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# Texture analysis using graphs generated by deterministic partially self-avoiding walks

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ABSTRACT

Texture is one of the most important visual attributes for image analysis. It has been widely used in image analysis and pattern recognition. A partially self-avoiding deterministic walk has recently been proposed as an approach for texture analysis with promising results. This approach uses walkers (called tourists) to exploit the gray scale image contexts in several levels. Here, we present an approach to generate graphs out of the trajectories produced by the tourist walks. The generated graphs embody important characteristics related to tourist transitivity in the image. Computed from these graphs, the statistical position (degree mean) and dispersion (entropy of two vertices with the same degree) measures are used as texture descriptors. A comparison with traditional texture analysis methods is performed to illustrate the high performance of this novel approach.



## 1. Introduction

Texture is a visual attribute widely used to describe patterns and characteristics of images. In fact, it is one of the most important visual attributes for image analysis and pattern recognition. Texture consists of the repetition of a gray-scale or color pattern on an image, or even the lack of repetition or pixel organization. On one hand, the definition of the texture concept can become vague and abstract, which leads to a lack of a formal definition in the literature [1]. On the other hand, its characteristics are straight connected with physical properties of an object surface [2–4], which make textures very attractive for a wide range of applications, such as medical images diagnose [5–7], remote sensing [8], geological images [9], microscope images [10,11], etc.

Texture analysis is supported by a wide variety of different descriptors proposed along the years. There are different approaches to deal with texture, some examples are properties obtained from spectral analysis (e.g., Fourier descriptors [12] and Gabor filters [13]), statistical analysis of the pixels (e.g., co-occurrence matrices [14], local binary pattern [15], feature-based interaction map [16]) and complexity of pixels distribution (e.g., fractal dimension [4,17–18]).

Recently, a partially self-avoiding deterministic walk (deterministic tourist walk, DTW) algorithm has emerged as a very promising approach for texture analysis [20–23]. It considers independent walkers leaving from each pixel to exploit an image characteristics. Each walker moves from one pixel to another according to a deterministic rule and a given memory. This results in partially self-avoiding trajectories, which can be separated into two parts: one, where the walker mainly explores new pixels and the other, where the walker is trapped in an attractor, a cycle of pixels from where the walker cannot escape. Image analysis using the tourist walks is usually performed through statistical analysis over the joint distribution of transient times and attractor periods [20–22]. As a result, the trajectory produced by each tourist is not taken into account during the image analysis step. The attractor and transient lengths are used to build histograms. Here, we present a new concept: instead of focusing on attractor and transient length histograms, we use the generated trajectories to build a graph. The generated graph is capable to characterize the image texture patterns.

This paper starts presenting a review about the deterministic partially self-avoiding walk in Section 2. In Section 3, we show how to build a graph from the trajectories engendered by the tourist, given a walking rule and memory. A signature capable to represent these graph properties and, as a consequence, characteristic from the original image is proposed in Section 4. Experiments using images extracted from the Brodatz album [24] are presented in Section 5. Section 6 presents the obtained results and a discussion. Finally, in Section 7, we conclude and propose future studies.

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## 2. Deterministic partially self-avoiding walk

The partially self-avoiding deterministic walk algorithm [25–29] can be understood as a tourist wishing to visit $N$ cities distributed in a map of $d$ dimension, where each city is visited at each time step according to the rule of walking to the nearest city which was not visited in the last $\mu$ steps. This rule produces a partially self-avoiding trajectory, which can be separated into two parts: a transient part of length $t$ and an attractor final part, which ends in a cycle with period $p$, $p \geq \mu+1$, and from where the tourist cannot escape. From the distribution of transient times and cycle periods one has been able to characterize thesaurus [30] and perform cluster analysis [31]. Also, stochastic versions of this algorithm have been addressed [32–34]. These walks have been used in other contexts, such as the modeling of the searching behavior of social monkeys [35], foraging of primates [36], emerging of power-laws in deterministic walks [37] and exploration of heterogeneous media by deterministic agents [38].

Recently, from these deterministic partially self-avoiding walks algorithm has emerged as a very promising approach for texture analysis [20–22]. Consider a digital image containing $N$ pixels with a gray-level scale ranging from 0 to 255 associated to each pixel. A traveler walks from one pixel to another belonging to its 8-neighborhood, according to the following rule: move to the nearest or furthest neighbor pixel (i.e., the one which differs in minimum or maximum gray-level, respectively, from current position) and that has not been visited in the last $\mu$ ($\mu \in [1,N]$) previous steps. This algorithm has been adapted to deal with color images [39].

Considering each image pixel as a starting point of the tourist walk, the joint distribution of transients $t$ and attractors $p$, $S^{(N)}_{\mu,2}(t,p)$ is achieved (Fig. 1). Studies have been performed over this joint distribution to provide a feasible signature for image textures [20–22]. It is also important to note that, once the walking rule is defined (to move to minimum or maximum difference), the rule must be considered for all pixel and cannot be changed along the trajectory.

## 3. Building graph from walks

Instead of considering the transient time and cycle period joint distribution to achieve a texture signature, we propose a novel approach to use the deterministic partially self-avoiding walk. We focus now on the behavior of the trajectories produced by each tourist during its walk on the texture image. Each trajectory consists of a set of transitions between two pixels performed by the tourist, for a given memory $\mu$ and walking rule. The motion from one pixel to another can be interpreted as a connection between these two pixels in the image. As a result, the trajectories can be used to build a graph, which describes the tourist transitivity and, as a consequence, the attractive regions in the image (attractors). Therefore, we propose to use these walks on gray scale images to generate a graph, which holds information about the texture pattern.

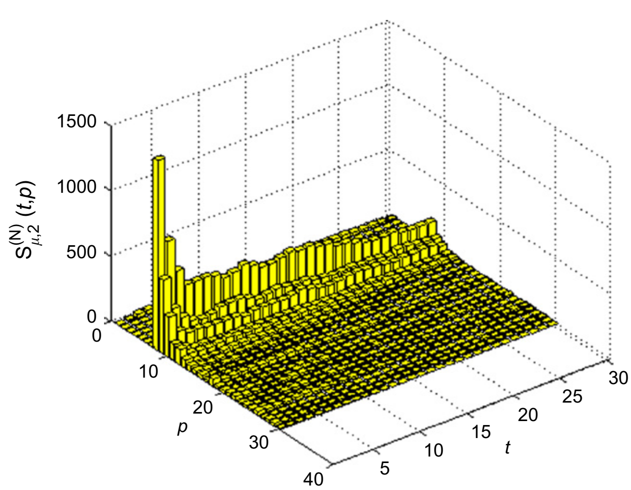


**Fig. 1.** Example of the transient time and cycle period joint distribution computed from the deterministic partially self-avoiding walk on a gray scale image.

Consider a graph $G_{\mu,rule} = (V,E)$, where $\mu$ is the memory and *rule* is the walking rule (minimum or maximum difference) used by the tourist to move over the image. Initially, each image pixel corresponds to a vertex in the graph (i.e., $N=V$) with no edges connecting them ($E=\{\}$). As the tourist moves from pixel $i$ to pixel $j$ on the image (Fig. 2), a non-directed edge $e_{i,j}$ is added to $E$. Note that this is performed only if $e_{i,j} \notin E$, so that, duplicated vertices are not added to the graph.

Taking each image pixel as a starting point for the tourist walk, a graph representing the deterministic self-avoiding trajectories found by the traveler is built (Fig. 2c). None of the vertices is disconnected from the graph. Once these trajectories depend on the gray-level distribution in an image region, vertices connections are altered according to different texture patterns. Thus, the graph comprehends important characteristics concerning the transitivity and attractive image regions. Studying its properties, a feasible signature for texture analysis is proposed as follows.

## 4. Proposed signature

The proposed approach performs texture characterization through properties of the graph generated from the deterministic partially self-avoiding walk. To accomplish this purpose, two graph measurements are considered: the graph *degree* and *joint degree*.

The *degree* of a vertex $v_i$, $d(v_i)$, represents the connectivity of that vertex in the graph. It is defined as the number of edges of the graph bound to $v_i$:

$$d(v_i) = |\{e \in E | v_i \in e\}| = |\{v_j \in V | \{v_i, v_j\} \in E\}| = |\partial v_i|, \tag{1}$$

where $\partial v_i = \{v_j \in S | (v_i, v_j) \in E\}$ represents the set of neighbor of $v_i$ and $|\cdot|$ denotes the cardinality of a set [40].

*Joint degree* is a measure of correlation between the degrees of two vertices connected by an edge [41]. Here we consider the probability of having a vertex $v_i$ connected to another vertex $v_j$. Since both

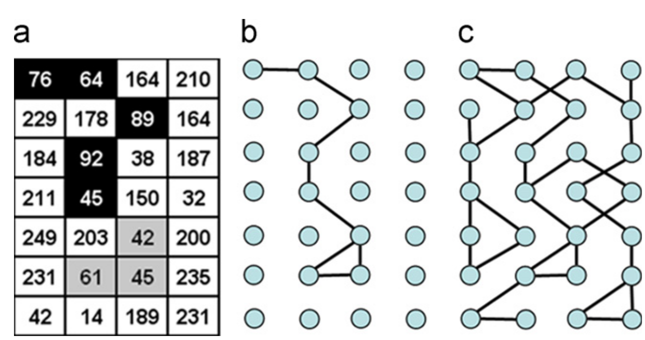


**Fig. 2.** (a) Example of a deterministic partially self-avoiding walk on a gray scale image using minimum contrast difference and the last visited pixel is not allowed ($\mu = 1$) (transient time in black, attractor cycle in gray). A walker leaves from pixel "76" and goes to the one of minimum contrast in the neighborhood "64". From this pixel, the walker repeats the search and find pixel "76", but this pixel is not allowed since it is in the memory window, so that the walker goes to the second minimum contrast pixel "89". The process is repeated, and the walker passes by the pixels "92" and "45" before being trapped the cycle with pixels: "42", "45" and "61". (b) Each pixel in (a) corresponds to a vertex in (b). Edges are added into the graph by following the walk. (c) The full graph is obtained with the tourist leaving from all the pixels.

vertices present the same degree $d(v_i)$, this probability is

$$P(v_i) = \frac{n(v_i)}{d(v_i)}, \tag{2}$$

where

$$n(v_i) = |\{v_j \in V | \{v_i, v_j\} \in E \wedge d(v_i) = d(v_j)\}| \tag{3}$$

is the number of vertices connected to $v_i$ which present its same degree value, $deg(v_i)$. Vertices with the same degree may indicate that the tourist walk has a similar behavior in that region of the graph and, as a consequence, in the texture. Thus, it is interesting to investigate the connection of these vertices.

Different measures can be computed from a graph $G_{\mu,rule}$ analyzing its degree and joint degree distributions over the vertices. For the proposed application, the following measurements were used: the *average degree*

$$D_{\mu,rule} = \sum_{v_i \in V} \frac{d(v_i)}{N}, \tag{4}$$

and the *entropy of the joint degree*

$$H_{\mu,rule} = -\sum_{i=1}^{N} P(v_i) \log_2 [P(v_i)]. \tag{5}$$

The statistical position and dispersion measurements were used to compose two feature vectors. These vectors represent the mean degree distribution and the dispersion of the same degree vertex pairs. They allow us to characterize the texture behavior according to the properties of the graphs generated for tourist walks considering different walking rules and memories:

$$\psi^{(rule)}_{\mu_1,\ldots,\mu_M} = [D_{\mu_1,rule}, D_{\mu_2,rule}, \ldots, D_{\mu_M,rule}], \tag{6}$$

and

$$\varphi^{(rule)}_{\mu_1,\ldots,\mu_M} = [H_{\mu_1,rule}, H_{\mu_2,rule}, \ldots, H_{\mu_M,rule}]. \tag{7}$$

## 5. Experiments

The proposed signatures were evaluated in a texture classification experiment which used images extracted from Brodatz album [24]. This album is a set of texture images widely used as benchmark for texture analysis methods. A total of 1110 images grouped into 111 classes of 10 samples each was considered. Each image has $200 \times 200$ pixels of size and 256 gray-levels, and it represents a subsection of a larger Brodatz image. Fig. 3 shows an example of each Brodatz texture class considered.

From each texture sample considered, the proposed signatures were computed. Statistical analysis of these signatures was carried out applying a linear discriminant analysis (LDA) [42,43]. The LDA is a well-known and supervised method which aims to find a linear combination of descriptors with good discriminative properties. It searches a combination of descriptors where the variance inter-classes is larger than intra-classes. The leave-one-out cross-validation (LOOCV) scheme was also used during the experiment. LOOCV is one of the simplest procedures for training and testing samples. Basically, each sample is used as a testing set, while the remaining samples are defined as training samples [44].

## 6. Results and discussion

### 6.1. Parameters evaluation

At first, each signature was evaluated in order to determine the set of memory values that best characterizes the texture. At this point, we also considered both walking rules: minimum and maximum difference. Tables 1 and 2 show the results obtained for degree and joint degree signatures, respectively.

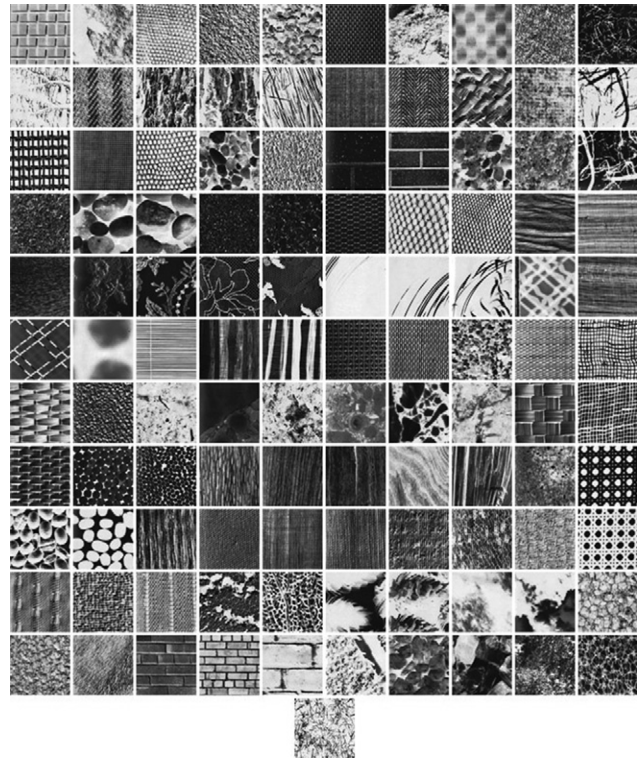

**Fig. 3.** Example of 111 Brodatz texture classes considered.

**Table 1**
Success rate (%) for the $\psi$ signatures combining different $\mu$ values in the Brodatz database.

| Memories used ($\mu$) | $\psi^{(min)}_{\mu_1,\ldots,\mu_M}$ | $\psi^{(max)}_{\mu_1,\ldots,\mu_M}$ | $[\psi^{(min)}_{\mu_1,\ldots,\mu_M}, \psi^{(max)}_{\mu_1,\ldots,\mu_M}]$ |
|---|---|---|---|
| {0, 1} | 29.37 | 39.64 | 71.08 |
| {0, 1, 2} | 44.05 | 60.63 | 80.18 |
| {0, 1, 2, 3} | 53.69 | 72.34 | 85.76 |
| {0, 1, 2, 3, 4} | 59.82 | 75.22 | 86.67 |
| {0, 1, 2, 3, 4, 5} | 60.99 | 77.93 | 88.20 |
| {0, 1, 2, 3, 4, 5, 6} | 63.69 | 79.10 | 87.84 |

**Table 2**
Success rate (%) for the $\varphi$ signatures combining different $\mu$ values in the Brodatz database.

| Memories used ($\mu$) | $\varphi^{(min)}_{\mu_1,\ldots,\mu_M}$ | $\varphi^{(max)}_{\mu_1,\ldots,\mu_M}$ | $[\varphi^{(min)}_{\mu_1,\ldots,\mu_M}, \varphi^{(max)}_{\mu_1,\ldots,\mu_M}]$ |
|---|---|---|---|
| {0, 1} | 24.59 | 34.23 | 59.01 |
| {0, 1, 2} | 27.66 | 54.59 | 68.83 |
| {0, 1, 2, 3} | 30.09 | 61.80 | 72.79 |
| {0, 1, 2, 3, 4} | 32.07 | 66.04 | 74.68 |
| {0, 1, 2, 3, 4, 5} | 33.69 | 69.37 | 76.40 |
| {0, 1, 2, 3, 4, 5, 6} | 34.59 | 69.64 | 77.21 |

In general, success rate yielded by each signature tends to increase as the number of memories considered increases. More memory values selected leads to more descriptors in the computed signature. Moreover, memories of different sizes influence the way the walks are performed by the tourist in an image. Previous studies [21] showed that the memory size affects the number of attractors found, as also the quantity of attractors, in the image. As the memory increases, the number of attractors is reduced, i.e., it is more difficult to find a set of pixels to compose an attractor satisfying the memory and the walking rule used. As a consequence, tourists are forced to perform longer walks to find an attractor. Thus, the use of different memories provides a better exploration of image context, and it enables us to capture texture details in both micro- and macro-scales. This mechanism improves the capacity of discrimination of the proposed signatures.

Independently of the memory set considered, descriptors computed from graphs generated using the maximum difference in the tourist walk achieve a better performance than the minimum one. An explanation for this behavior lies in the fact that tourists guided to the minimum difference tend to locate attractors where the homogeneity in the image is higher, thus avoiding remarkable characteristics of the image, such as edges. Otherwise, tourists guided to the maximum difference search for attractor where the changes in image are more abrupt (illumination and texture pattern changes or presence of edges). This difference between walking rules is reflected on the generated graph and also on its properties. Thus, these signatures emphasize different texture characteristics and, therefore, they present different performances.

Once different walking rules generate signatures with distinguished characteristics of the image (homogeneous and heterogeneous regions), it is convenient to consider the concatenation of the signatures computed for both rules in the image classification. As expected, the results show that this approach yields a signature with superior performance, independent on the graph property (degree or joint degree) considered.

With respect to the graph characteristic used to compose a signature, degree signatures ($\psi$) present a superior performance than joint degree signatures ($\varphi$). This difference of performances indicates that degree measures are more effective than measures based on probability distributions (joint degree) in the characterization of graph based structures. As the nature of these signatures is different, it can be convenient to combine them. Table 3 presents the results for this signature combination. The results show, as expected, an increase in the success rate.

However, as the number of memories increases, smaller is the increase in the success rate, which may indicate that this approach is more effective only when few memory values are considered. However, the more memory values we use, the smaller is the increase in the success rate. This may indicate that this approach is more effective only when few memory values are considered.

### 6.2. Comparison with other texture analysis methods

To provide a better evaluation of the proposed approach, a comparison with other texture methods found in literature was performed. For this comparison, the best result achieved by our approach was considered. Thus, our signature consists of both graph signatures (degree $\psi$ and joint degree $\varphi$ signatures), each computed for the memory set $\mu = \{0,1,2,3,4,5\}$ and for both minimum and maximum walking rules. This makes a total of 24 descriptors.

**Table 3**
Success rate (%) for the $\psi$ and the $\varphi$ signatures combining different $\mu$ values in the Brodatz database.

| Memories used ($\mu$) | $[\psi^{(min)}_{\mu_1,\ldots,\mu_M}, \psi^{(max)}_{\mu_1,\ldots,\mu_M}, \varphi^{(min)}_{\mu_1,\ldots,\mu_M}, \varphi^{(max)}_{\mu_1,\ldots,\mu_M}]$ |
|---|---|
| {0, 1} | 85.49 |
| {0, 1, 2} | 87.48 |
| {0, 1, 2, 3} | 89.55 |
| {0, 1, 2, 3, 4} | 90.00 |
| {0, 1, 2, 3, 4, 5} | 91.89 |
| {0, 1, 2, 3, 4, 5, 6} | 91.80 |

The methods used in this comparison experiment are: co-occurrence matrices [14], Fourier descriptors [12], Gabor filters [13,45,46] and multilevel fractal dimension [4]. The following paragraphs briefly describe these texture descriptors.

*Co-occurrence matrices*: Each matrix represents the joint probability distributions between the gray-level values of pairs of pixels at a pre-determined distance and orientation. The feature vector is composed of energy and entropy values computed from the matrices obtained for distances of 1 and 2 pixels, with angles of $-45^{\circ}$, $0^{\circ}$, $45^{\circ}$, $90^{\circ}$, what makes a total of 16 descriptors. A non-symmetric implementation of the matrices is considered.

*Fourier descriptors*: A vector containing 99 coefficients is computed from the image spectrum, which is obtained through Fourier transform applied over the input image. Each coefficient is the sum of the spectrum absolute values from a given radial distance from the center of the transformation.

*Gabor filters*: Each filter is a bi-dimensional gaussian function moduled with an oriented sinusoid in a determined frequency and direction. In this paper, a total of 16 filters (four rotation and four scale filters), with 0.01 for lower frequency and 0.3 for upper frequency were used. Feature vector is composed of energy values computed from the image resulting from the convolution of each Gabor filter over the input image.

*Wavelet descriptors*: Four dyadic decompositions with daubechies 4 are performed over a given image using the multilevel 2D wavelet decomposition. Energy, entropy and mean features are measured for horizontal, diagonal and vertical details, what makes a total of 36 features [47–49].

*Multilevel fractal dimension*: It is a complexity-based method which analyzes an image by the complexity changes in its different gray-levels. It uses a set of threshold values to achieve a set of binary versions of the original image. The fractal dimension is estimated from each binary image, thus resulting in a feature vector characteristic for the image. In this paper, a total of 70 thresholds was used.

*Tamura features*: It is a set of six texture features corresponding to human visual perception, developed by Tamura et al. [50]. These features are coarseness, contrast, directionality, line-likeness, regularity, and roughness. The first three features are more correlated with the human perception.

The proposed approach was also compared to the original tourist walk method [22]. This approach computes a texture signature directly from the joint distribution of transient and attractors. Histograms from these walks are computed from the joint probability distribution achieved for different $\mu$ values and walking rules. Thus, descriptors selected from these histograms are combined to compose the feature vector which represents the texture pattern under analysis. A total of four histogram descriptors were considered to compose the feature vectors signature. Memory values $\mu = \{0,1,2,3,4,5\}$, for both minimum and maximum walk rules, were considered to compute the histograms.

Table 4 shows the results obtained by each method. Signatures computed from the graph presented the highest success rate, overcoming all compared methods. On one hand, it is important to note that the proposed approach also uses fewer descriptors than Fourier descriptors and multilevel fractal dimension methods. On the other hand, co-occurrence matrices and Gabor filters methods uses fewer descriptors than our approach. Moreover, Gabor filters presented a success rate close to ours using only 16 descriptors. However, it is interesting to recall the result achieved when using the degree and joint degree signatures, $[\psi^{(min)}_{\{0,1,2,3\}}, \psi^{(max)}_{\{0,1,2,3\}}, \varphi^{(min)}_{\{0,1,2,3\}}, \varphi^{(max)}_{\{0,1,2,3\}}]$, in Table 3. In this case, the signature is composed of only 16 descriptors, and it yields a success rate of 89.55%, a result superior than the one achieved by Gabor filters and using the same number of descriptors, what corroborates the effectiveness of the approach for texture analysis and

**Table 4**
Comparison results for different texture methods.

| Method | Images correctly classified | Success rate (%) |
|---|---|---|
| Co-occurrence matrices | 968 | 87.21 |
| Fourier descriptors | 888 | 80.00 |
| Gabor filters | 992 | 89.37 |
| Wavelet descriptors | 1001 | 90.18 |
| Multilevel fractal dimension | 1016 | 91.53 |
| Histogram based tourist walk [22] | 992 | 89.37 |
| Tamura features | 734 | 66.13 |
| Proposed approach | 1020 | 91.89 |

classification. As the processing bottleneck of the proposed algorithm is the walk over the image and the attractor detection, its computational complexity is the same of the original tourist walk method [22]. In Ref. [22], the computational complexities of the tourist walk were studied and estimated. It depends on the nature of the image and can vary from $O(N^4)$ (worst case) to $O(N^2)$ (best case). The worst case is very rare and occurs when no attractor is found in the trajectories. Since attractors are found in the great majority of trajectories, in practical terms the computational complexities of the algorithm is $O(N^2)$. The algorithm has a good performance. It is not so fast as the multilevel fractal dimension, which is based on the box counting (which can achieve $O(N\log N)$ in an optimized implementation), but its performance is similar to the Fourier transform, wavelet discrete transform and co-occurrence matrix ($O(N^2)$) and better than Gabor ($O(N^2\log N)$), which allowed a competitive time consuming performance to the proposed algorithm.

## 7. Conclusion

We presented a novel approach of texture feature extraction based on deterministic tourist walks and graph theory. The tourist walk is a method which uses a traveler to explore an image according to a given memory, resulting in partially self-avoiding trajectories, which are modeled as a graph. The behavior of this graph depends on the walking rule, memory size and the image context. From the study of the graph degree and joint degree distributions obtained from different walking rules and memories, the proposed approach yields a signature. This signature comprehends important characteristics concerning about the transitivity and attractive regions on an image and was evaluated in an experiment using linear discriminant analysis to classify a set of Brodatz textures. The results show the great potential of the method as a feasible texture analysis methodology.

As for short term future work, we plan to investigate the discrimination power of other features of the graph, such as clustering coefficient, average path length and others [51,52]. Moreover, we intend to investigate the application of the method to texture segmentation, with special interest in the connection among graph vertices, more specifically, groups of vertices that have a high density of edges within them, and thus given origin to community structure in the graph and, as a consequence, different texture regions in the image.

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