Pattern Recognition Letters xxx (2010) xxx-xxx

Contents lists available at ScienceDirect

ELSEVIER



journal homepage: www.elsevier.com/locate/patrec



² Texture analysis based on maximum contrast walker

André Ricardo Backes^a, Alexandre Souto Martinez^b, Odemir Martinez Bruno^{c,*}

^a Instituto de Ciências Matemáticas e de Computação (<u>ICMC–USP</u>), Universidade de São <mark>Paulo,</mark> Av. Trabalhador São Carlense, 400 13560-970, São Carlos, SP, <mark>Brazil</mark> ^b Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto (FFCLRP–USP), Universidade de São <u>Paulo,</u> Avenida Bandeirantes, 3900 14040-901, Ribeirão Preto, SP, <mark>Brazil</mark> ^c Instituto de Física de São Carlos (JFSC–USP), Universidade de São <u>Paulo,</u> Av. Trabalhador São Carlense, 400 13560-970, São Carlos, SP, <u>Brazil</u>

ARTICLE INFO

2 9
10 Article history:
11 Received 13 May 2009
12 Available online xxxx
13 Communicated by Y.J. Zhang
14 Keywords:
15 Texture analysis
16 Image analysis

17 Deterministic walk

18 Agents

6

7

19 Tourist walk 20

31

32 1. Introduction

33 Texture is an important visual attribute which is presented in 34 the most real world images. Although this attribute is naturally 35 processed by natural vision and easily comprehended by humans, 36 there is no formal definition for it. Indeed, textures are complex visual patterns formed by arrangements of pixels, regions or even 37 set of patterns formed by other visual attributes, such as shape or 38 39 color. These patterns can be composed by completely distinct fac-40 tors, such as pixel organization or even its disorganization. In fact, 41 depending of the context, the noise can be considered as a sort of texture. These characteristics of the texture attribute make it spe-42 cial and hard to be well defined. A detailed description of the tex-43 ture perception and its applications to machine vision can be found 44 45 in (Tuceryan and Jain, 1993)

There are many approaches for texture analysis and segmenta-46 47 tion. Some consider different aspects of the visual attribute as also 48 use different mathematics to handle it. Most popular approaches are based on spectral analysis of the image pixels (e.g., Fourier 49 descriptors (Azencott et al., 1997), Wavelets and Gabor filters (Jain 50 and Farrokhnia, 1991)), statistical analysis of the pixels (e.g., co-51 52 occurrence matrices (Haralick, 1979), local binary pattern, fea-53 ture-based interaction map (Chetverikov, 1999) and complexity 54 analysis by fractal dimension (Chaudhuri and Sarkar, 1995; Emer-55 son et al., 1999; Kasparis et al., 2001).

ABSTRACT

Recently, the deterministic tourist walk has emerged as a novel approach for texture analysis. This method employs a traveler visiting image pixels using a deterministic walk rule. Resulting trajectories provide clues about pixel interaction in the image that can be used for image classification and identification tasks. This paper proposes a new walk rule for the tourist which is based on contrast direction of a neighborhood. The yielded results using this approach are comparable with those from traditional texture analysis methods in the classification of a set of Brodatz textures and their rotated versions, thus confirming the potential of the method as a feasible texture analysis methodology. Q1

© 2010 Published by Elsevier B.V.

22

23

24

25

26

27

28

29

30

56

57

58

59

60

61

62

63

64

65

66

67

68

69

70

71

72

73

74

75

76

77

78

79

80

81

82

83

84

85

Recently, we have proposed a novel approach to texture analysis based on deterministic walks (Campiteli et al., 2006; Backes et al., 2006), which overcomes the most popular and state of art texture analysis methods, specially for uniform biological textures (Backes et al., 2010). Although it is not so thoroughly investigated as random walks on regular latices and random media (Fisher, 1984; Metzler and Klafter, 2000; Derrida, 1997), deterministic walks in regular (Freund and Grassberger, 1992; Bunimovich and Troubetzkoy, 1992; Gale et al., 1995) and disordered media (Bunimovich, 2004) have also presented very interesting results. While the deterministic walk appears in computer science literature as intelligent agents, our approach explores trajectories inside the image using a statistical strategy. Thus, it brings a novel approach to explore walkers in pattern recognition and image analysis.

The deterministic tourist walks (DTW) was introduced in Lima et al. (2001) to study the models of deterministic walks. On images, the DTW is adapted to consider each pixel as a city with 8-connected neighbours. The distance between the cities is determined by the difference in pixel intensity. In this approach, there are some situations where some neighbours may present the same pixel intensity and a rule must be incorporated to choose just one of them. Special situations arise from this choice, and it can compromise with the accuracy of the DTW texture analysis. To correct this, we propose a different approach to model images for the DTW. In the new DTW image analysis, the connection between the pixels is established by vectors and a deterministic rule is determined by the vector arithmetic. It guarantees that there is just one direction for the walker to choose, even when some cities present the same distance. This new model is simple, efficient, and it improves considerably the DTW. These paper details the method

^{*} Corresponding author. Tel.: +55 16 3373 8728; fax: +55 16 3373 9879.

E-mail addresses: backes@icmc.usp.br (A.R. Backes), asmartinez@ffclrp.usp.br (A.S. Martinez), bruno@ifsc.usp.br (O.M. Bruno).

^{0167-8655/\$ -} see front matter \circledcirc 2010 Published by Elsevier B.V. doi:10.1016/j.patrec.2010.05.022

PATREC 4882

31 May 2010

115

116

117

118

119

120

121

2

A.R. Backes et al./Pattern Recognition Letters xxx (2010) xxx-xxx

and presents comparative experiments that demonstrate the
advantages of this approach to the usual DTW and the performance
of the method.

89 This paper starts by presenting an overview of the deterministic 90 tourist walk in Section 2. In Section 3, the method is detailed for image applications as well as the problem of detecting an attractor 91 during a walk. A new walk rule is proposed to improve the algo-92 93 rithm efficiency. In Section 4, a study of the dynamics of the tourist 94 walk on texture images is presented. We also show how to build texture signatures vectors from the transient time and cycle period 95 96 joint probability distributions. Experiments using synthetic and 97 natural texture images are proposed in Section 5. The obtained re-98 sults are presented in Section 6. Finally, in Section 7, conclusions and improvements of the method are discussed. 99

100 **2. Deterministic tourist walk (DTW)**

101 The deterministic tourist walk algorithm can be understood as 102 a traveler wishing to visit N cities distributed on a map of d

dimensions. Starting from a given city, the tourist moves accord-103 ing to the following rule: go to the nearest city, which was not vis-104 ited in the last μ steps (Lima et al., 2001; Stanley and Buldyrev, 105 2001; Kinouchi et al., 2002; Tertariol and Martinez, 2005; Terca-106 riol et al., 2007). This partially self-avoiding walk consists of a 107 transient part of length t (where new cities can be visited) and 108 a final cycle of period $p \ge \mu + 1$, called attractor, and where 109 new cities are not visited any longer (Fig. 1). The tourist's move-110 ments are entirely performed based on its neighbourhood and its 111 trajectory depends on the starting point and memory μ . Trajecto-112 ries which start at different points can end in the same attractor 113 of period *p*. 114

For image applications (d = 2), the tourist walk algorithm considers each pixel as a city in a two-dimensional map. Each pixel interacts only with its 8 nearest neighbor pixels. The tourist moves according to the deterministic rule of going to the pixel which presents the nearest intensity in comparison with the current pixel intensity. Also, this pixel must have not been visited in the preceding μ steps. For a given memory μ , the transient time and cycle

15	4	214	149	127	1	5	4	214	149	127		15	4	214	149	127
90	190	5	174	229	9	0	190	5	174	229		90	190	5	174	229
207	138	77	174	210	20	07	138	77	174	210		207	138	77	174	210
3	238	97	138	164	3	3	238	97	138	164		3	238	97	138	164
35	119	98	38	209	3	5	119	98	38	209		35	119	98	38	209
52	107	128	178	168	5	2	107	128	178	168		52	107	128	178	168
Λ	1em	noru		2		Me	emo	ru =	= {1	$\{5\}$		M	em	oru	= {	4}
		(9)						(h)		.,				(c)	, L	-)
		(a)						(0)						(\mathbf{c})		
15	4	214	149	127	1	5	4	214	149	127	Í	15	4	214	149	127
90	190	5	174	229	9	0	190	5	174	229		90	190	5	174	229
207	138	77	174	210	20	07	138	77	174	210		207	138	77	174	210
3	238	97	138	164	3	3	238	97	138	164		3	238	97	138	164
35	119	98	38	209	3	5	119	98	38	209		35	119	98	38	209
52	107	128	178	168	5	2	107	128	178	168		52	107	128	178	168
M	em	oru	= {	5}	1	Me	emo	ry =	= {7	77}	9 JA	M	emo	ry =	= {9)7}
		(d)	·	,				(e)		,				(f)	L.	,
		(-)						(-)						<u> </u>		
15	4	214	149	127	1	5	4	214	149	127		15	4	214	149	127
90	190	5	174	229	9	0	190	5	174	229		90	190	5	174	229
207	138	77	174	210	20	07	138	77	174	210		207	138	77	174	210
3	238	97	138	164	3	3	238	97	138	164		3	238	97	138	164
35	119	98	38	209	3	5	119	98	38	209		35	119	98	38	209
52	107	128	178	168	5	2	107	128	178	168		52	107	128	178	168
M	eme	ory	= {!	98}		M	emo	ory	= {;	38}			Er	nd w	alk	
		1	10	APP.				(1)	10	2				(.)		

Fig. 1. Example of a tourist walk over an image using $\mu = 1$. (a)–(h) Tourist's current position in red, previous steps in gray; (i) The transient part, t = 4, is in gray, while the attractor part, p = 3, is in black. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

A.R. Backes et al./Pattern Recognition Letters xxx (2010) xxx-xxx

122 period are computed for all starting points of the image, thus 123 resulting in the joint distribution of transient and attractor of the 124 image $S_{2,\mu}^{(N)}(t,p)$, where $S_{2,\mu}^{(N)}(t,p)$ is a bi-dimensional histogram 125 which represents the number of times that a walk presents tran-126 sient size *t* and attractor size *p* when walking on a image contain-127 ing *N* pixels. An example is depicted in Fig. 2.

The joint distribution can efficiently be used as features for im-128 129 age analysis and texture characterization purposes (Backes et al., 130 2006, 2010). This is due to the fact that the joint distribution behavior is a result of the different changes in the tourist trajectory 131 during its walk. These changes in trajectory depend on the image 132 context of the image, and therefore, it takes into account local 133 134 and global information of the image. As a result, the texture infor-135 mation is stored in the joint probability distribution, which can be 136 used for texture characterization and classification.

The drawback of the DTW on image is the presence of two or
more directions complying with the tourist walking rule. To solve
this problem, we propose the following strategy presented in the
next section: the *maximum contrast direction*.

141 **3. Maximum contrast direction**

142 Consider a vector in the Cartesian space $V = \{v_x, v_y\}$, where v_x 143 and v_y represent its components along the *x* and *y* axis, respec-144 tively. Given an image pixel g_0 , which here we consider as the pixel 145 where the tourist is current placed, each one of its neighboring pix-146 els g_{i} , i = 1, ..., 8, has its gray level intensity mapped into a vector V_i 147 according to its relative position to pixel g_0 . From this mapping, 148 three types of vectors arise:

• Horizontal vectors:
$$\vec{V}_3 = \{g_3, 0\}$$
 and $\vec{V}_7 = \{-g_7, 0\}$.

• Vertical vectors:
$$V_1 = \{0, -g_1\}$$
 and $V_5 = \{0, g_5\}$.

• Diagonal vectors:
$$V_2 = \frac{\sqrt{2g_2}}{2} \{1, -1\}, V_4 = \frac{\sqrt{2g_4}}{2} \{1, 1\}, V_6 = \frac{\sqrt{2g_6}}{2} \{-1, 1\}, V_8 = \frac{\sqrt{2g_6}}{2} \{-1, -1\}.$$

From the sum of the vectors achieved, it is possible to compute the maximum contrast direction relative to pixel g_0 :

157
$$\vec{V}_r = \{r_x, r_y\} = \sum_{i=1}^8 \vec{V}_i$$
. (1)

By normalizing the components of vector V_r , we are able to determine the maximum contrast direction in the discreet space, which is the case of image pixels. This normalization is performed by dividing each component of vector V_r by its absolute value, thus resulting in:

$$\vec{V}_{d} = \{d_{x}, d_{y}\} = \left\{\frac{r_{x}}{|r_{x}|}, \frac{r_{y}}{|r_{y}|}\right\},$$
(2)
164

where $|\underline{r}_x|$ and $|r_y| \neq 0$. The components $d_x, d_y \in \{-1, 0, +1\}$ of the vector V_d represent the coordinates of the image pixel where the tourist must move into, relative to the pixel g_0 , and which coincides with the maximum contrast direction at pixel g_0 (see Fig. 3).

The maximum contrast direction V_d is computed at each step of the tourist walk, and this rule is applied until the tourist finds an attractor and ends it walk. However, when the transient time reaches the number of cities/pixels of the image (which means that the tourist already visited all the cities on the map without finding a cycle) or when $V_d = \{0,0\}$ (the tourist found a homogeneous region in the image where there is no contrast direction), the tourist is not able to find an attractor, and so it stops its walk.

4. Texture signature with DTW

As the tourist walks on an image, its trajectory changes according to the image context. These changes during the trajectory reflect on the behavior of the transient time and attractor period computed for each tourist walk throughout the joint distribution probability. Thus, measurements computed from the joint distribution probabilities can efficiently be used as features for texture analysis and characterization (Backes et al., 2006).

Let us consider the transient time $[ht_{\mu}(n)]$, attractor period $[hp_{\mu}(n)]$ and walking $[hw_{\mu}(n)]$ histograms as feasible texture signatures. These histograms are computed from the joint distribution as follows:



Fig. 3. Calculus of the direction where the tourist must move into during its walk from a pixel g_0 : (a) Neighbor pixels and relative positions (d_x, d_y) ; (b) vectors computed considering the gray level intensity and the relative position to pixel g_0 ; maximum contrast direction V_d computed. In this case, tourist must move from g_0 to g_2 .



Fig. 2. Example of texture image and the tourist walk transient time t and cycle period p joint distribution, for $\mu = 1$.

Please cite this article in press as: Backes, A.R., et al. Texture analysis based on maximum contrast walker. Pattern Recognition Lett. (2010), doi:10.1016/ j.patrec.2010.05.022

3

165

166

172

177

178

179

180

181

182

183

184

185

186

187

188

4

190

191

192

193

194

195

196

197

2

ARTICLE IN PRESS

A.R. Backes et al./Pattern Recognition Letters xxx (2010) xxx-xxx

$$ht_{\mu}(n) = \sum_{p} S_{2,\mu}^{(N)}(n,p), \qquad (3)$$

$$hp_{\mu}(n) = \sum_{p} S_{2,\mu}^{(N)}(t,n), \qquad (4)$$

$$hw_{\mu}(n) = \sum_{n=t+p}^{t} S_{2,\mu}^{(N)}(t,p),$$
(5)

where *n* represents, respectively, the distribution of transient time, attractor period and walk length on the image.

As the texture pattern and the memory μ employed changes, new joint distributions are achieved. Each joint distribution has a particular behavior which reflects in the histograms computed. It makes these histograms useful tools for texture analysis (see Fig. 4).

It is important to note that the attractors have period $p \ge \mu + 1$, 198 199 unlike the transient time, which starts in t = 0. Thus, the first 200 descriptor selected for the attractor and walking histograms neces-201 sarily has a size μ + 1. The feature vector is built by concatenating 202 the *n* first descriptors selected from one histogram for a given spe-203 cific memory:

$$\vec{\psi}_{\mu}^{ht}(n) = [ht_{\mu}(0), ht_{\mu}(1), \dots, ht_{\mu}(n-1)], \tag{6}$$

$$\vec{\psi}_{\mu}^{hp}(n) = [hp_{\mu}(\mu+1), hp_{\mu}(\mu+2), \dots, hp_{\mu}(\mu+n)], \tag{7}$$

205
$$\vec{\psi}_{\mu}^{hw}(n) = [hw_{\mu}(\mu+1), hw_{\mu}(\mu+2), \dots, hw_{\mu}(\mu+n)].$$
 (8)

206 As the transient time and cycle period joint distribution depends on 207 μ value, a feature vector which considers different μ values is also 208 evaluated:

$$\mathbf{10} \qquad \varphi^{h}_{\mu_{1},\dots,\mu_{M}}(n) = \Big[\psi^{h}_{\mu_{1}}(n),\psi^{h}_{\mu_{2}}(n),\dots,\psi^{h}_{\mu_{M}}(n)\Big], \tag{9}$$

211 where *h* is the histogram adopted (*ht*, *hp* or *hw*).

212 This φ^h feature vector enables us to characterize a texture pat-213 tern considering different scales, where each scale is represented by a different μ value. 214

5. Experiments 215

216 Our proposed approach was evaluated using transient, attractor and walking histograms for different μ values in a texture analysis 217 and classification context. A database containing 111 textures ob-218 219 tained from Brodatz texture album (Brodatz, 1966) was used. Bro-220 datz textures are broadly used in computer vision and image 221 processing literature as benchmark for texture analysis. A total of 10 samples of 200×200 size and 256 grey levels were considered 222 for each texture class, which makes a total of 1110 texture images 223 in the database. Fig. 5 shows an example of each texture class con-224 sidered while Fig. 6 shows examples of texture variability inside a 225 226

Statistical analysis was performed by applying linear discriminant analysis (LDA) in a cross-validation scheme over the signatures computed for each texture sample considered. LDA enables us to estimate a linear subspace with good discriminative properties, i.e., a linear subspace where the variance between classes is larger than the variance within classes. As LDA is a supervised method, the class definition is necessary during its estimation process (Everitt and Dunn, 2001; Fukunaga, 1990).

To perform a better evaluation of the method, a comparison with traditional texture analysis methods was also performed. Thus, Fourier descriptors (Azencott et al., 1997), co-occurrence matrices (Haralick, 1979) and Gabor filters (Jain and Farrokhnia, 1991; Daugman and Downing, 1995; Idrissa and Acheroy, 2002) were tested with the proposed database. A brief description of each method is presented as follows:

Fourier descriptors: the Fourier Transform is applied over the image and, after a shifting operation, a feature vector is built containing the sum of the spectrum absolute values at a specific radius distance, thus resulting in a total of 99 descriptors.

Co-occurrence matrices: basically, the co-occurrence matrices are the joint probability distributions between pairs of pixels at a pre-specific distance and direction. During the experiments, distances of 1 and 2 pixels with angles of -45° , 0°, 45°, 90° were used. Descriptors of energy and entropy were computed from resulting matrices, thus resulting in a feature vector containing 16 descriptors. A non-symmetric version has been adopted in experiments.

Gabor filters: an input image is convolved by a family of Gabor filters. Each Gabor filter is a bi-dimensional gaussian function moduled with an oriented sinusoid in a determined frequency and direction. During the experiments, the best results were yielded by using a family of 16 filters (4 rotation and 4 scale filters), with lower and upper frequency equal to 0.01 and 0.3, respectively. Descriptors of energy were computed for each computed filter. Definition of the individual parameters of each filter follows mathematical model presented in Manjunath and Ma (1996).

To evaluate the rotation invariance of the method, an additional database containing 10 different orientations for each texture class was considered. It is important to emphasize that some Brodatz patterns cannot be freely rotated during the extraction of a



Fig. 4. Examples of the walking histogram for different texture patterns and μ values.

238 239 240

227

228

229

230

231

232

233

234

235

236

237

241 242

243

254



262

263

264

265

A.R. Backes et al./Pattern Recognition Letters xxx (2010) xxx-xxx



Fig. 5. One example of each of the 111 Brodatz's classes considered. Each image has 200 × 200 pixels and 256 grey levels.

 order to avoid this problem, a single region containing a well-de-
fined pattern was considered during the extraction of the rotated
samples. Fig. 7 shows examples of a given texture under different
orientations.270
271
272

Please cite this article in press as: Backes, A.R., et al. Texture analysis based on maximum contrast walker. Pattern Recognition Lett. (2010), doi:10.1016/ j.patrec.2010.05.022

5

297

298

299

300

301

302

303

304

305

306

307

308

309

310

311

312

313

314

315

316

6





Fig. 6. Examples of variability in texture of two Brodatz classes.

274 6. Results

276 277

278

279 280

275 6.1. Comparison with other methods

Table 1 shows the yielded results of each method compared. For this comparison, we employed the parameters of the tourist walk which leads to the best result. Thus, the tourist signature here employed is the concatenation of the feature vectors $\varphi_{0,1,2,3,4,5}^{ha}(3)$, $\varphi_{0,1,2,3,4,5}^{ht}(5)$ and $\varphi_{0,1,2,3,4,5}^{hw}(8)$, totalizing 96 descriptors.

Although results yielded for proposed method show a superior 281 282 performance over Fourier descriptors and co-occurrence matrices, Gabor filters presented a similar result for the considered database 283 284 (89.37%). This result confirms that the combination of different features extracted from joint distribution, as also the use of different 285 286 memory values, produces a texture signature with great discrimi-287 nation power, which is also capable of dealing with a large number 288 of texture patterns. Notice, however, that Gabor filters use only 16 289 descriptors while the tourist signatures need 96 descriptors to obtain the same result. 290

291 6.2. Rotation tolerance

An interesting and desirable characteristic in texture recognition applications is the ability of the method to recognize a texture
pattern independent of its orientation.

In the proposed approach, the tourist walks on a texture image according to the maximum contrast direction from the current step. Rotation transform does not affect pixel intensities, and it does not change the neighborhood of a pixel, both items considered when computing the subsequent tourist step. Nevertheless, images are discreet structures, and they cannot be freely rotated. A small variation in the computed direction may occur for a given texture sample depending on the chosen rotation angle. However, for 90°-rotated versions of an image, the maximum contrast direction is maintained perfectly unaltered. This indicates that the proposed texture features are insensitive for rotation multiples of 90° (Fig. 8).

Table 2 shows the results yielded when the method is applied over rotated textures. Results show a superiority of the proposed approach over Gabor filter when dealing with different rotated versions of a texture pattern. As in the previous experiment, the tourist walk also presented a performance similar other methods (in this case, with the Fourier descriptors). It indicates that the proposed approach presents a performance similar to Gabor filter and Fourier descriptors for image analysis and rotation tolerance, respectively.

6.3. Computational complexity

To understand the computational complexity of the tourist 317 walk, we must consider that the method considers each image pixel as a starting point. Thus, for an image of $N \times N$ size, this leads to N^2 walks. Each resulting walk consists of a transient part, of size t, 320 and, an attractor of size $p \ge \mu + 1$, which may not be present. In 321



Fig. 7. Examples of Brodatz rotated samples: (a) 15°; (b) 30°; (c) 45°; (d) 60°; (e) 75°; (f) 90°; (g) 105°; (h) 120°; (i) 135°; (j) 150°.

7

357

358

359

360

361

362 363

364

365

366

367

368 369

370

371

372

373

374

375 376

377

378

379

380

381 382

383

384 385

386

387 388

389

390

391

392

393

394

395

396

397

398

399 400

401

402

403

404

405

406

407

408

409

410

411

412

413 414

415

416 417

418

419

420

351

352

A.R. Backes et al. / Pattern Recognition Letters xxx (2010) xxx-xxx

Table 1

Comparison with traditional texture analysis methods.

Method	No. of	Images correctly	Success rate
	descriptors	classified	(%)
Gabor filters	16	992	89.37
Fourier descriptors	99	888	80.00
Co-occurrence	16	665	59.91
matrices Tourist walk	96	992	89.37

$V_{\cdot}=$	= {0.	+1}	V.:	= {+	1.0}	V.:	= {0	-1}	V.:	= {-]	1.0}
25	45	50	50	75	25	25	45	25	25	35	25
35	50	75	45	50	45	75	50	35	45	50	45
25	45	25	25	35	25	50	45	25	25	75	50

Fig. 8. For 90°-based rotation the maximum contrast direction is constant.

 Table 2

 Comparison with traditional texture analysis methods using rotated textures.

Method	No. of descriptors	Images correctly classified	Success rate (%)
Gabor filters	16	885	79.73
Fourier descriptors	99	966	87.02
Co-occurrence matrices	16	105	9.46
Tourist walk	96	966	87.02

this case, the transient part is considered as having its size equal to the number of image pixels, i.e., $t = N \times N$ and p = 0. All this considered, the computational complexity of the tourist walk is stated as $O(N^2(t + p))$, where (t + p) is the size of a tourist walk.

326 Both best and worst case of the method are achieved only in 327 special cases of image context or memory. The best case occurs when all walks already start on an attractor (t = 0) and only if the 328 attractor presents the minimum size possible. The attractor's min-329 imum size depends on the memory μ and it is achieved for μ = 0, 330 331 which minimizes the attractor size to p = 1. Therefore, in this case, the computational complexity of the method is $O(N^2)$. The worst 332 333 case occurs when the attractor is never found during the walk. Thus, independent of the memory size μ , the tourist walk presents 334 size $t + p = N^2$, which leads to complexity $O(N^4)$. However, it is 335 important to emphasize that both cases, specially the worst case, 336 337 are very rare cases. On one hand, the best case is easily found for some walks in a common image. On the other hand, the worst case 338 requires a very specific configuration of pixels in the image, so that, 339 340 even a random generated image does not produce this special case 341 of walk.

7. Conclusion

In this paper, we proposed a different approach to compute the 343 344 direction during the deterministic tourist walk in order to explorer 345 an image in a given scale (memory). Instead of using a simple dif-346 ference of intensities between pixels, we proposed to use the 347 intensities and relative positions of the neighbor pixels to compute 348 the maximum contrast direction of a pixel. This direction points to the neighbor pixel that a traveler must go in the next step of the 349 Tourist Walk to find an attractor. 350

Signatures computed from joint distribution computed using this walk were tested in image classification experiments. Linear discriminant analysis was employed to classify a set of Brodatz textures and their rotated versions. Comparison with other methods shows a great potential of the method as a feasible texture analysis methodology.

Acknowledgments

A.R.B. acknowledges support from FAPESP (2006/54367-9). O.M.B. acknowledges support from CNPq (303746/2004-1 and 484474/2007-3) and FAPESP (2006/54367-9). A.S.M. acknowledges support from CNPq (303990/2007-4 and 476862/2007-8).

References

- Azencott, R., Wang, J.-P., Younes, L., 1997. Texture classification using windowed Fourier filters. IEEE Trans. Pattern Anal. Machine Intell. 19 (2), 148–153.
- Backes, A.R., Bruno, O.M., Campiteli, M.G., Martinez, A.S., 2006. Deterministic tourist walks as an image analysis methodology based. In: CIAPR. Lecture Notes in Computer Science, vol. 4225. Springer, pp. 784–793.
- Backes, A.R., Gontalves, W.N., Martinez, A.S., Bruno, O.M., 2010. Texture analysis and classification using deterministic tourist walk. Pattern Recognition 43 (3), 685– 694.
- Brodatz, P., 1966. Textures: A Photographic Album for Artists and Designers. Dover Publications, New York.
- Bunimovich, L.A., 2004. Deterministic walks in random environments. Phys. D 187, 20–29.
- Bunimovich, L.A., Troubetzkoy, S.E., 1992. Recurrence properties of Lorentz lattice gas cellular automata. J. Stat. Phys. 67, 289–302.
- Campiteli, M.G., Martinez, A.S., Bruno, O.M., 2006. An image analysis methodology based on deterministic tourist walks. In: IBERAMIA-SBIA. Lecture Notes in Computer Science, vol. 4140. Springer, pp. 159–167.
- Chaudhuri, B.B., Sarkar, N., 1995. Texture segmentation using fractal dimension. IEEE Trans. Pattern Anal. Machine Intell. 17 (1).
- Chetverikov, D., 1999. Texture analysis using feature-based pairwise interaction maps. Pattern Recognition 32 (3), 487–502.
- Daugman, J., Downing, C., 1995. Gabor wavelets for statistical pattern recognition. In: Arbib, M.A. (Ed.), The Handbook of Brain Theory and Neural Networks. MIT Press, Cambridge, MA, pp. 414–419.
- Derrida, B., 1997. From random walks to spin glasses. Phys. D 107 (2–4), 186–198. Emerson, C.W., Lam, N.N., Quattrochi, D.A., 1999. Multi-scale fractal analysis of image texture and patterns. Photogrammet. Eng. Remote Sensing 65 (1), 51–62.
- Everitt, B.S., Dunn, G., 2001. Applied Multivariate Analysis, 2nd ed. Arnold.
- Fisher, D.S., 1984. Random walks in random environments. Phys. Rev. A 30 (2), 960– 964.
- Freund, H., Grassberger, P., 1992. The Red Queens walk. Phys. A 190 (3–4), 218–237. Fukunaga, K., 1990. Introduction to Statistical Pattern Recognition, 2nd ed. Academic Press.
- Gale, D., Propp, J., Sutherland, S., Troubetzkoy, S., 1995. Further travels with my ant. Math. Intell. 17, 48–56.
- Haralick, R.M., 1979. Statistical and structural approaches to texture. Proc. IEEE 67 (5), 768–804.
- Idrissa, M., Acheroy, M., 2002. Texture classification using Gabor filters. Pattern Recognition Lett. 23 (9), 1095–1102.
 Jain, A.K., Farrokhnia, F., 1991. Unsupervised texture segmentation using Gabor
- filters. Pattern Recognition 24 (12), 1167–1186. Kasparis, T., Charalampidis, D., Georgiopoulos, M., Rolland, J.P., 2001. Segmentation
- of textured images based on fractals and image filtering. Pattern Recognition 34 (10).
- Kinouchi, O., Martinez, A.S., Lima, G.F., Lourento, G.M., Risau-Gusman, S., 2002. Deterministic walks in random networks: An application to thesaurus graphs. Phys. A 315 (3/4), 665–676.
- Lima, G.F., Martinez, A.S., Kinouchi, O., 2001. Deterministic walks in random media. Phys. Rev. Lett. 87 (1), 010603.
- Manjunath, B.S., Ma, W.-Y., 1996. Texture features for browsing and retrieval of image data. IEEE Trans. Pattern Anal. Machine Intell. 18 (8), 837–842.
- Metzler, R., Klafter, J., 2000. The random walk's guide to anomalous diffusion: A fractional dynamics approach. Phys. Rep. 339 (1), 1–77. URL http://dx.doi.org/ 10.1016/S0370-1573(00)00070-3.
- Stanley, H.E., Buldyrev, S.V., 2001. Statistical physics the salesman and the tourist. Nature (London) 413 (6854), 373–374.
- Tertariol, C.A., Martinez, A.S., 2005. Analytical results for the statistical distribution related to a memoryless deterministic walk: Dimensionality effect and mean-field models. Phys. Rev. E, 72.
- Tercariol, C.A.S., González, R.S., Martinez, A.S., 2007. Exact analytical calculation for the percolation crossover in deterministic partially self-avoiding walks in onedimensional random media. Phys. Rev. E 75, 061117.
- Tuceryan, M., Jain, A.K., 1993. Texture analysis. Handbook of Pattern Recognition and Computer Vision, 235–276.