Harris, Warner & Pravec 2010).

Object	Period (h)	Region
1999 CU3	3.7829	NEA
Hektor	6.294	JT
Ceres	9.074170	MB (dwarf planet)
Sedna	10.273	TNO
Pluto	153.2935	TNO (dwarf planet)

Table 2. Small fast rotators NEAs at the approximately flat region of Fig. 1. Data from Binzel et al. (2002).

Object	H	Diameter (km)	Period (h)
1998 KY26	25.6	0.04	0.1784
2000 DO8	24.8	0.04	0.022
1999 SF10	24.0	0.06	0.0411
1999 TY2	23.1	0.08	0.1213

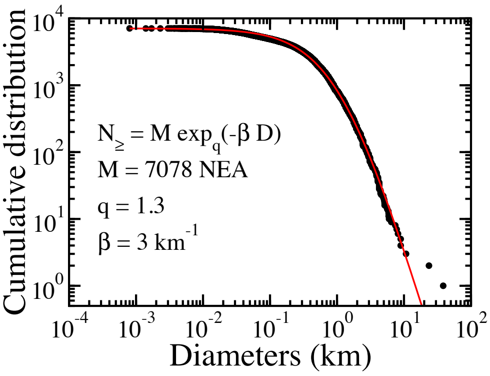

Fig. 3. Decreasing cumulative distribution of diameters of NEAs (dots) in log-log plot. Red curve is a q exponential, Eq. (5), with $q = 1.3, \beta = 3$ km^{-1}, and $M = 7078$ is the number of NEAs.

4. Distribution of periods

Fig. 1 shows the decreasing cumulative distribution of 3567 periods and a superposed q-gaussian ($N_{\geq}(h) = M \exp_q(-\beta_q x^2)$) and it is seen that it quite satisfactorily describes almost all the data with $q = 2.6$ and $\beta = 0.028$ h^{-2} ($M = 3567$ is the number of objects). This suggests that the distribution does not depend on (i) the diameters; (ii) the mineralogical composition; (iii) the region of the SS in which the object is found. The latter is particularly important once the sample includes NEAs, trans-netunian objects (TNO), asteroids from the main belt (MB), Jupiter Trojans (JT) and dwarf planets like Ceres and Pluto. Some asteroids are identified in the Figure and in Tables 1 and 2. The value of the entropic index $q = 2.6$ — rather distant from unit, that is, distant from the Maxwellian distribution — indicates that long-range interactions play an essential role in the distribution of periods of rotation.

The spin-barrier period of 2.2 h is close to the intersection of the asymptotic power-law and the approximately flat region of Fig. 1. This result suggests that in the approximately flat region we may find monolithic fast-rotating asteroids (Whiteley, Hergenrother & Tholen 2002) with absolute magnitude $H > 23$ and $D < 0.2$ km. Some of these asteroids are identified in Fig. 1 and in Table 2. Consequently in the asymptotic power-law we probably find gravitational bound objects.

The distributions for each complex taken separately (C, S, X) is shown in Fig. 2. They can be satisfactorily approximated by q-gaussians ($q = 2.7$ for S, $q = 2.0$ for C and $q = 2.0$ for X). For C and X complexes, the fittings can be improved if we admit that the number of objects are 10% higher than that found in the sample.

5. Distribution of Diameters of near-Earth asteroids

The decreasing cumulative distribution of diameters of NEAs is not described by Eq. (3) once it is only asymptotically a power law. We have tried to fit a q-exponential to the diameters of 7078 NEAs ($N_{\geq}(D) = M \exp_q(-\beta_q D)$) and the result, shown in Fig. 3, is quite good for the entire range of the data.

The value of $q = 1.3$ (different from one) is an indication that not only collisional processes are implied in the formation of these objects. Other mechanisms may also be present: the YORP effect may lead to the decrease of the period of rotation up to the point of rupture. This fragmentation process may yield the formation of binary or multiple systems. About (15 ± 4)% of NEAs with $D \geq 0.3$ km and periods of rotation between 2 and

3 h possibly are binary systems (Pravec, Harris & Warner 2007). According to the distribution we have found there are 1240 ± 180 NEAs with $D \geq 1$ km, that is superior than the 1090 asteroids found by Stuart & Binzel (2004). The analysis of the distributions of diameters of MBA and TNO are welcome.

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References

Alvarez, L. W., Alvarez, F., Asaro, F., Michel, H. V. 1980, Science, 208, 1095
Bernui, A., Tsallis, C., Villela, T. 2006, Phys. Lett. A, 356, 426
Bernui, A., Tsallis, C., Villela, T. 2007, Europhys. Lett. 78, 19001
Binzel, R. P., Lupishko, D., di Martino, M., Whiteley, R. J., Hahn, G. J. 2002, in Asteroids III. Bottke, W. F., Cellino, A., Paolicchi, P., Binzel, R. P. (eds.), Univ. Arizona Press, Tucson, p. 255
Binzel, R. P., Rivkin, A. S., Stuart, J. S., Harris, A. W., Bus, S. J., Burbine, T. H. 2004, Icarus, 170, 259
Bottke, W. F., Morbidelli, A., Jedicke, R., Petit, J.-M., Levison, H. F., Michel, P., Metcalfe, T. S. 2002, Icarus, 156, 399
Bowell, E., Hapke, B., Domingue, D., Lumme, K., Peltoniemi, J., Harris, A. W. 1989, in Asteroids II. Binzel, R. P., Gehrels, T., Matthews, M. S. (eds.), Univ. Arizona Press, Tucson, p. 524
Burlaga, L. F., Vinas, A. F. 2005, Physica A, 356, 375.
Burlaga, L. F., Ness, N. F. 2009, Astrophys. J. 703, 311
Burlaga, L. F., Ness, N. F. 2010, Astrophys. J. 725, 1306
Bus, S. J., Binzel, R. P. 2002, Icarus, 158, 146
Curado, E. M. F., Tsallis, C. 1991, J. Phys. A 24, L69; Corrigenda: 1991, 24, 3187 and 1992, 25, 1019
Davis, D. R., Chapman, C. R., Greenberg, R., Weidenschilling, S. J., Harris, A. W. 1979, in Asteroids. Gehrels, T. (ed.), Univ. Arizona Press, Tucson, p. 253
Harris, A. W., Young, J. W. 1983, Icarus, 54, 59
Harris, A. W. 1996, in Lunar and Planetary Institute Conference Abstracts, 27, Lunar and Planetary Inst. Technical Report, 493
Harris, A. W., Warner, B. D., Pravec, P. (eds) 2010, Asteroid Lightcurve Derived Data V11.0. EAR-A-5-DDR-DERIVED-LIGHTCURVE-V11.0. NASA Planetary Data System
Landsberg, P. T. 1990, Thermodynamics and Statistical Mechanics (Dover)
Lima, J. A. S., R. Silva, R., Santos, J. 2002, A&A 396, 309
Michelson, R. 2004, PhD thesis, University of Copenhagen
Minor Planet Center 2010, MPCORB, `http://www.minorplanetcenter.org/iau/MPCORB.html`
Paolicchi, P., Burns, J. A., Weidenschilling, S. J. 2002, in Asteroids III. Bottke, W. F., Cellino, A., Paolicchi, P., Binzel, R. P. (eds.), Univ. Arizona Press, Tucson, p. 517
Plastino, A. R., Plastino, A. 1993, Phys. Lett. A 174, 384
Prato, D., Tsallis, C. 1999, Phys. Rev. E 60, 2398
Pravec, P., Harris, A. W. 2000, Icarus, 148, 12
Pravec, P., Harris, A. W., Warner, B. D. 2007, in Proceedings if IAU Symposium 236, Near Earth Objects, our Celestial Neighbors: Opportunity and Risk, eds. Valsecchi, G. B., Vokrouhlicky, D. (eds), Cambridge University Press, Cambridge, p. 167
Rubincam D. P. 2000, Icarus, 148, 2
Scheeres, D. J., Marzari, F., Rossi, A. 2004, Icarus, 170, 312
Shoemaker, E. M., Williams, J. G., Helin, E. F., Wolfe, R. F. 1979, in Asteroids. Gehrels, T. (ed.), Univ. Arizona Press, Tucson, p. 253
Soares, B. B., Carvalho, J. C., do Nascimento, J. D., Jr., de Medeiros, J. R. 2006, Physica A 364, 413
Stuart, J. S. 2003, PhD thesis, MIT
Stuart, J. S., Binzel, R. P. 2004. Icarus, 170, 295
Trilling, D. E., Mueller, M., Hora, J. L., Harris, A. W., Bhattacharya, B., Bottke, W. F., Chesley, S., Delbo, M., Emery, J. P., Fazio, G., Mainzer, A., Penprase, B., Smith, H. A., Spahr, T. B., Stansberry, J. A., Thomas, C. A. 2010, AJ, 140, 770
Tsallis, C. 1988, J. Stat. Phys. 52, 479
Tsallis, C. 1994, Quimica Nova 17, 468
Tsallis, C., Levy, S. V. F., de Souza, A. M. C., Maynard, R. 1995, Phys. Rev. Lett, 75, 3589; erratum: 1996, 77, 5442
Tsallis, C., Anjos, J. C., Borges, E. P. 2003, Phys. Lett. A, 310, 372
Tsallis, C. 2009, Introduction to Nonextensive Statistical Mechanics — Approaching a Complex World (Springer, New York)
Warner, B. D., Harris, A. W., Pravec, P. 2009, Icarus, 202, 134
Whiteley, R. J., Hergenrother, C. W., Tholen, J. J. 2002, Proceedings of Asteroids, Comets, Meteors (ACM 2002) 29 July – 2 August 2002, Technical University Berlin, Berlin, Germany (ESA-SP-500, November 2002)
Wuensche, C. A., Ribeiro, A. L. B,. Ramos, F. M., Rosa, R. R. 2004, Physica A, 344, 743