MINI-CURSO INTENSIVO - Centro Brasileiro de Pesquisas Físicas

Coordenação de Formação Científica

Rio de Janeiro, 2 a 6 de abril de 2007

MECÂNICA ESTATÍSTICA NÃO EXTENSIVA:

Aspectos Teóricos, Experimentais, Observacionais e Computacionais

Coordenação: Prof. Constantino Tsallis

Destinado a estudantes de pós-graduação e pesquisadores de Física, Matemática, Química, Economia, Biologia, Ciências Computacionais e áreas correlatas.
Na definição da CAPES, vale 2 créditos de Pós-Graduação.
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$$S_q = k\,\frac{1-\sum_i p_i^q}{q-1}$$

1ª. parte (16 horas, em Português)
Prof. Constantino Tsallis (CBPF, Brasil)

2ª. parte (8 horas, em Inglês)
Prof. Alberto Robledo (UNAM, México)

3ª. parte (8 horas, em Inglês)
Prof. Andrea Rapisarda (Universidade de Catânia, Itália)

Rua Dr. Xavier Sigaud, 150 - Rio de Janeiro - RJ - 22290-180

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CAPES

MECANICA ESTATISTICA NAO EXTENSIVA

ASPECTOS TEORICOS, EXPERIMENTAIS, OBSERVACIONAIS E COMPUTACIONAIS

Constantino Tsallis

Centro Brasileiro de Pesquisas Fisicas

Rio de Janeiro - Brasil

Rio de Janeiro, CBPF, Abril 2007

EXTENSIVITY OF THE NONADDITIVE ENTROPY Sq

and q - GENERALIZED CENTRAL LIMIT THEOREM:

C. T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sci (USA) **102**, 15377 (2005)

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EXPERIMENTAL VERIFICATION IN COLD ATOMS:

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Sabir Umarov

Stanly Steinberg

Miguel Fuentes

Yuzuru Sato

Ugur Tirnakli

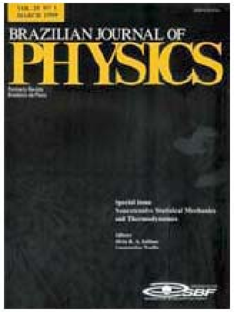
Silvio M.D. Queiros

Luis Moyano
John Marsh
Paul Rivkin
Kenric Nelson
William Thistleton

Christian Beck

E.G.D. Cohen

NONEXTENSIVE STATISTICAL MECHANICS AND THERMODYNAMICS



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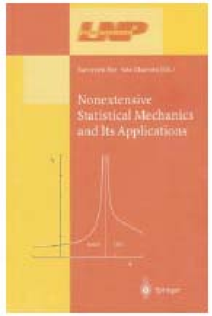

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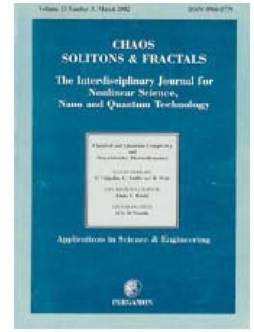

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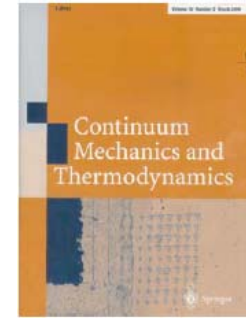

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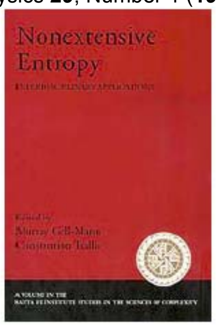

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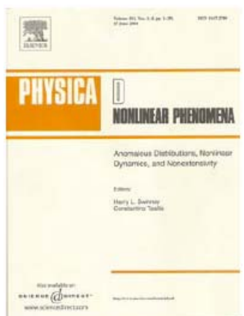

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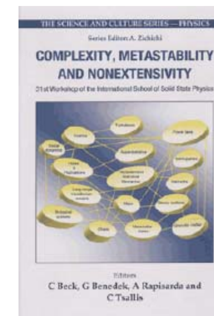
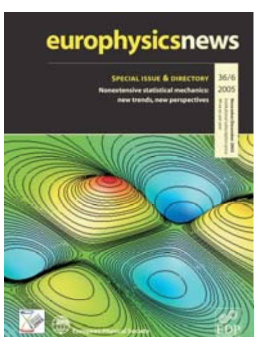


Complexity, Metastability and Nonextensivity, C Beck, G Benedek, A Rapisarda and C Tsallis, eds, (World Scientific, Singapore, **2005**)



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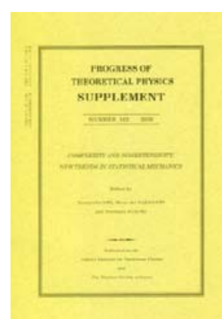
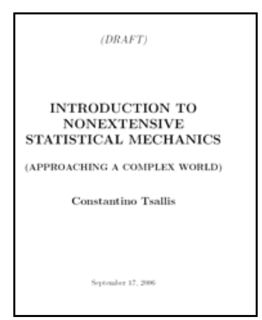
Fundamental Problems of Modern Statistical Mechanics, G Kaniadakis, A Carbone and M Lissia, eds, Physica A **365**, Issue 1 (**2006**)



Complexity and Nonextensivity: New Trends in Statistical Mechanics, S Abe, M Sakagami and N Suzuki, eds, Progr. Theoretical Physics Suppl **162** (**2006**)



Introduction to Nonextensive Statistical Mechanics - Approaching a Complex World, C. Tsallis (in preparation)

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

2,107 articles (done by 1,550 scientists from 60 countries) which led to

> 7,520 citations of papers

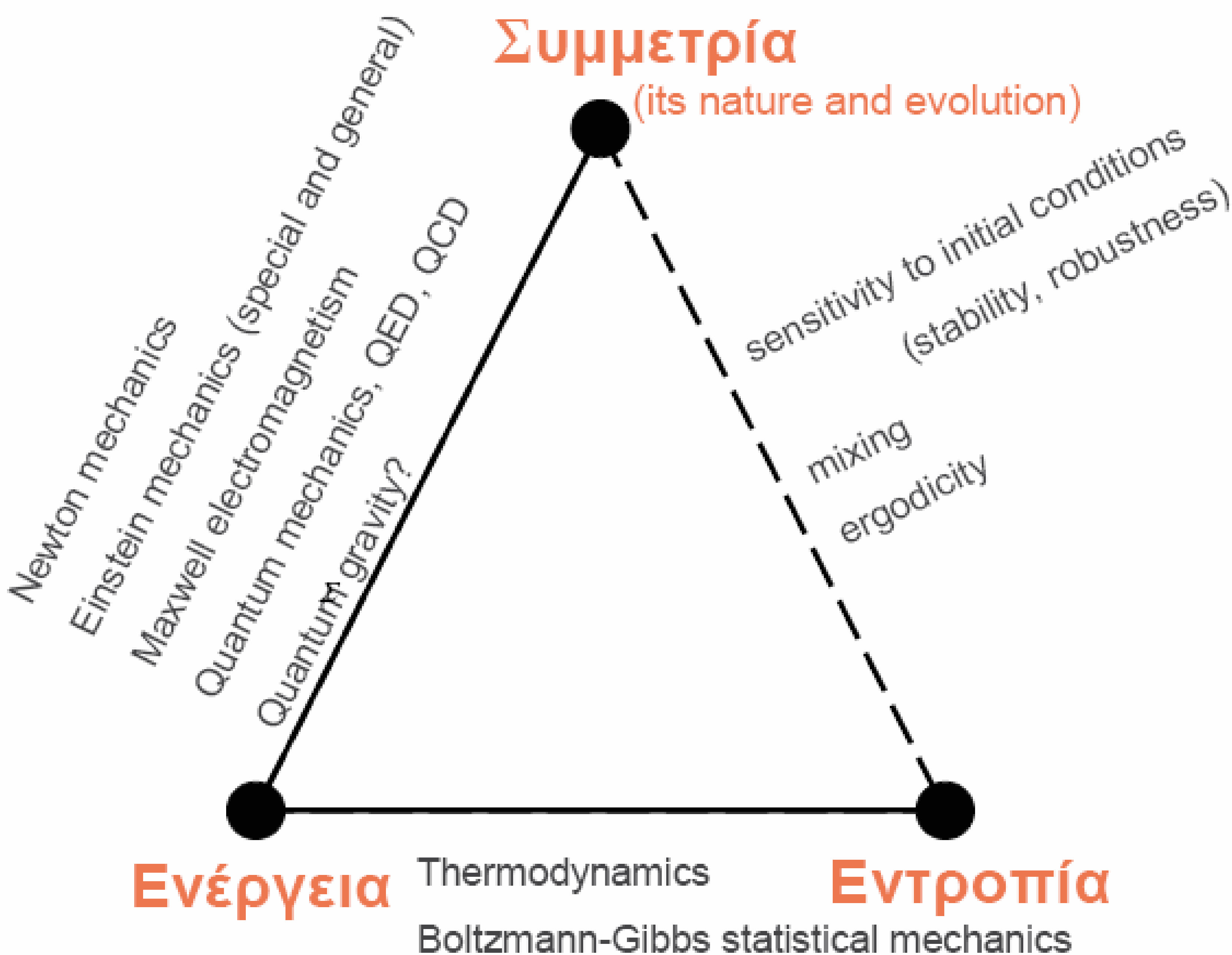
(including > 1,407 citations of the 1988 paper)

> 957 nominal citations

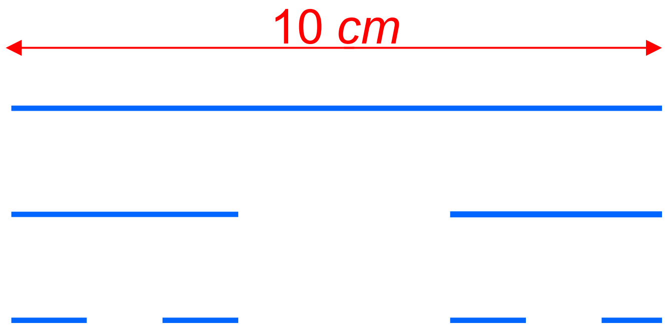
[31 March 2007]

CONTRIBUTORS **(2107 MANUSCRIPTS)** [Updated 31 March 2007]

BRAZIL	303	NETHERLANDS	11	SLOVAK	3
USA	217	BELGIUM	11	CROATIA	3
ITALY	116	GREECE	11	IRELAND	3
JAPAN	105	AUSTRALIA	8	BOLIVIA	2
FRANCE	92	PORTUGAL	8	CZECK	2
CHINA	88	SOUTH AFRICA	7	FINLAND	2
ARGENTINA	79	CUBA	7	KAZAKSTAN	2
SPAIN	57	HUNGARY	7	MOLDOVA	2
GERMANY	55	IRAN	7	PHILIPINES	2
ENGLAND	40	VENEZUELA	6	PUERTO RICO	2
POLAND	37	CHILE	6	ARMENIA	1
RUSSIA	31	NORWAY	5	INDONESIA	1
TURKEY	27	SINGAPORE	4	JORDAN	1
INDIA	23	SWEDEN	4	MALAYSIA	1
CANADA	23	TAIWAN	4	SAUDI ARABIA	1
AUSTRIA	21	URUGUAY	4	SERBIA	1
MEXICO	19	ROMENIA	4	SRI LANKA	1
UKRAINE	18	EGYPT	4	UZBEKISTAN	1
SWITZERLAND	17	DENMARK	4		
ISRAEL	13	SLOVENIA	4	60 COUNTRIES	1550 SCIENTISTS
KOREA	12	BULGARIA	3		


Συμμετρία
(its nature and evolution)
Newton mechanics
Einstein mechanics (special and general)
Maxwell electromagnetism
Quantum mechanics, QED, QCD
Quantum gravity?
sensitivity to initial conditions
(stability, robustness)
mixing
ergodicity
Ενέργεια
Thermodynamics
Boltzmann-Gibbs statistical mechanics
Εντροπία

TRIADIC CANTOR SET:



$$d_F = \frac{\ln 2}{\ln 3} = 0.6309...$$

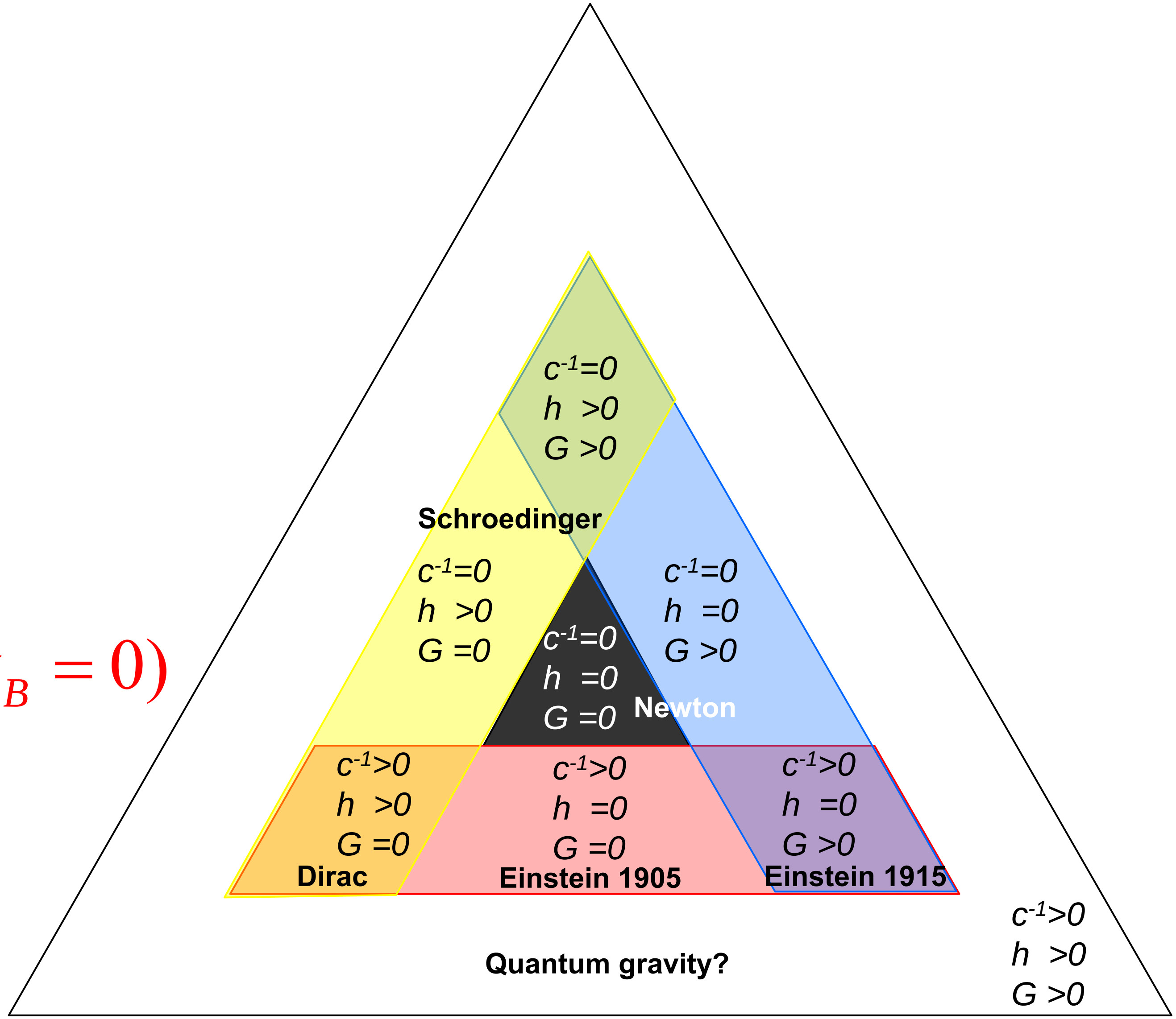
Hence the interesting measure is

$$(10\ cm)^{0.6309...} \simeq 4.275\ cm^{0.6309}$$

It is the natural (or artificial or social) system itself which, through its geometrical-dynamical properties, indicates the specific informational tool --- entropy --- to be meaningfully used for the study of its thermostatistical and thermodynamical properties.

The four independent universal constants of contemporary physics: $G,\ c^{-1},\ h,\ k_B^{-1}$

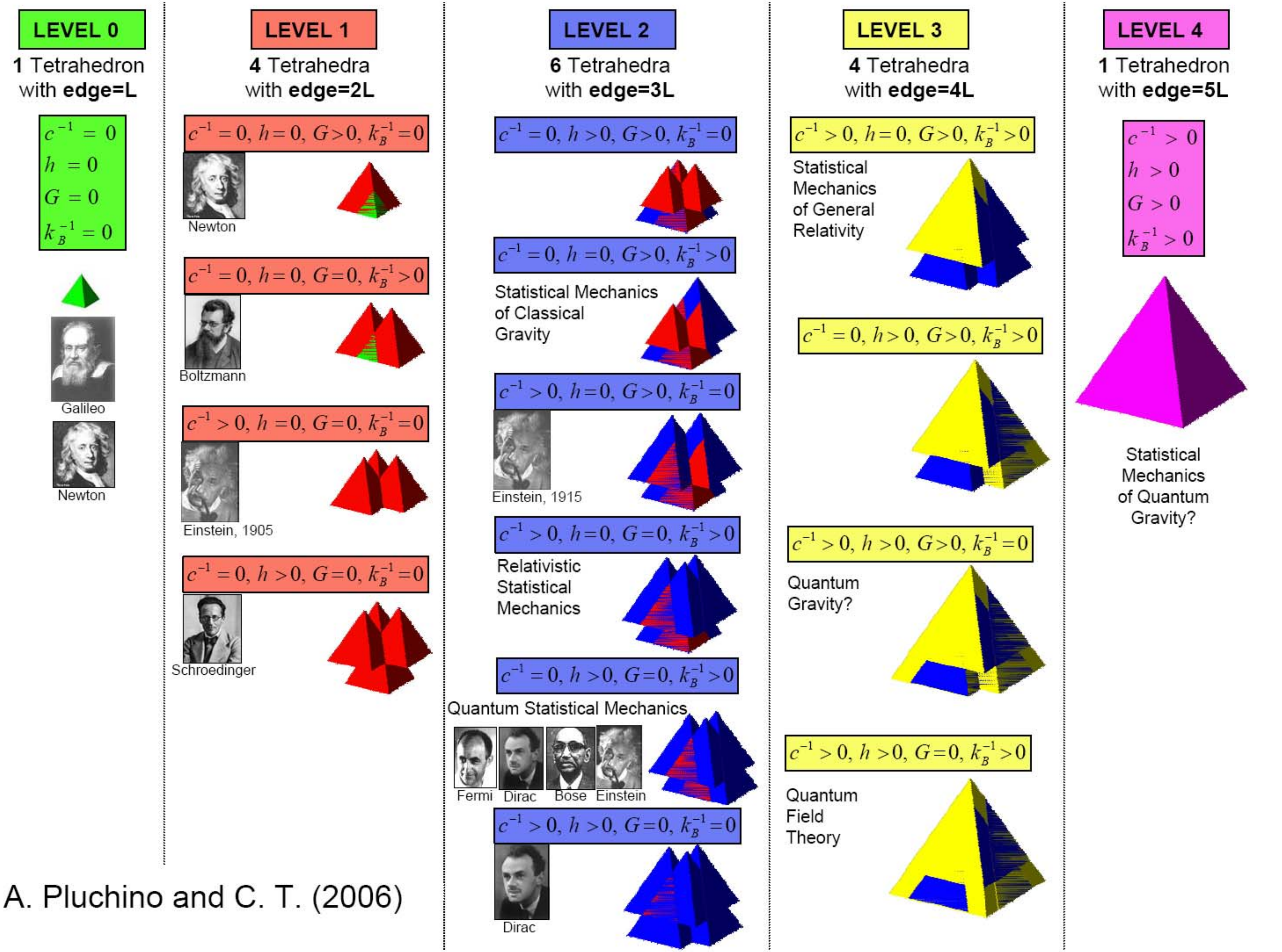
$(1/k_B = 0)$



The full tetrahedron $c^{-1} > 0;\ h > 0;\ G > 0;\ k_B^{-1} > 0$ *corresponds to the*

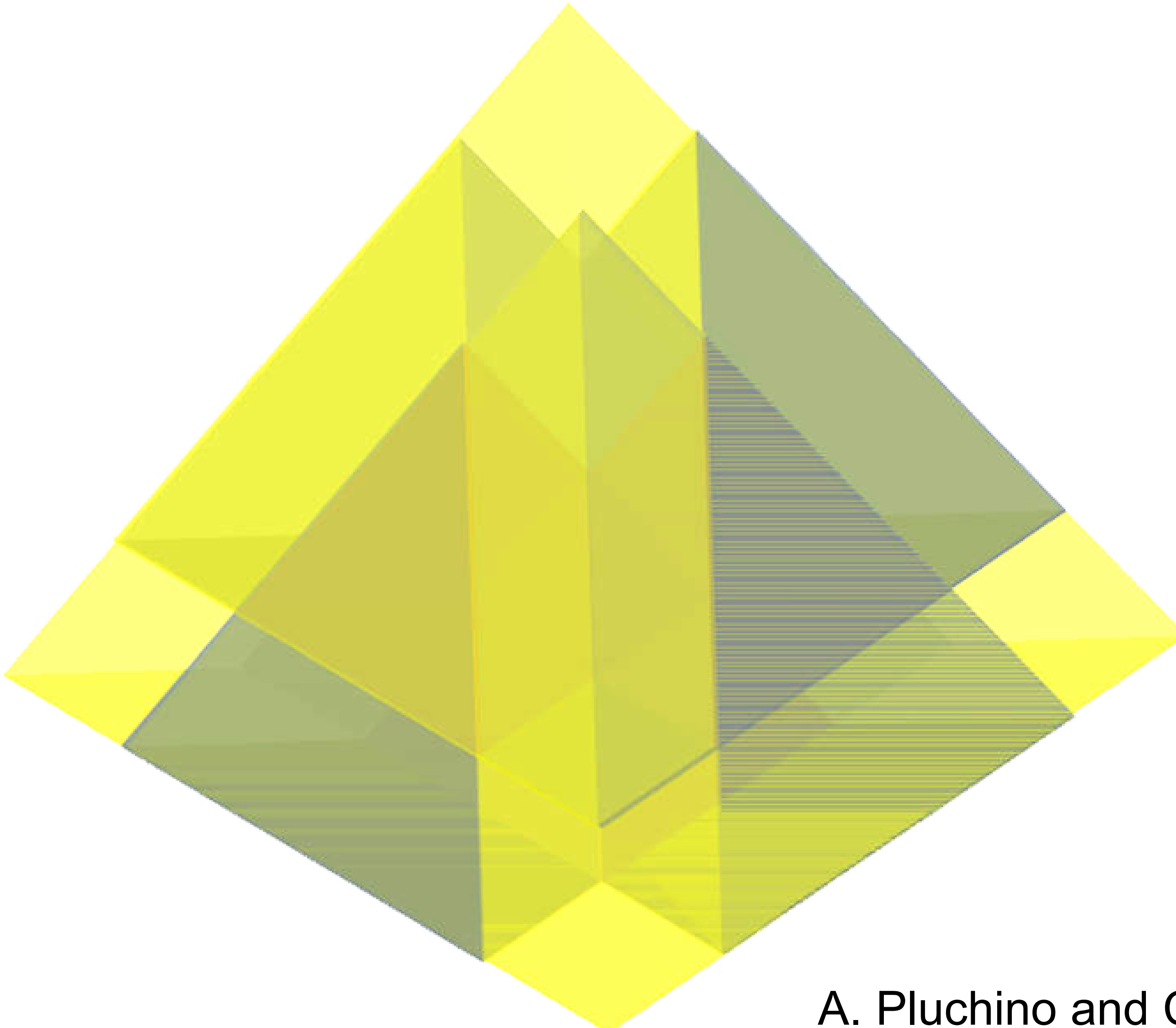
statistical mechanics of quantum gravity (*at its center*: $c^{-1} = h = G = k_B^{-1} = 0$)

C.T., *Introduction to Nonextensive Statistical Mechanics-Approaching a Complex World* (in progress)



A. Pluchino and C. T. (2006)

Full Tetrahedron



A. Pluchino and C. T. (2006)

HISTORICAL BACKGROUND AND PHYSICAL MOTIVATIONS FOR ATTEMPTING TO GENERALIZE BOLTZMANN-GIBBS STATISTICAL MECHANICS

ALONG THE LAST 135 YEARS…

Ludwig BOLTZMANN

Vorlesungen uber Gastheorie (Leipzig, 1896)
Lectures on Gas Theory, transl. S. Brush
(Univ. California Press, Berkeley, 1964), page 13

The forces that two molecules impose one onto the other during an interaction can be completely arbitrary, only assuming that their sphere of action is very small compared to their mean free path.

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, 1981), page 35

> *In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92)* [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

Ettore MAJORANA

The value of statistical laws in physics and social sciences.
Original manuscript in Italian published by G. Gentile Jr. in *Scientia* **36**, 58 (1942); translated into English by R. Mantegna (2005).

This is mainly because entropy is an additive quantity as the other ones. In other words, the entropy of a system composed of several independent parts is equal to the sum of entropy of each single part. [...]
Therefore one considers all possible internal determinations as equally probable. This is indeed a new hypothesis because the universe, which is far from being in the same state indefinitively, is subjected to continuous transformations. We will therefore admit as an extremely plausible working hypothesis, whose far consequences could sometime not be verified, that all the internal states of a system are a priori equally probable in specific physical conditions. Under this hypothesis, the statistical ensemble associated to each macroscopic state A turns out to be completely defined.

Claude Elwood SHANNON

The Mathematical Theory of Communication
(University of Illinois Press, Urbana, 1949)

It is practically more useful. [...]
It is nearer to our intuitive feeling as to the proper measure. [...]
It is mathematically more suitable. [...].}

This theorem and the assumptions required for its proof, are in no way necessary for the present theory. It is given chiefly to lend a certain plausibility to some of our later definitions. The real justification of these definitions, however, will reside in their implications.

Laszlo TISZA

Generalized Thermodynamics
(MIT Press, Cambridge, Massachusetts, 1961)

The situation is different for the additivity postulate Pa2, the validity of which cannot be inferred from general principles. We have to require that the interaction energy between thermodynamic systems be negligible. This assumption is closely related to the homogeneity postulate Pd1. From the molecular point of view, additivity and homogeneity can be expected to be reasonable approximations for systems containing many particles, provided that the intramolecular forces have a short range character.

Radu BALESCU

Equilibrium and Nonequilibrium Statistical Mechanics
(John Wiley and Sons, 1975, New York)

It therefore appears from the present discussion that the mixing property of a mechanical system is much more important for the understanding of statistical mechanics than the mere ergodicity. [...] A detailed rigorous study of the way in which the concepts of mixing and the concept of large numbers of degrees of freedom influence the macroscopic laws of motion is still lacking.

Peter LANDSBERG

Thermodynamics and Statistical Mechanics (1978)

The presence of long-range forces causes important amendments to thermodynamics, some of which are not fully investigated as yet.

Is equilibrium always an entropy maximum?
J. Stat. Phys. **35**, 159 (1984)

[...] in the case of systems with long-range forces and which are therefore nonextensive (in some sense) some thermodynamic results do not hold. [...] The failure of some thermodynamic results, normally taken to be standard for black hole and other nonextensive systems has recently been discussed. [...] If two identical black holes are merged, the presence of long-range forces in the form of gravity leads to a more complicated situation, and the entropy is nonextensive.

David RUELLE

Thermodynamical Formalism -
The Mathematical Structures of Classical Equilibrium Statistical Mechanics
(page 1 of both 1978 and 2004 editions)

The formalism of equilibrium statistical mechanics -- which we shall call thermodynamic formalism -- has been developed since J.W. Gibbs to describe the properties of certain physical systems. [...] While the physical justification of the thermodynamic formalism remains quite insufficient, this formalism has proved remarkably successful at explaining facts.
The mathematical investigation of the thermodynamic formalism is in fact not completed: the theory is a young one, with emphasis still more on imagination than on technical difficulties. This situation is reminiscent of pre-classic art forms, where inspiration has not been castrated by the necessity to conform to standard technical patterns.

(page 3) *The problem of why the Gibbs ensemble describes thermal equilibrium (at least for "large systems") when the above physical identifications have been made is deep and incompletely clarified.*

[The first equation is dedicated to define the *BG* entropy form. It is introduced after the words "we define its *entropy*" without any kind of justification or physical motivation.]

Nico van KAMPEN

Stochastic Processes in Physics and Chemistry
(North-Holland, Amsterdam, 1981)

Actually an additional stability criterion is needed, see M.E. Fisher, Archives Rat. Mech. Anal. ***17****, 377 (1964); D. Ruelle, Statistical Mechanics: Rigorous Results (Benjamin, New York 1969). A collection of point particles with mutual gravitation is an example where this criterion is not satisfied, and for which therefore no statistical mechanics exists.*

Roger BALIAN

From Microphysics to Macrophysics
(Springer-Verlag, Berlin, 1991), p. 205 and 206; French edition (1982).

These various quantities are connected with one another through thermodynamic relations which make their extensive or intensive nature obvious, as soon as one postulates, for instance, for a fluid, that the entropy, considered as a function of the volume Omega and of the constants of motion such as U and N, is homogeneous of degree 1: S(x Omega, x U, x N)=x S(Omega, U, N) (Eq. 5.43). [...] Two counter-examples will help us to feel why extensivity is less trivial than it looks. [...] A complete justification of the Laws of thermodynamics, starting from statistical physics, requires a proof of the extensivity (5.43), a property which was postulated in macroscopic physics. This proof is difficult and appeals to special conditions which must be satisfied by the interactions between the particles.

L.G. TAFF

Celestial Mechanics
(John Wiley, New York, 1985)

This means that the total energy of any finite collection of self-gravitating mass points does not have a finite, extensive (e.g., proportional to the number of particles) lower bound. Without such a property there can be no rigorous basis for the statistical mechanics of such a system (Fisher and Ruelle 1966). Basically it is that simple. One can ignore the fact that one knows that there is no rigorous basis for one's computer manipulations; one can try to improve the situation, or one can look for another job.

W.C. SASLAW

Gravitation Physics of Stellar and Galactic Systems
(Cambridge University Press, Cambridge, 1985)

When interactions are important the thermodynamic parameters may lose their simple intensive and extensive properties for subregions of a given system. [...] Gravitational systems, as often mentioned earlier, do not saturate and so do not have an ultimate equilibrium state.

John MADDOX

When entropy does not seem extensive
Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

A.C.D. van ENTER, R. FERNANDEZ and A.D. SOKAL, *Regularity Properties and Pathologies of Position-Space Renormalization-Group Transformations: Scope and Limitations of Gibbsian Theory* [J. Stat. Phys. **72**, 879-1167 (1993)]

We provide a careful, and, we hope, pedagogical, overview of the theory of Gibssian measures as well as (the less familiar) non-Gibbsian measures, emphasizing the distinction between these two objects and the possible occurrence of the latter in different physical situations.

Toward a Non-Gibbsian Point of View: Let us close with some general remarks on the significance of (non-)Gibbsianness and (non)quasilocality in statistical physics. Our first observation is that Gibbsianness has heretofore been ubiquitous in equilibrium statistical mechanics because it has been put in *by hand*: nearly all measures that physicists encounter are Gibbsian because physicists have *decided* to study Gibbsian measures! However, we now know that natural operations on Gibbs measures can sometimes lead out of this class. [...] It is thus of great interest to study which types of operations preserve, or fail to preserve, the Gibbsianness (or quasilocality) of a measure. This study is currently in its infancy. [...] More generally, in areas of physics where Gibbsianness is not put in by hand, one should expect non-Gibbsianness to be ubiquitous. This is probably the case in nonequilibrium statistical mechanics. Since one cannot expect all measures of interest to be Gibbsian, the question then arises whether there are *weaker* conditions that capture some or most of the “good” physical properties characteristic of Gibbs measures. For example, the stationary measure of the voter model appears to have the critical exponents predicted (under the hypothesis of Gibbsianness) by the Monte Carlo renormaliztion group, even though this measure is provably non-Gibbsian. One may also inquire whether there is a classification of non-Gibbsian measures according to their “degree of non-Gibbsianness”.

Floris TAKENS

Structures in Dynamics – Finite Dimensional Deterministic Studies

Eds. H.W. Broer, F. Dumortier, S.J. van Strien and F. Takens, p. 253 (North-Holland, Amsterdam, 1991)

The values of p_i *are determined by the following dogma: if the energy of the system in the i-th state is* E_i *and if the temperature of the system is T then:*

$$p_i = \frac{e^{-E_i/kT}}{Z(T)}, \text{ where } Z(T) = \sum_i e^{-E_i/kT} \text{ (this last constant is taken so that } \sum_i p_i = 1).$$

This choice of p_i *is called the Gibbs distribution. We shall give no justification for this dogma; even a physicist like Ruelle disposes of this question as "deep and incompletely clarified".*

> ***The advantages of the method of postulation are great; they are the same as the advantages of theft over honest toil.***
>
> (Bertrand Russell)

ENTROPY

ENTROPIC FORMS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \ (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1\right)$
***BG* entropy** **(*q* =*1*)**	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* **(*q real*)**	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

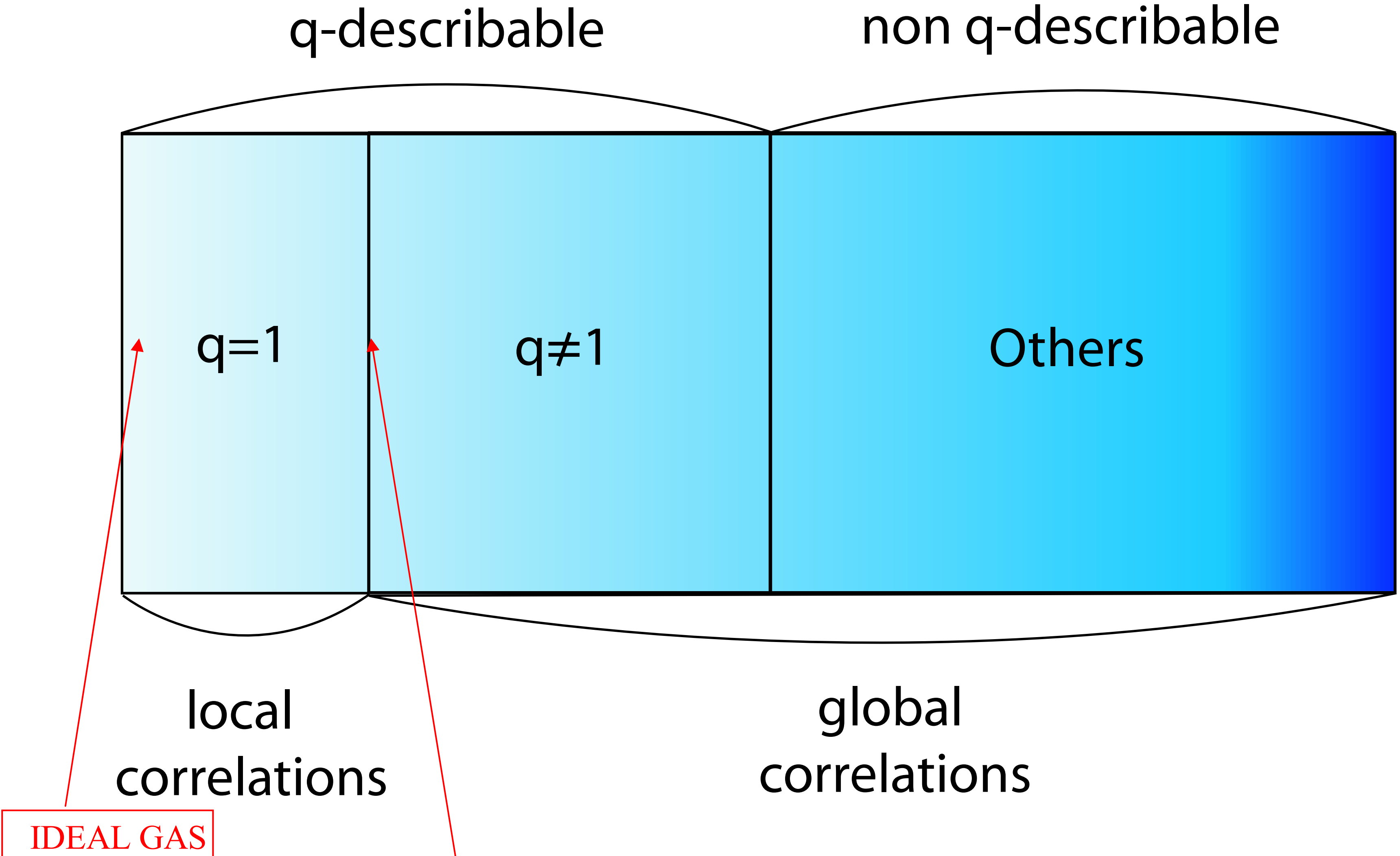
DEFINITION : q − logarithm :

$$\ln_q x \equiv \frac{x^{1-q} - 1}{1-q} \qquad (x > 0)$$

$$\ln_1 x = \ln x$$

Hence, the entropies can be rewritten :

	equal probabilities	*general probabilities*
BG entropy $(q = 1)$	$k \ \ln W$	$k \sum_{i=1}^{W} p_i \ln \ (1/p_i)$
entropy S_q $(q \in R)$	$k \ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \ (1/p_i)$

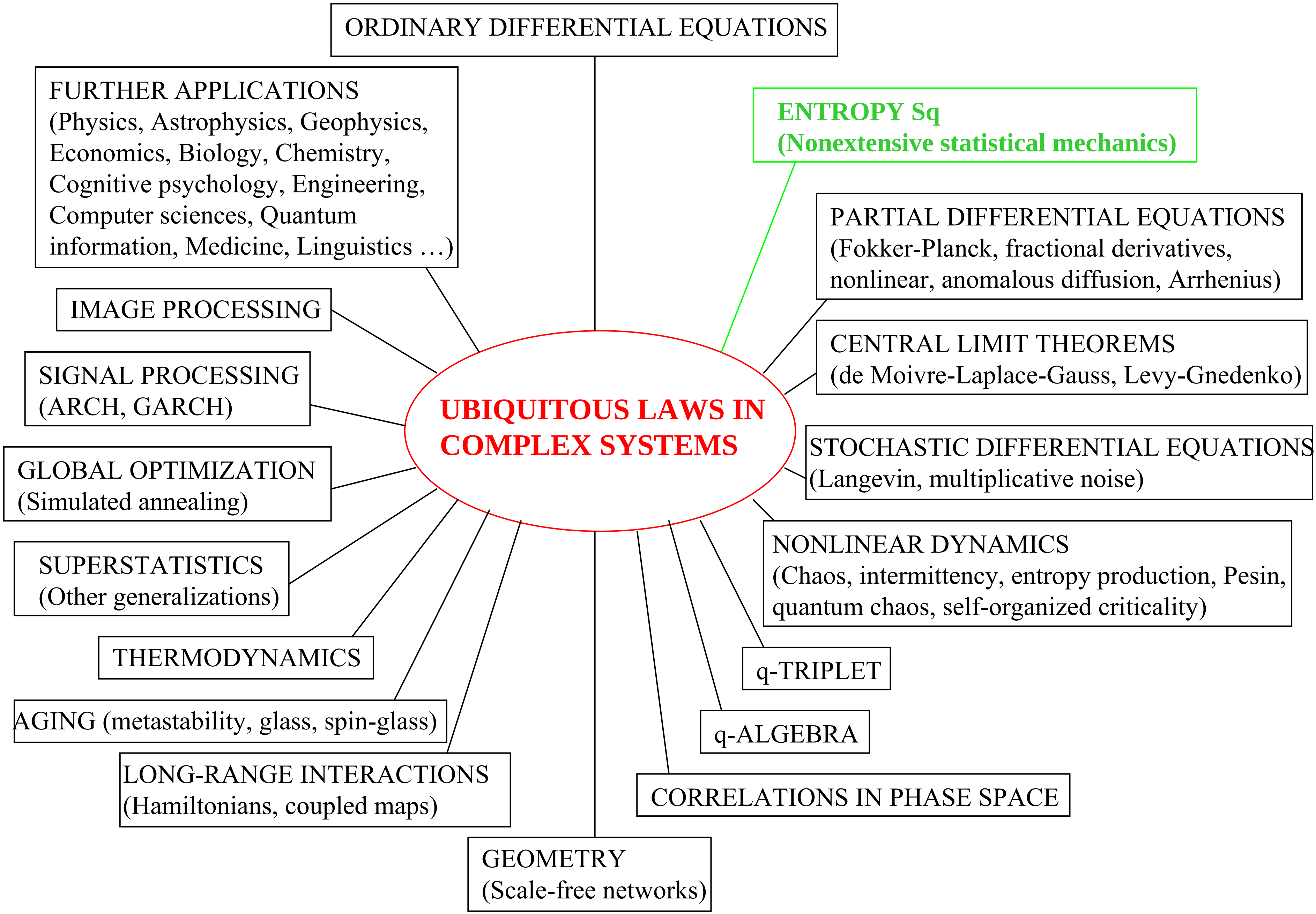


CRITICAL PHENOMENA

$$q = \frac{1+\delta}{2}$$ (A. Robledo, Mol Phys 103 (2005) 3025)

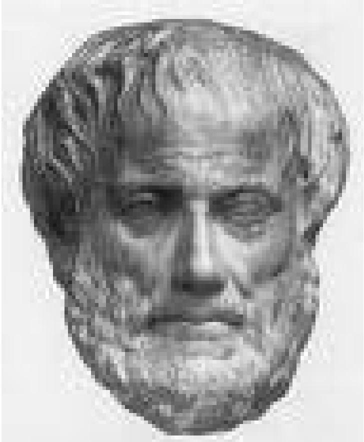
$$q = \frac{\sqrt{9+2c^2} - 3}{c}$$ (F. Caruso and C. T., 2006)

C.T., M. Gell-Mann and Y. Sato
Europhysics News **36** (6), 186
(European Physical Society, 2005)



ORDINARY DIFFERENTIAL EQUATIONS
FURTHER APPLICATIONS (Physics, Astrophysics, Geophysics, Economics, Biology, Chemistry, Cognitive psychology, Engineering, Computer sciences, Quantum information, Medicine, Linguistics …)
ENTROPY Sq (Nonextensive statistical mechanics)
PARTIAL DIFFERENTIAL EQUATIONS (Fokker-Planck, fractional derivatives, nonlinear, anomalous diffusion, Arrhenius)
IMAGE PROCESSING
CENTRAL LIMIT THEOREMS (de Moivre-Laplace-Gauss, Levy-Gnedenko)
SIGNAL PROCESSING (ARCH, GARCH)
UBIQUITOUS LAWS IN COMPLEX SYSTEMS
STOCHASTIC DIFFERENTIAL EQUATIONS (Langevin, multiplicative noise)
GLOBAL OPTIMIZATION (Simulated annealing)
NONLINEAR DYNAMICS (Chaos, intermittency, entropy production, Pesin, quantum chaos, self-organized criticality)
SUPERSTATISTICS (Other generalizations)
THERMODYNAMICS
q-TRIPLET
AGING (metastability, glass, spin-glass)
q-ALGEBRA
LONG-RANGE INTERACTIONS (Hamiltonians, coupled maps)
CORRELATIONS IN PHASE SPACE
GEOMETRY (Scale-free networks)

ARISTOTLE (384 – 322 BC)



[Αριστοτέλη, Περί Ποιητικής, 1459$^{\alpha}$]

Ἔστιν δὲ μέγα μὲν τὸ ἑκάστωι τῶν εἰρημένων πρεπόντως χρῆσθαι, καὶ διπλοῖς ὀνόμασι καὶ γλώτταις, πολὺ δὲ μέγιστον τὸ μεταφορικὸν εἶναι. Μόνον γὰρ τοῦτο οὔτε παρ᾽ ἄλλου ἔστι λαβεῖν εὐφυίας τε σημεῖόν ἐστι· τὸ γὰρ εὖ μεταφέρειν τὸ τὸ ὅμοιον θεωρεῖν ἐστιν.

Το πιο σημαντικό από όλα τα παραπάνω είναι η δεξιοτεχνική χρήση της μεταφοράς. Διότι μόνο αυτό δεν μπορεί να διδαχθεί, ενώ είναι δείγμα ευφυΐας καθώς μια σωστή μεταφορά υποδηλώνει την ικανότητα να διακρίνει κανείς ομοιότητες ανάμεσα σε ανόμοια πράγματα.

"By far the greatest thing is to be a master of metaphor. It is the one thing that cannot be learned from others. It is a sign of genius, for a good metaphor implies an intuitive perception of similarity among dissimilars."—Aristotle

ABOUT ORDINARY DIFFERENTIAL EQUATIONS

$(i)\quad \frac{dy}{dx} = 0 \quad \text{with} \quad y(0) = 1$

$\Rightarrow y = 1$

Inverse function: $x = 1$ ← $(x \rightleftarrows y \ \ symmetry)$

$(ii)\quad \frac{dy}{dx} = 1 \quad \text{with} \quad y(0) = 1$

$\Rightarrow y = 1 + x$

Inverse function: $x = y - 1$

$(iii)\quad \frac{dy}{dx} = y \quad \text{with} \quad y(0) = 1$

$\Rightarrow y = e^x$ (EXPONENTIAL)

Inverse function: $x = \ln y$

Property: $\ln(y_A\, y_B) = \ln y_A + \ln y_B$

(iv) Unification:

$\frac{dy}{dx} = y^q \quad (q \in \mathcal{R}) \quad \text{with} \quad y(0) = 1$

$\Rightarrow y = [1 + (1-q)x]^{\frac{1}{1-q}} \equiv e_q^x$ (POWER-LAW)

Inverse function: $x = \frac{y^{1-q} - 1}{1-q} \equiv \ln_q y$

Property: $\ln_q(y_A\, y_B) = \ln_q y_A + \ln_q y_B + (1-q)(\ln_q y_A)(\ln_q y_B)$

[$q = 1$, $q = 0$ and $q \to \infty$ recover the three previous cases]

ABOUT MEAN VALUES

(i) Equiprobability: $p_i = \frac{1}{W} \quad (\forall i)$

$S_{BG} = k \ln W \quad (k = \text{positive constant})$

Naturally generalized into:

$S_q = k \ln_q W$

(ii) General: (not necessarily equiprobability)

$$S_{BG}(\{p_i\}) = -k \sum_{i=1}^{W} p_i \ln p_i = k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i} \equiv \langle k \ln \frac{1}{p_i} \rangle$$

"surprise" or "unexpectedness"

[We verify that $p_i = \frac{1}{W} \quad (\forall i)$ recovers $S_{BG} = k \ln W$]

Naturally generalized into:

$$S_q(\{p_i\}) \equiv \langle k \ln_q \frac{1}{p_i} \rangle = k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$$

"q-surprise" or "q-unexpectedness"

hence: $S_q(\{p_i\}) = k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

[We verify that $p_i = \frac{1}{W} \quad (\forall i)$ recovers $S_q = k \ln_q W$]

Property: $p_{ij}^{A+B} = p_i^A p_j^B \Rightarrow$

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q) S_q(A) S_q(B)$$

$[0 < p_i < 1 \quad (\forall i)]$

$$p_i^q > p_i \quad if \quad q < 1$$
$$< p_i \quad if \quad q > 1$$
$$= p_i \quad if \quad q = 1 \quad (BG)$$

ABOUT BIAS

(i) $S_q = (\{p_i\})$ *should be invariant under permutation.*
The simplest manner is to be

$$S_q(\{p_i\}) = f(\sum_{i=1}^{W} p_i^q)$$

(ii) *The simplest function* $f(x)$ *is*

$$S_q(\{p_i\}) = a + b\sum_{i=1}^{W} p_i^q$$

(iii) *Certainty must correspond to* $S_q = 0$
In other words $p_i = 1 \quad for \; i = i_0$
$= 0 \quad otherwise$

hence $a + b = 0$

hence $S_q(\{p_i\}) = a(1 - \sum_{i=1}^{W} p_i^q)$

(iv) *For* $q \to 1$ *we must recover* $S_{BG}(\{p_i\})$.
Using $p_i^{q-1} \sim 1 + (1-q)\ln p_i$ we obtain

$$S_q(\{p_i\}) \sim -a(q-1)\sum_{i=1}^{W} p_i \ln p_i$$

consequently, by identifying $a(q-1) = k$ we obtain

$$S_q(\{p_i\}) = k\,\frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$$

ABOUT REACTION UNDER BIAS

S. Abe
Phys Lett A **224**, 326 (1997)

(i) *Testing the function under* **translation** *of the bias x:*

$$S_{BG}(\{p_i\}) = -k \sum_{i=1}^{W} p_i \ln p_i = -k \left[\frac{d}{dx} \sum_{i=1}^{W} p_i^x\right]_{x=1}$$

(ii) *Testing the function under* **dilatation** *of the bias x:*

We replace $\frac{d}{dx}$ by Jackson's 1909 generalized derivative

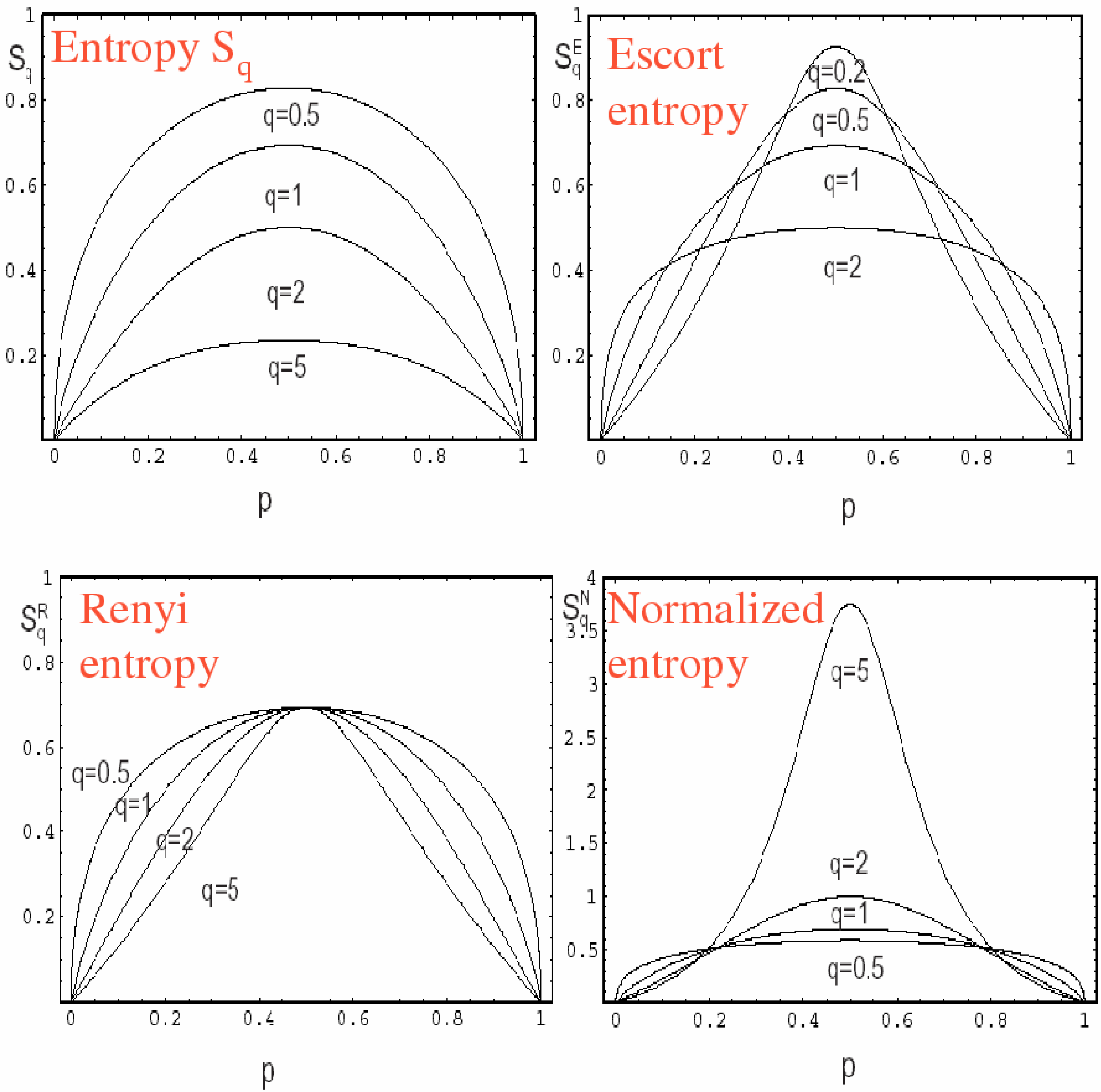
$$D_q\, h(x) \equiv \frac{h(qx)-h(x)}{qx-x} \quad \left[D_1 h(x) = \frac{dh(x)}{dx}\right]$$

and obtain

$$S_q(\{p_i\}) = -k \left[D_q \sum_{i=1}^{W} p_i^x\right]_{x=1}$$

hence

$$S_q(\{p_i\}) = k\, \frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$$


Entropy S_q
S_q
q=0.5
q=1
q=2
q=5
p
Escort entropy
S_q^E
q=0.2
q=0.5
q=1
q=2
p
Renyi entropy
S_q^R
q=0.5
q=1
q=2
q=5
p
Normalized entropy
S_q^N
q=5
q=2
q=1
q=0.5
p

SANTOS THEOREM: RJV Santos, J Math Phys **38**, 4104 (1997)

(q -generalization of Shannon 1948 theorem)

IF $\quad S(\{p_i\})$ *continuous function of* $\{p_i\}$

AND $\quad S(p_i = 1/W, \forall i)$ *monotonically increases with* W

AND $$\frac{S(A+B)}{k} = \frac{S(A)}{k} + \frac{S(B)}{k} + (1-q)\frac{S(A)}{k}\frac{S(B)}{k} \quad (\text{with } p_{ij}^{A+B} = p_i^A p_j^B)$$

AND $$S(\{p_i\}) = S(p_L, p_M) + p_L^q\, S(\{p_l / p_L\}) + p_M^q\, S(\{p_m / p_M\}) \quad (\text{with } p_L + p_M = 1)$$

THEN AND ONLY THEN

$$S(\{p_i\}) = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad \left(q = 1 \Rightarrow S(\{p_i\}) = -k\sum_{i=1}^{W} p_i \ln p_i\right)$$

CE SHANNON (The Mathematical Theory of Communication):

"This theorem, and the assumptions required for its proof, **are in no way necessary** *for the present theory. It is given chiefly to lend a certain plausibility to some of our later definitions.*

The real justification of these definitions, however, will reside in their implications.

ABE THEOREM: S Abe, Phys Lett A **271**, 74 (2000)

(q -generalization of Khinchin 1953 theorem)

IF $S(\{p_i\})$ *continuous function of* $\{p_i\}$

AND $S(p_i = 1/W,\ \forall i)$ *monotonically increases with* W

AND $S(p_{1,} p_2, ..., p_W, 0) = S(p_{1,} p_2, ..., p_W)$

AND $$\frac{S(A+B)}{k} = \frac{S(A)}{k} + \frac{S(B|A)}{k} + (1-q)\frac{S(A)}{k}\frac{S(B|A)}{k}$$

THEN AND ONLY THEN

$$S(\{p_i\}) = k\frac{1-\sum_{i=1}^{W} p_i^{\,q}}{q-1} \quad \left(q = 1 \Rightarrow S(\{p_i\}) = -k\sum_{i=1}^{W} p_i \ln p_i \right)$$

The possibility of such theorem was conjectured

by AR Plastino and A Plastino (1996, 1999).

STABILITY

or CONTINUITY

or EXPERIMENTAL ROBUSTNESS

B. Lesche
J Stat Phys 27, 419 (1982)

The entropy S is said stable iff, for any given $\varepsilon > 0$, a $\delta_\varepsilon > 0$ exists such that, independently from W,

$$\sum_{i=1}^{W} | p_i - p'_i | \leq \delta_\varepsilon \quad \Rightarrow \quad \left| \frac{S(\{p_i\}) - S(\{p_i'\})}{S_{\max}} \right| < \varepsilon$$

Hence $$\lim_{\delta \to 0} \lim_{W \to \infty} \frac{S(\{p_i\}) - S(\{p_i'\})}{S_{\max}} = 0$$

S_{BG} and S_q $(\forall q > 0)$ are stable

S. Abe, Phys Rev E **66**, 046134 (2002)

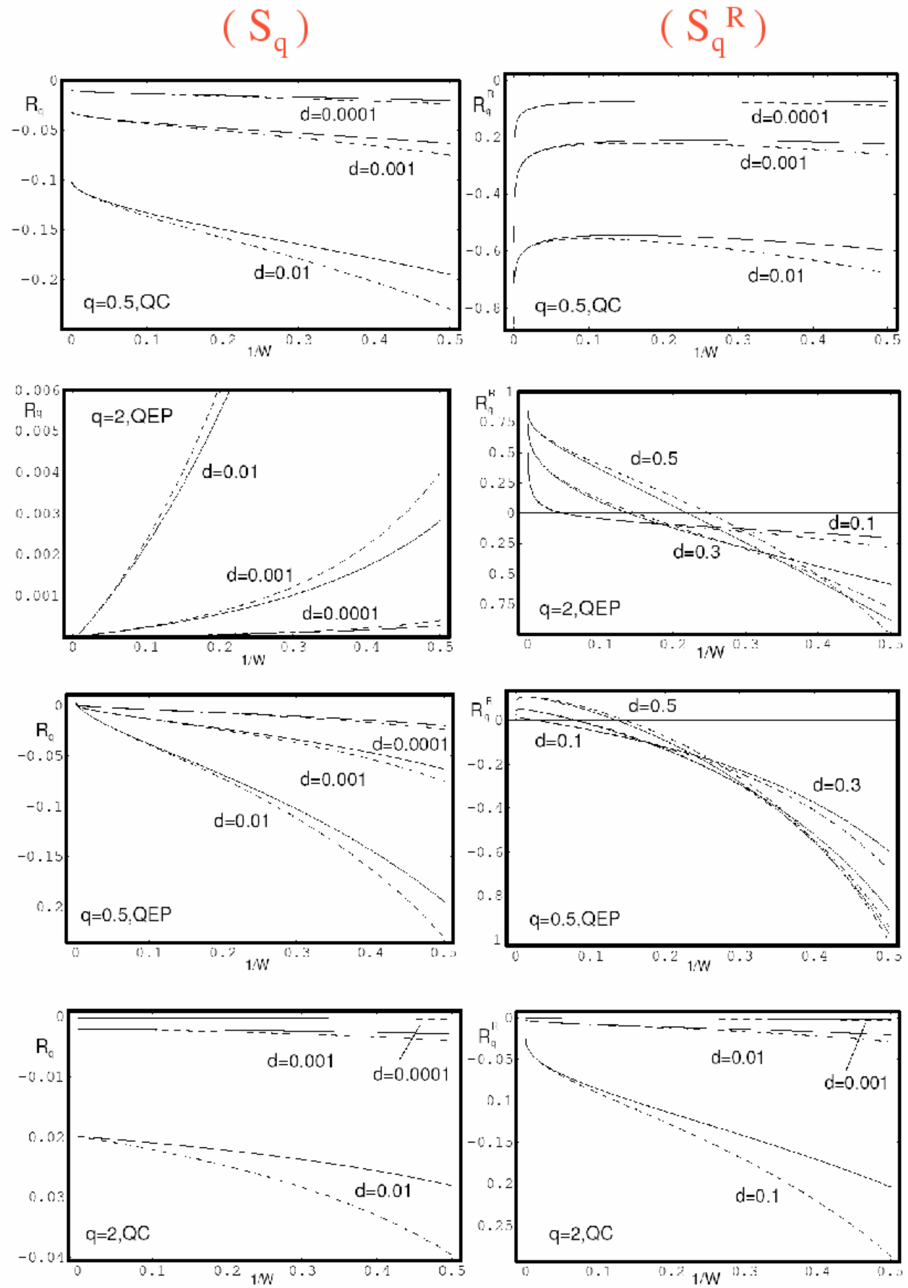
$$S_q^R(\{p_i\}) \equiv \frac{\ln \sum_{i=1}^{W} p_i^q}{q-1} \qquad \text{(Renyi entropy)}$$

$$S_q^N(\{p_i\}) \equiv \frac{S_q(\{p_i\})}{\sum_{i=1}^{W} p_i^q} \qquad \text{(Normalized entropy)}$$

$$S_q^E(\{p_i\}) \equiv \frac{1 - \left(\sum_{i=1}^{W} p_i^{1/q} \right)^{-q}}{q-1} \qquad \text{(Escort entropy)}$$

are unstable

B. Lesche (1982); S. Abe (2002); C.T. and E. Brigatti (2003)



QEP = quasi equal probabilities

QC = quasi certainty

C.T. and E. Brigatti (2003)

DEFINITIONS :

$q-logarithm:$

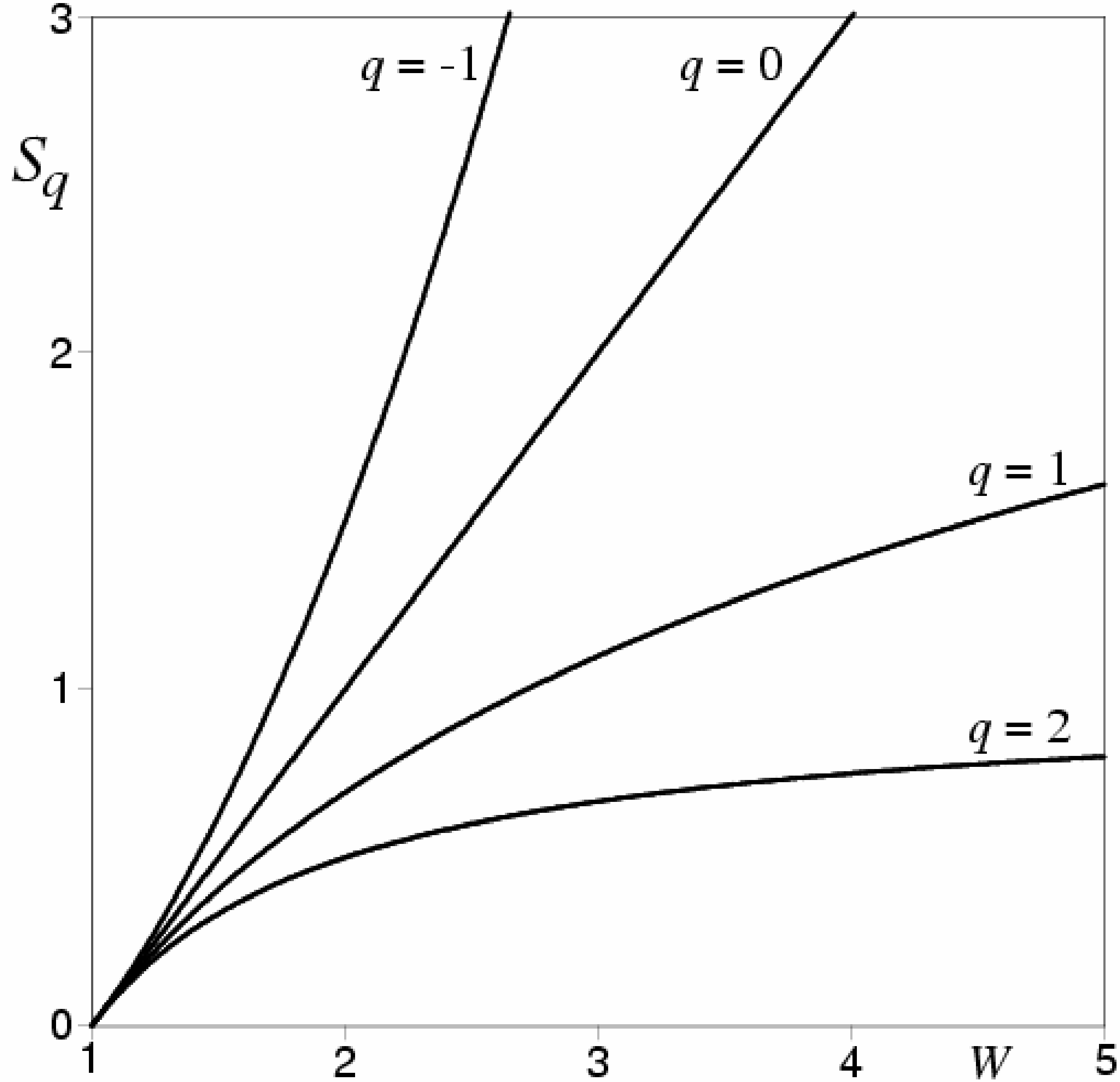
$$\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0)$$

$$\ln_1 x = \ln x$$

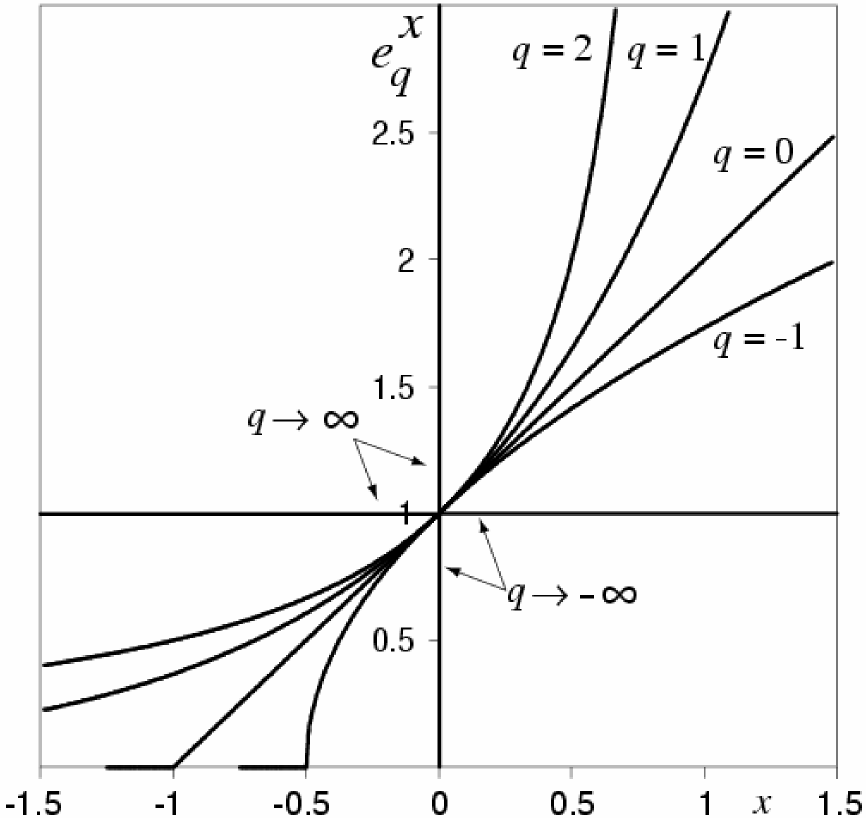
q - $exponential:$

$$e_q^x \equiv \begin{cases} [1+(1-q)x]^{\frac{1}{1-q}} & if \;\; 1+(1-q)x \geq 0 \\ 0 & otherwise \end{cases}$$

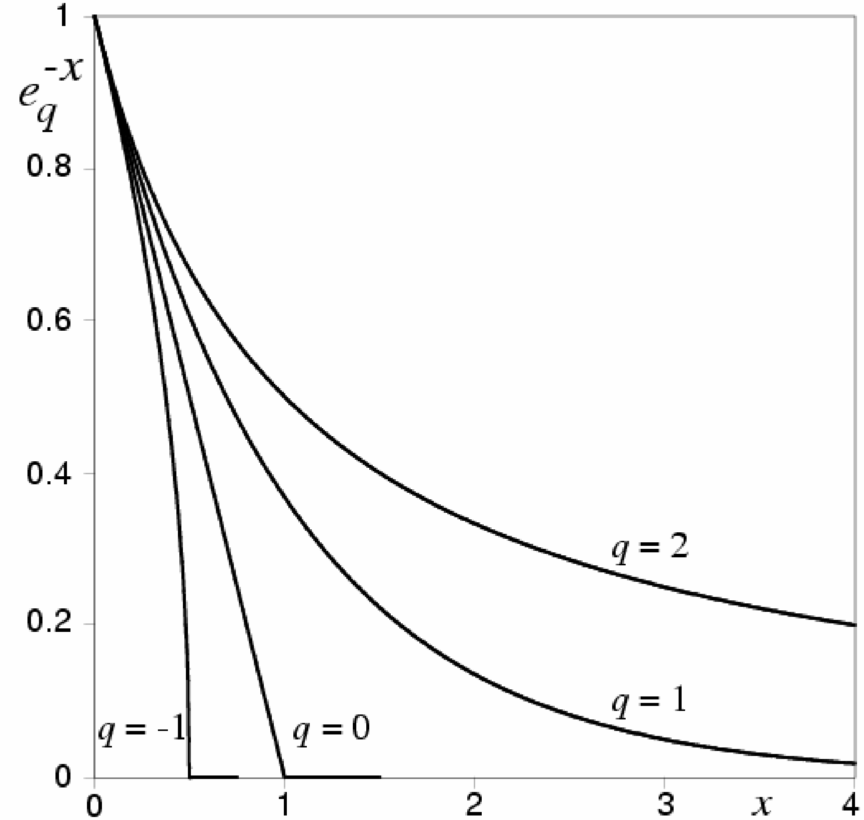
$$e_1^x = e^x$$



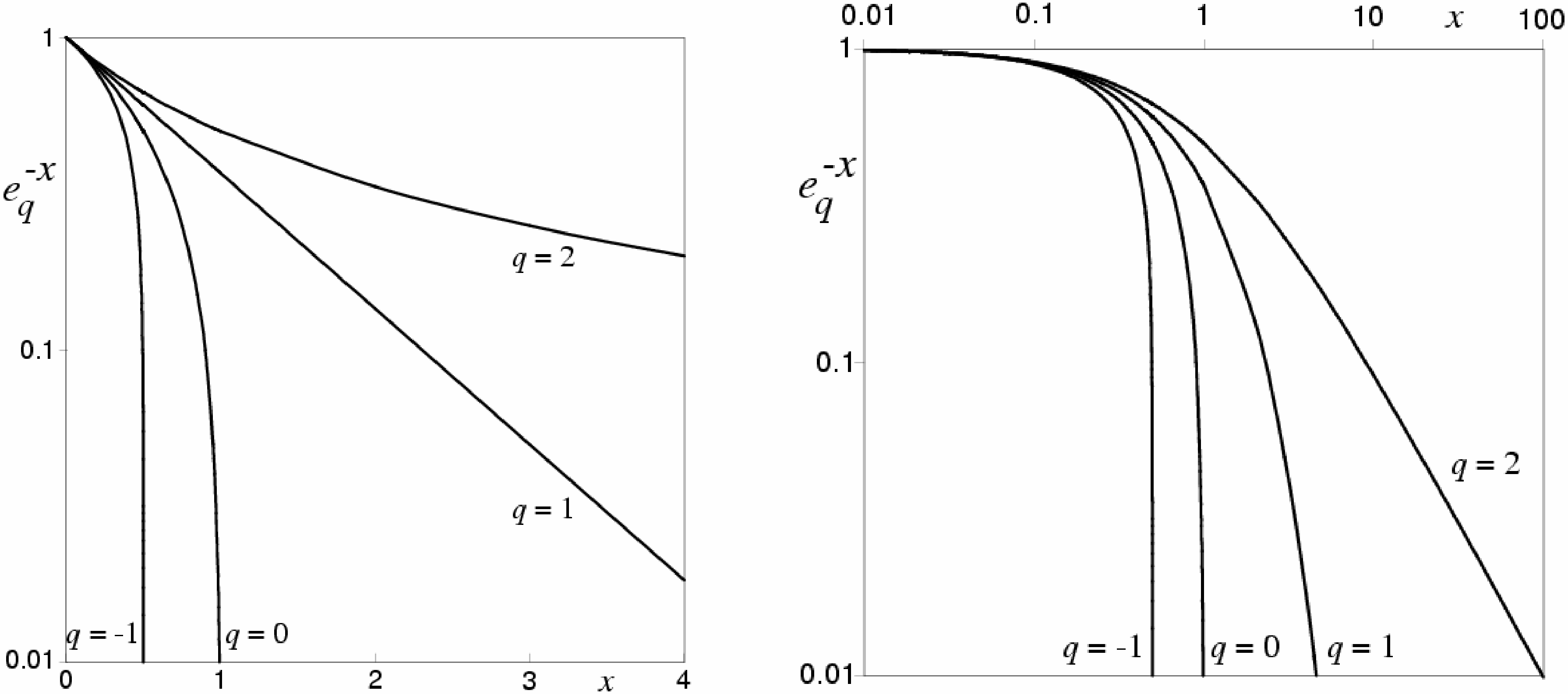
3
2
1
0
S_q
$q = -1$
$q = 0$
$q = 1$
$q = 2$
1
2
3
4
W
5



e_q^x
$q = 2$
$q = 1$
$q = 0$
$q = -1$
$q \to \infty$
$q \to -\infty$
2.5
2
1.5
1
0.5
-1.5
-1
-0.5
0
0.5
1
1.5
x



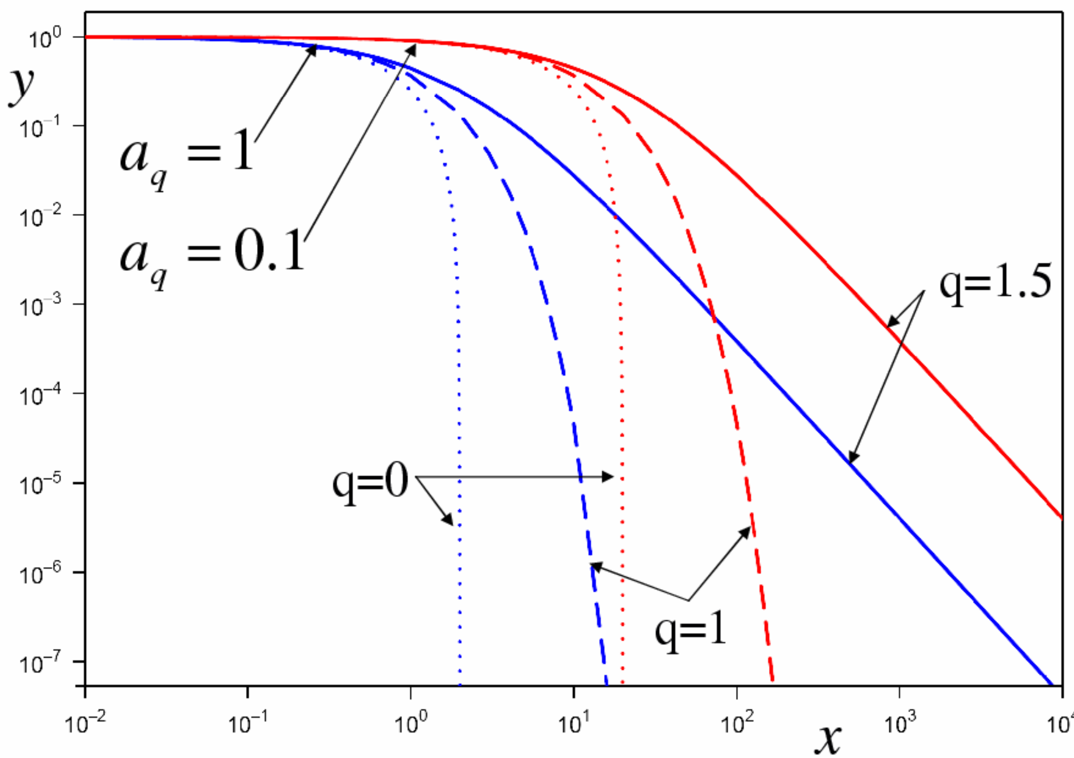
e_q^{-x}
$q = 2$
$q = 1$
$q = 0$
$q = -1$
1
0.8
0.6
0.4
0.2
0
1
2
3
4
x

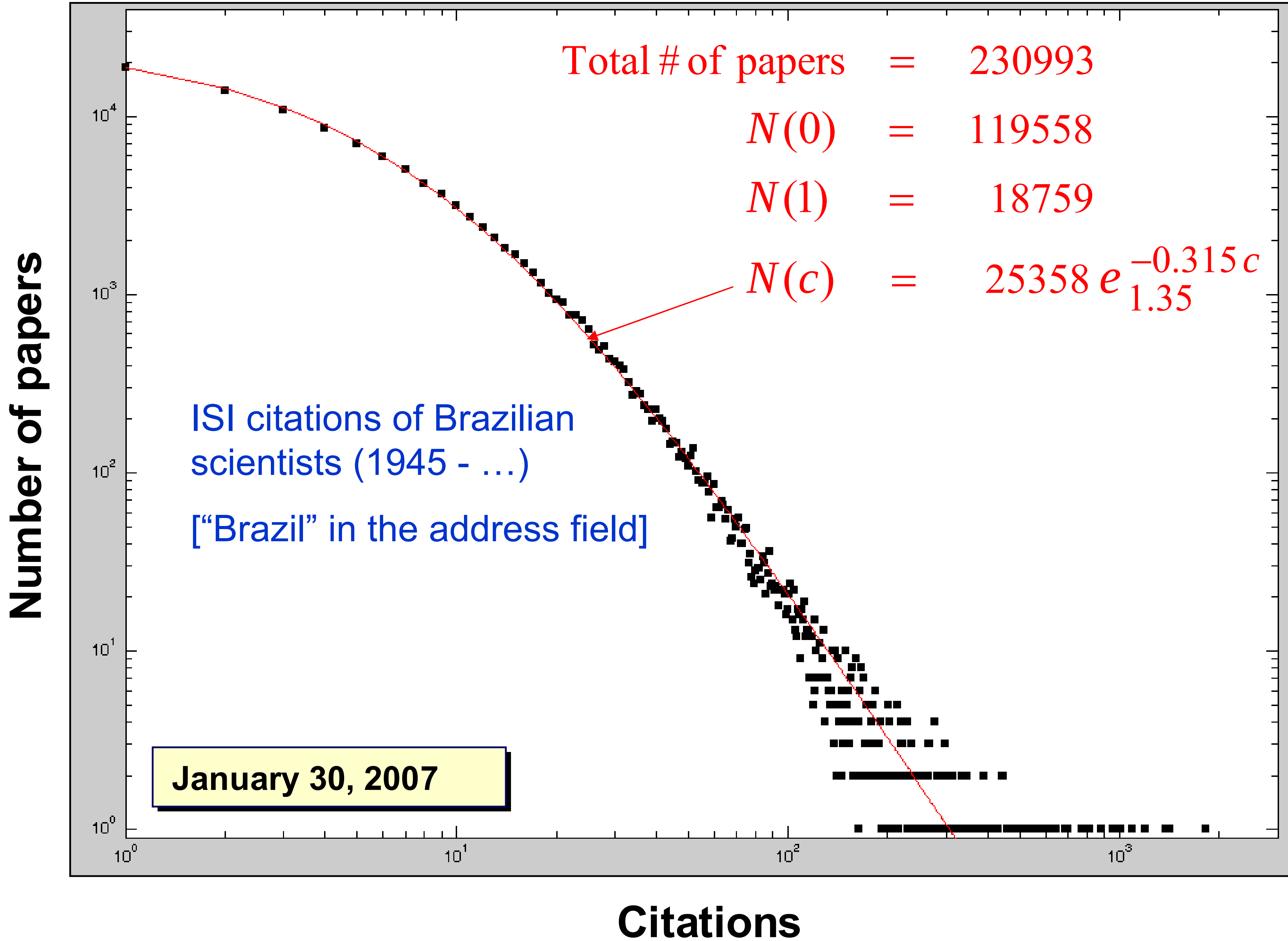


1
0.1
0.01
0
1
2
3
4
x
e_q^{-x}
q = 2
q = 1
q = -1
q = 0
0.01
0.1
1
10
x
100
1
0.1
0.01
e_q^{-x}
q = 2
q = -1
q = 0
q = 1

$$\frac{dy}{dx} = -a_q y^q \quad with \quad y(0) = 1$$

$$\Rightarrow y = \frac{1}{[1+(q-1)a_q x]^{\frac{1}{q-1}}} \equiv e_q^{-a_q x}$$





M.P de Albuquerque and D.B. Mussi (2007)

PREDECESSORS

RENYI ENTROPY $\propto \ln \sum_i p_i^q$:

M.P. Schutzenberger, Publ. Inst. Statist. Univ. Paris (1954) [*according to I. Csiszar* (1974,1978)]

A. Renyi, Proc. 4^{th} *Berkeley Symposium* (1969)

ENTROPY $\propto 1-\sum_i p_i^q$:

J. Harvda and F. Charvat, Kybernetica 3, 30 (1967)

I. Vajda, Kybernetica 4, 105 (1968)

Z. Daroczy, Inf. Control 16, 36 (1970)

J. Lindhard and V. Nielsen, Det Kongelige Danske Videnskabernes Selskab Matematisk - fysiske Meddelelser (*Denmark*) 38 (9), 1 (1971)

B.D. Sharma and D.P. Mittal, J. Math. Sci. 10, 28 (1975) [*unification of both previous entropic forms*]

A. Wehrl, Rev. Mod. Phys. 50, 221 (1978)

$q-$*GAUSSIANS* :

Gaussian distribution	*Abraham de Moivre* (1733)
	Pierre Simon de Laplace (1774)
	Robert Adrain (1808)
	Carl Friedrich Gauss (1809)
Cauchy – Lorentz – Breit	*Agustin Louis Cauchy* (~1821)
– Wigner distribution	*Hendric Antoon Lorentz* (~1880)
Student's t – distribution	*William Sealy Gosset* (1908)

THE VARIOUS FORMS OF THE NONADDITIVE q-ENTROPY Sq:

DISCRETE CLASSICAL STATES (Shannon for $q=1$):

$$\frac{S_q}{k} = \frac{1-\sum_{i=1}^{W} p_i^q}{q-1} = \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i} = -\sum_{i=1}^{W} p_i^q \ln_q p_i \quad \left(with \sum_{i=1}^{W} p_i = 1\right)$$

CONTINUOUS CLASSICAL STATES (Boltzmann, Gibbs for $q=1$):

$$\frac{S_q}{k} = \frac{1-\int dx\, [p(x)]^q}{q-1} = \int dx\, p(x)\, \ln_q \frac{1}{p(x)} = -\int dx\, [p(x)]^q\, \ln_q p(x) \quad \left(with \int dx\, p(x) = 1\right)$$

Particular case: $p(x) = \sum_{i=1}^{W} p_i\, \delta(x - x_i) \Rightarrow \frac{S_q}{k} = \frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$

QUANTUM STATES (von Neumann for $q=1$):

$$\frac{S_q}{k} = \frac{1-Tr\,\rho^q}{q-1} = Tr\,(\rho\, \ln_q \rho^{-1}) = -Tr\,(\rho^q\, \ln_q \rho) \quad (with\ Tr\,\rho = 1)$$

Particular case: $\rho_{ij} = p_i\, \delta_{ij} \Rightarrow \frac{S_q}{k} = \frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$

$$S_q(N,t)\ versus\ \ t$$

DISSIPATIVE MAPS:

- Strongly chaotic (i.e., maximal Lyapunov exponent > 0)
- Weakly chaotic (i.e., maximal Lyapunov exponent = 0)

CONSERVATIVE MAPS:

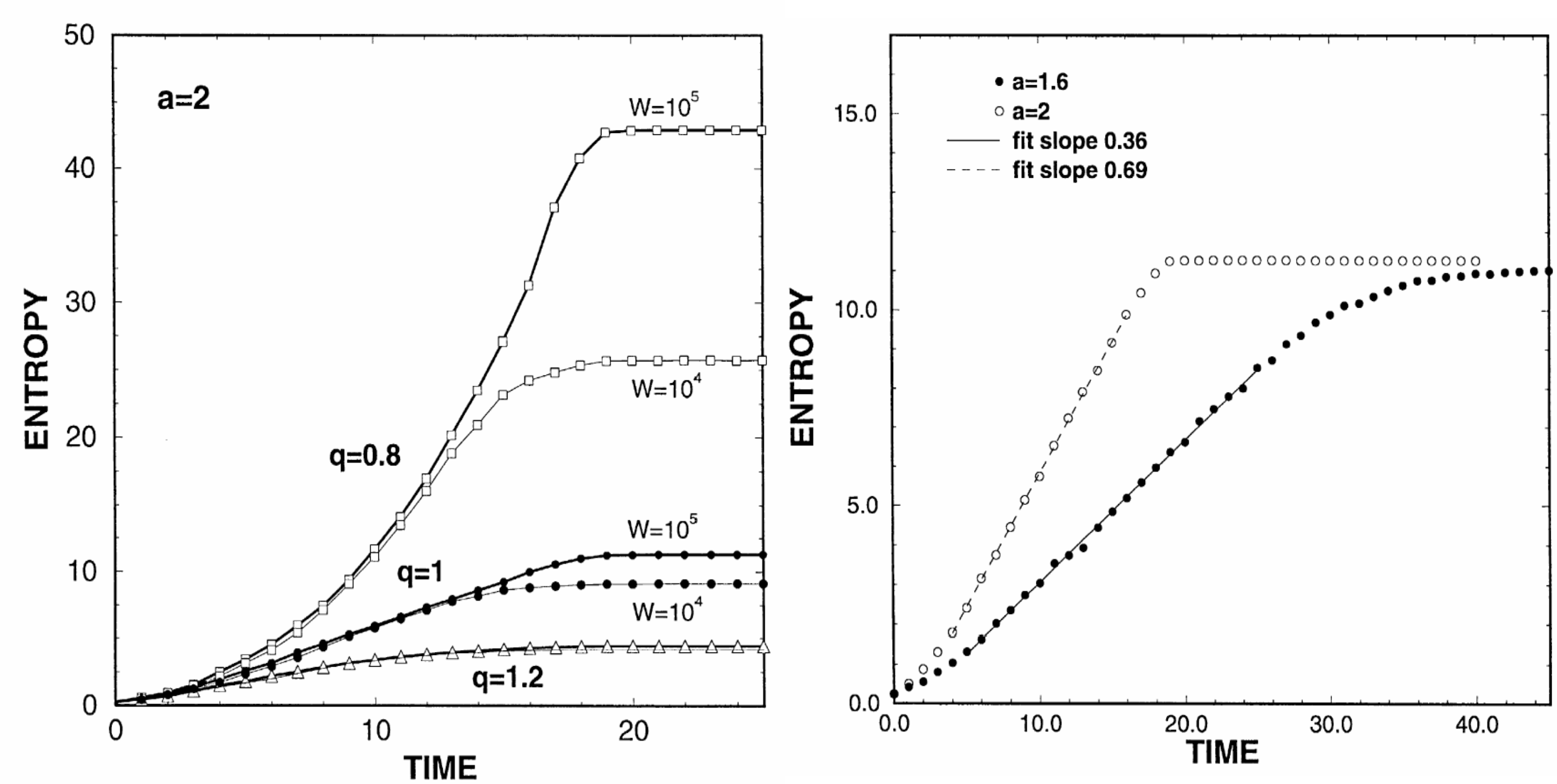
- Strongly chaotic (i.e., maximal Lyapunov exponent > 0)
- Weakly chaotic (i.e., maximal Lyapunov exponent = 0)

DISSIPATIVE MAPS

LOGISTIC MAP:

$$x_{t+1} = 1 - a\,x_t^2 \quad (0 \le a \le 2;\; -1 \le x_t \le 1;\; t = 0,1,2,\ldots)$$

(strong chaos, i.e., **positive** Lyapunov exponent)



V. Latora, M. Baranger, A. Rapisarda and C. T., Phys. Lett. A **273**, 97 (2000)

We verify

$$K_1 = \lambda_1 \quad (Pesin-like\ identity)$$

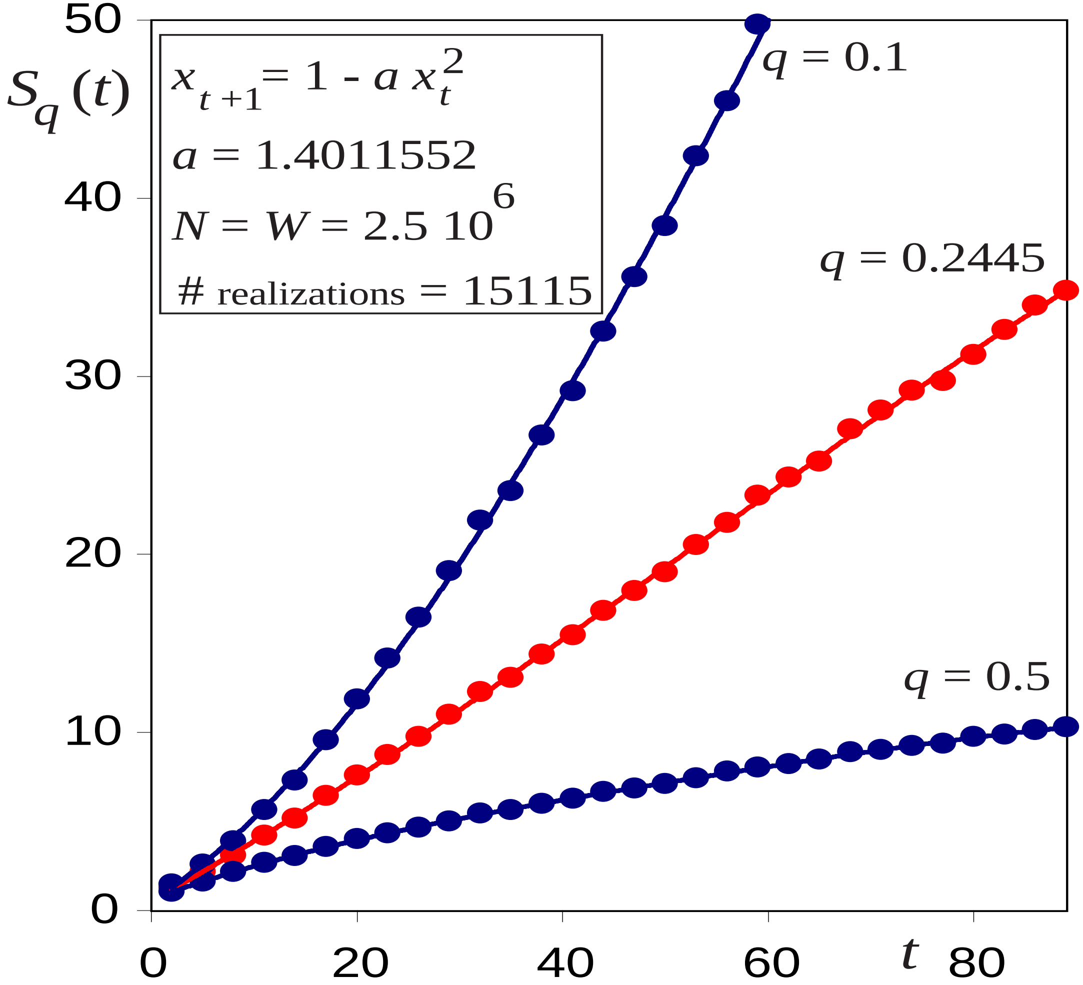
where

$$K_1 \equiv \lim_{t\to\infty} \frac{S_1(t)}{t}$$

and

$$\xi(t) \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)} = e^{\lambda_1 t}$$

(weak chaos, i.e., **zero** Lyapunov exponent)



C. T. , A.R. Plastino and W.-M. Zheng, Chaos, Solitons & Fractals **8**, 885 (1997)
M.L. Lyra and C. T. , Phys. Rev. Lett. **80**, 53 (1998)
V. Latora, M. Baranger, A. Rapisarda and C. T. , Phys. Lett. A **273**, 97 (2000)
E.P. Borges, C. T. , G.F.J. Ananos and P.M.C. Oliveira, Phys. Rev. Lett. **89**, 254103 (2002)
F. Baldovin and A. Robledo, Phys. Rev. E **66**, R045104 (2002) and **69**, R045202 (2004)
G.F.J. Ananos and C. T. , Phys. Rev. Lett. **93**, 020601 (2004)
E. Mayoral and A. Robledo, Phys. Rev. E **72**, 026209 (2005), and references therein

It can be proved that

$$K_q = \lambda_q \quad (q-generalized \;\; Pesin-like\ identity)$$

where

$$K_q \equiv \lim_{t\to\infty} \sup\left\{\frac{S_q(t)}{t}\right\}$$

and

$$\xi(t) \equiv \sup\left\{\lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)}\right\} = e_q^{\lambda_q\, t}$$

with

$$\frac{1}{1-q} = \frac{1}{\alpha_{\min}} - \frac{1}{\alpha_{\max}} = \frac{\ln \alpha_F}{\ln 2} \quad and \quad \lambda_q = \frac{1}{1-q}$$

$$\left[x_{t+1} = 1 - a\,|\,x_t\,|^z \;\;\Rightarrow\;\; \frac{1}{1-q(z)} = \frac{1}{\alpha_{\min}(z)} - \frac{1}{\alpha_{\max}(z)} = (z-1)\frac{\ln \alpha_F(z)}{\ln 2} \right]$$

EDGE OF CHAOS OF THE LOGISTIC MAP:

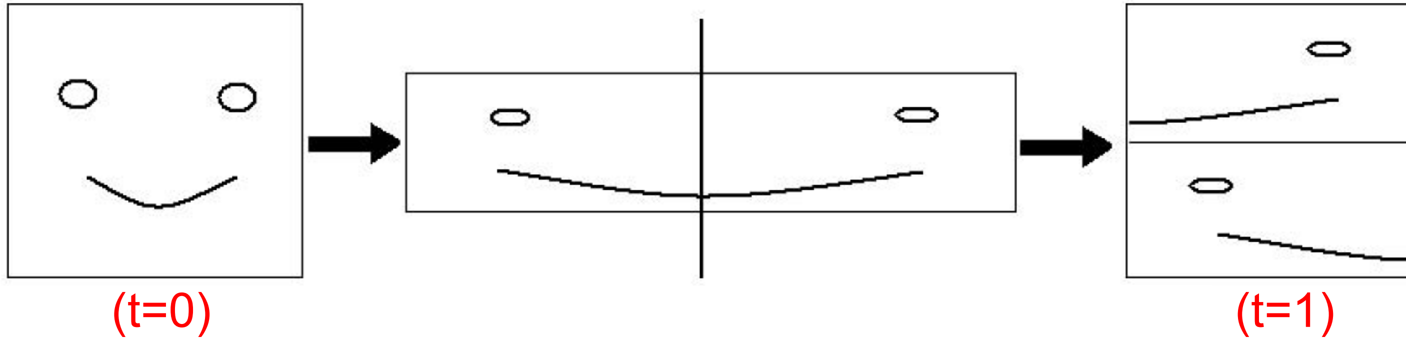
(Using result in http://pi.lacim.uqam.ca/piDATA/feigenbaum.txt)

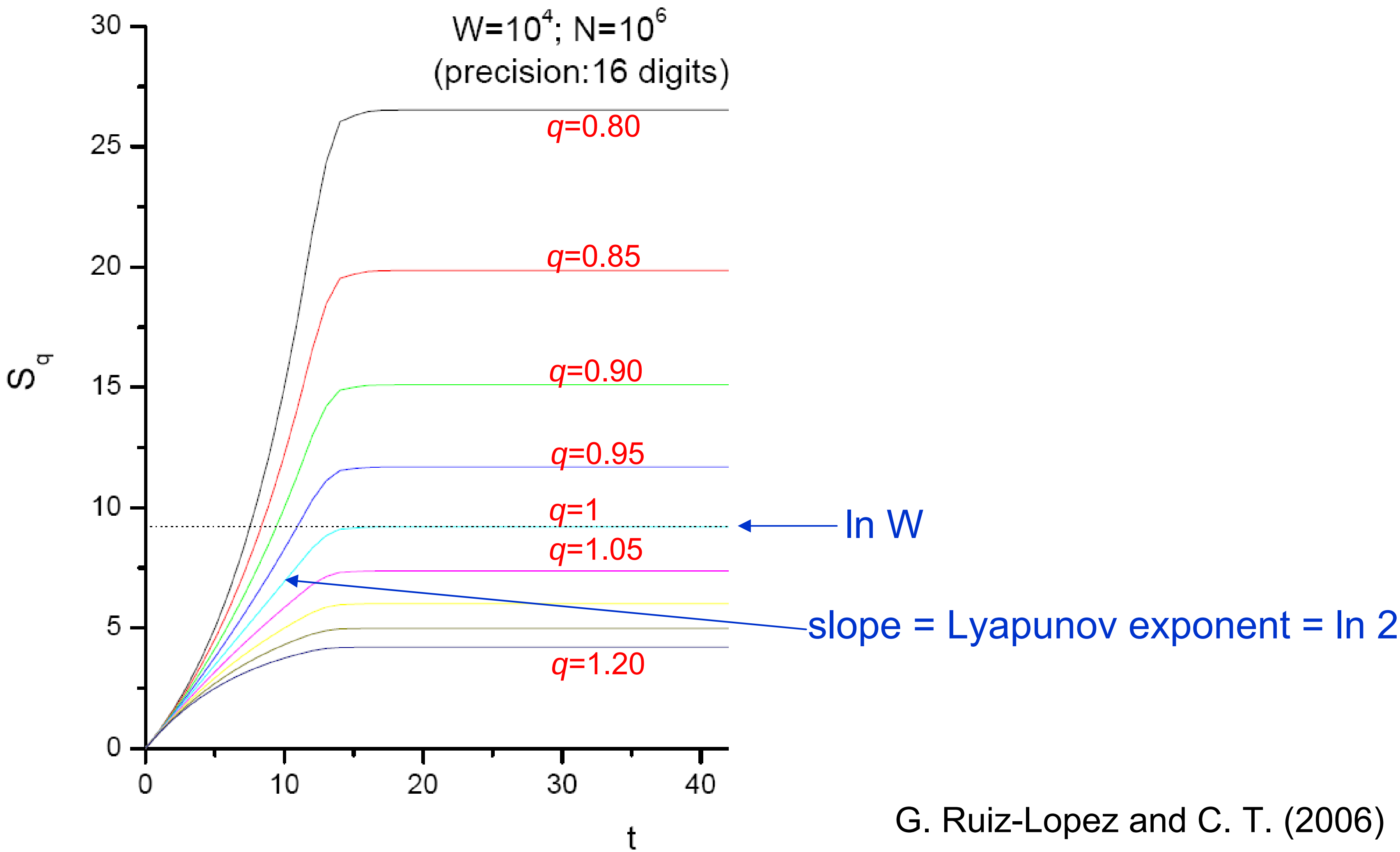
q =

0.244487701341282066198770423404680405234446935490057673670365
986327749672766558665755156226857540706288349640382728306063600
1937303318189645513410812778097921943860270831944900524658135215
031745349520749404481654609490874483340567236224664880833330721
4231898714587299268154849677460786482183456906337020594682046189
9021675321457546117438305008496860408846969491704367478991506
01664649106021783482788999381838252255458233803811311803180544823
675794499039707439546614634081555316878853503011382149141126624
6328940130370152354936571471269917921021622688833029675405780
630706822368810432015790352123740735444602970006055250423142028
0891935788112397319779748442351524560409264467095795703046586141
2956647966668774368324049202275739300475089531185517955872048399
269689682755585244502443652682560942378012803309487795440354252
4859043379761802711830004573585550738941136758784400629135630
42167454169409213569860320785908819985935900731933680106996749670
79044560924186321120541305473939857955444103476122225921368462
19346009360… (1018 meaningful digits)

CONSERVATIVE MAPS

BAKER MAP:





G. Ruiz-Lopez and C. T. (2006)

THE CASATI-PROSEN TRIANGLE MAP:

G. Casati and T. Prosen,
Phys. Rev. Lett. **83**, 4729 (1999) and **85**, 4261 (2000)

"While exponential instability is sufficient for a meaningful statistical description, it is not known whether or not it is also necessary."

$$y_{t+1} = y_t + \alpha \ \mathrm{sgn}(x_t) + \beta \quad \pmod 2$$

$$x_{t+1} = x_t + y_{t+1} \quad \pmod 2$$

(α and β independent irrationals)

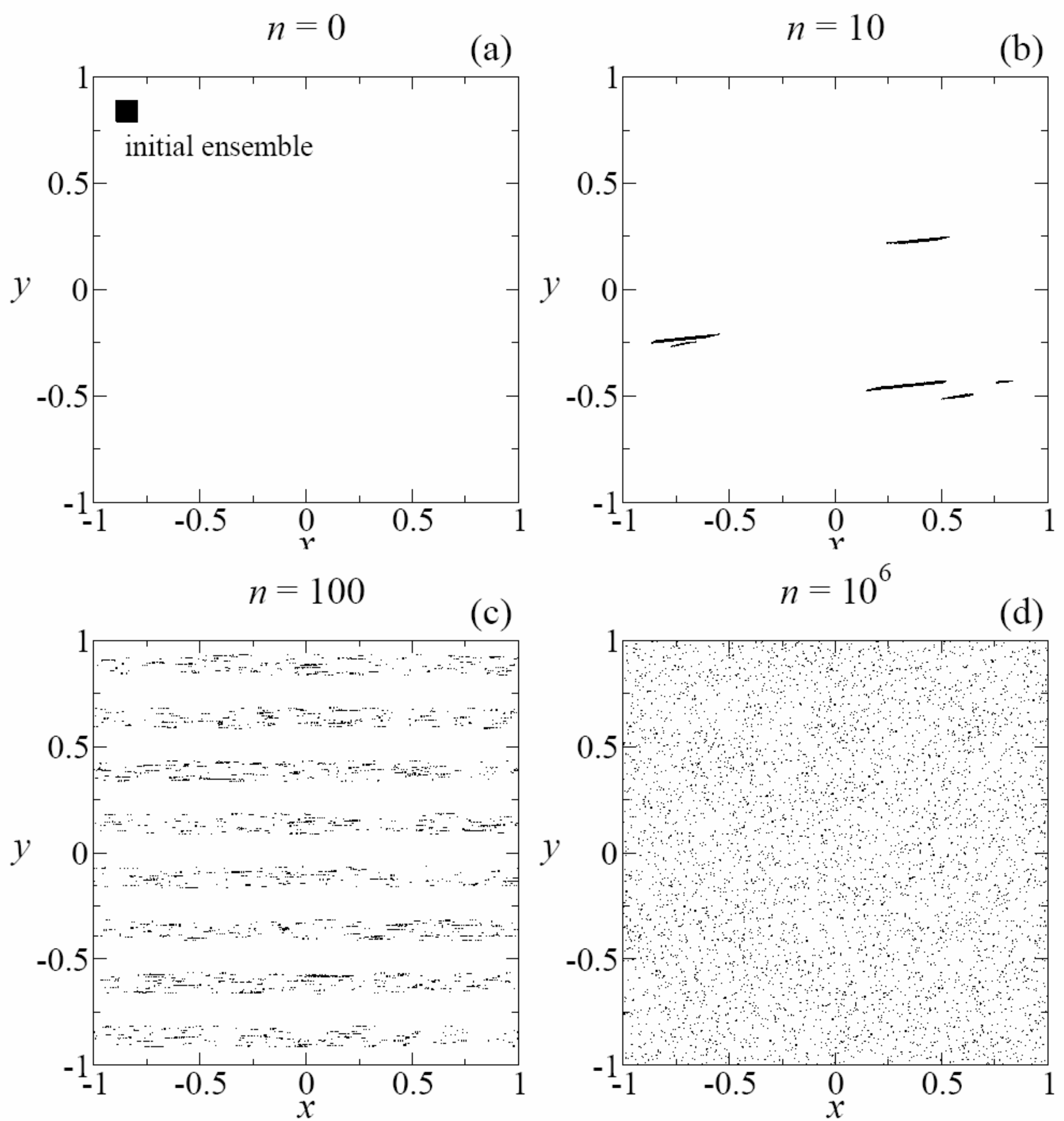
e.g., $(\alpha,\beta)=\left((1/2)(\sqrt{5}-1)-(1/e),\ (1/2)(\sqrt{5}-1)+(1/e)\right)$

This map is conservative, mixing, ergodic and nevertheless with zero Lyapunov exponent!

Furthermore $\xi \equiv \lim_{\Delta X(0)\to 0} \dfrac{\Delta X(t)}{\Delta X(0)} \propto t$

CASATI-PROSEN TRIANGLE MAP [Casati and Prosen, Phys Rev Lett **83**, 4729 (1999) and **85**, 4261 (2000)]

(two-dimensional, conservative, mixing, ergodic, vanishing maximal Lyapunov exponent)



G. Casati, C. T. and F. Baldovin, Europhys. Lett. **72**, 355 (2005)

NONEXTENSIVITY OF THE CASATI-PROSEN MAP:

[G. Casati, C.T. and F. Baldovin, Europhys Lett **72**, 355 (2005)]

Answer to the above equation:

It is not necessary: a meaningful statistical description is possible with zero Lyapunov exponent!

[Essentially because an integrable system has zero Lyapunov exponent but the opposite is not true]

In general, $\xi = [1+(1\text{-q})\lambda_q t]^{1/(1-q)}$

hence, $\xi \propto t \Rightarrow q = 0$

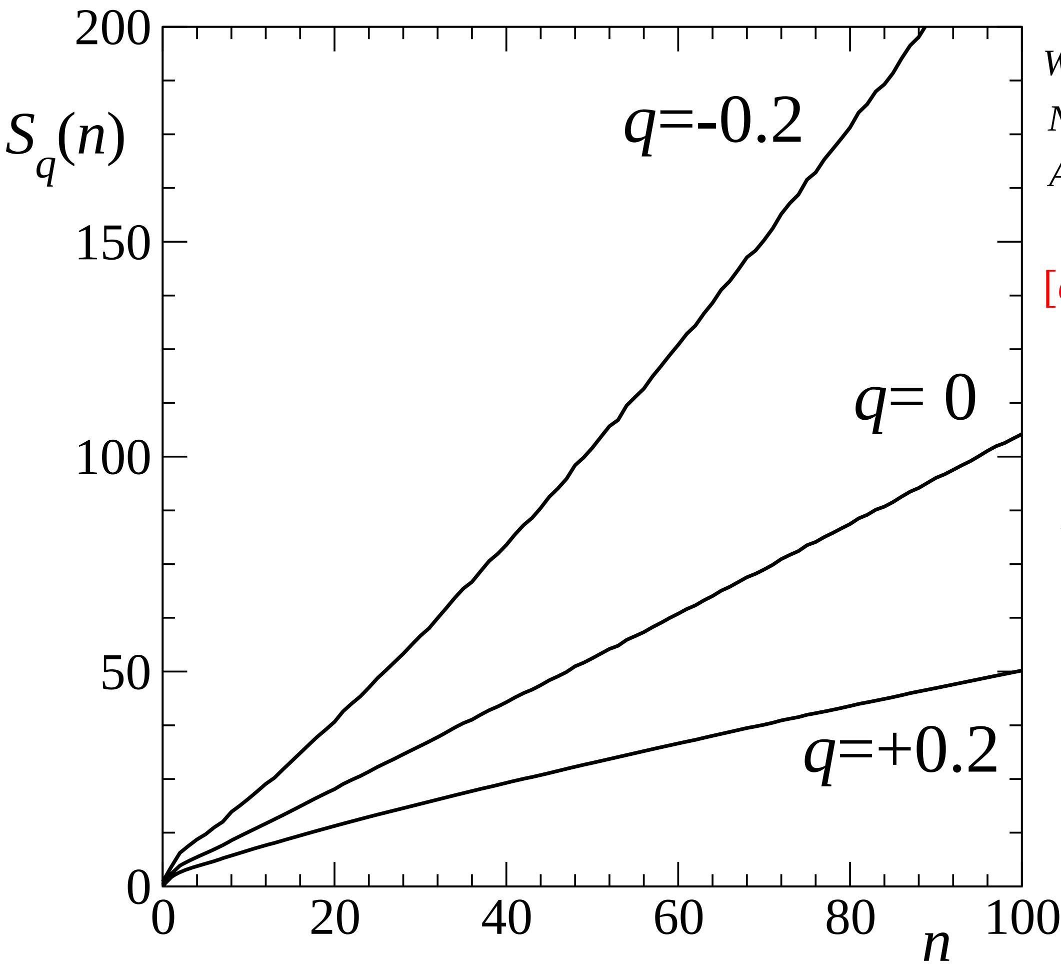
Consistently, we expect

$$(i)\ S_q(t) \equiv \frac{1-\sum_{i=1}^{W}[p_i(t)]^q}{q-1} \propto t \text{ only for } q=0$$

$$(ii) K_q \equiv \lim \frac{S_q(t)}{} = \lambda \qquad \text{for } a=0$$

CASATI-PROSEN TRIANGLE MAP [Casati and Prosen, Phys Rev Lett **83**, 4729 (1999) and **85**, 4261 (2000)]

(two-dimensional, conservative, mixing, ergodic, vanishing maximal Lyapunov exponent)



W = 4000 × 4000 *cells*

N = 1000 *initial conditions randomly chosen in one cell*

Average done over 100 initial cells

[$q = 0 \rightarrow$ *linear correlation* = 0.99993]

Also $\xi = e_0^{\lambda_0 t}$

with $\lambda_0 = \lim_{n\to\infty} \dfrac{S_0(n)}{n} = 1$

q - generalization of Pesin (- like) theorem

G. Casati, C. T. and F. Baldovin, Europhys. Lett. **72**, 355 (2005)

$$S_q(N,t) \; versus \; N$$

HYBRID PASCAL - LEIBNITZ TRIANGLE

$$(N=0) \quad 1 \times \frac{1}{1}$$

$$(N=1) \quad 1 \times \frac{1}{2} \quad 1 \times \frac{1}{2}$$

$$(N=2) \quad 1 \times \frac{1}{3} \quad 2 \times \frac{1}{6} \quad 1 \times \frac{1}{3}$$

$$(N=3) \quad 1 \times \frac{1}{4} \quad 3 \times \frac{1}{12} \quad 3 \times \frac{1}{12} \quad 1 \times \frac{1}{4}$$

$$(N=4) \quad 1 \times \frac{1}{5} \quad 4 \times \frac{1}{20} \quad 6 \times \frac{1}{30} \quad 4 \times \frac{1}{20} \quad 1 \times \frac{1}{5}$$

$$(N=5) \quad 1 \times \frac{1}{6} \quad 5 \times \frac{1}{30} \quad 10 \times \frac{1}{60} \quad 10 \times \frac{1}{60} \quad 5 \times \frac{1}{30} \quad 1 \times \frac{1}{6}$$

$$\Sigma = 1 \quad (\forall \; N)$$

Blaise **Pascal** (1623-1662)
Gottfried Wilhelm **Leibnitz** (1646-1716)
Daniel **Bernoulli** (1700-1782)

$(N=2)$

A \ B	1	2	
1	$p^2+\kappa$	$p(1-p)-\kappa$	p
2	$p(1-p)-\kappa$	$(1-p)^2+\kappa$	1- p
	p	1- p	1

<u>EQUIVALENTLY:</u>

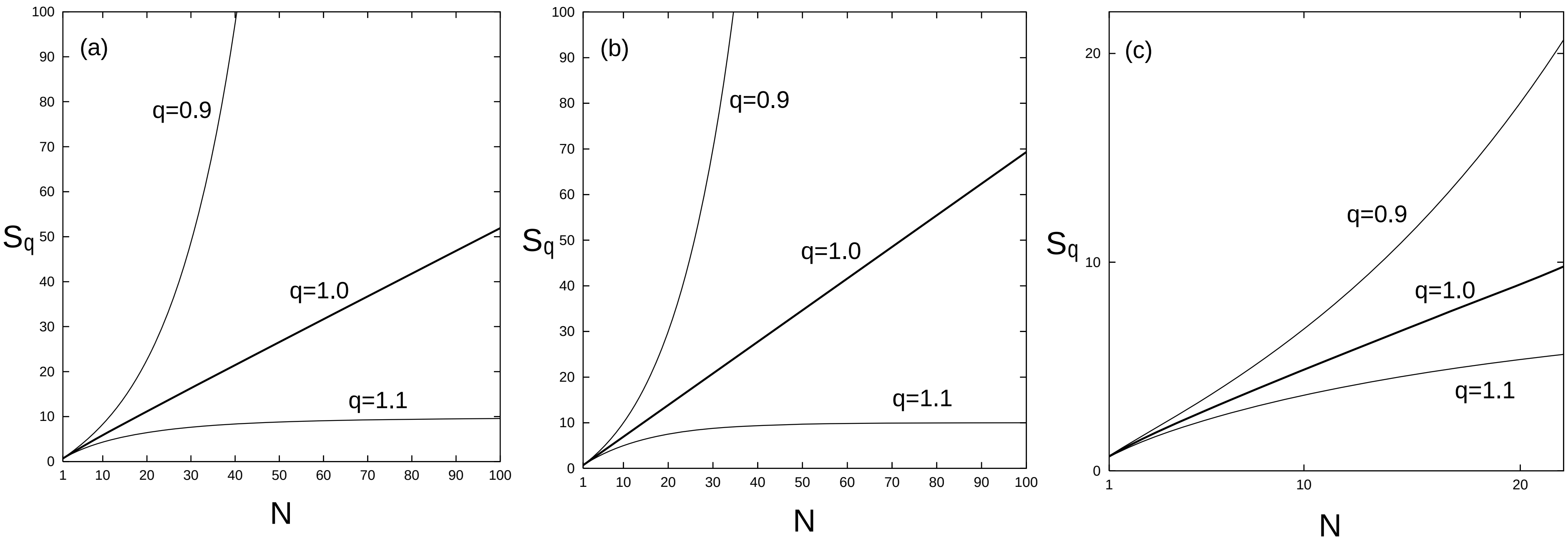
$(N=0) \qquad 1\times 1$

$(N=1) \qquad 1\times p \qquad 1\times(1-p)$

$(N=2) \qquad 1\times[p^2+\kappa] \qquad 2\times[p(1-p)-\kappa] \qquad 1\times[(1-p)^2+\kappa]$

$q=1$ SYSTEMS

i.e., such that $S_1(N) \propto N \quad (N \to \infty)$



Leibnitz triangle

$$\left(p_{N,0} = \frac{1}{N+1} \right)$$

N independent coins

$$\left(\begin{array}{l} p_{N,0} = p^N \\ \\ \text{with } p = 1/2 \end{array} \right)$$

Stretched exponential

$$\left(\begin{array}{l} p_{N,0} = p^{N^\alpha} \\ \\ \text{with } p = \alpha = 1/2 \end{array} \right)$$

(All three examples **strictly** satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato

Proc Natl Acad Sc USA **102**, 15377 (2005)

Asymptotically scale-invariant (d=2)

$(N=0)$	1				
$(N=1)$	1/2	1/2			
$(N=2)$	1/3	1/6	1/3		
$(N=3)$	3/8	5/48	5/48	0	
$(N=4)$	2/5	3/40	1/20	0	0

d+1

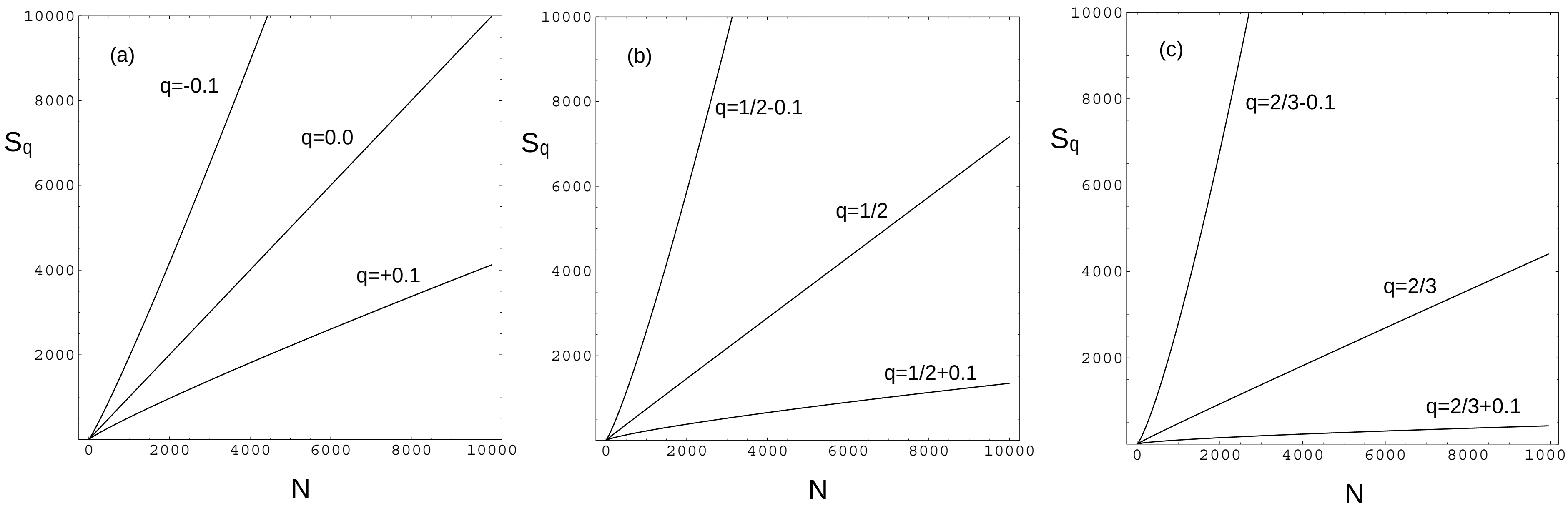
(It **asymptotically** satisfies the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato
Proc Natl Acad Sc USA **102**, 15377 (2005)

$q \neq 1$ SYSTEMS

i.e., such that $S_q(N) \propto N \quad (N \to \infty)$

(d =1) (d = 2) (d = 3)

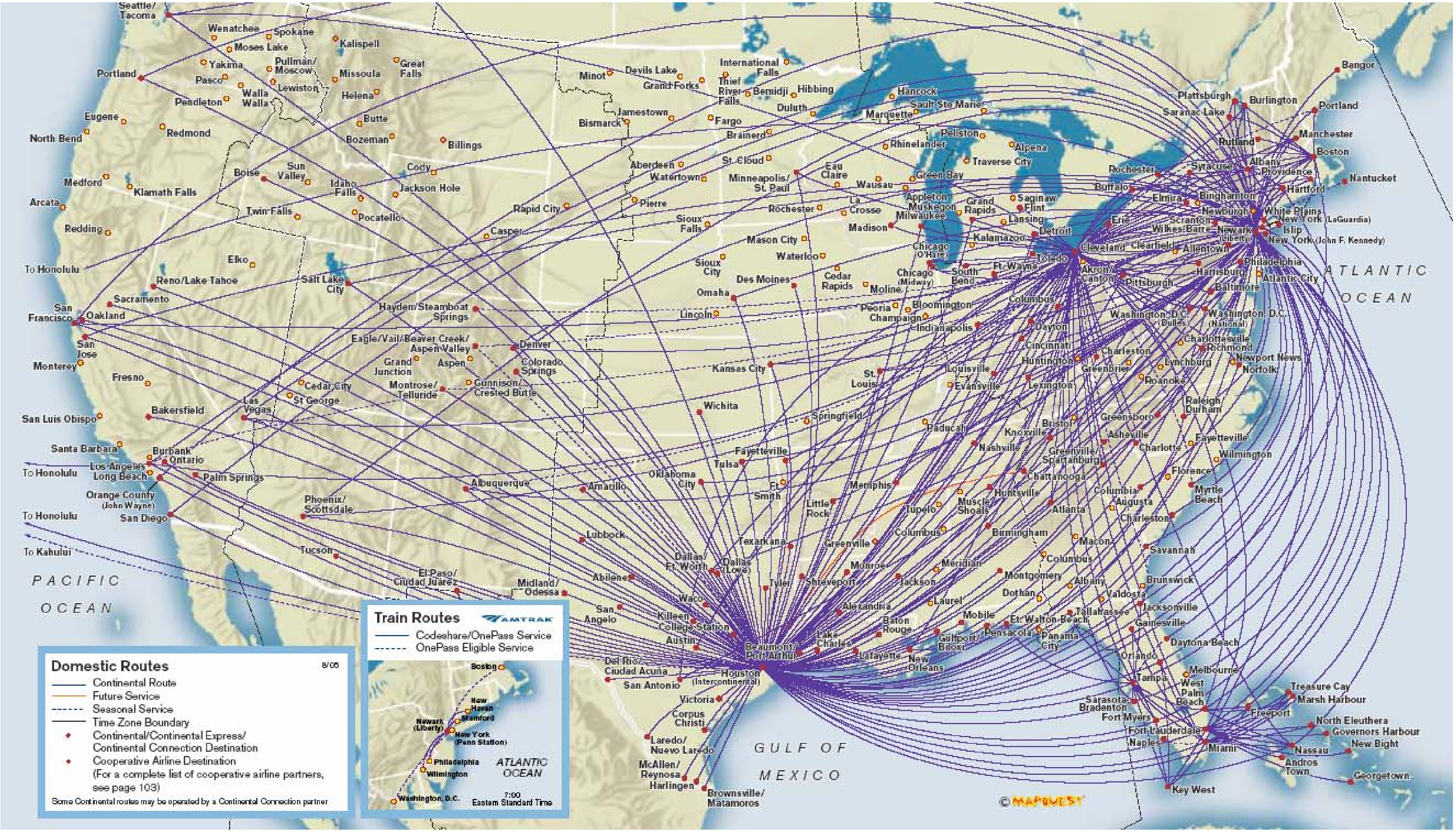


$$q = 1 - \frac{1}{d}$$

(All three examples **asymptotically** satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato

Proc Natl Acad Sc USA **102**, 15377 (2005)



Continental Airlines

If A and B are independent,

i.e., if $p_{ij}^{A+B} = p_i^{A}\, p_j^{B}$,

then

$$S_{BG}(A+B) = S_{BG}(A) + S_{BG}(B)$$

whereas

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{1-q}{k_B}\, S_q(A)\, S_q(B)$$

$$\neq S_q(A) + S_q(B) \quad (if\;\; q \neq 1)$$

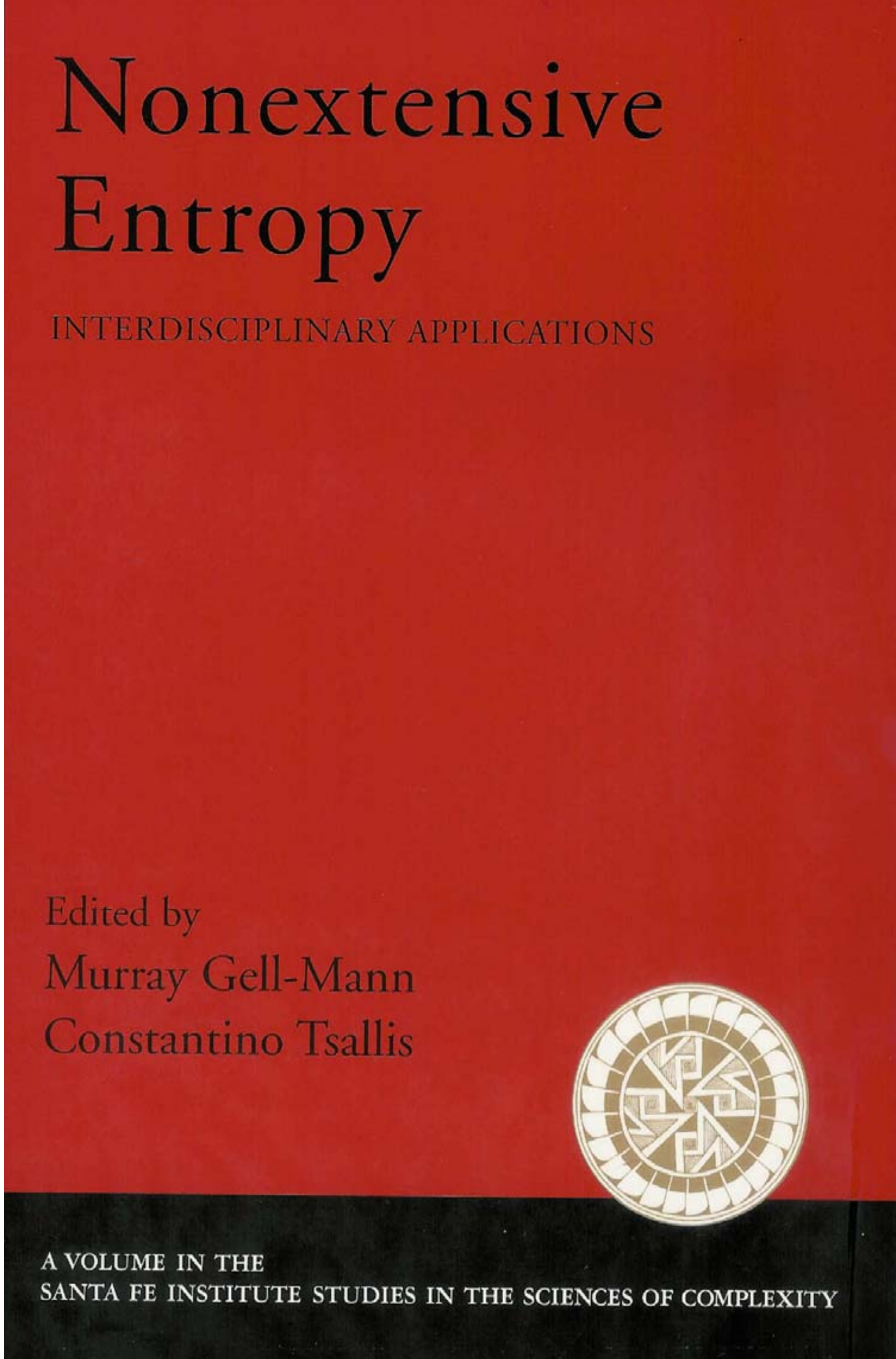
But if A and B are especially (globally) correlated

then

$$S_q(A+B) = S_q(A) + S_q(B)$$

whereas

$$S_{BG}(A+B) \neq S_{BG}(A) + S_{BG}(B)$$



ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

Hence, S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N. An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \text{ , } i.e., \ S(N) \propto N \ \ (N \to \infty)$$

CONSEQUENTLY:

The additive entropies S_{BG} and S_q^{Renyi} are extensive if and only if the N subsystems are (strictly or asymptotically) independent; otherwise, S_{BG} and S_q^{Renyi} are nonextensive.

The nonadditive entropy S_q $(q \neq 1)$ is extensive for special values of q if the subsystems are specially (globally) correlated.

MEPHISTOPHELES:

Denn eben wo Begriffe fehlen,

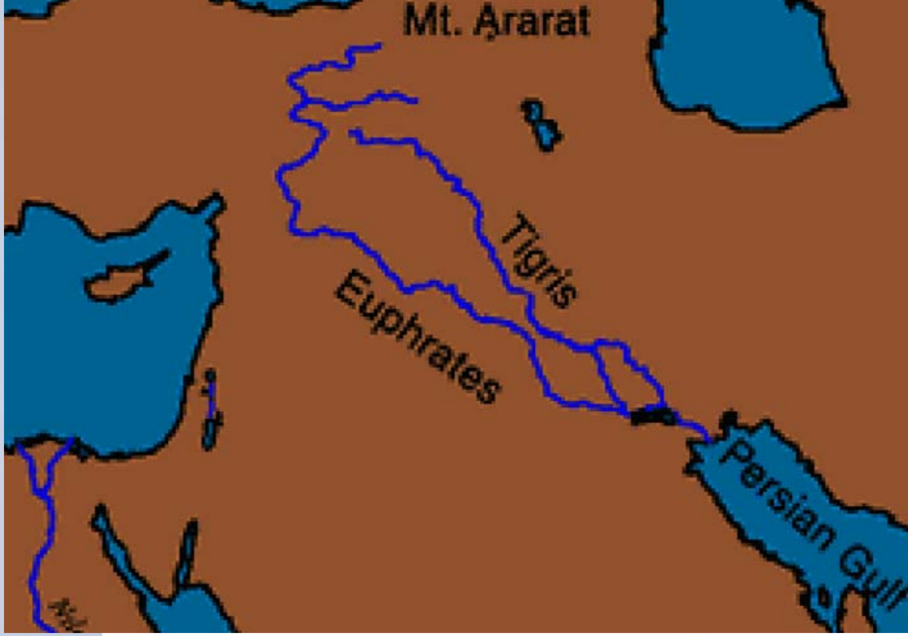
Da stellt ein Wort zur rechten Zeit sich ein.

Wolfgang von Goethe

[*Faust I, Vers 1995, Schuelerszene (1808)*]

For at the point where concepts fail,

At the right time a word is thrust in there.



King Thutmosis III
18th Dynasty
c. 1460 B. C.

TYPICAL EXAMPLES FOR THE CASE OF EQUAL PROBABILITIES:

1) *If* $W(N) \sim A\,\mu^{N}\;\; (A>0,\; \mu>1,\; N\to\infty)$

then $\frac{S_{BG}(N)}{k} = \ln\,[W(N)] \sim (\ln\mu)\,N$, *i.e.*, S_{BG} *is extensive!*

whereas $\frac{S_q(N)}{k} = \ln_q\,[W(N)] = \frac{[W(N)]^{1-q}-1}{1-q} \sim \frac{A^{1-q}}{1-q}\,\mu^{N(1-q)} \neq \textit{constant}\; N,$

i.e., S_q *is nonextensive for* $q<1$ (*and neither is for* $q>1$).

2) *If* $W(N) \sim B\,N^{\rho}\;\; (B>0,\; \rho>0,\; N\to\infty)$

then $\frac{S_{BG}(N)}{k} = \ln\,[W(N)] \sim \rho\;\ln N$, *i.e.*, S_{BG} *is nonextensive*

whereas $\frac{S_q(N)}{k} = \ln_q\,[W(N)] = \frac{[W(N)]^{1-q}-1}{1-q} \sim \frac{B^{1-q}}{1-q}\,N^{\rho(1-q)}$, *i.e.*, $S_{1-\frac{1}{\rho}}$ *is extensive!*

3) *If* $W(N) \sim C\,\mu^{N^{\gamma}}\;\; (C>0,\; \mu>1,\; \gamma\neq 1,\; N\to\infty)$

Hence, there is no value of q (*neither* $q=1$ *nor* $q\neq 1$) *for which* S_q *is extensive.*

HOW IT WORKS? S. Abe, Phys Lett A **271**, 74 (2000) and Physica A **368**, 430 (2006)

We consider generic subsystems A and B:

$$p_{ij}(A+B) = p_i(A)\,p_j(B\,|\,A) = p_i(A\,|\,B)\,p_j(B) \quad \text{(Bayes theorem)}$$

implies

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B\,|\,A)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B\,|\,A)}{k}$$

$$= \frac{S_q(B)}{k} + \frac{S_q(A\,|\,B)}{k} + (1-q)\frac{S_q(B)}{k}\frac{S_q(A\,|\,B)}{k}$$

hence

$$S_q(A+B) = S_q(A) + S_q(B\,|\,A) + \frac{1-q}{k} S_q(A)\, S_q(B\,|\,A)$$

$$= S_q(B) + S_q(A\,|\,B) + \frac{1-q}{k} S_q(B)\, S_q(A\,|\,B)$$

A special class of correlations exists such that, for a special value of q,

$$S_q(A\,|\,B) + \frac{1-q}{k} S_q(B)\, S_q(A\,|\,B) = S_q(A)$$

and

$$S_q(B\,|\,A) + \frac{1-q}{k} S_q(A)\, S_q(B\,|\,A) = S_q(B)$$

hence $S_q(A+B) = S_q(A) + S_q(B)$ (extensivity!)

A MANY-BODY HAMILTONIAN ILLUSTRATION OF THE EXTENSIVITY OF S_q FOR ANOMALOUS VALUES OF q

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow$ *Ising ferromagnet*

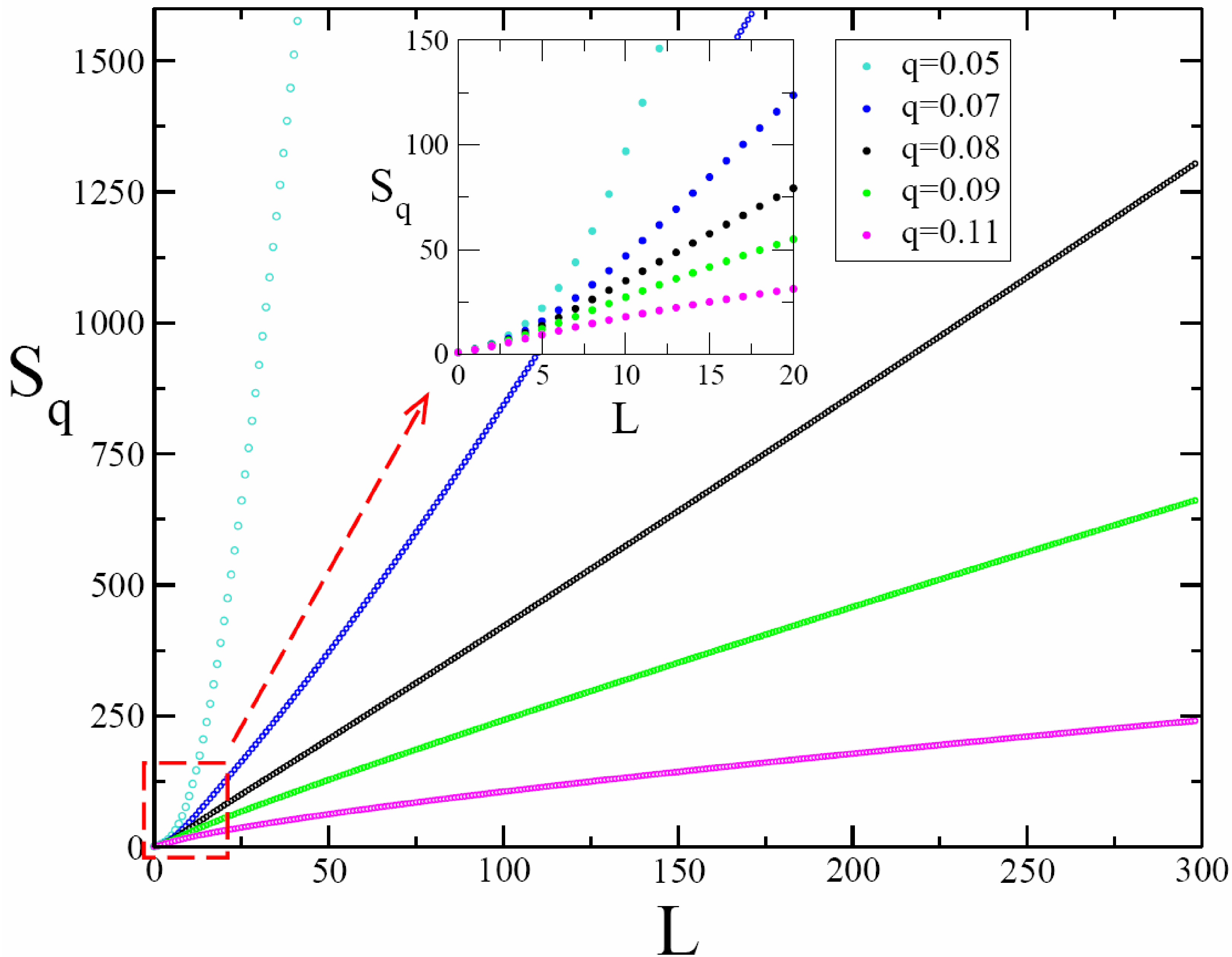
$0 < |\gamma| < 1 \quad \rightarrow$ *anisotropic XY ferromagnet*

$\gamma = 0 \quad \rightarrow$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

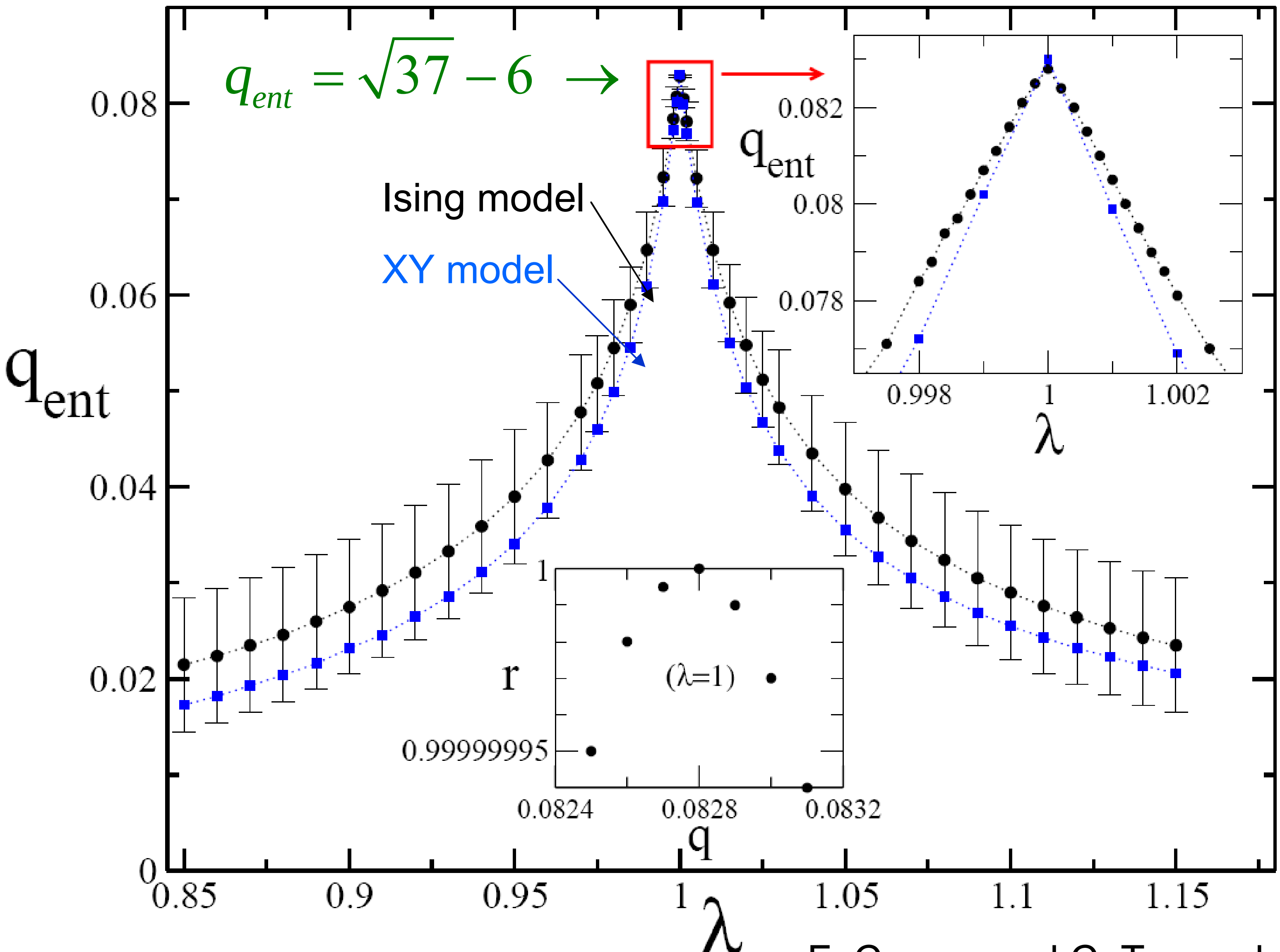
$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., cond-mat/0612032



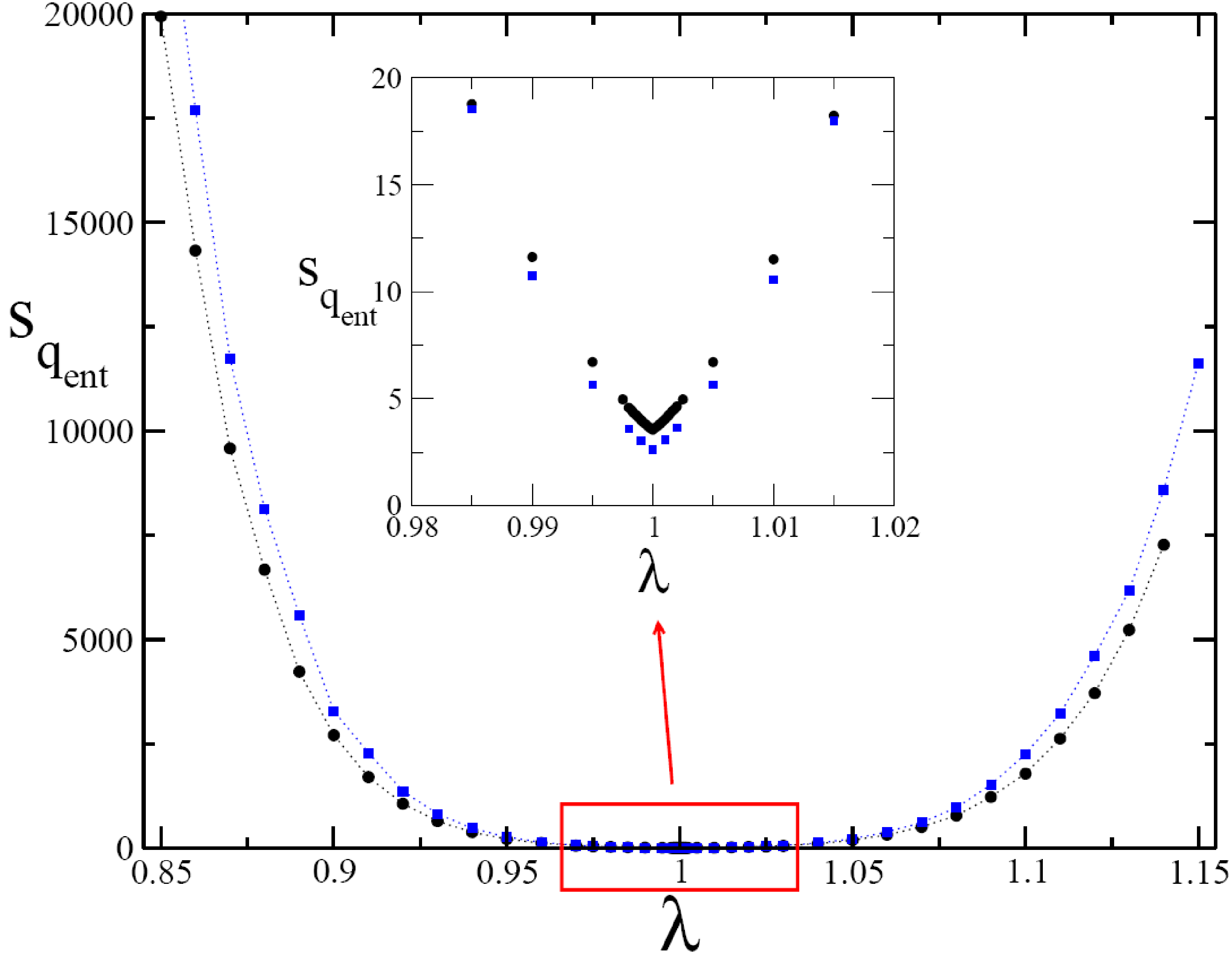
F. Caruso and C. T., cond-mat/0612032

$$S_{q_{ent}}(L) \sim s_{q_{ent}} L \qquad (L \to \infty)$$



F. Caruso and C. T., cond-mat/0612032

$$\lambda = 1 \Rightarrow S_{q_{ent}}(L) \sim s_{q_{ent}} L \quad (L \to \infty) \quad \textit{with} \quad s_{q_{ent}} = 3.56 \pm 0.03$$



F. Caruso and C. T., cond-mat/0612032

Using a Quantum Field Theory result

in P. Calabrese and J. Cardy, JSTAT P06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

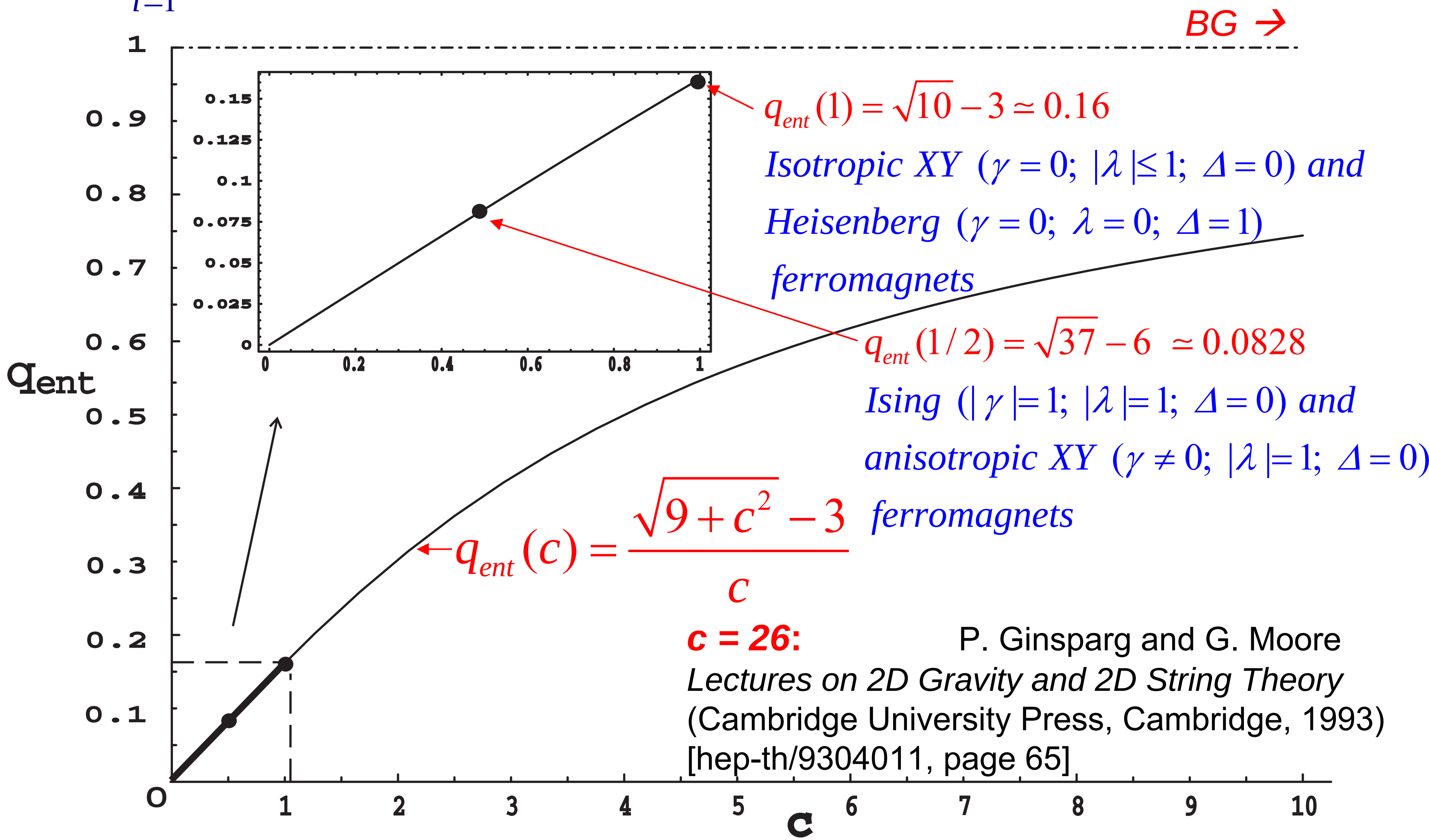
Ising and anisotropic XY ferromagnets $\Rightarrow\ c = \frac{1}{2}\ \Rightarrow\ q_{ent} = \sqrt{37}-6\ \simeq 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow\ c = 1\ \Rightarrow\ q_{ent} = \sqrt{10}-3 \simeq 0.16$

F. Caruso and C. T., cond-mat/0612032

$$H=-\sum_{i=1}^{N-1}\left[(1+\gamma)\ \sigma_i^x\sigma_{i+1}^x+(1-\gamma)\ \sigma_i^y\sigma_{i+1}^y+\Delta\ \sigma_i^z\sigma_{i+1}^z+2\lambda\ \sigma_i^z\right]$$



F. Caruso and C. T., cond-mat/0612032

In other words,

$$S_{\left[\sqrt{9+c^2}-3\right]c^{-1}}(L) \propto L \qquad \text{(extensive!)}$$

whereas

$$S_{BG}(L) \propto \ln L \qquad \text{(nonextensive!)}$$

REVISITING THE DIFFUSION OF ONE ELECTRON IN A MANY-BODY QUANTUM HAMILTONIAN

1D ANDERSON MODEL WITH LONG-RANGE CORRELATED DISORDER (METAL-INSULATOR TRANSITION):

F.A.B.F. de Moura and M.L. Lyra, Phys Rev Lett **81**, 3735 (1998)

One electron in a disordered linear chain [the disorder comes from a randomly-correlated potential characterized by a spectral density $s(k) \propto k^{-\alpha}$ $(\alpha \geq 0)$]:

$$H = \sum_{n=1}^{N} \varepsilon_n |n><n| + t_{\mathrm{H}} \sum_{n} \left[|n><n+1| + |n><n-1| \right] \qquad (t_{\mathrm{H}} = 1)$$

$$with \quad \varepsilon_n = \sum_{k=1}^{N/2} \left[k^{-\alpha} |2\pi / N|^{1-\alpha} \right]^{1/2} \cos\left(\frac{2\pi nk}{N} + \Phi_k \right)$$

where Φ_k *are* $N/2$ *independent random phases uniformly distributed in* $[0, 2\pi]$

$\alpha = 2\mathrm{H} + 1$ (H $\equiv$ Hurst exponent)

$\alpha = 0 \quad \Rightarrow$ *standard Anderson model [white noise spectrum, i.e., no correlation from site to site), i.e.,* $<\varepsilon_n \varepsilon_{n'}> = <\varepsilon_n{}^2> \delta_{n,n'}$]

$\alpha \to \infty \Rightarrow$ *crystal (no disorder)*

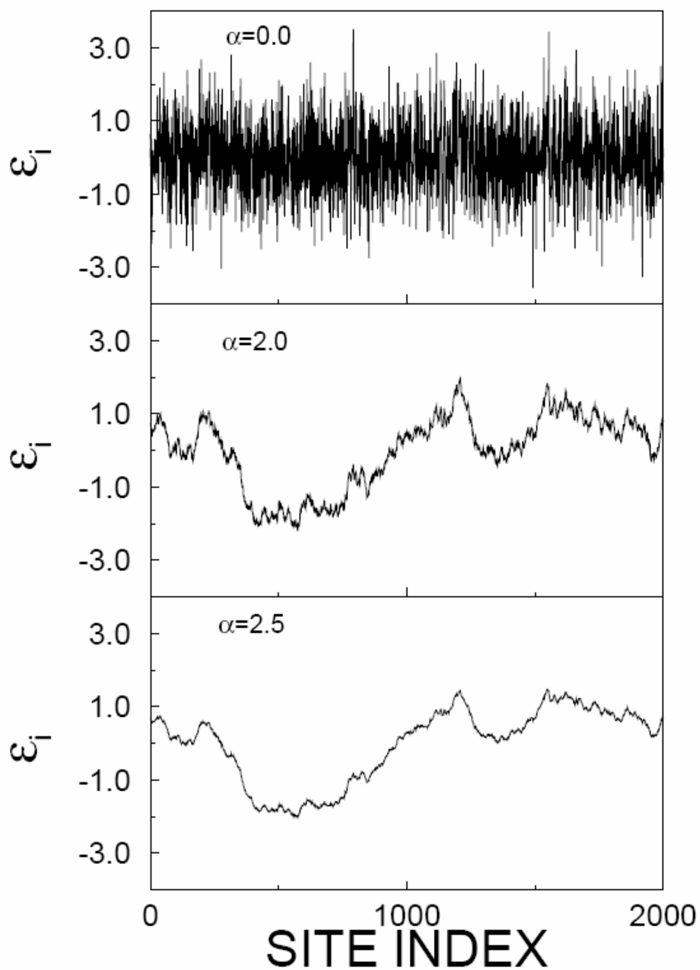


FIG. 1. Typical on-site energy landscapes generated from relation (2) with $N = 4096$: $\alpha = 0.0$: uncorrelated random sequency; $\alpha = 2.0$: trace of a usual Brownian motion; $\alpha = 2.5$: trace of a fractional Brownian motion with persistent increments. Notice the smoothening of the energy landscape for increasing values of α.

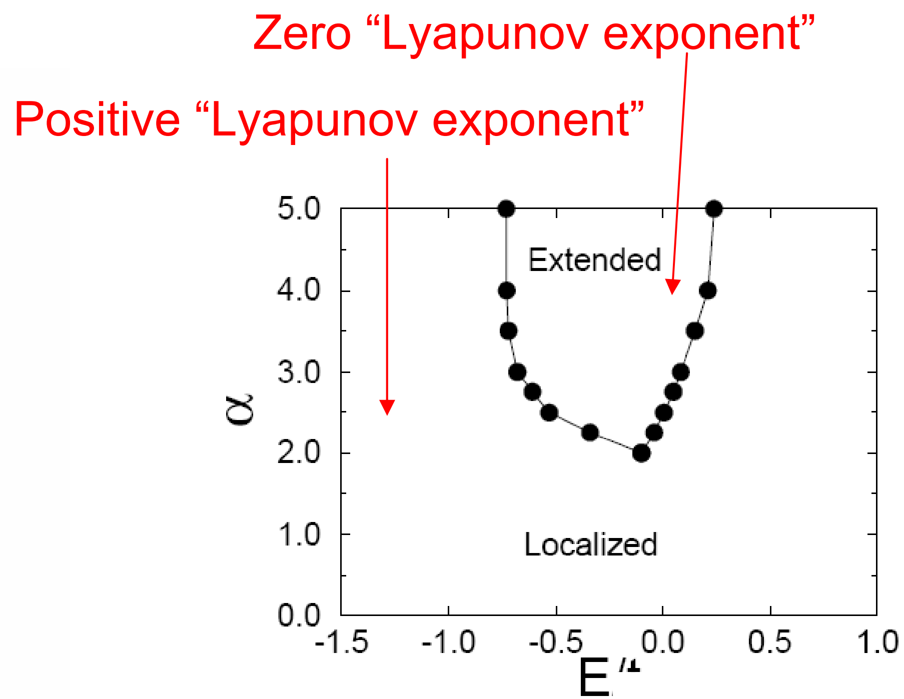
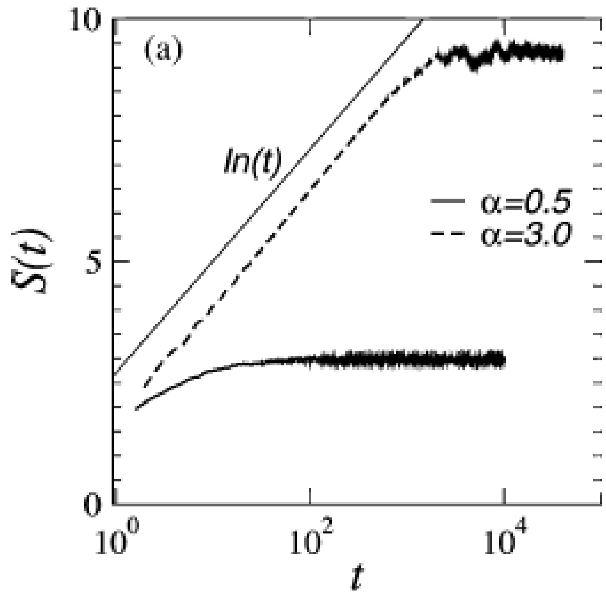


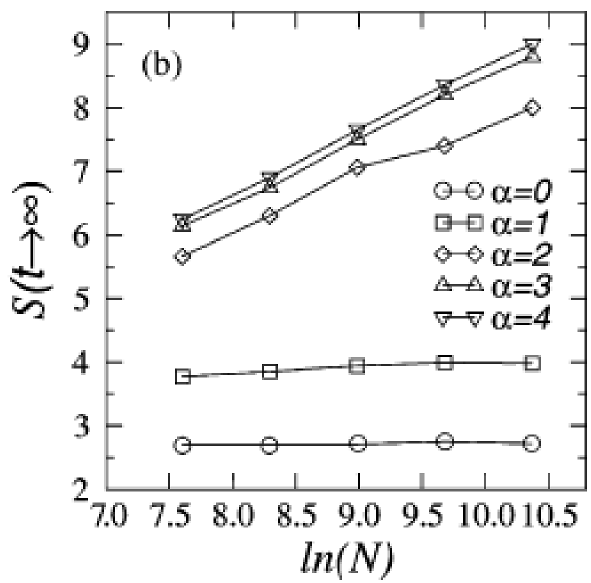
FIG. 5. Phase diagram in the $(E/t, \alpha)$ plane. Data were obtained from chains with 10^4 sites, $\Delta\epsilon/t = 1.0$, and the same random phases sequency. The phase of extended states emerges for $\alpha > 2$, and its width saturates as $\alpha \to \infty$. The band of allowed states ranges approximately from $-4.0 < E < 4.0$ and is independent of α by construction (see text).

F.A.B.F. de Moura and M.L. Lyra, Phys Rev Lett **81**, 3735 (1998)

$S_{BG}(8000,t)$

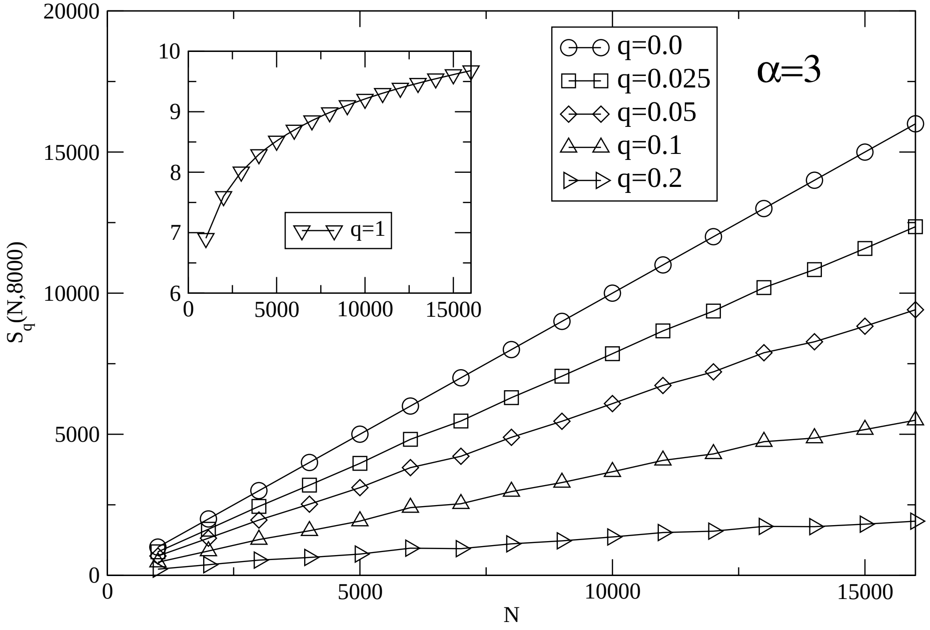


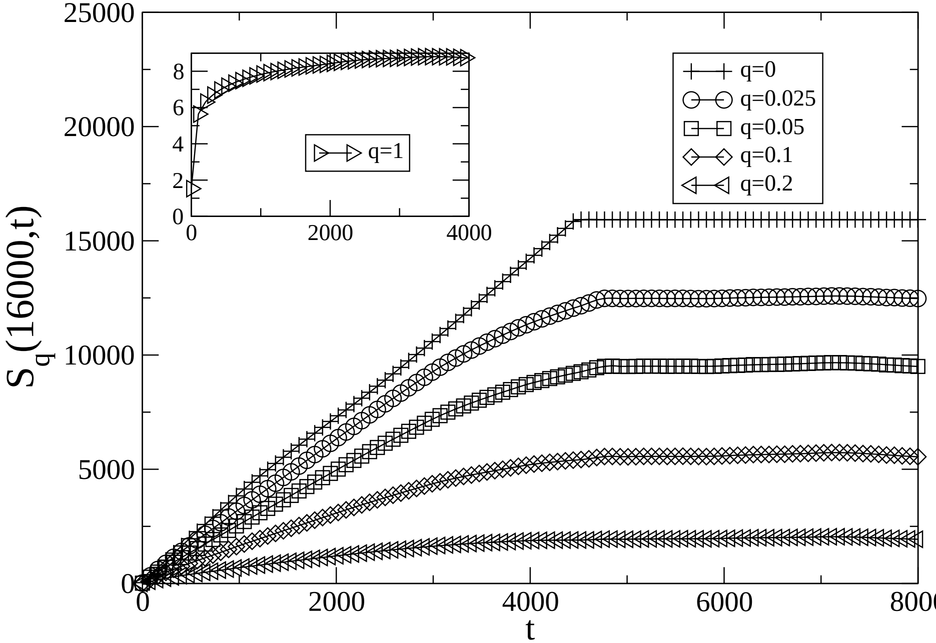
$S_{BG}(N,t>>1)$

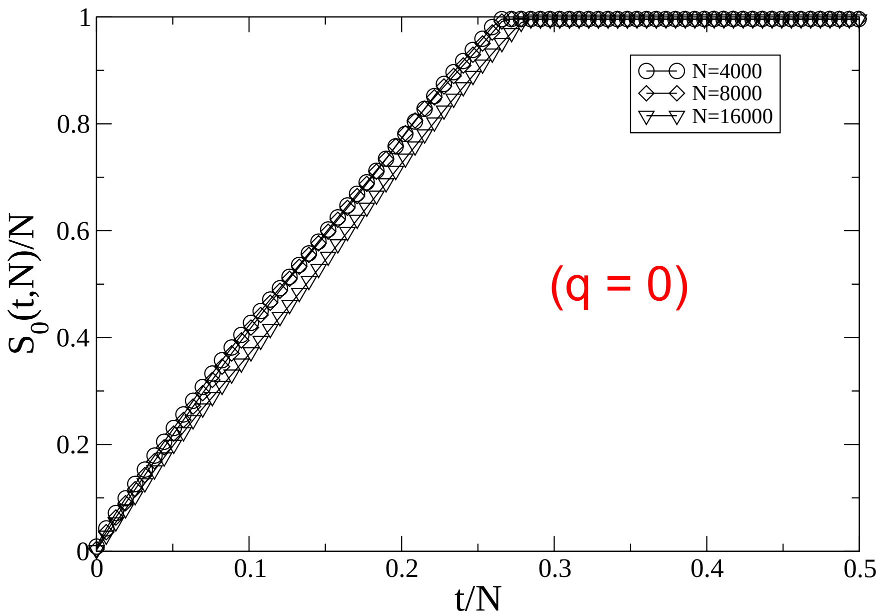


B. Santos, L.P. Viana, M.L. Lyra and F.A.B.F. de Moura, Sol State Comm **138**, 585 (2006)

REVISITING THE PREVIOUS RESULTS:



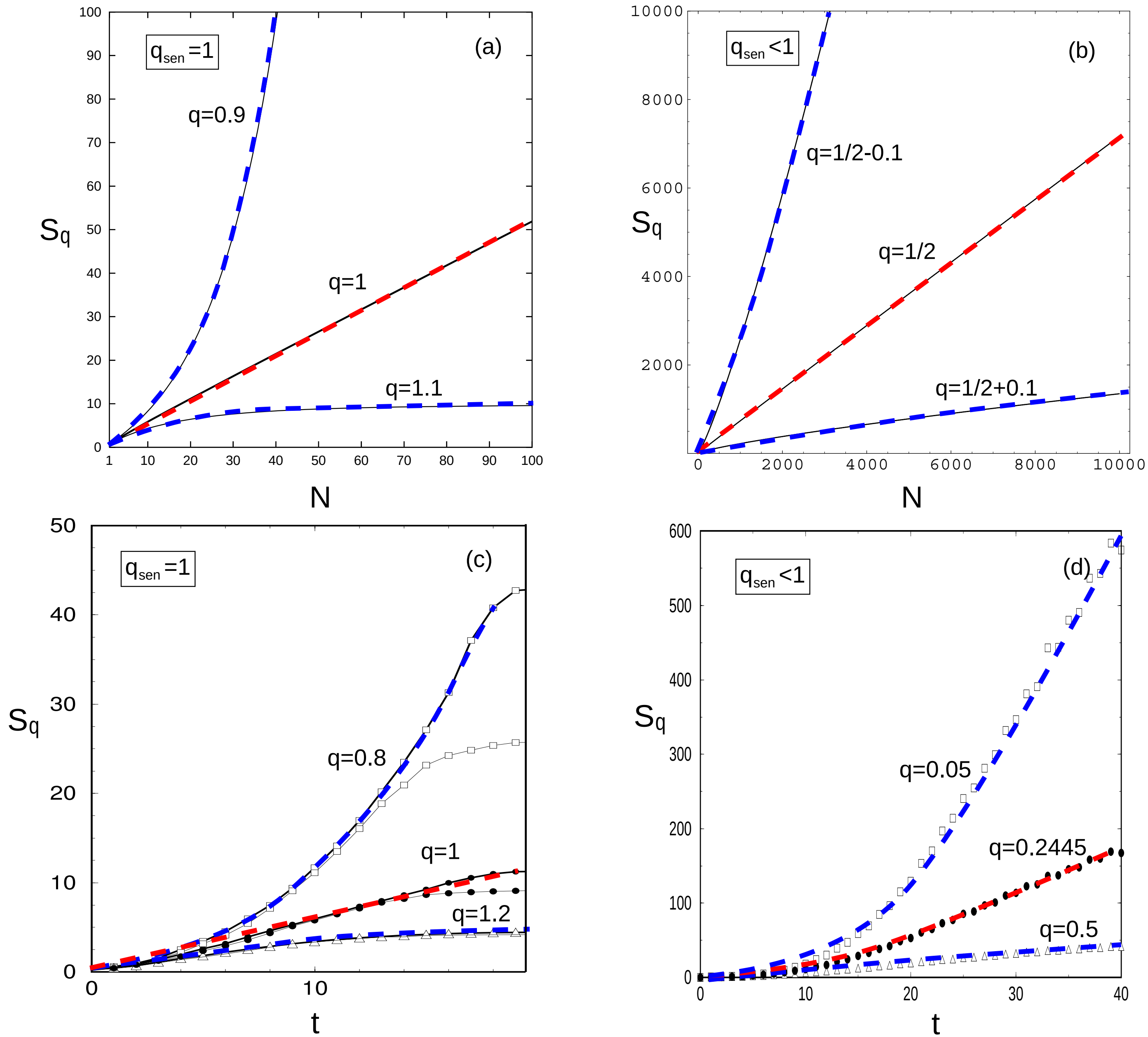




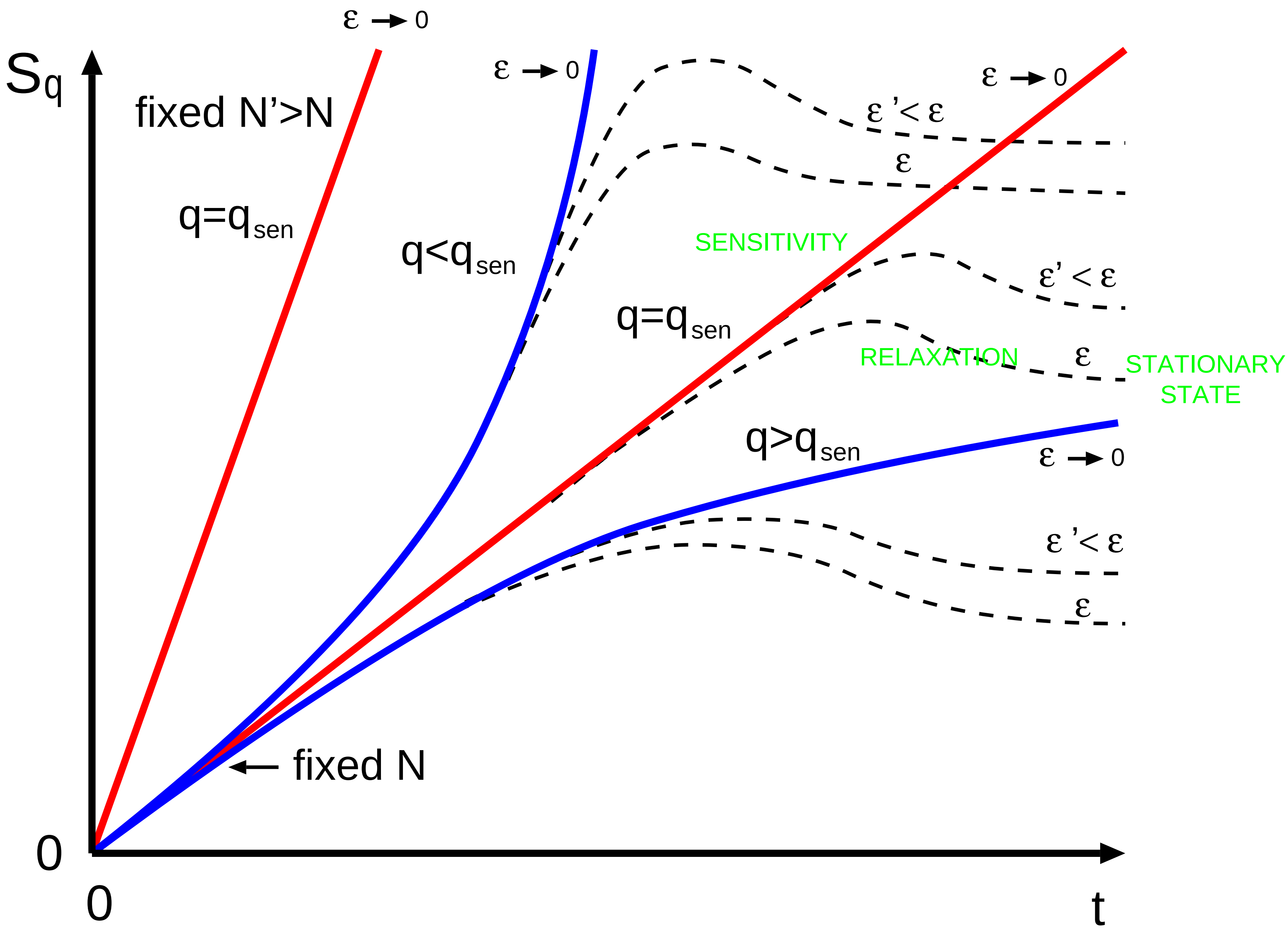
$(\alpha = 3)$

F.A.B.F. de Moura and M.L. Lyra (2007)

$$S_q(N,t)\ versus\ (t,N)$$



C.T., M. Gell-Mann and Y. Sato, Europhysics News **36** (6), 186 (2005) [European Physical Society]



C.T., M. Gell-Mann and Y. Sato, Europhysics News **36** (6) (Nov-Dec 2005)
Special Issue *Nonextensive Statistical Mechanics – New Trends, New Perspectives*
(European Physical Society)

A conjecture for $S_q(N,t)$:

For $q = q_{sen}$, $N \to \infty$ *and* $t \to \infty$ *play essentially the same role.*
In particular,

*i) Under conditions of **infinitely fine** graining in phase space,*

$$S_{q_{sen}}(N,t) \sim K_{q_{sen}}(N)\ t \propto N\ t$$

$$\left(q_{sen} = 1 \Rightarrow K_1 = \sum_{j/\lambda_1^{(j)}>0} \lambda_1^{(j)},\ \text{i.e., Pesin}-\textit{like identity for finite } N\right)$$

*ii) Under conditions of **finite** graining in phase space,*

$$\lim_{t\to\infty} S_{q_{sen}}(N,t) \propto N$$

(Clausius)

C.T., M. Gell-Mann and Y. Sato
Europhysics News **36**, 186 (2005)

NONEXTENSIVE STATISTICAL MECHANICS
AND
THERMODYNAMICS

C. T.
Possible generalization of Boltzmann-Gibbs statistics
J Stat Phys **52**, 479 (1988)

E.M.F. Curado and C. T.
Generalized statistical mechanics: connection with thermodynamics
J Phys A **24**, L69 (1991)
[Corrigenda: **24**, 3187 (1991) and **25**, 1019 (1992)]

C. T., R.S. Mendes and A.R. Plastino
The role of constraints within generalized nonextensive statistics
Physica A **261**, 534 (1998)

NONEXTENSIVE STATISTICAL MECHANICS AND THERMODYNAMICS (CANONICAL ENSEMBLE):

Extremization of the functional

$$S_q[p_i] \equiv k \frac{1-\sum_{i=1}^{W} p_i^q}{q-1}$$

with the constraints

$$\sum_{i=1}^{W} p_i = 1 \quad \text{and} \quad \frac{\sum_{i=1}^{W} p_i^q E_i}{\sum_{i=1}^{W} p_i^q} = U_q$$

yields

$$p_i = \frac{e_q^{-\beta_q (E_i - U_q)}}{\mathbb{Z}_q}$$

with

$$\beta_q \equiv \frac{\beta}{\sum_{i=1}^{W} p_i^q}\,, \quad \beta \equiv \textit{energy Lagrange parameter}, \quad \text{and} \quad \mathbb{Z}_q \equiv \sum_{i=1}^{W} e_q^{-\beta_q (E_i - U_q)}$$

We can rewrite $p_i = \dfrac{e_q^{-\beta_q' E_i}}{Z_q'}$

with $\beta_q' \equiv \dfrac{\beta_q}{1+(1-q)\beta_q U_q}$, *and* $Z_q' \equiv \sum_{i=1}^{W} e_q^{-\beta_q' E_i}$

And we can prove

(*i*) $\dfrac{1}{T} = \dfrac{\partial S_q}{\partial U_q}$ *with* $T \equiv \dfrac{1}{k\beta}$

(*ii*) $F_q \equiv U_q - TS_q = -\dfrac{1}{\beta}\ln_q Z_q$ *where* $\ln_q Z_q = \ln_q \mathbb{Z}_q - \beta U_q$

(*iii*) $U_q = -\dfrac{\partial}{\partial \beta}\ln_q Z_q$

(*iv*) $C_q \equiv T\dfrac{\partial S_q}{\partial T} = \dfrac{\partial U_q}{\partial T} = -T\dfrac{\partial^2 F_q}{\partial T^2}$

(*i.e., the Legendre structure of Thermodynamics is q-invariant!*)

SOME FORM-INVARIANT RELATIONS (arbitrary q)

CLAUSIUS INEQUALITY AND BOLTZMANN H-THEOREM
(macroscopic time irreversibility)

Mariz, Phys Lett A **165** (1992) 409; Ramshaw, Phys Lett A **175** (1993) 169;
Abe and Rajagopal, Phys Rev Lett **91** (2003)

$$\beta \, \delta Q_q \leq \delta S_q \quad ; \qquad q \frac{d S_q}{d t} \geq 0$$

EHRENFEST THEOREM (correspondence principle)

Plastino and Plastino, Phys Lett A **177** (1993) 177

$$\frac{d}{d t} \langle \hat{O} \rangle_q = \frac{i}{\hbar} \langle [\hat{H} , \hat{O}] \rangle_q$$

FACTORIZATION OF LIKELIHOOD FUNCTION
(Einstein's 1910 reversal of Boltzmann's formula; thermodynamically independent systems)

Caceres and Tsallis, unpublished (1993); Chame and Mello,
J Phys A **27** (1994) 3663; Tsallis, Chaos, Solitons and Fractals **6** (1995) 539

$$W_q(A+B) = W_q(A)\, W_q(B)$$

ONSAGER RECIPROCITY THEOREM
(microscopic time reversibility)

Caceres, Physica A **218** (1995) 471; Rajagopal, Phys Rev Lett **76** (1996) 3469;
Chame and Mello, Phys Lett A **228** (1997) 159

$$L_{jk} = L_{kj}$$

KRAMERS AND KRONIG RELATION (causality)

Rajagopal, Phys Rev Lett **76** (1996) 3469

PESIN EQUALITY
(mixing; Kolmogorov-Sinai entropy and Lyapunov exponent)

Tsallis, Plastino and Zheng, Chaos, Solitons and Fractals **8** (1997) 885;
Baldovin and Robledo, Phys. Rev. E **69**, 045202(R) (2004).

$$K_q = \begin{cases} \lambda_q & \text{if } \lambda_q > 0 \\ 0 & \text{otherwise} \end{cases}$$

WHY USING ESCORT DISTRIBUTIONS FOR THE CONSTRAINTS?

1) The optimizing probability distribution is automatically invariant with regard to uniform translation of the energy eigenvalues (zero-point invariance).

2) The constraints $\sum_i p_i = 1$ and $\sum_i P_i\ E_i = constant$, where $P_i \equiv \frac{p_i^q}{\sum_j p_j^q}$ and $p_i \equiv \frac{P_i^{1/q}}{\sum_j P_j^{1/q}}$, are finite up to a common upper bound for q ($e.g,$ for $E(x) \propto x^2$, it must be $q<3$).

3) $\frac{de_q^x}{dx} = (e_q^x)^q \neq e_q^x$ yields $\{P_i\}$ instead of $\{p_i\}$ in the steepest-descent-method calculation of the stationary-state distribution [*Abe and Rajagopal*, *J Phys A* 33, 8733 (2000)].

4) The conditional entropy naturally appears [*Abe*, *Phys Lett A* 271, 74 (2000)] as a q-expectation value without involving any optimization principle.

5) The principle of minimal relative entropy is consistent as a rule of statistical inference (1980 Shore-Johnson axioms) only if [*Abe and Bagci*, *Phys Rev E* 71, 016139 (2005)] we select the q-expectation values if we use the entropy S_q.

GENERALIZATION OF THE CENTRAL LIMIT THEOREM

ONE OF MANY CONNECTIONS OF THE CENTRAL LIMIT THEOREM WITH BOLTZMANN-GIBBS STATISTICAL MECHANICS

Optimization of

$$S = - k\int dx\, p(x)\; \ln[p(x)]$$

with

$$\int dx\, p(x) = 1$$

and

$$\langle E(x)\rangle \equiv \int dx\, p(x)\, E(x) = constant$$

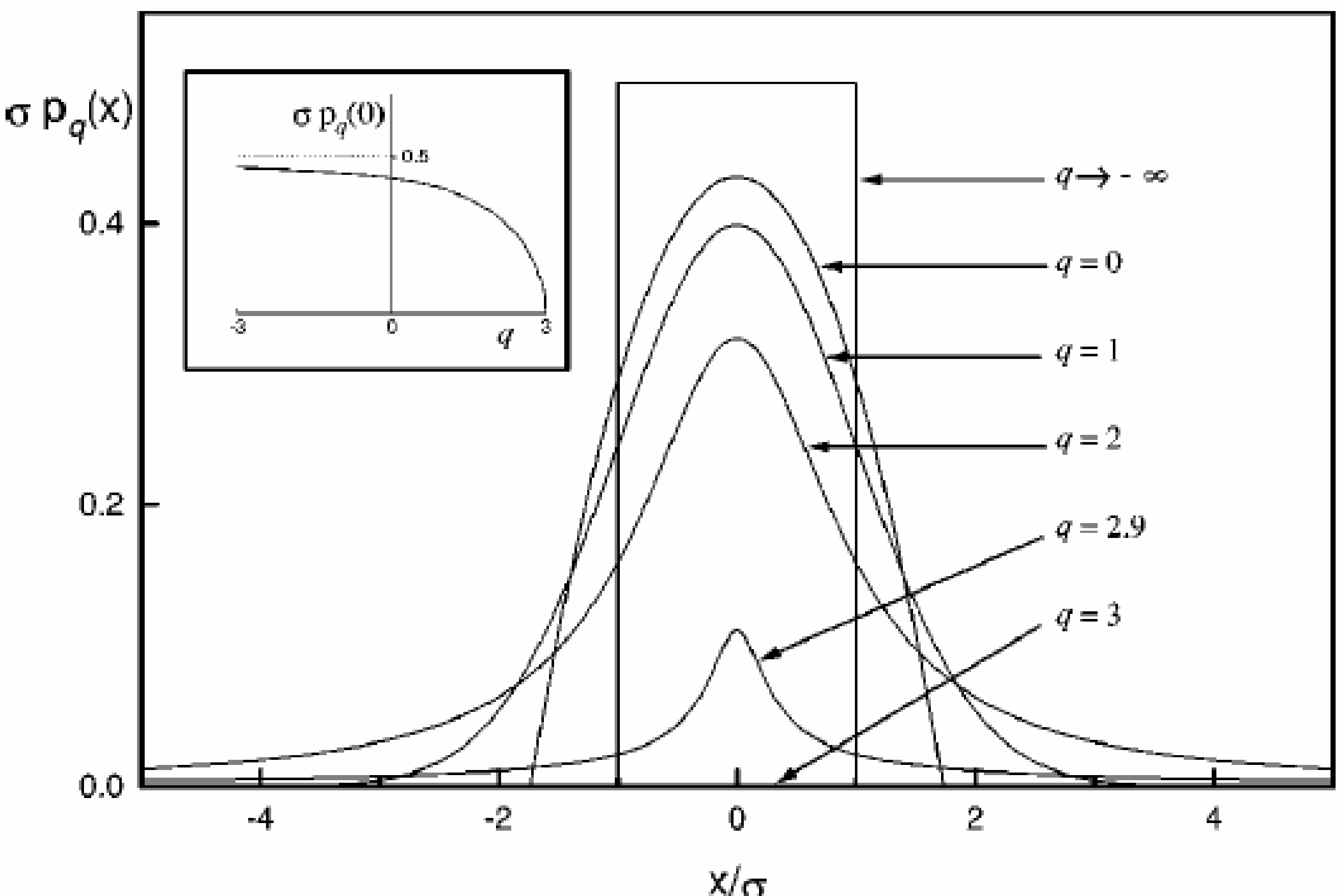
yields

$$p(x) = \frac{e^{-\beta E(x)}}{\int dy\; e^{-\beta E(y)}}$$ (Boltzmann-Gibbs distribution for thermal equilibrium)

Example: $\langle x\rangle = 0$ *and* $\langle x^2\rangle = constant$ *yields*

$$p(x) = \frac{e^{-\beta x^2}}{\int dy\; e^{-\beta y^2}}$$ (Gaussian distribution)

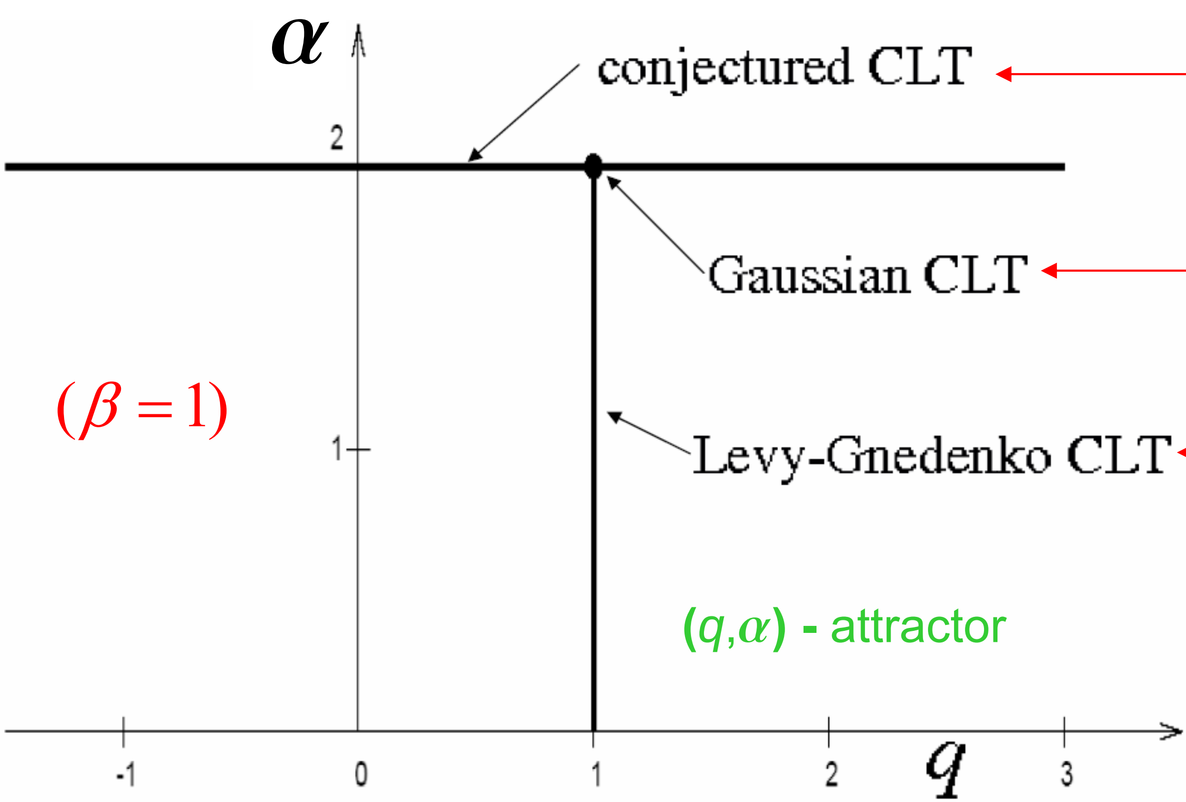
q-GAUSSIANS: $p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}}$ $(q<3)$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

q - CENTRAL LIMIT THEOREM: (conjecture)

$$\frac{\partial^{\beta} p(x,t)}{\partial t^{\beta}} = D \frac{\partial^{\alpha} [p(x,t)]^{2-q}}{\partial |x|^{\alpha}} \quad (0 < \alpha \leq 2;\ q < 3)$$



globally correlated variables;
finite q-variance;
q -Gaussian attractor

independent variables;
finite variance;
Gaussian attractor

independent variables;
divergent variance;
Levy attractor

M. Bologna, C. T. and P. Grigolini, Phys. Rev. E **62**, 2213 (2000)

C. T., Milan J. Math. **73**, 145 (2005)

q - PRODUCT: L. Nivanen, A. Le Mehaute and Q.A. Wang, Rep. Math. Phys. **52**, 437 (2003)
E.P. Borges, Physica A **340**, 95 (2004)

$$For\ x \geq 0\ and\ y \geq 0, \quad x \otimes_q y \equiv \begin{cases} \left[x^{1-q} + y^{1-q} - 1 \right]^{\frac{1}{1-q}} & if\ x^{1-q} + y^{1-q} > 1,\ \forall q \\ 0 & if\ x^{1-q} + y^{1-q} \leq 1\ and\ q < 1 \end{cases}$$

$$\left(It\ can\ be\ extended\ to\ all\ (x,y)\ through\ x \otimes_q y = sign(xy) \left[|x|^{1-q} + |y|^{1-q} - 1 \right]^{\frac{1}{1-q}} \right)$$

Properties:

$$i) \quad x \otimes_1 y = x\, y$$

$$ii) \quad \ln_q (x \otimes_q y) = \ln_q x + \ln_q y$$

$$[whereas\ \ln_q (x\, y) = \ln_q x + \ln_q y + (1-q)(\ln_q x)(\ln_q y)]$$

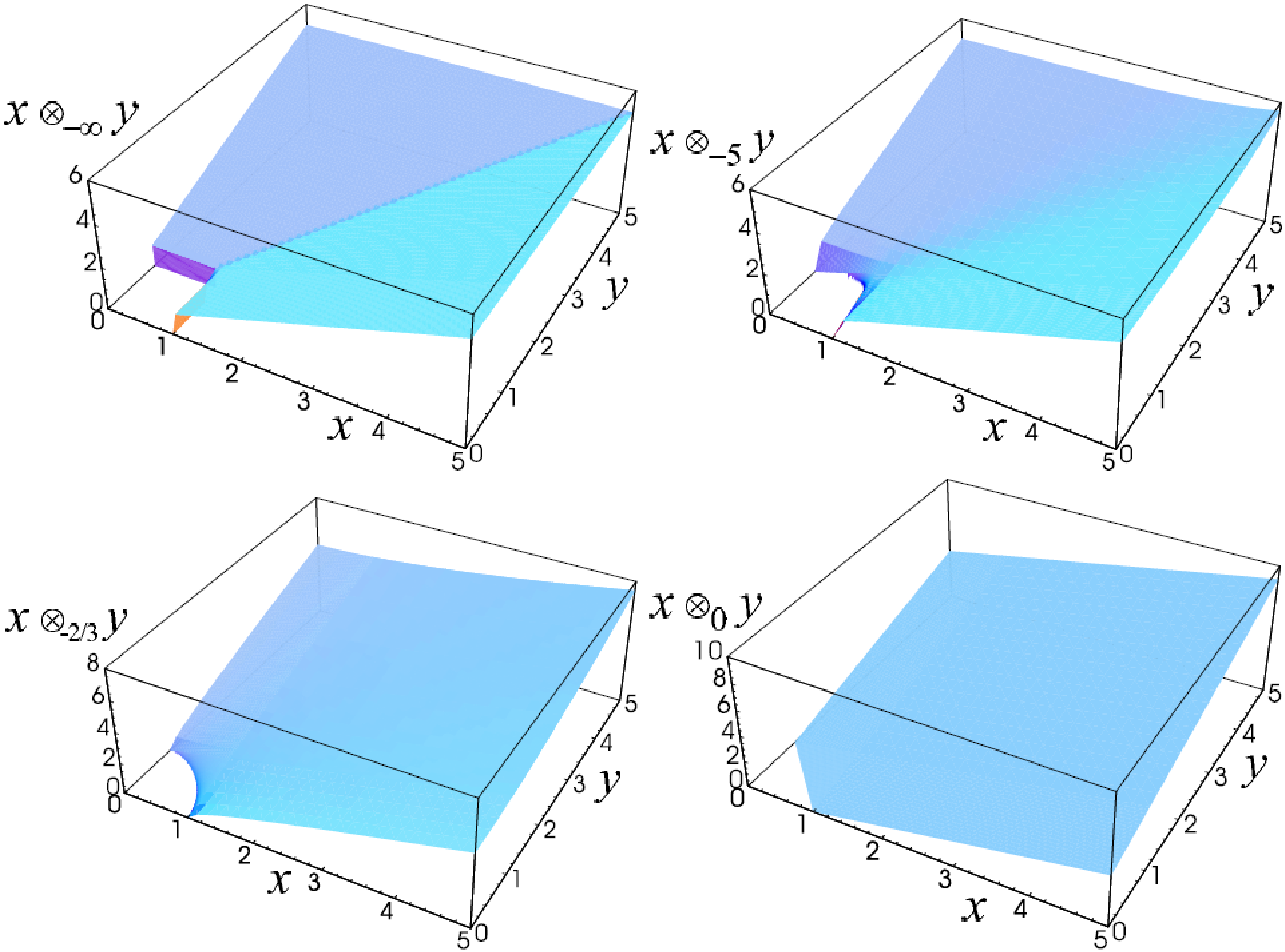
$$iii) \quad \frac{1}{x \otimes_q y} = \frac{1}{x} \otimes_{2-q} \frac{1}{y}$$

$$iv) \quad x \otimes_q y = y \otimes_q x$$

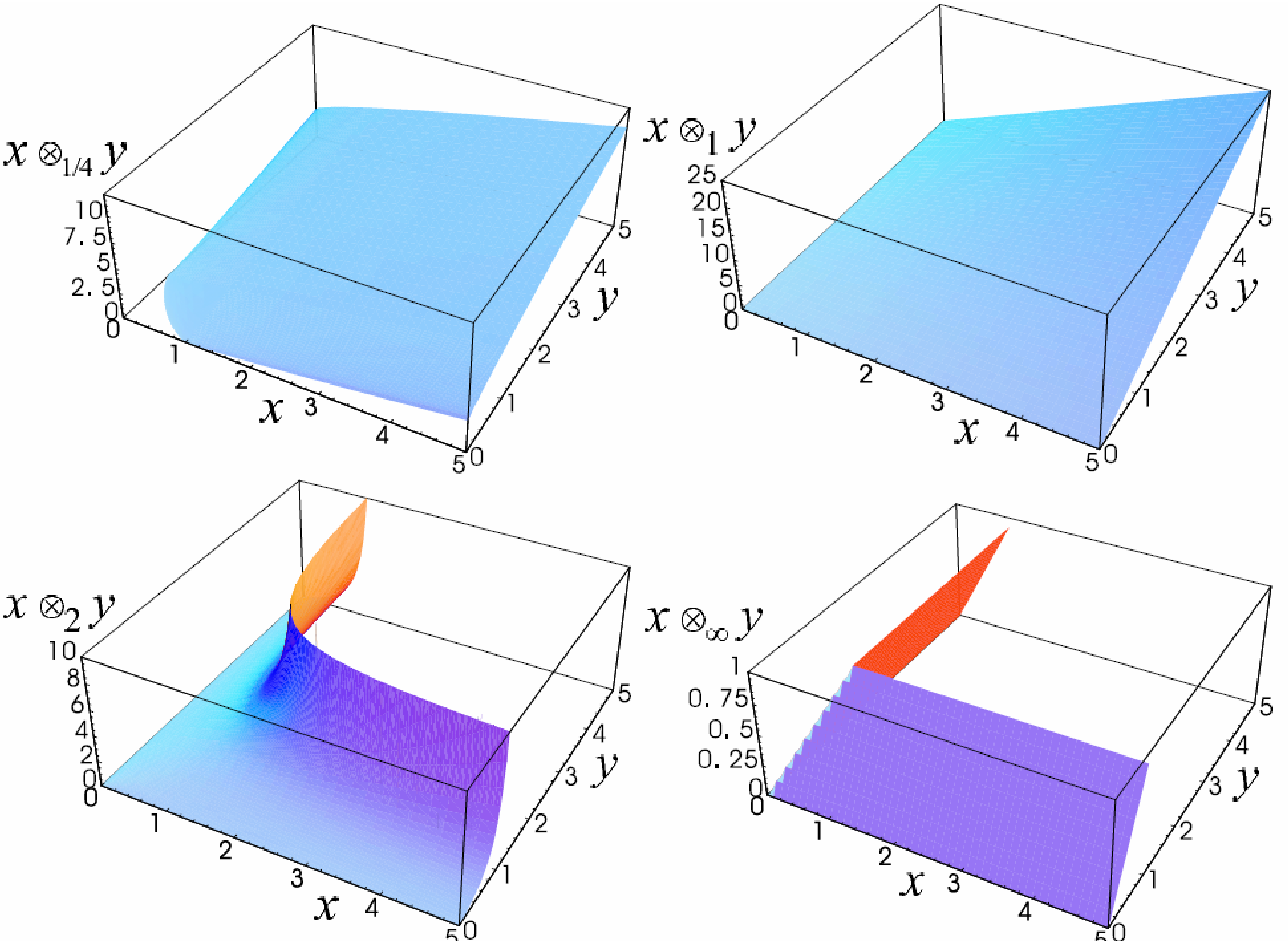
$$v) \quad (x \otimes_q y) \otimes_q z = x \otimes_q (y \otimes_q z) = x \otimes_q y \otimes_q z = \left[x^{1-q} + y^{1-q} + z^{1-q} - 2 \right]^{\frac{1}{1-q}}$$

$$vi) \quad x \otimes_q 1 = x$$

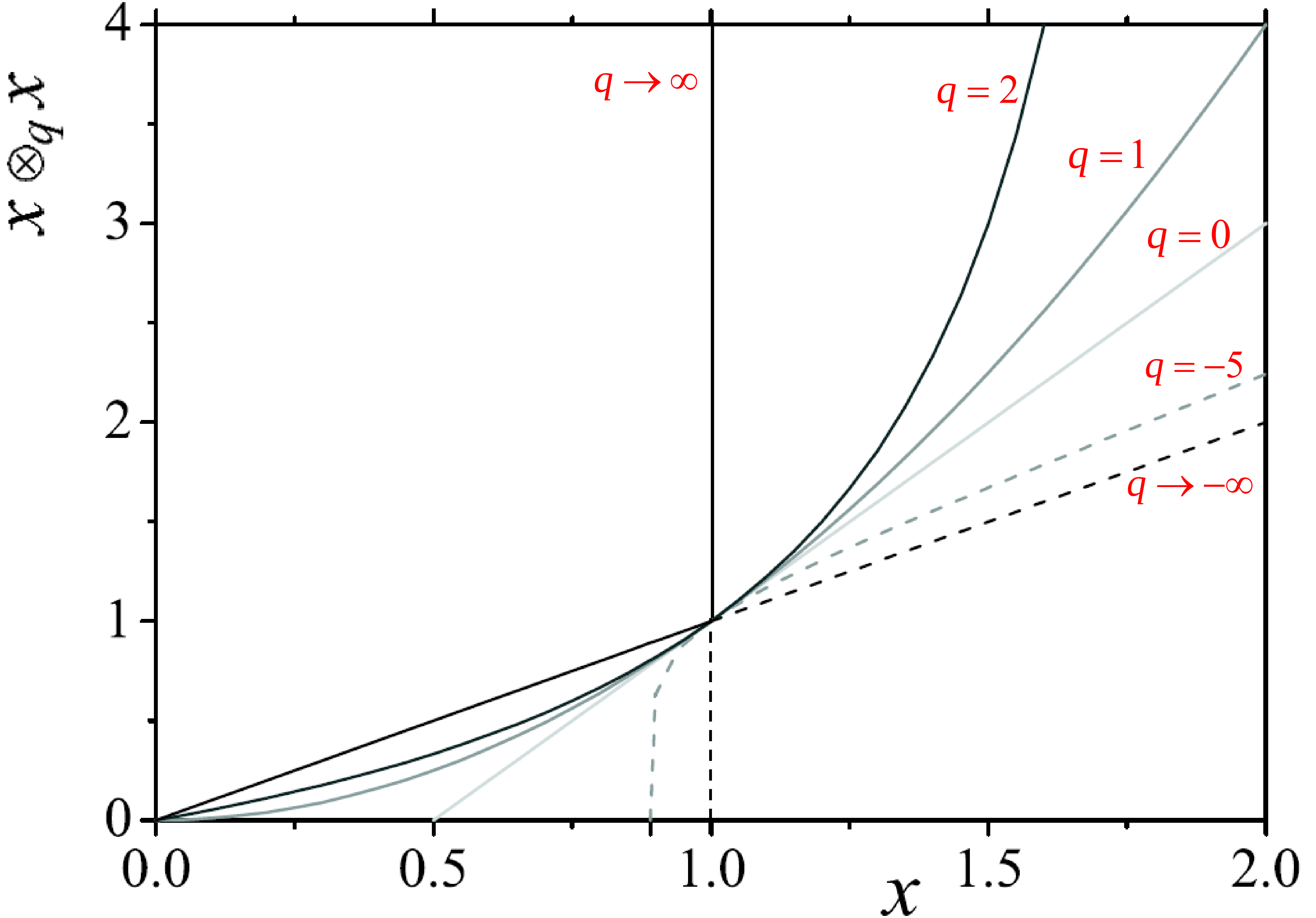
$$vii) \quad x \otimes_q 0 = \begin{cases} 0 & if\ (q \geq 1\ and\ x \geq 0)\ or\ if\ (q < 1\ and\ 0 \leq x \leq 1) \\ (x^{1-q} - 1)^{\frac{1}{1-q}} & otherwise \end{cases}$$



C. T. and S.M.D. Queiros (2007)



C. T. and S.M.D. Queiros (2007)



C. T. and S.M.D. Queiros (2007)

q - CENTRAL LIMIT THEOREM (q-product and de Moivre-Laplace theorem):

The q - product is defined as follows:

$$x \otimes_q y \equiv \left[x^{1-q} + y^{1-q} - 1 \right]^{\frac{1}{1-q}}$$

$$Properties:$$

$$i) \quad x \otimes_1 y = x\, y$$

$$ii) \quad \ln_q (x \otimes_q y) = \ln_q x + \ln_q y$$

$$[whereas \;\; \ln_q (x\, y) = \ln_q x + \ln_q y + (1-q)(\ln_q x)(\ln_q y)]$$

The de Moivre-Laplace theorem can be constructed with

$$p_{N,0} = p^N \quad with \quad p = 1/2$$

and

Leibnitz rule

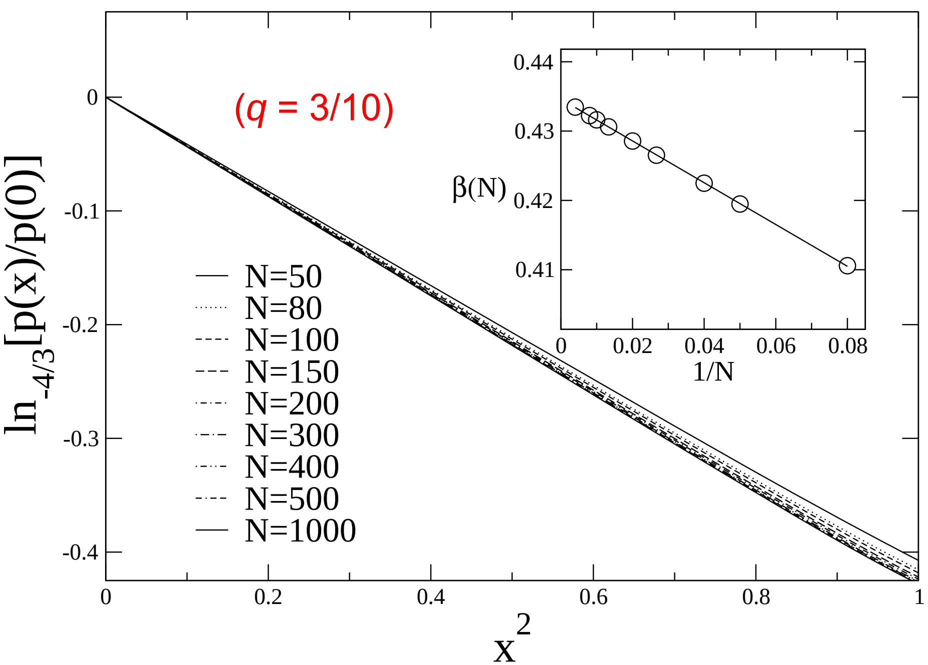
q - CENTRAL LIMIT THEOREM: (numerical indications)

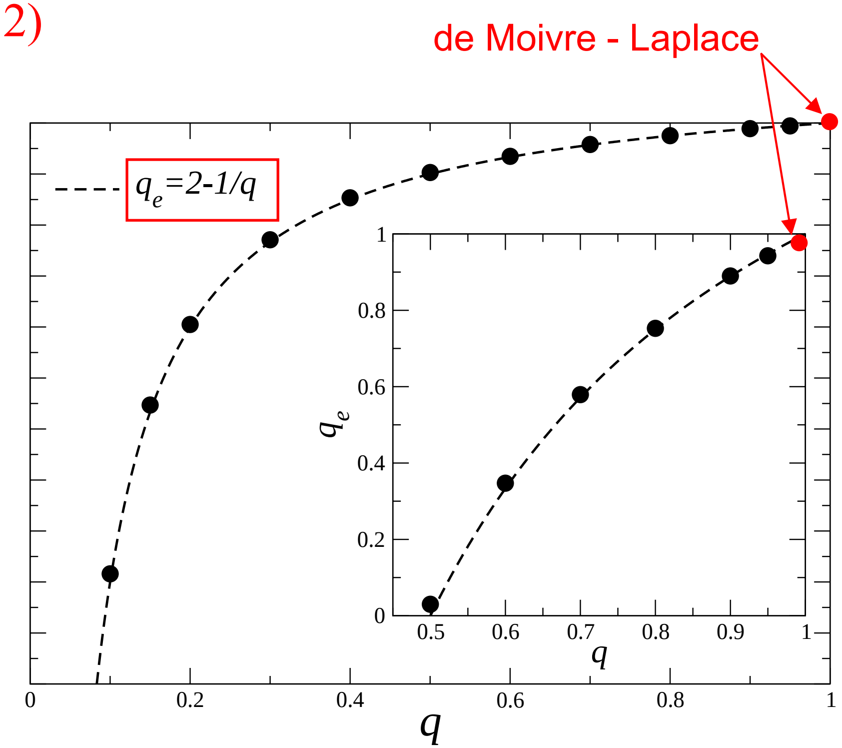
We q – generalize the de Moivre – Laplace theorem with

$$\frac{1}{p_{N,0}} = \left(\frac{1}{p}\right) \otimes_q \left(\frac{1}{p}\right) \otimes_q \dots \left(\frac{1}{p}\right) \quad (N\ terms)$$

i.e.,

$$p_{N,0} = \left[N\, p^{q-1} - (N-1)\right]^{\frac{1}{q-1}} \quad (with\ p = 1/2)$$





[Hence q → $2 - q$ (additive duality) and q → $1/q$ (multiplicative duality) are involved]

L.G. Moyano, C. T. and M. Gell-Mann, Europhys Lett **73**, 813 (2006)

$$(1/r_{N,0}) = (1/p) \otimes_q (1/p) \otimes_q (1/p) \otimes_q \ldots \otimes_q (1/p)\,,$$

$$r_{N,0} = p \otimes_{2-q} p \otimes_{2-q} p \otimes_{2-q} \cdots \otimes_{2-q} p = 1/\left[N p^{q-1} - (N-1)\right]^{1/(1-q)}$$

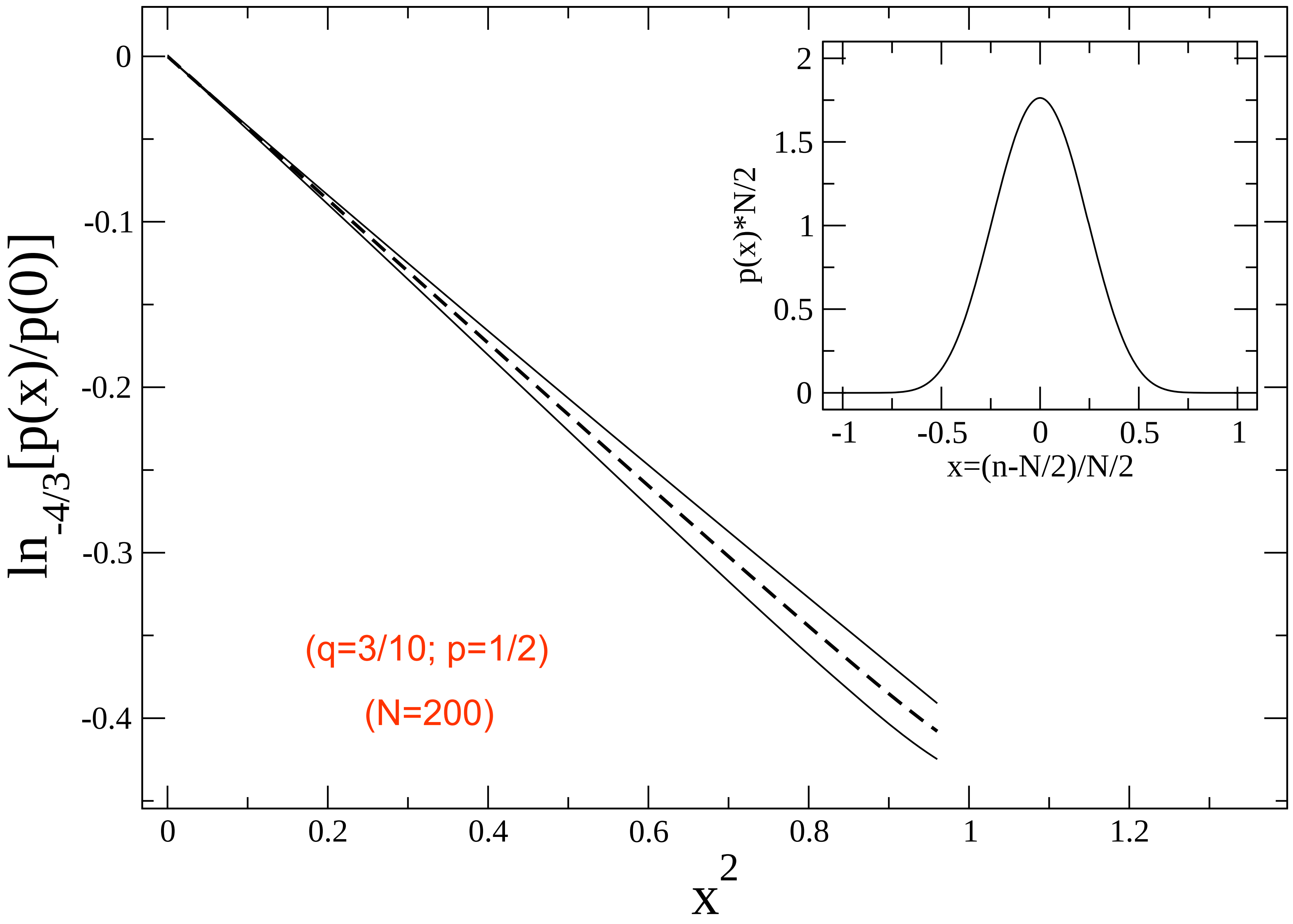
For $0 < p < 1$ we see that

$$r_{N,0} = p^N = e^{-N \ln(1/p)} \text{ if } q = 1$$

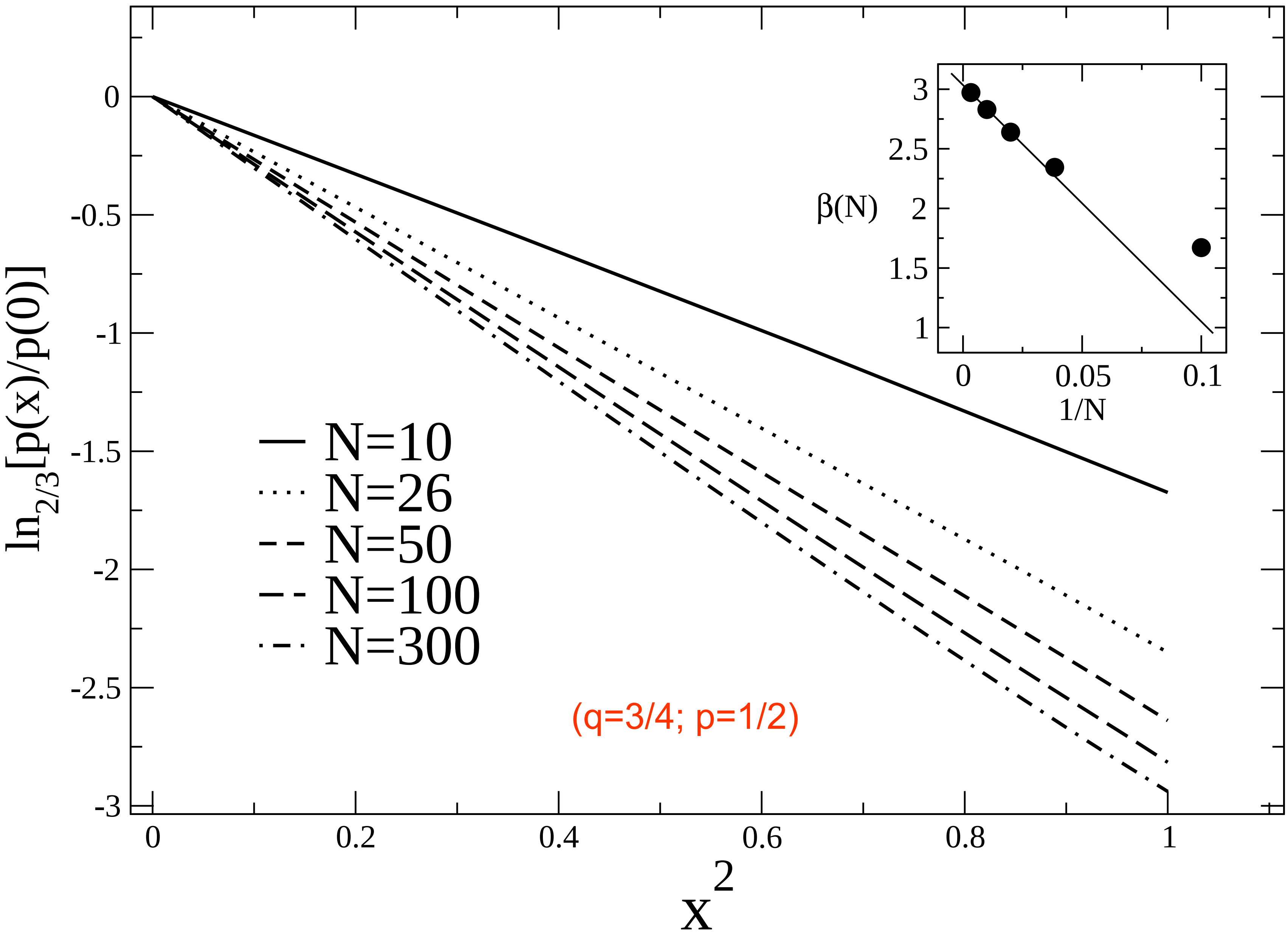
$$r_{N,0} \sim \frac{1}{[(1/p)^{1-q} - 1]^{1/(1-q)}} \frac{1}{N^{1/(1-q)}}$$

$$\propto 1/N^{1/(1-q)} \; (N \to \infty) \text{ for } q < 1$$

L.G. Moyano, C. T. and M. Gell-Mann, Europhys Lett **73**, 813 (2006)



L.G. Moyano, C. T. and M. Gell-Mann, Europhys Lett **73**, 813 (2006)



L.G. Moyano, C. T. and M. Gell-Mann, Europhys Lett **73**, 813 (2006)

q - GENERALIZED CENTRAL LIMIT THEOREM: (mathematical proof)

S. Umarov, C.T. and S. Steinberg [cond-mat/0603593]

q-Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x)\, dx = \int_{-\infty}^{\infty} e_q^{\frac{ix\xi}{[f(x)]^{1-q}}} f(x)\, dx \qquad \text{(nonlinear!)}$$

q_1-independence:

Two random variables X [*with density* $f_X(x)$] *and* Y [*with density* $f_Y(y)$] *having zero* $q-$*mean values are said* q_1-*independent if*

$$F_q[X+Y](\xi) = F_q[X](\xi) \otimes_{q_1} F_q[Y](\xi) \qquad \left(q_1 \equiv \frac{1+q}{3-q}\right),$$

i.e., if

$$\int_{-\infty}^{\infty} dz\, e_q^{iz\xi} \otimes_q f_{X+Y}(z) = \left[\int_{-\infty}^{\infty} dx\, e_q^{ix\xi} \otimes_q f_X(x)\right] \otimes_{(1+q)/(3-q)} \left[\int_{-\infty}^{\infty} dy\, e_q^{iy\xi} \otimes_q f_Y(y)\right],$$

with $f_{X+Y}(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, h(x,y)\, \delta(x+y-z) = \int_{-\infty}^{\infty} dx\, h(x, z-x) = \int_{-\infty}^{\infty} dy\, h(z-y, y)$

where $h(x,y)$ *is the joint density.*

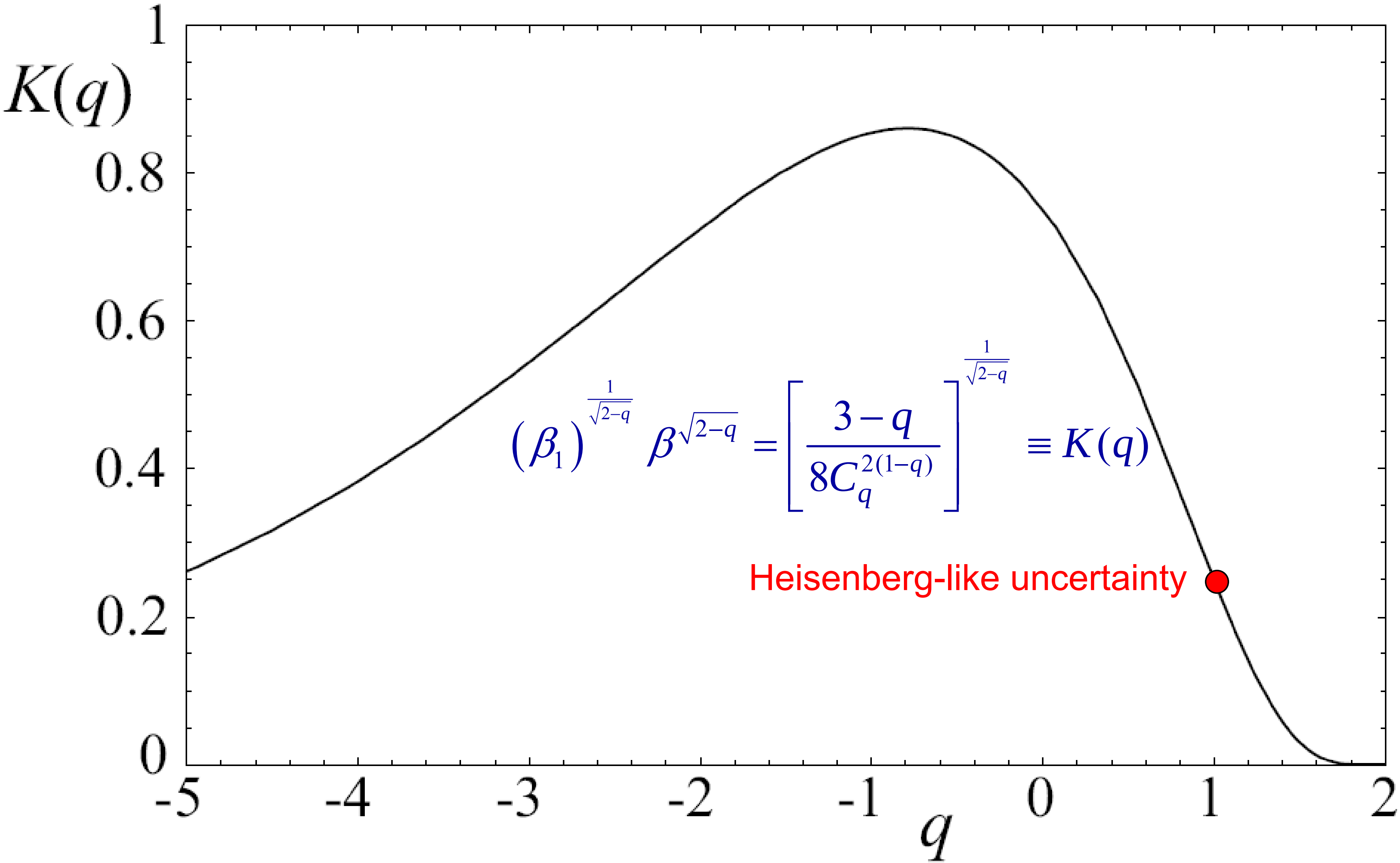
$$\left(\begin{array}{llll} q\text{-independence means} & \text{independence} & \text{if } q=1, \text{ i.e.,} & h(x,y) = f_X(x)\, f_Y(y) \\ & \text{global correlation} & \text{if } q \neq 1, \text{ hence} & h(x,y) \neq f_X(x)\, f_Y(y) \end{array} \right)$$

$$q-FourierTransform\left[\frac{\sqrt{\beta}}{C_q}e_q^{-\beta\ t^2}\right]=e_{q_1}^{-\beta_1\ \omega^2}$$

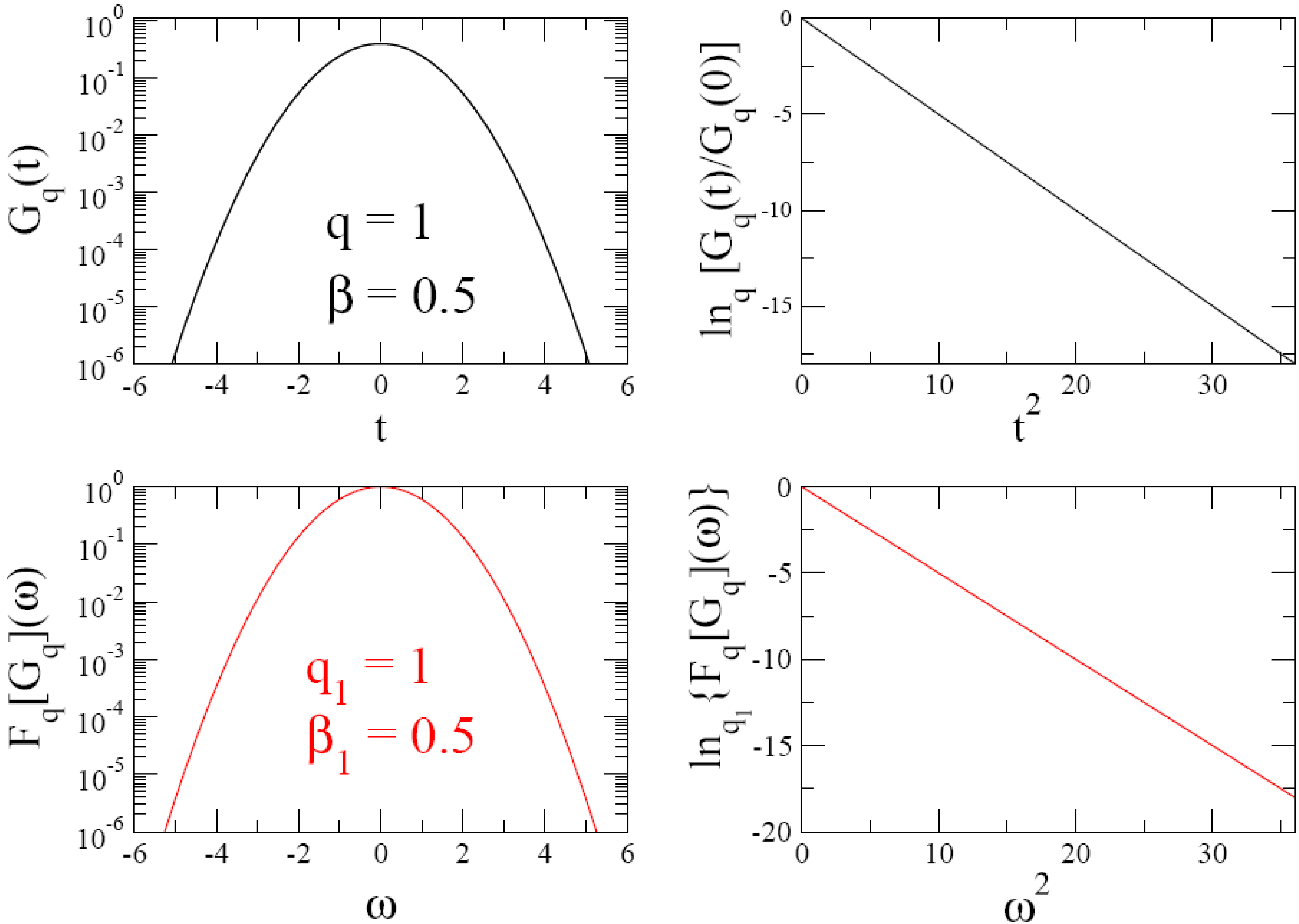
$$\text{where}\qquad q_1=\frac{1+q}{3-q}$$

$$\text{and}\qquad \beta_1=\frac{3-q}{8\beta^{2-q}C_q^{2(1-q)}}\quad\Leftrightarrow\quad(\beta_1)^{\frac{1}{\sqrt{2-q}}}\ \beta^{\sqrt{2-q}}=\left[\frac{3-q}{8C_q^{2(1-q)}}\right]^{\frac{1}{\sqrt{2-q}}}\equiv K(q)$$

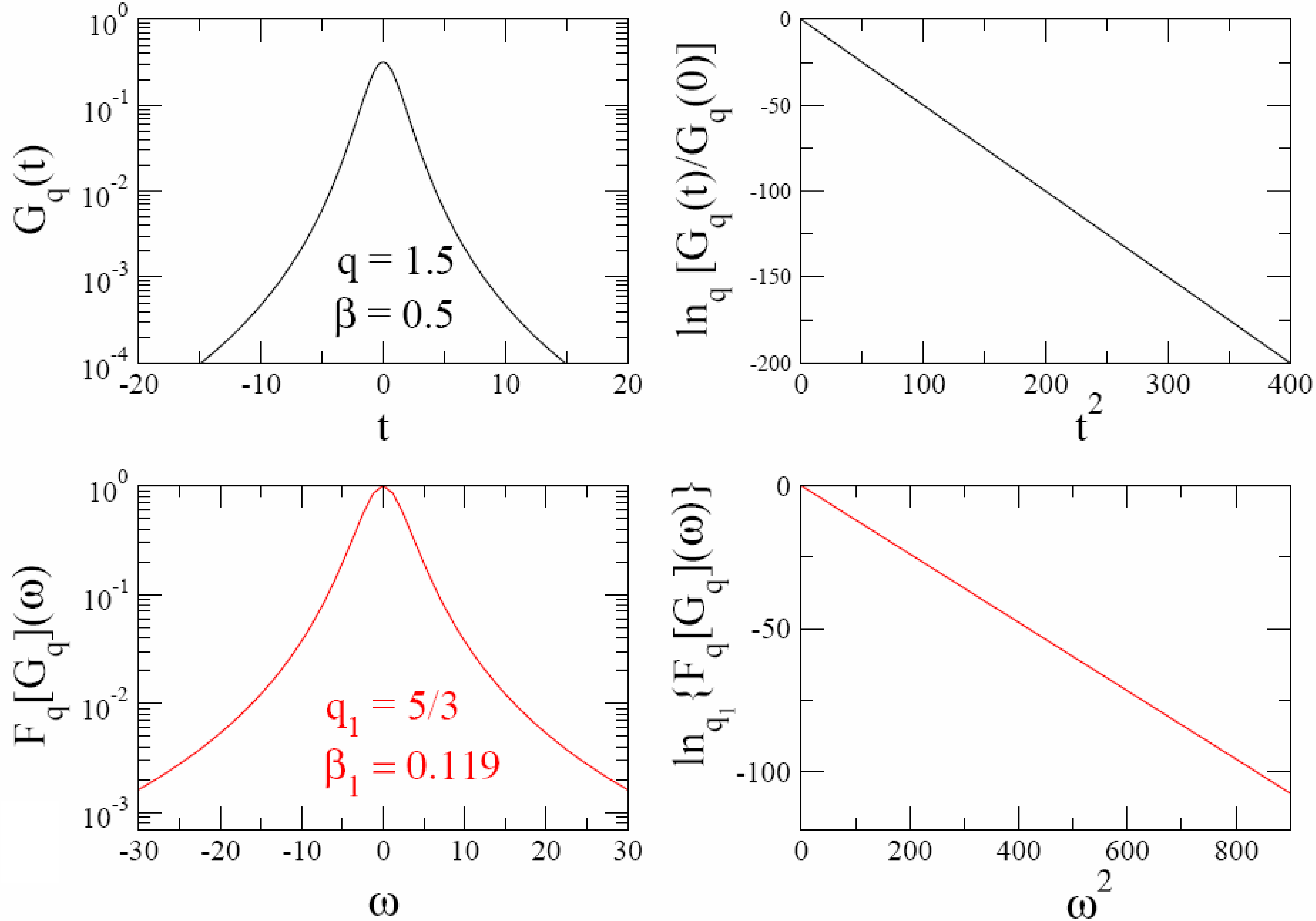
$$\text{with}\qquad C_q=\begin{cases}\dfrac{2\sqrt{\pi}\,\Gamma\left(\dfrac{1}{q-1}\right)}{(3-q)\sqrt{(1-q)}\,\Gamma\left(\dfrac{3-q}{2(1-q)}\right)} & \text{if}\quad q<1\\ \sqrt{\pi} & \text{if}\quad q=1\\ \dfrac{\sqrt{\pi}\,\Gamma\left(\dfrac{3-q}{2(q-1)}\right)}{\sqrt{q-1}\,\Gamma\left(\dfrac{1}{q-1}\right)} & \text{if}\quad 1<q<3\end{cases}$$



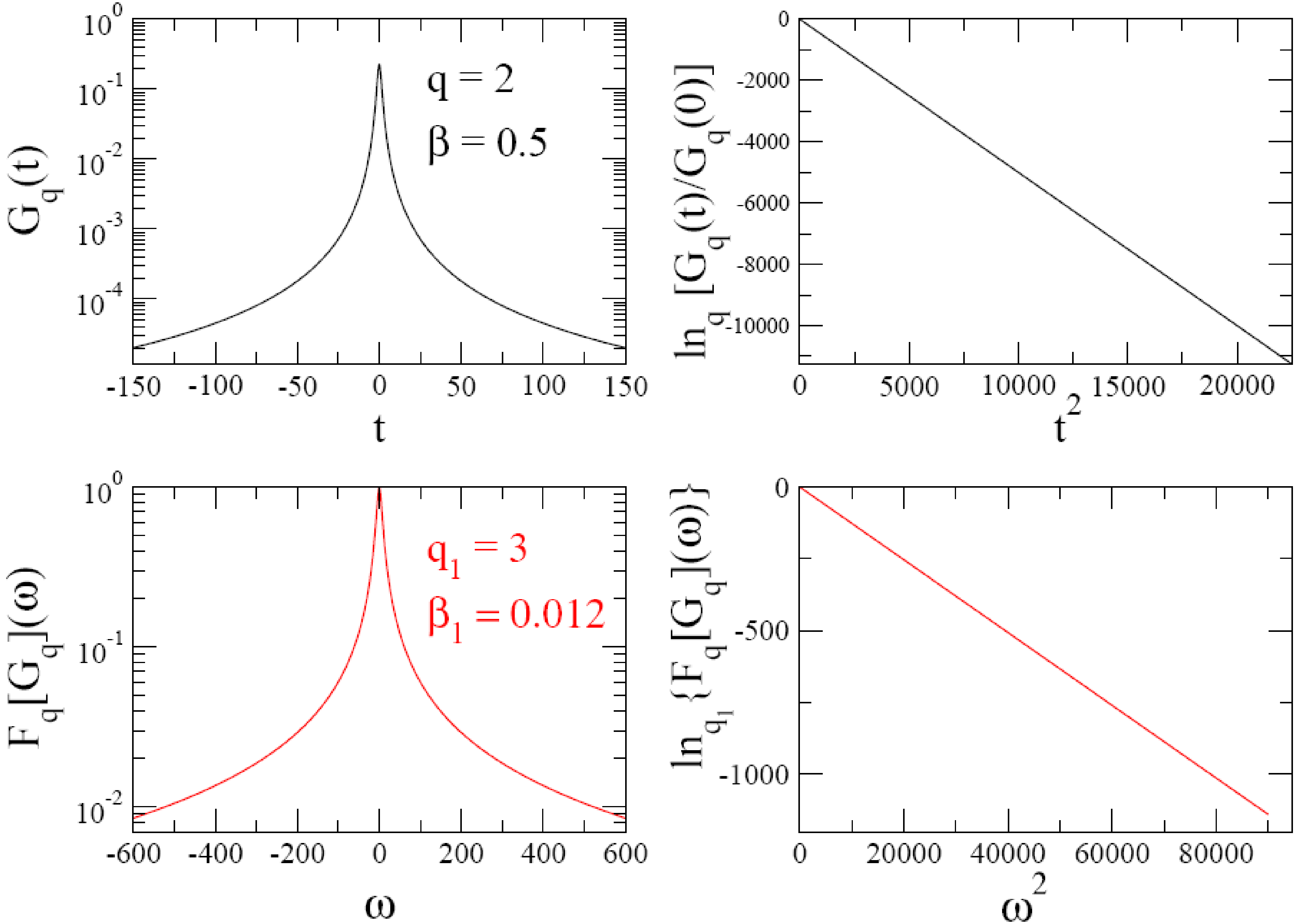
C. T. and S.M.D. Queiros (2007)



$G_q(t)$
$q = 1$
$\beta = 0.5$
t
$\ln_q [G_q(t)/G_q(0)]$
t^2
$F_q[G_q](\omega)$
$q_1 = 1$
$\beta_1 = 0.5$
ω
$\ln_{q_1}\{F_q[G_q](\omega)\}$
ω^2



$G_q(t)$
$q = 1.5$
$\beta = 0.5$
t
$\ln_q[G_q(t)/G_q(0)]$
t^2
$F_q[G_q](\omega)$
$q_1 = 5/3$
$\beta_1 = 0.119$
ω
$\ln_{q_1}\{F_q[G_q](\omega)\}$
ω^2



$G_q(t)$
$q = 2$
$\beta = 0.5$
t
$\ln_q[G_q(t)/G_q(0)]$
t^2
$F_q[G_q](\omega)$
$q_1 = 3$
$\beta_1 = 0.012$
ω
$\ln_{q_1}\{F_q[G_q](\omega)\}$
ω^2

A random variable X is said to have a (q,α)*-stable distribution* $L_{q,\alpha}(x)$

if its q-Fourier transform has the form $a\ e_{q_1}^{-b\,|\xi|^\alpha}$

$[a>0,\ b>0,\ 0<\alpha\le 2,\ q_1 \equiv (q+1)/(3-q)]$

i.e., if

$$F_q[L_{q,\alpha}](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q L_{q,\alpha}(x)\,dx = \int_{-\infty}^{\infty} e_q^{\frac{ix\xi}{[L_{q,\alpha}(x)]^{1-q}}} L_{q,\alpha}(x)\,dx = a\ e_{q_1}^{-b\,|\xi|^\alpha}$$

$L_{1,2}(x) \equiv G(x)$ *(Gaussian)*

$L_{1,\alpha}(x) \equiv L_\alpha(x)$ (α *– stable Levy distribution)*

$L_{q,2}(x) \equiv G_q(x)$ *(q – Gaussian)*

S. Umarov, C. T., M. Gell-Mann and S. Steinberg (2006)

cond-mat/0606038 and cond-mat/0606040

CLOSURE: *The q-Fourier transform of a q-Gaussian is a $z(q)$-Gaussian with*

$$z(q) = \frac{1+q}{3-q} \in (-\infty, 3)$$

ITERATION: $q_n \equiv z_n(q) \equiv z(z_{n-1}(q)) = \dfrac{2q + n(1-q)}{2 + n(1-q)}$ $(n = 0, \pm 1, \pm 2, ...; \; q_0 = q)$

(as in R.S. Mendes and C.T. [Phys Lett A 285, 273 (2001)] calculating marginal probabilities!)

hence

(i) $q_n(1) = 1 \; (\forall n), \quad q_{\pm\infty}(q) = 1 \; (\forall q),$

(ii) $q_{n-1} = 2 - \dfrac{1}{q_{n+1}}$,

(as in L.G. Moyano, C.T. and M. Gell-Mann (2005)!)

(as in A. Robledo [Physica D 193, 153 (2004)] for pitchfork and tangent bifurcations!)

(iii) $n = 2m = 0, \pm 2, \pm 4, ...$ *yields* $q_{(m)} \equiv q_{2m} = \dfrac{q + m(1-q)}{1 + m(1-q)}$

(as in C.T., M. Gell-Mann and Y. Sato [Proc Natl Acad Sci (USA) 102, 15377 (2005)], by combining *only* additive and multiplicative dualities, and which was conjectured to be a possible explanation for the NASA-detected *q*-triangle for m = 0, ± 1!)

$$q_{stat} \to q_{k-1} \xrightarrow{k \equiv n-1} q_{n-2} \quad (\sim 1.75 \text{ for solar wind})$$

$$q_{rel} \to q_{k+1} \xrightarrow{k \equiv n-1} q_{n} \quad (\sim 4 \text{ for solar wind})$$

$$q_{sen} \to q_{k+3} \xrightarrow{k \equiv n-1} q_{n+2} \quad (\sim -0.5 \text{ for solar wind})$$

For $\alpha = 2$, *the following properties are satisfied* :

$$q_{stat} + \frac{1}{q_{rel}} = 2$$

$$q_{rel} + \frac{1}{q_{sen}} = 2$$

The scaling of the sum of the random variables is given by

$$N^{\frac{1}{\alpha(2-q_{k-1})}} = N^{\frac{1}{\alpha(2-q_{n-2})}},$$

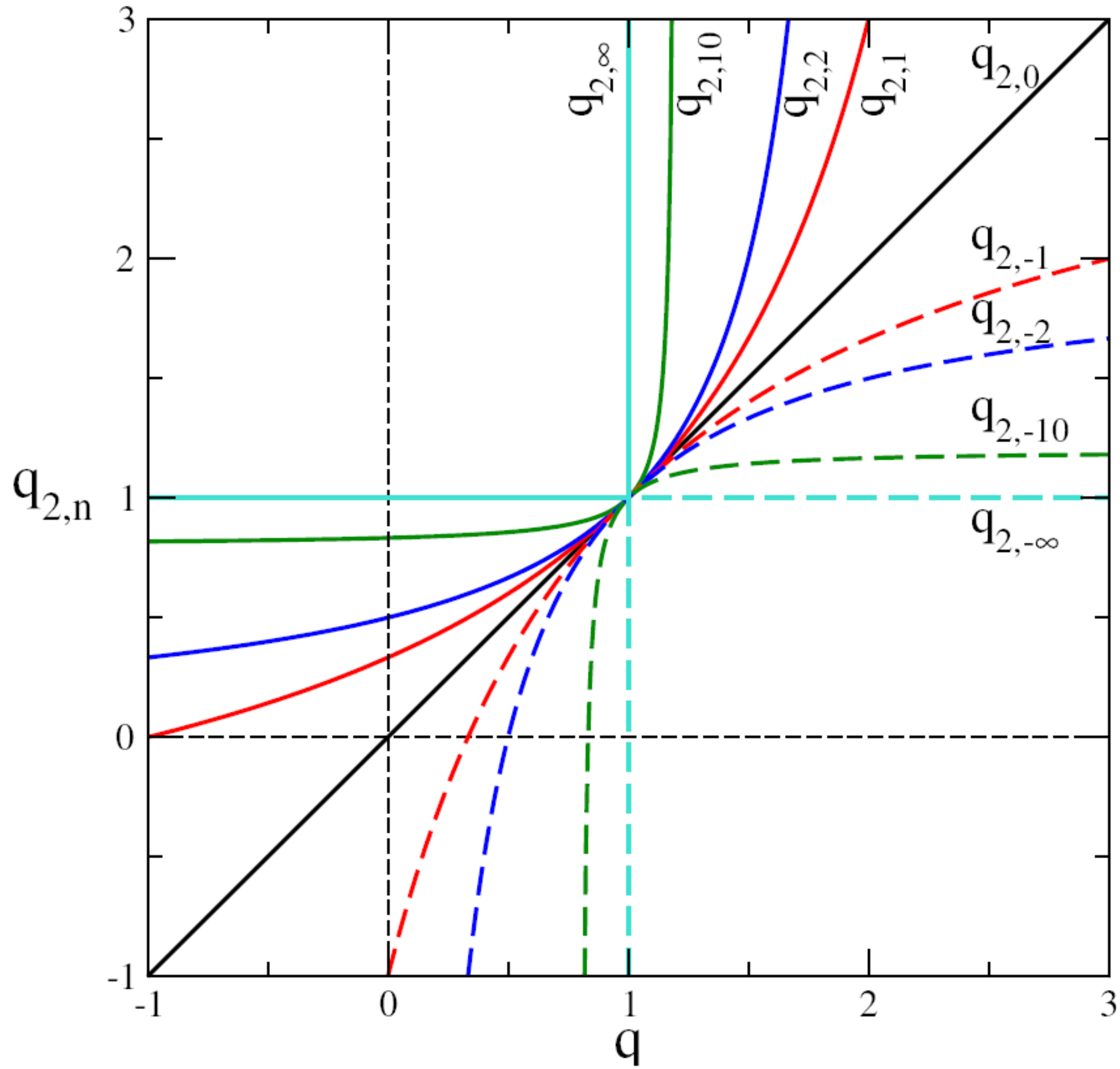
hence, for $\alpha = 2$,

$$N^{\frac{1}{2(2-q_{k-1})}} = N^{\frac{q_{k+1}}{2}} = N^{\frac{q_n}{2}}$$

ALGEBRA ASSOCIATED WITH q-GENERALIZED CENTRAL LIMIT THEOREMS:

$$\frac{\alpha}{1-q_{\alpha,n}} = \frac{\alpha}{1-q} + n$$

$$(n = 0, \ \pm 1, \ \pm 2, \ldots)$$



S. Umarov, C.T., M. Gell-Mann and S. Steinberg (2006), cond-mat/0606040

CENTRAL LIMIT THEOREM

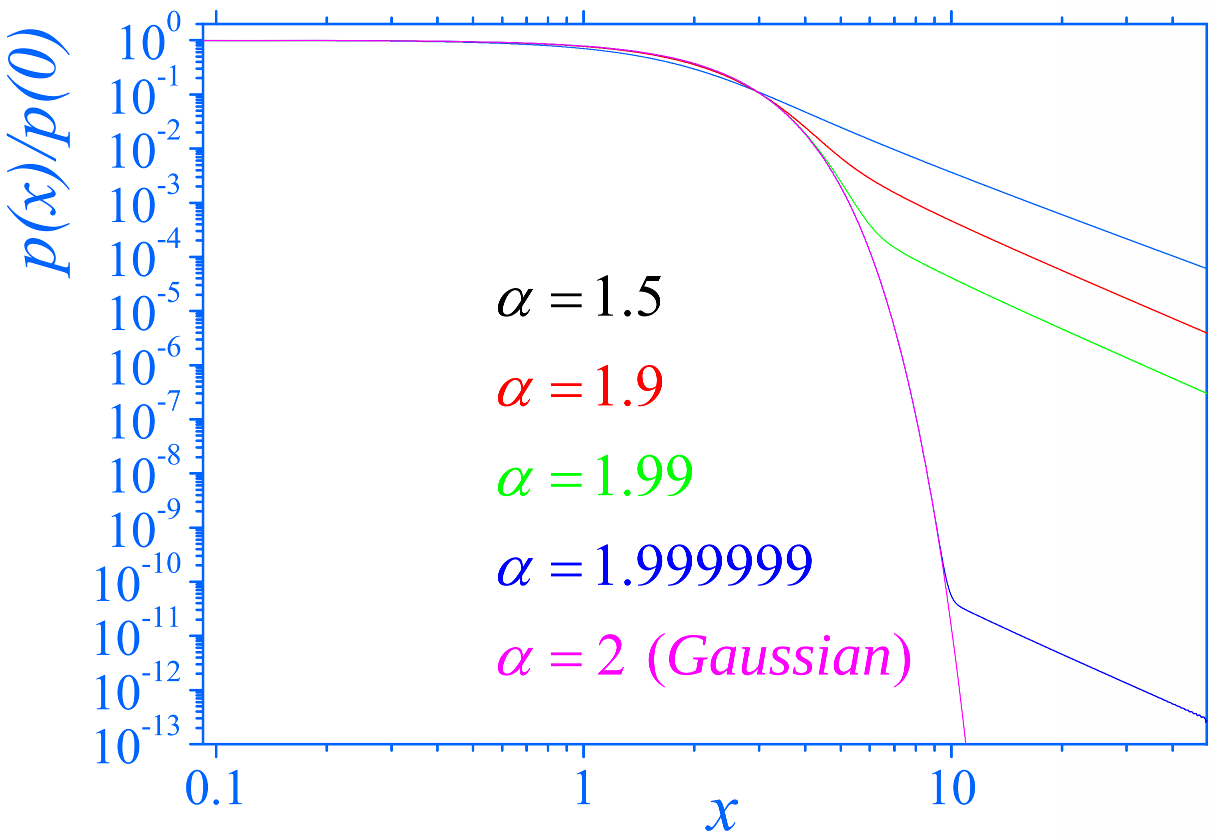
$N^{1/[\alpha(2-q)]}$ - scaled attractor $\mathbb{F}(x)$ when summing $N \to \infty$ q_1 - independent identical random variables

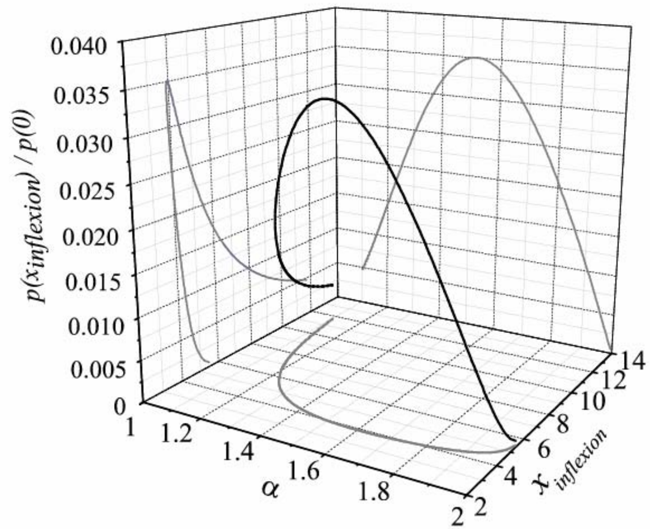
with symmetric distribution $f(x)$ with $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q \quad \left(Q \equiv 2q-1,\; q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ *[independent]*	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) *[globally correlated]*
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \begin{cases} \simeq G(x) & \text{if } \lvert x \rvert \ll x_c(q,2) \\ \sim f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg (2006) [cond-mat/0603593]
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \begin{cases} \simeq G(x) \\ \quad \text{if } \lvert x \rvert \ll x_c(1,\alpha) \\ \sim f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* *with* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\, \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(q-1)}} \\ \textit{(intermediate regime)} \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\, 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \textit{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg (2006) [cond-mat/0606038] and [cond-mat/0606040]

STANDARD AND LEVY-GNEDENKO CENTRAL LIMIT THEOREMS:

$$p(x) \propto Fourier^{-1}\left[e^{-|\xi|^{\alpha}} \right]$$





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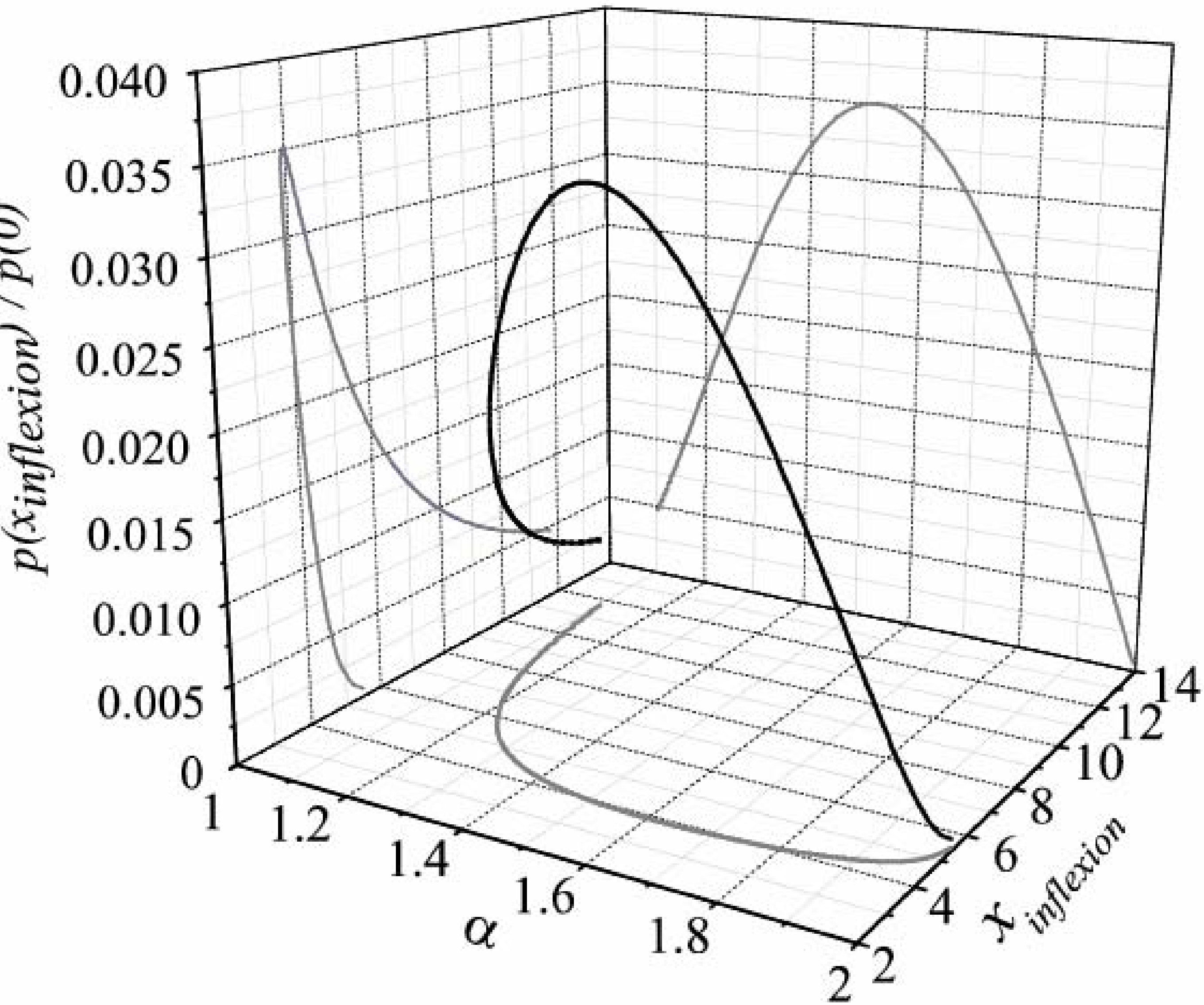
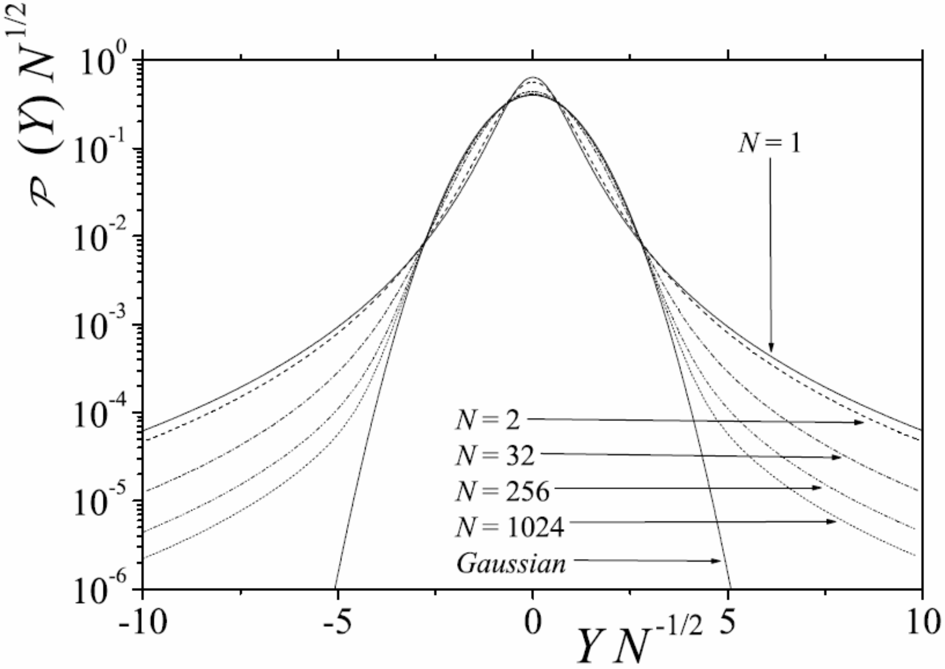
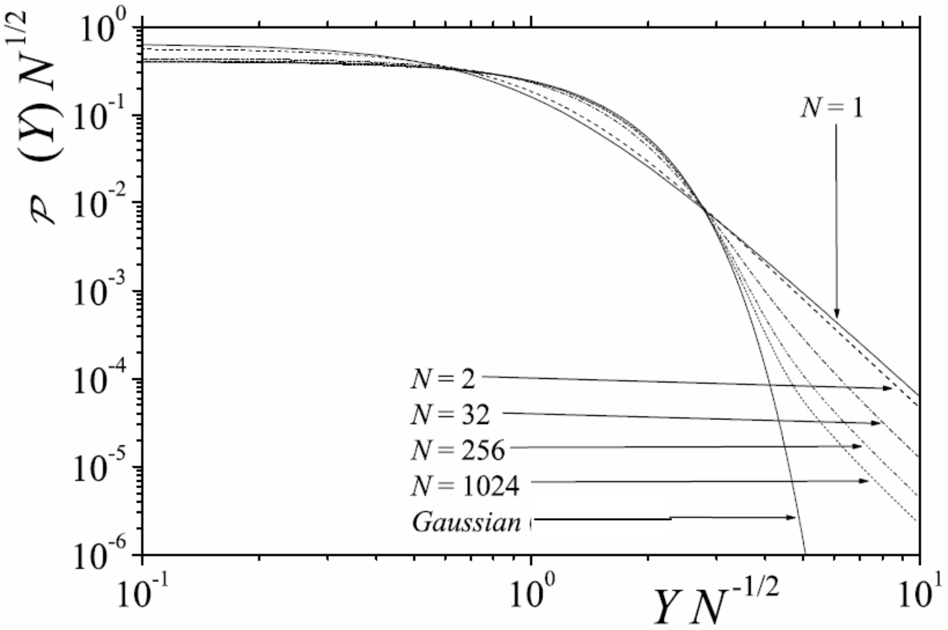


0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0
p(xinflexion) / p(0)
1
1.2
1.4
1.6
1.8
2
α
2
4
6
8
10
12
14
x inflexion

N INDEPENDENT RANDOM VARIABLES

N=1: q-Gaussian with q = 3/2 = 1.5

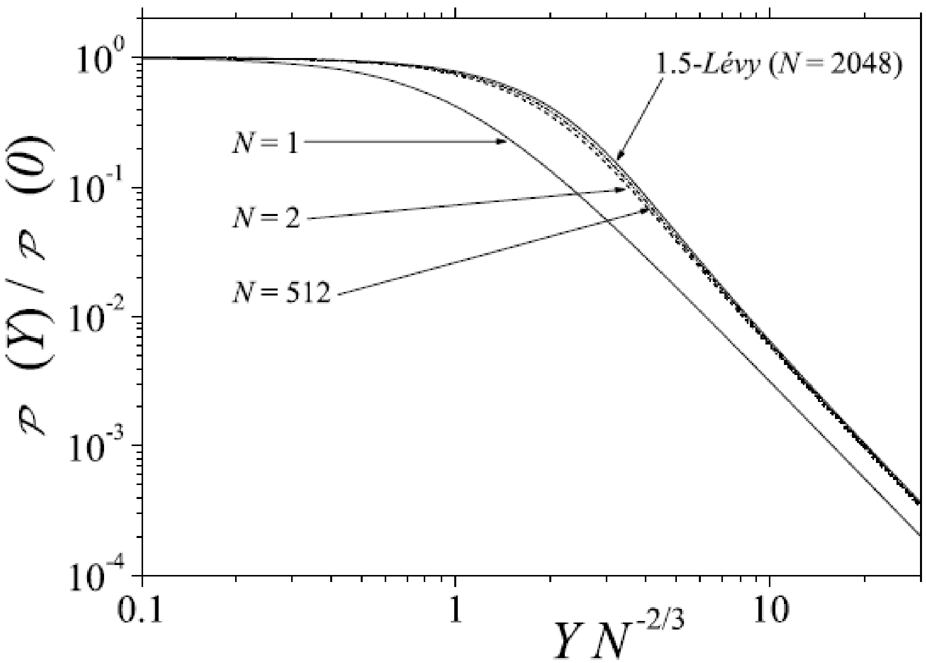


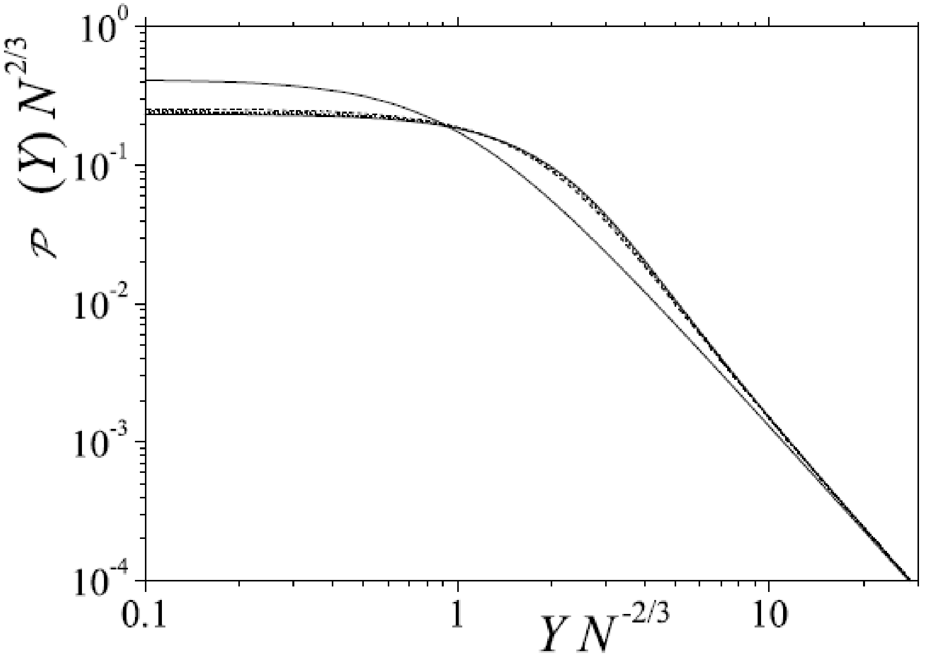


C. T. and S.M.D. Queiros (2007)

N INDEPENDENT RANDOM VARIABLES

N=1: q-Gaussian with q = 9/5 = 1.8

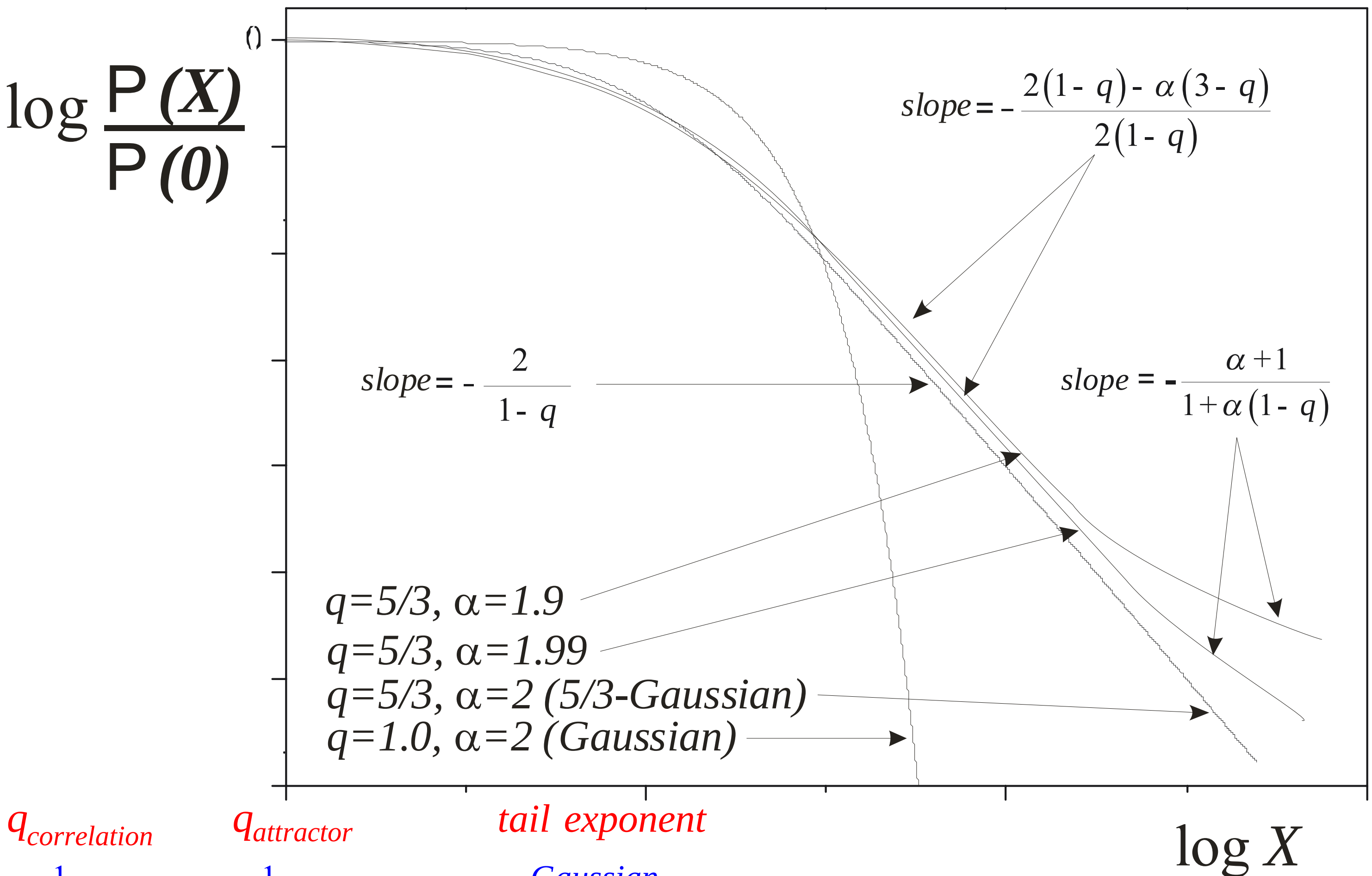




C. T. and S.M.D. Queiros (2007)

q-CENTRAL LIMIT THEOREMS:

$$p(x) = L_{q,\alpha}(x) = (q\text{-}Fourier)^{-1}\left[b e_{q_1}^{-\beta|\xi|^{\alpha}} \right]$$

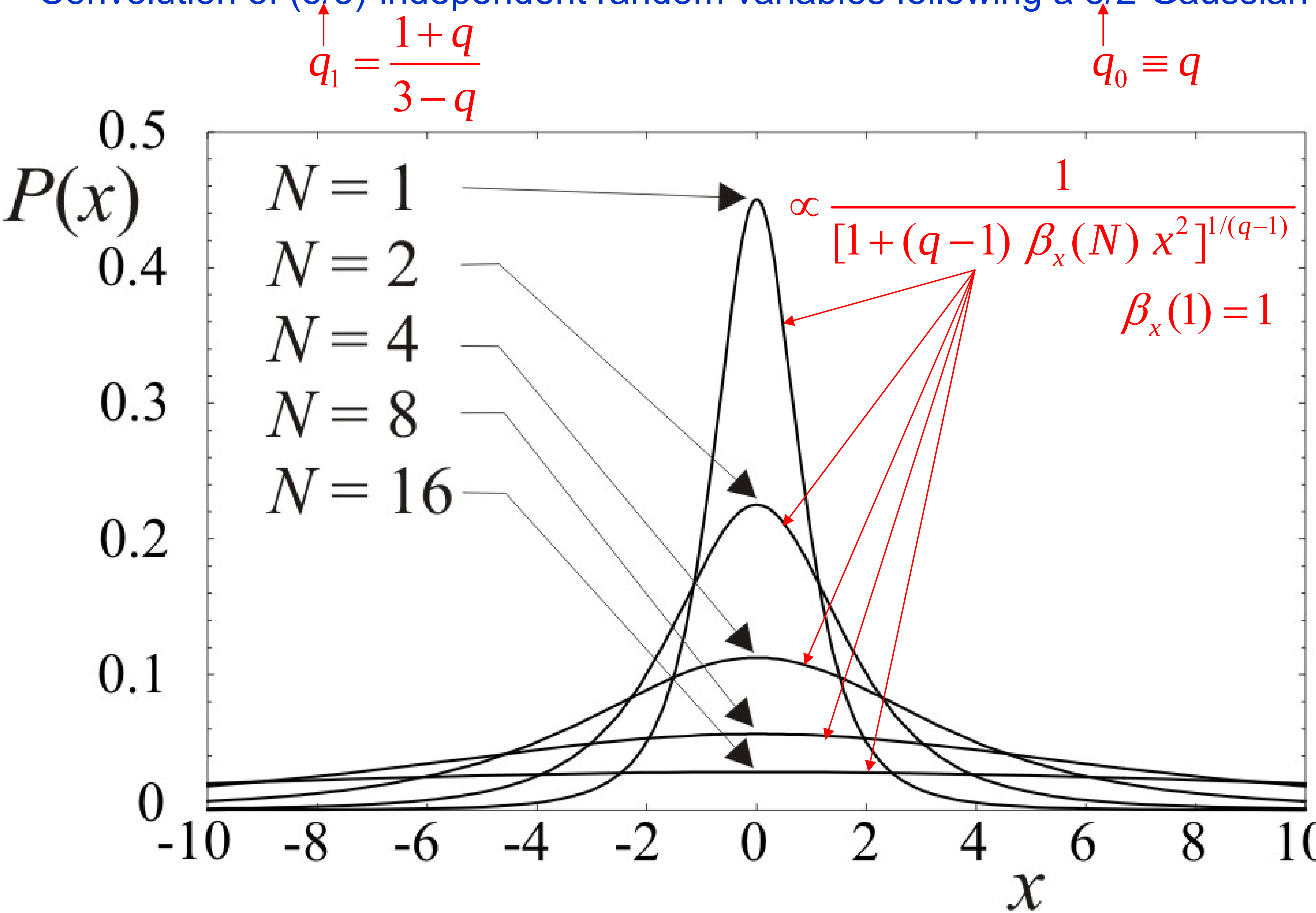


α	$q_{correlation}$	$q_{attractor}$	tail exponent	
2	1	1	*Gaussian*	
2	$\frac{q+1}{3-q}$	q	$\frac{2}{q-1}$	
α	$\frac{q+1}{3-q}$	$\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)}$	$\frac{2(1-q)-\alpha(3-q)}{2(1-q)}$	(intermediate regime)
		$\frac{2\alpha q-\alpha+3}{\alpha+1}$	$\frac{1+\alpha}{1+\alpha q-\alpha}$	(distant regime)

$$\frac{2}{(q-1)} \geq \frac{2(1-q)-\alpha(3-q)}{2(1-q)} > \frac{1+\alpha}{1+\alpha q-\alpha}$$

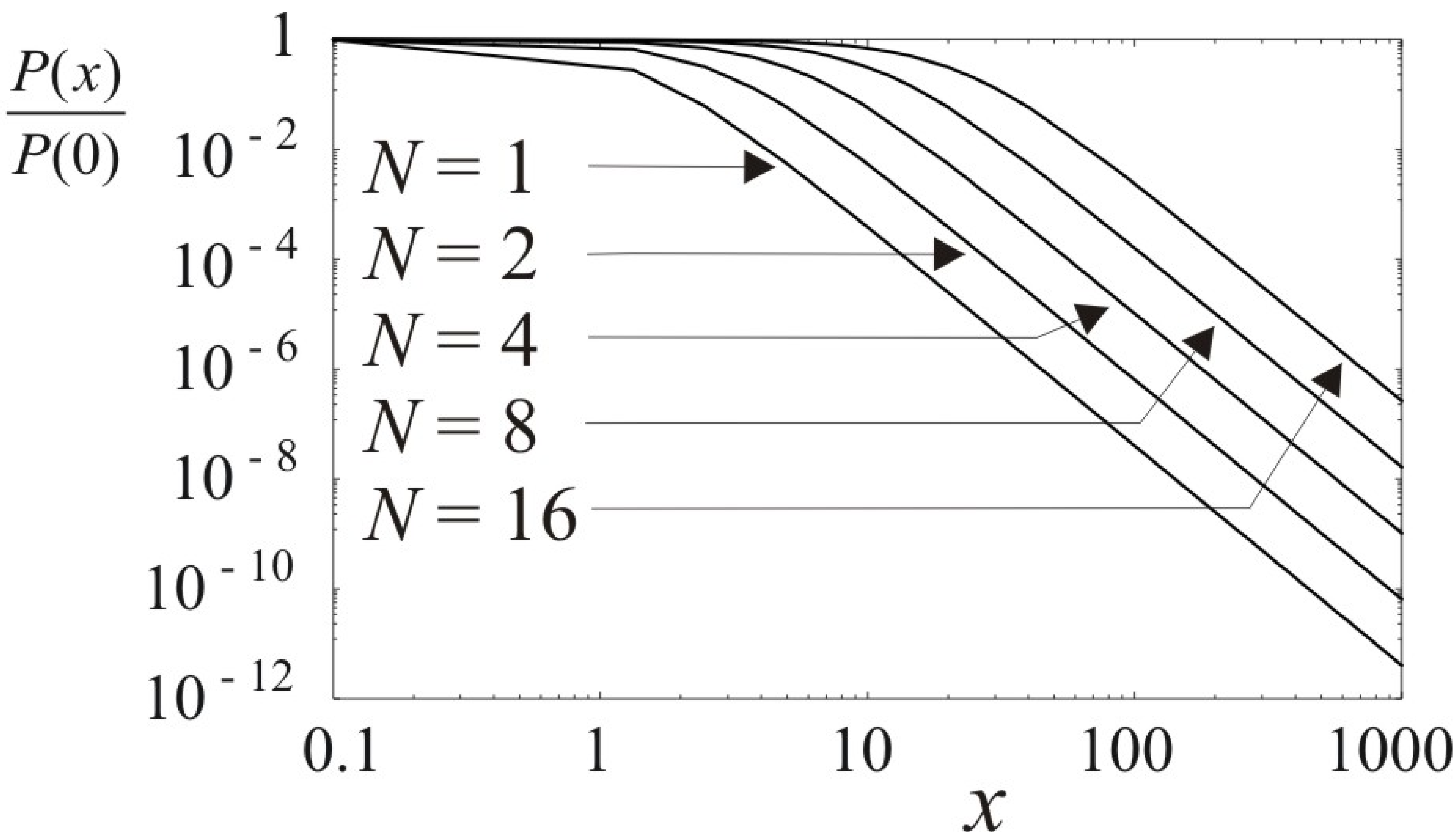
C. T. and S.M.D. Queiros (2007)

Convolution of (5/3)-independent random variables following a 3/2-Gaussian



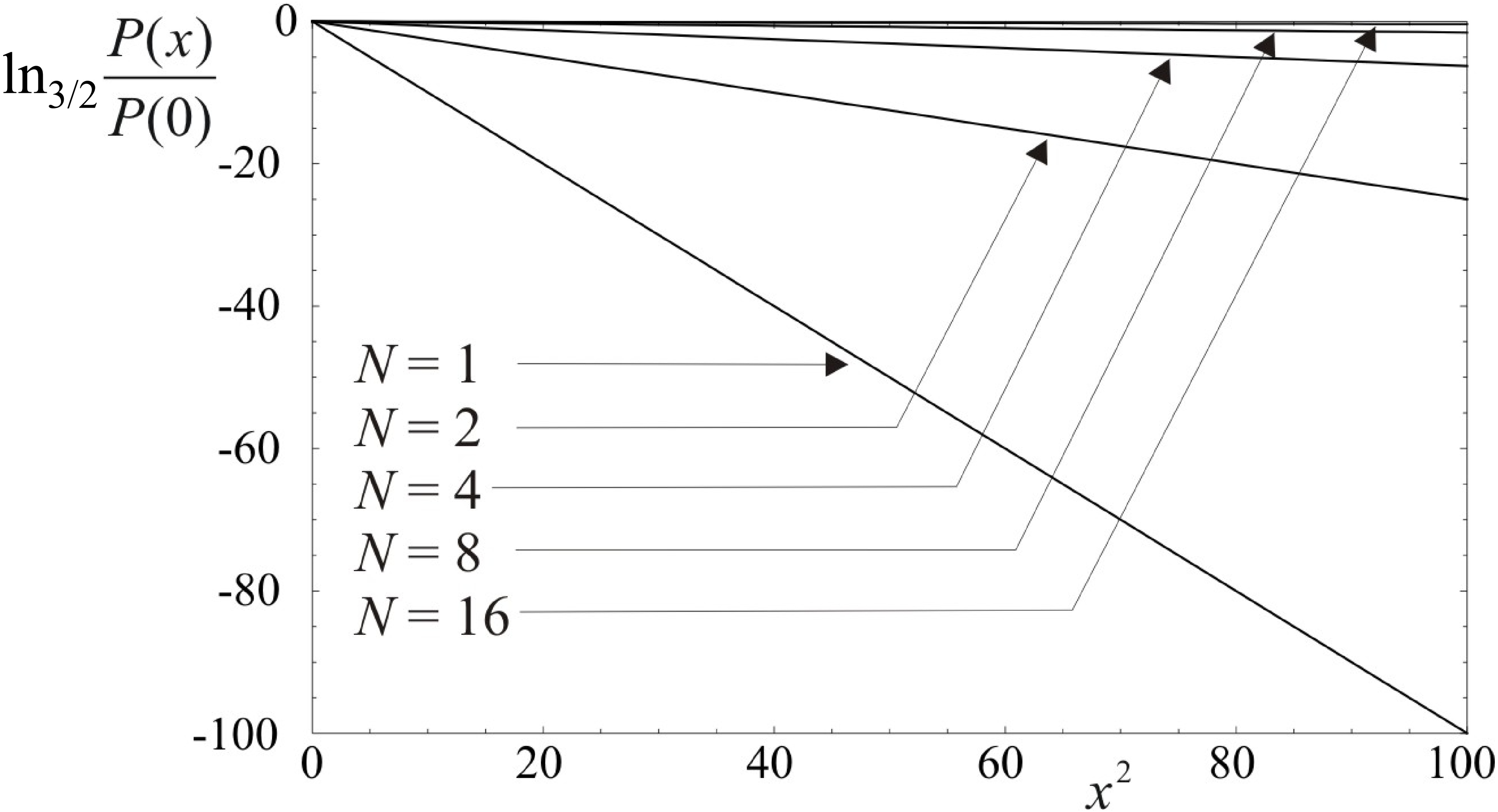
C. T. and S.M.D. Queiros (2007)

Convolution of (5/3)-independent random variables following a 3/2-Gaussian

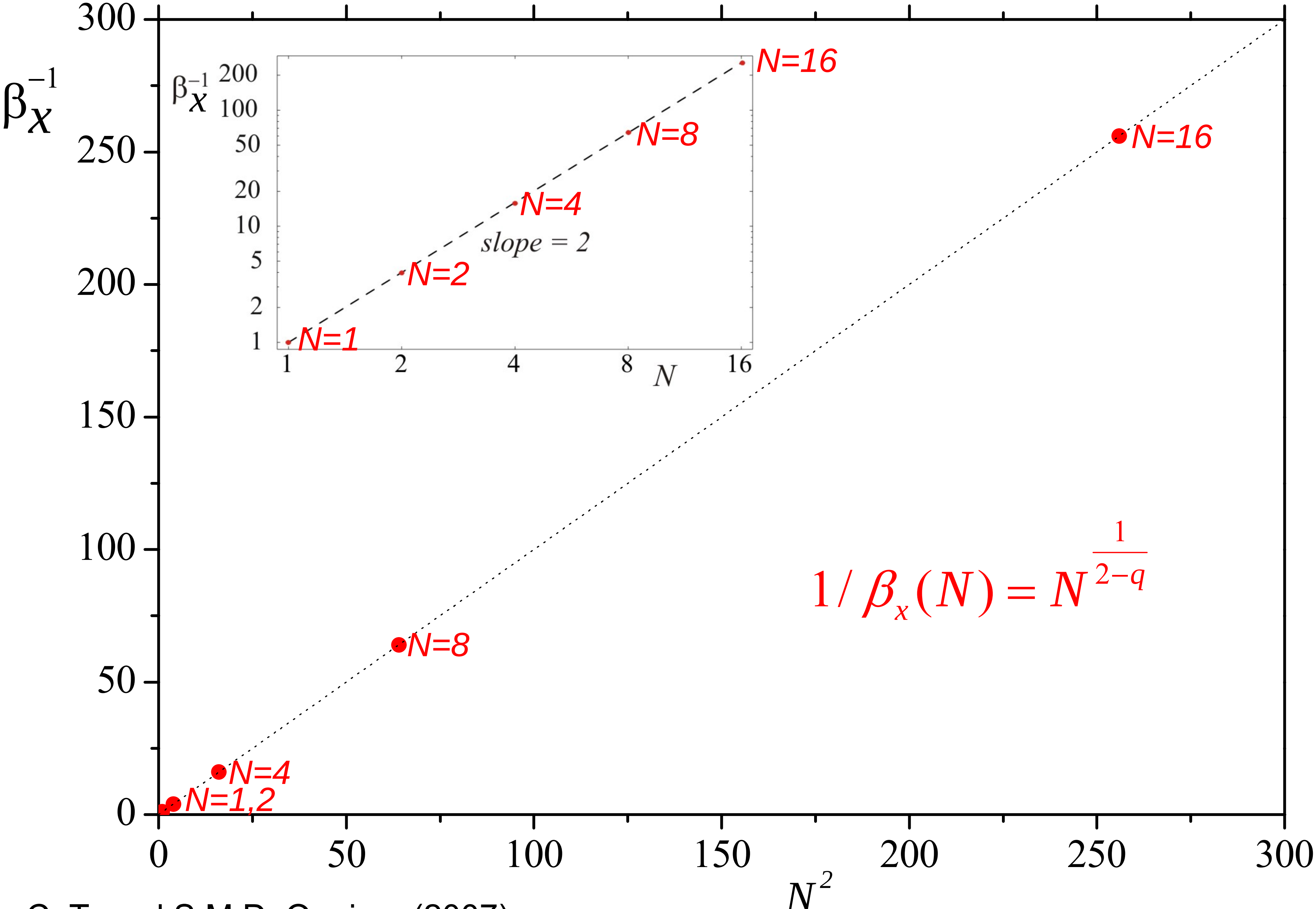


C. T. and S.M.D. Queiros (2007)

Convolution of (5/3)-independent random variables following a 3/2-Gaussian

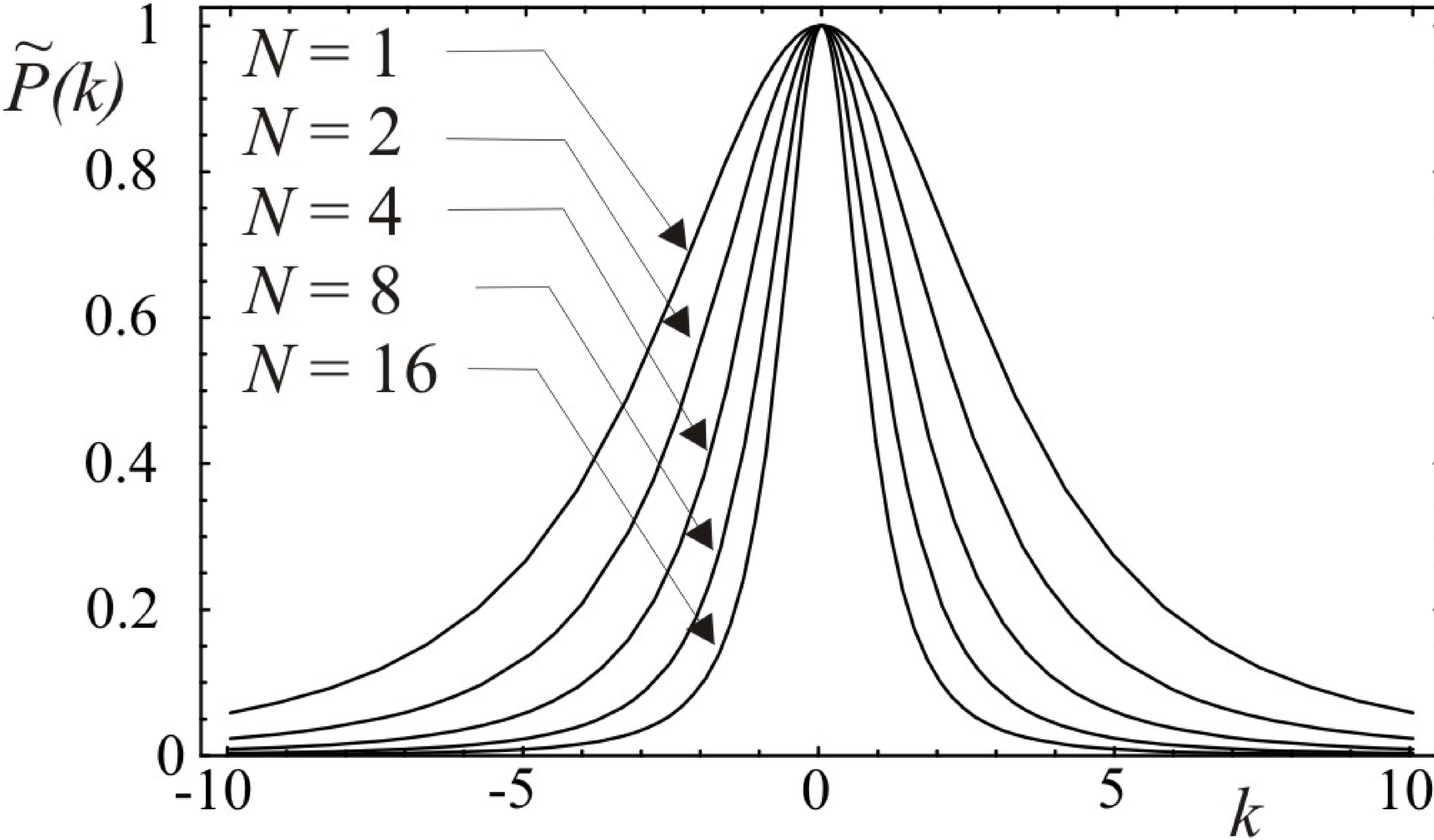


C. T. and S.M.D. Queiros (2007)



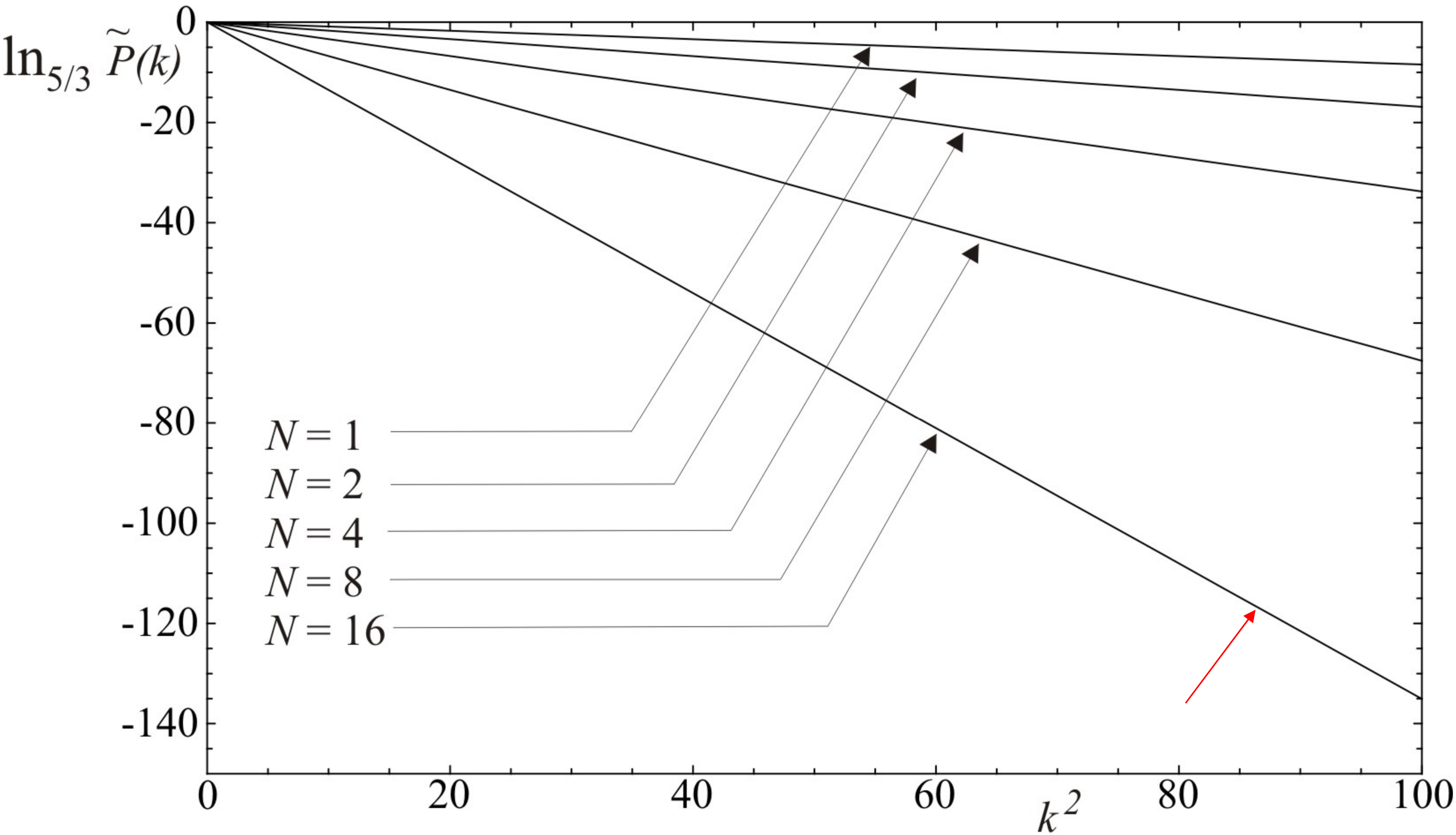
C. T. and S.M.D. Queiros (2007)

3/2-Fourier Transform of $P_N(x)$



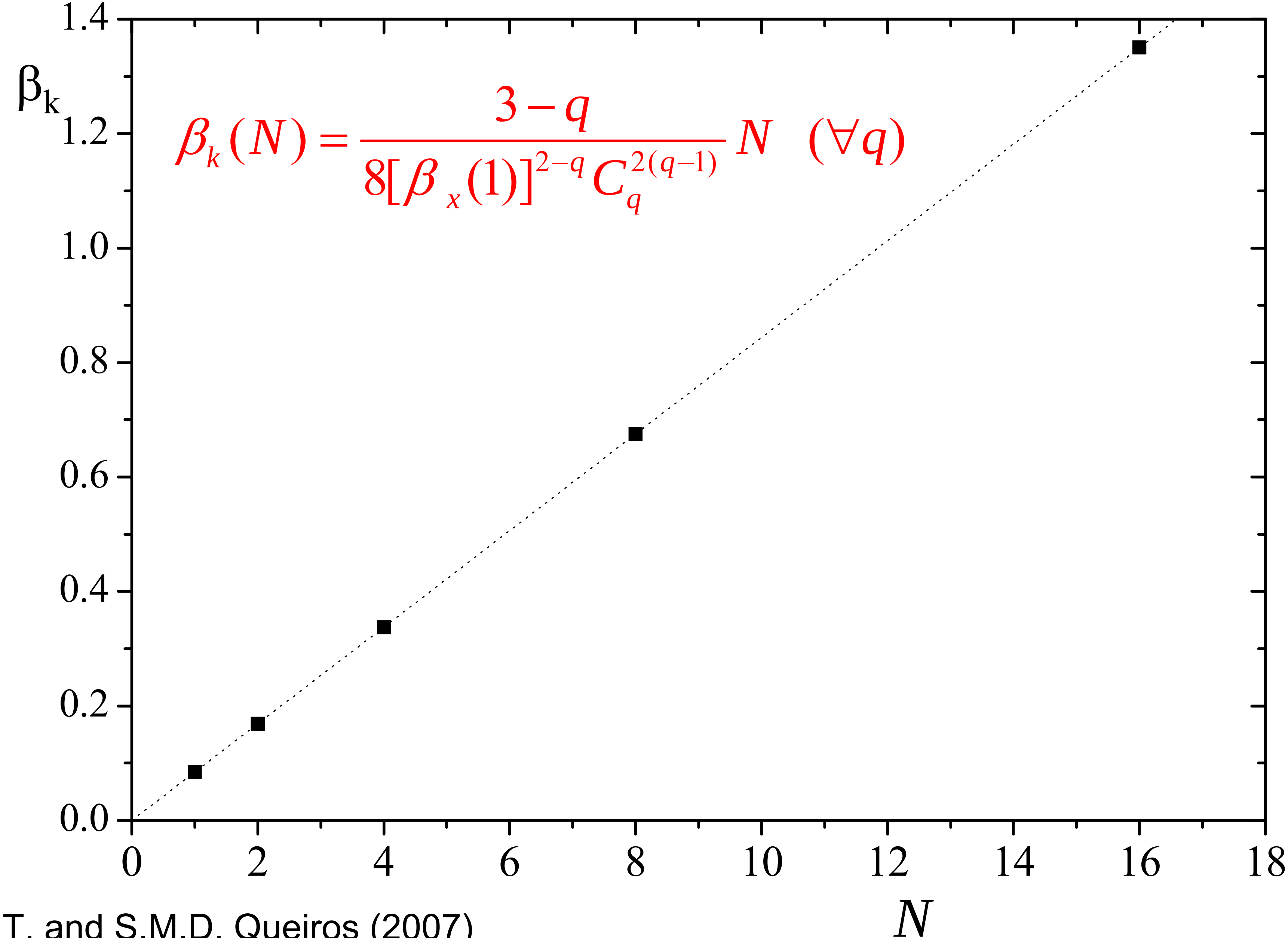
C. T. and S.M.D. Queiros (2007)

3/2-Fourier Transform of $P_N(x)$



C. T. and S.M.D. Queiros (2007)

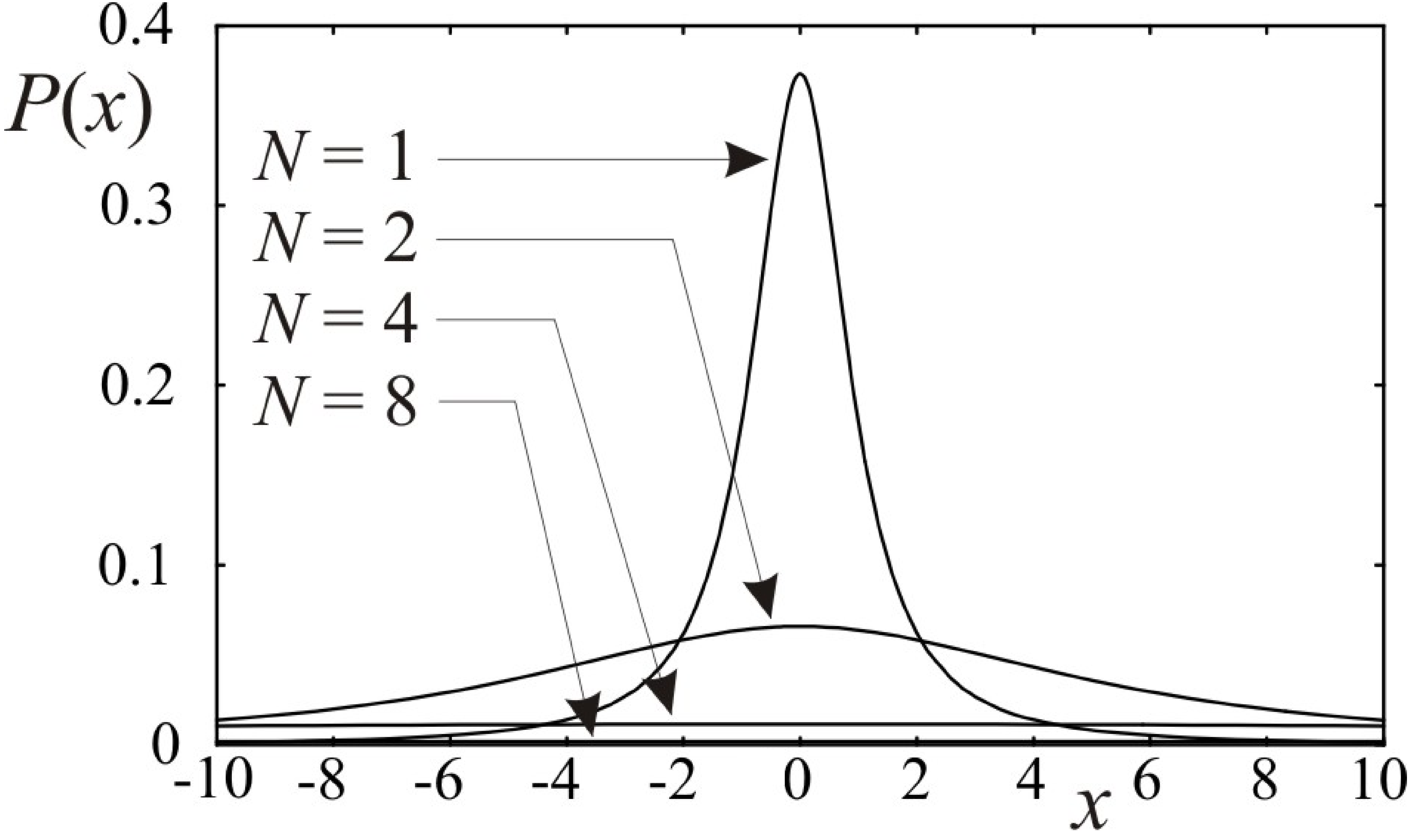
β_k for the 3/2-Fourier Transform



$$\beta_k(N) = \frac{3-q}{8[\beta_x(1)]^{2-q} C_q^{2(q-1)}} N \quad (\forall q)$$

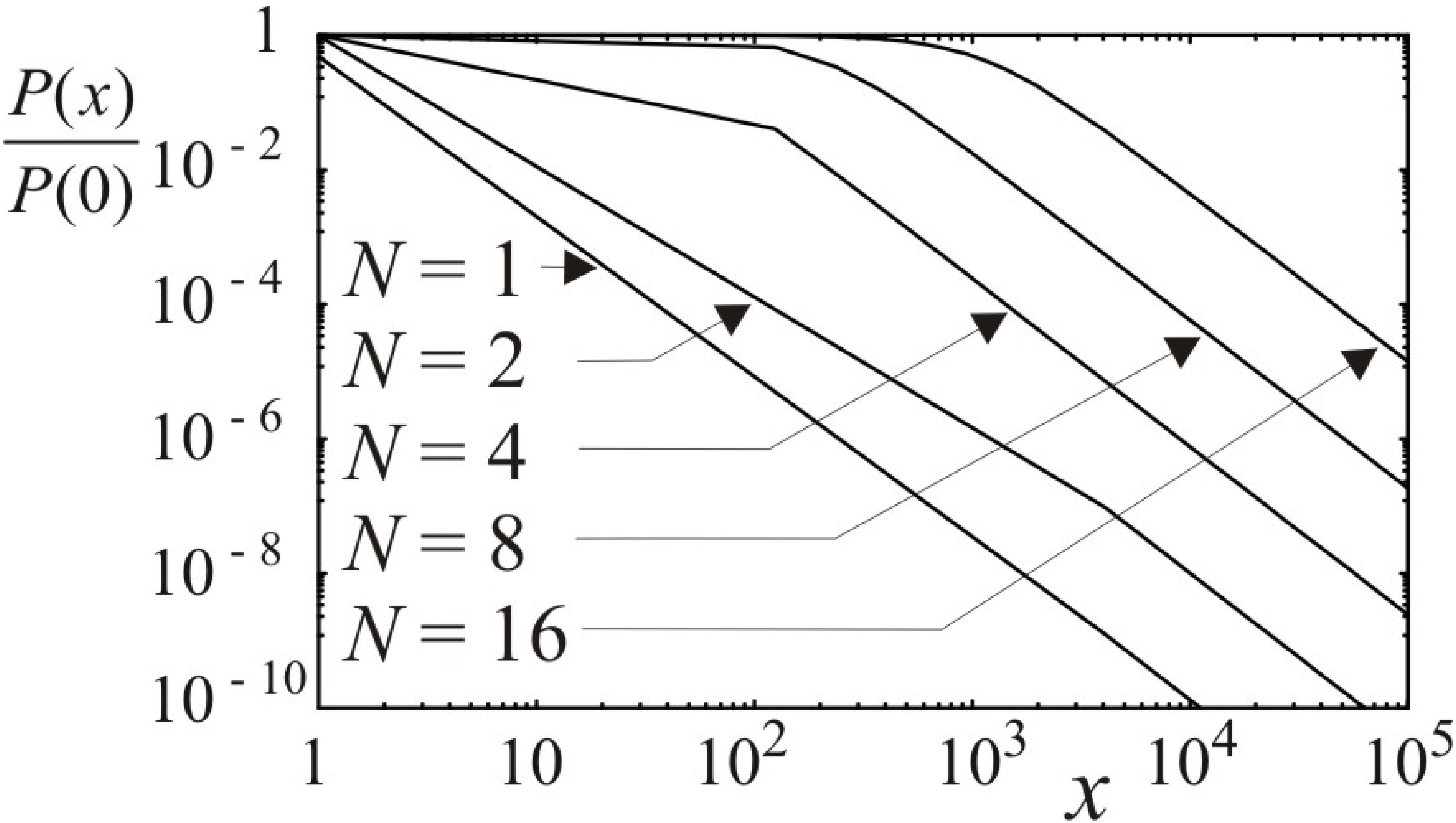
C. T. and S.M.D. Queiros (2007)

Convolution of (7/3)-independent random variables following a 9/5-Gaussian

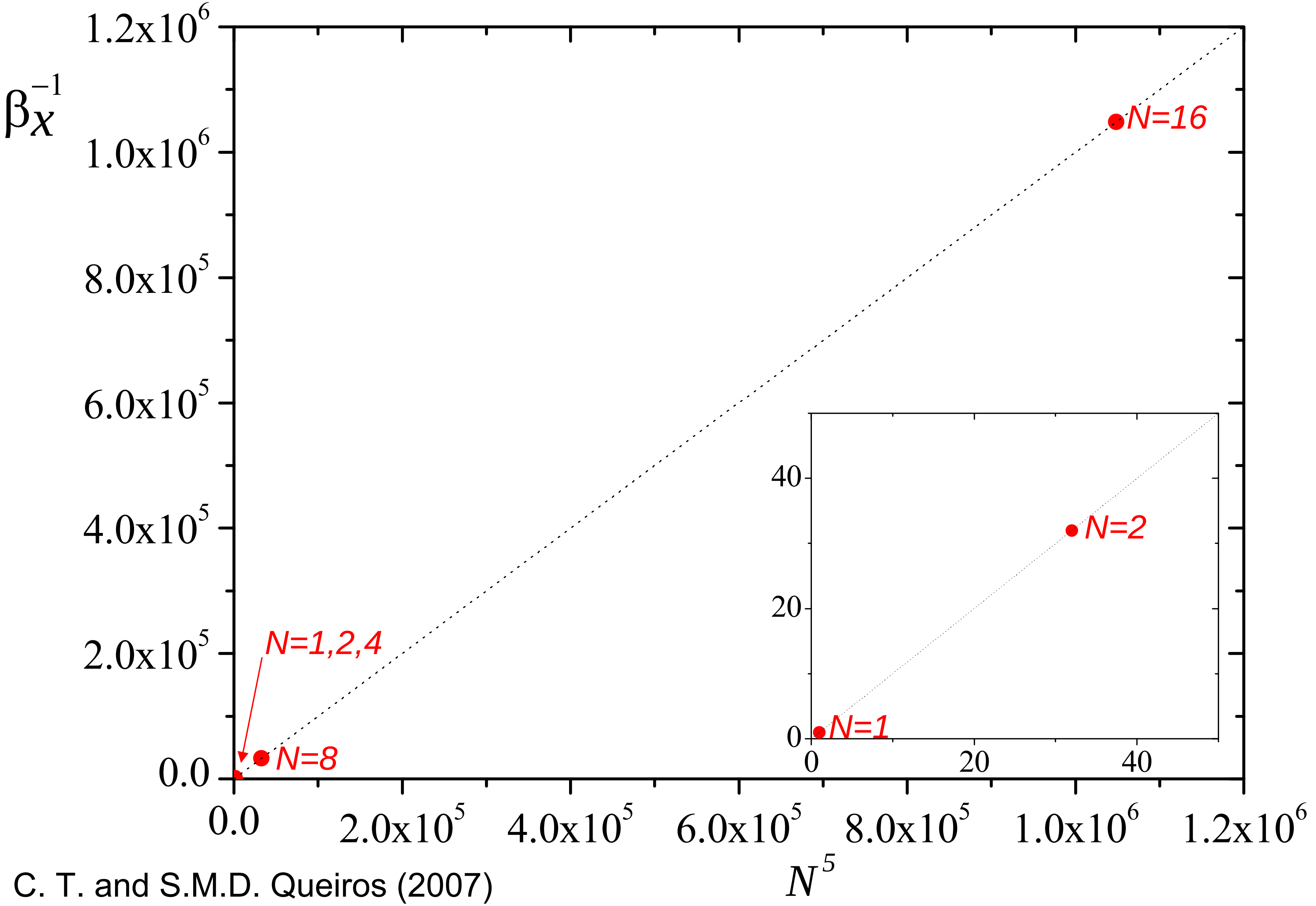


C. T. and S.M.D. Queiros (2007)

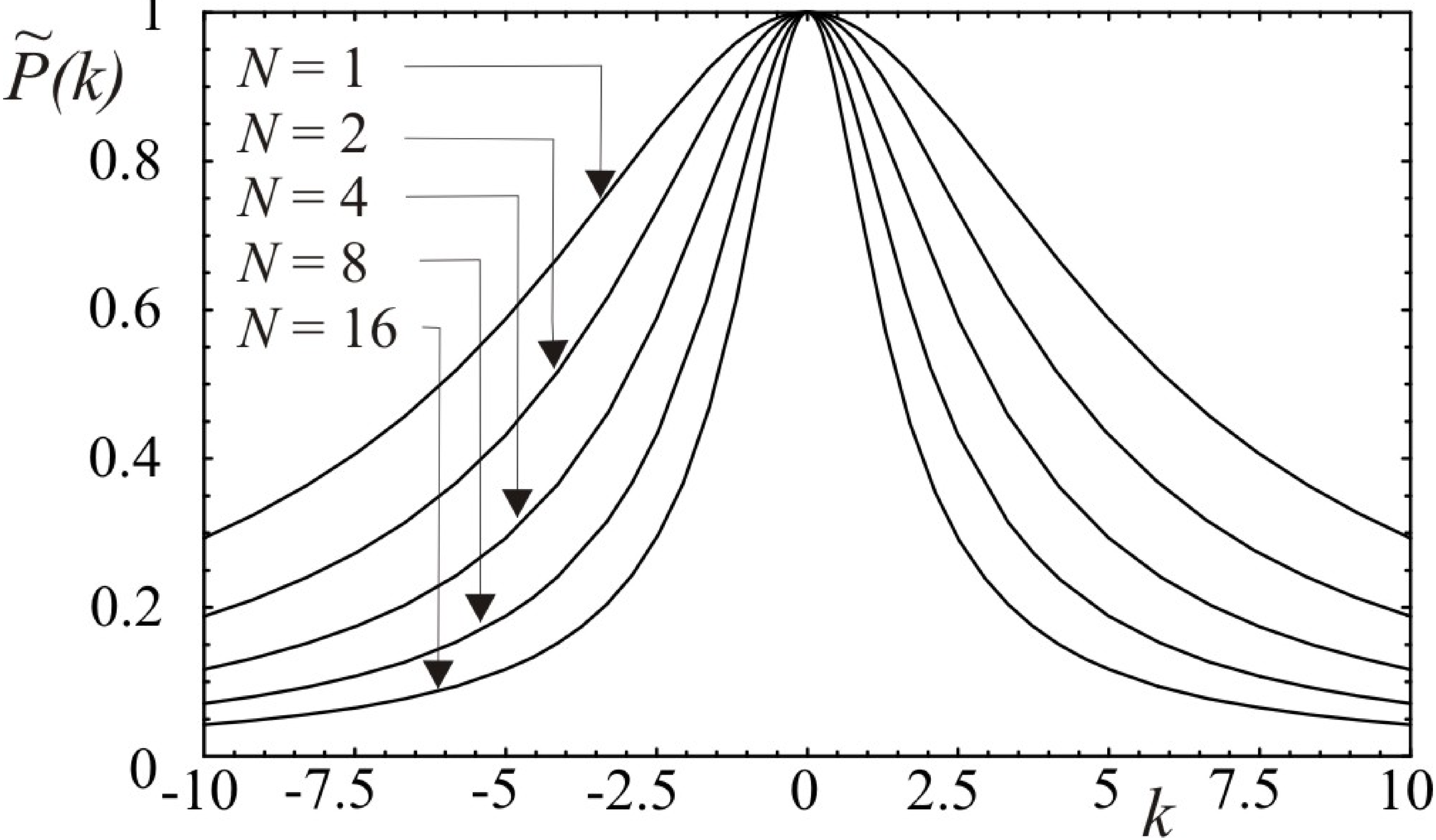
Convolution of (7/3)-independent random variables following a 9/5-Gaussian



C. T. and S.M.D. Queiros (2007)

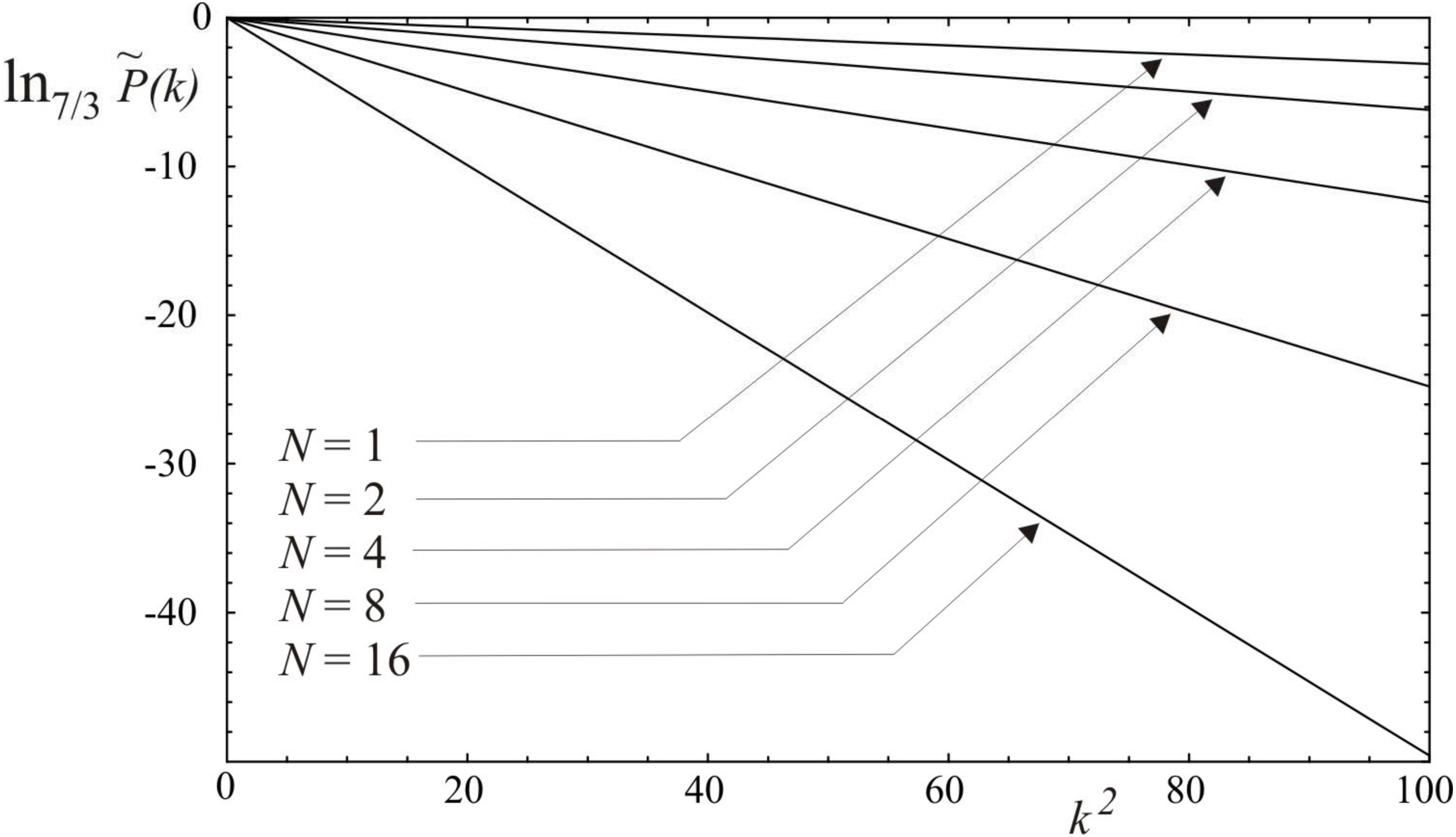

1.2x10^6
1.0x10^6
8.0x10^5
6.0x10^5
4.0x10^5
2.0x10^5
0.0
β_χ^−1
N=16
N=1,2,4
N=8
40
20
0
N=2
N=1
0.0
2.0x10^5
4.0x10^5
6.0x10^5
8.0x10^5
1.0x10^6
1.2x10^6
N^5
C. T. and S.M.D. Queiros (2007)

7/3-Fourier Transform of $P_N(x)$



C. T. and S.M.D. Queiros (2007)

7/3-Fourier Transform of $P_N(x)$



C. T. and S.M.D. Queiros (2007)

$q-$INDEPENDENCE : IT CORRESPONDS TO WHAT ?

It appears to be (no proof available yet)

N − body joint probability

$$\int dx_N \; h(x_1, x_2, \ldots, x_N) = h(x_1, x_2, \ldots, x_{N-1})$$

i.e., scale invariance!

MORE ON THE NATURE OF *q*-CORRELATION:

W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

Let us consider N correlated uniform random variables

$$f(x)=\begin{cases}1 & \text{if } \; -1/2\leq x\leq 1/2\\ 0 & \text{otherwise}\end{cases}$$

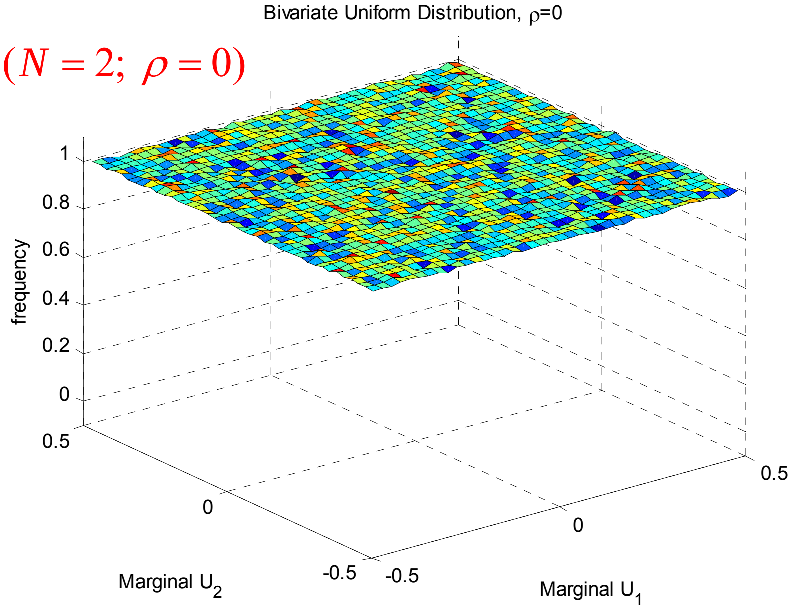
Correlation emerges through the following multivariate Gaussian $N\times N$ *covariance matrix, and probability integral transform (component by component):*

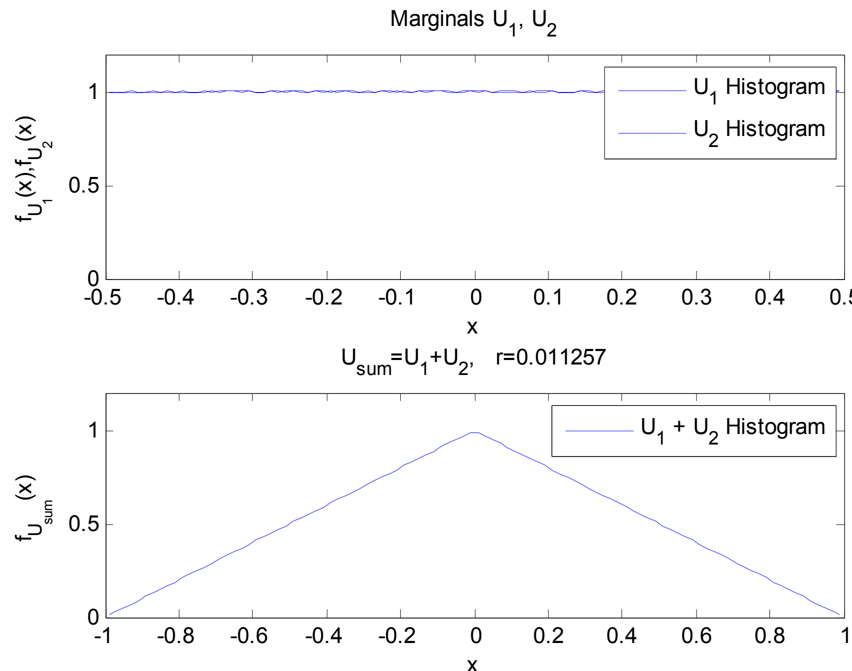
$$\begin{bmatrix}1 & \rho & \cdots & \cdots & \rho\\ \rho & 1 & \ddots & \ddots & \vdots\\ \vdots & \ddots & 1 & \ddots & \vdots\\ \vdots & \ddots & \ddots & 1 & \rho\\ \rho & \cdots & \cdots & \rho & 1\end{bmatrix} \qquad (-1\leq\rho\leq 1)$$

$\rho=0 \;\Rightarrow\;$ *independence*

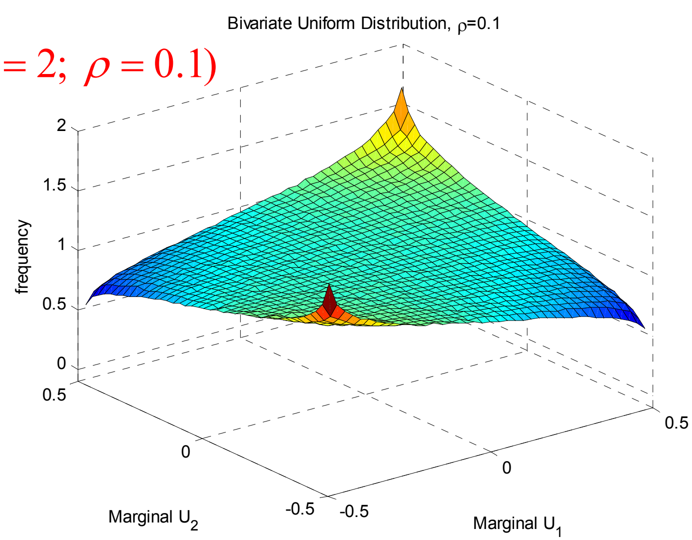
$\rho=1 \;\Rightarrow\;$ *full correlation*

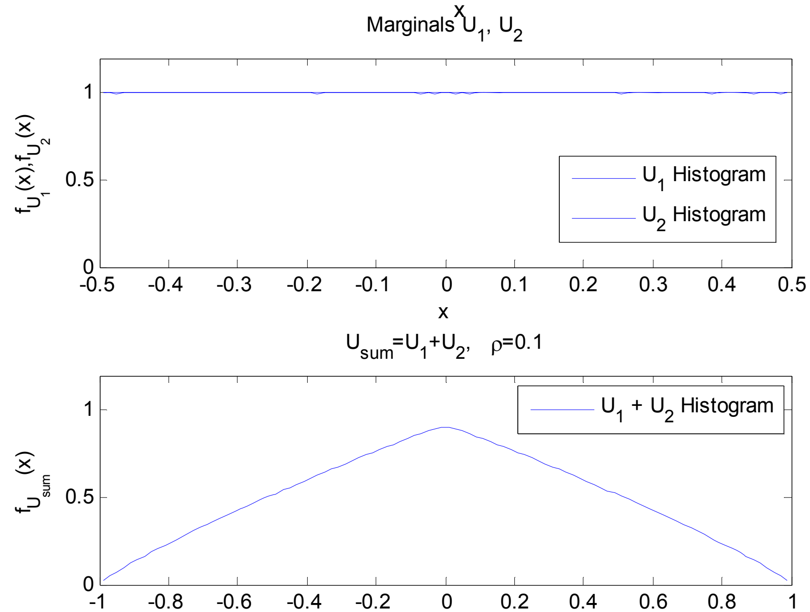
$(N = 2;\ \rho = 0)$





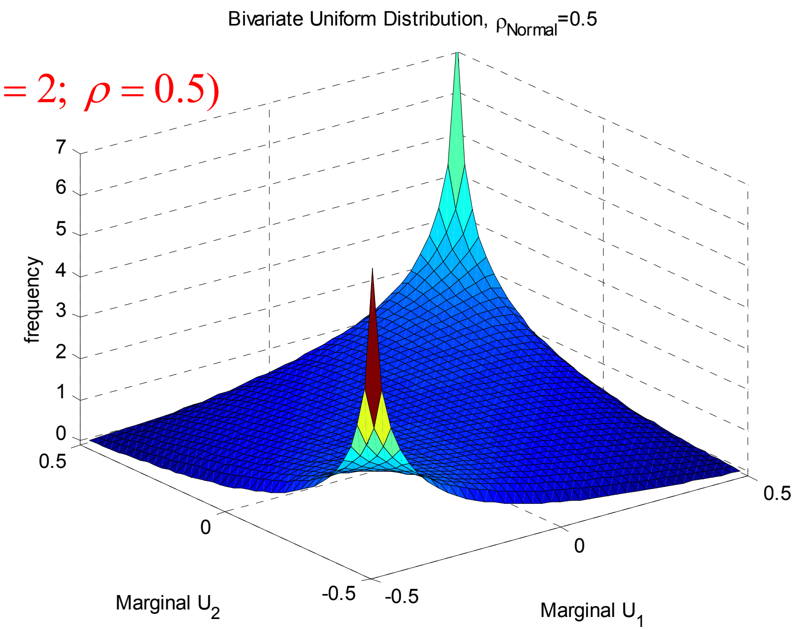
$(N = 2;\ \rho = 0.1)$

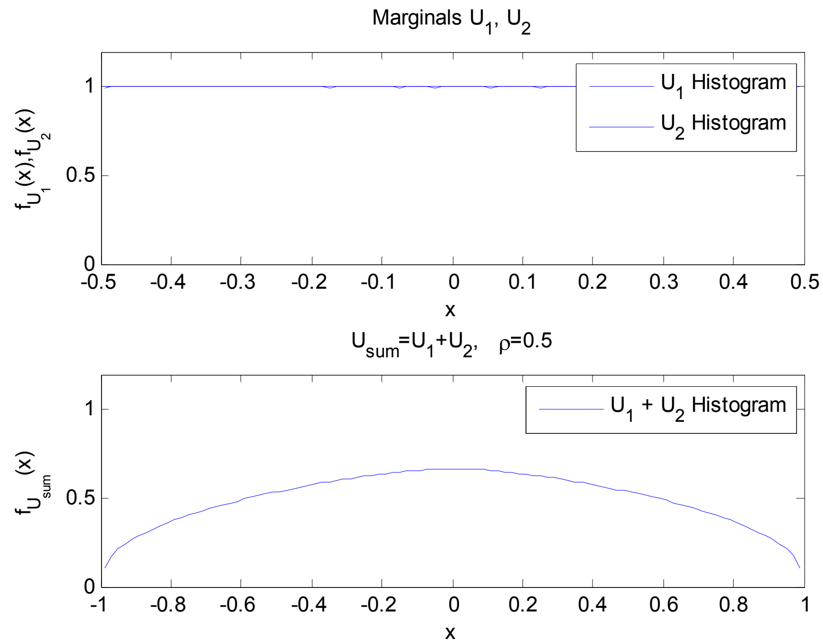




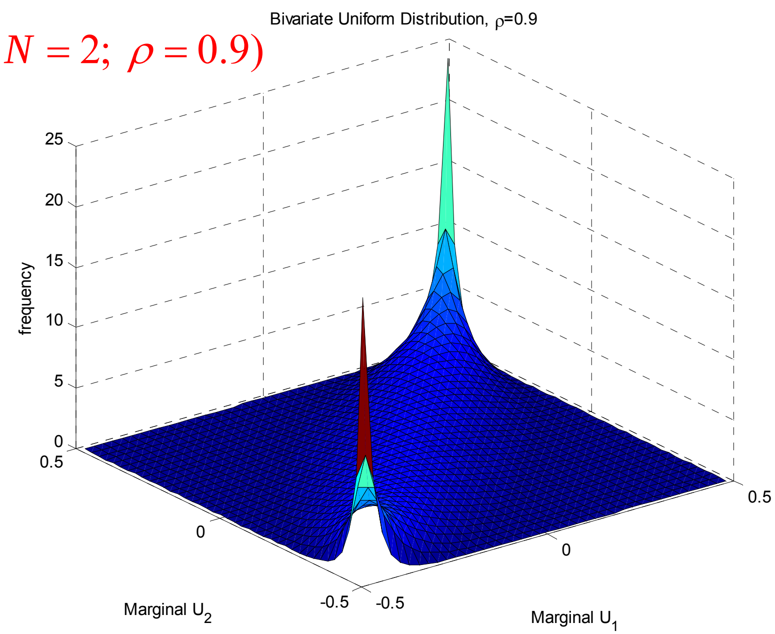
W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

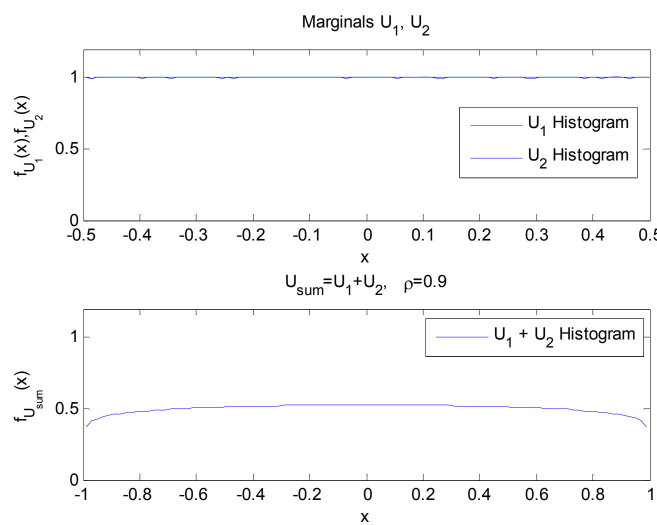
$(N = 2;\ \rho = 0.5)$



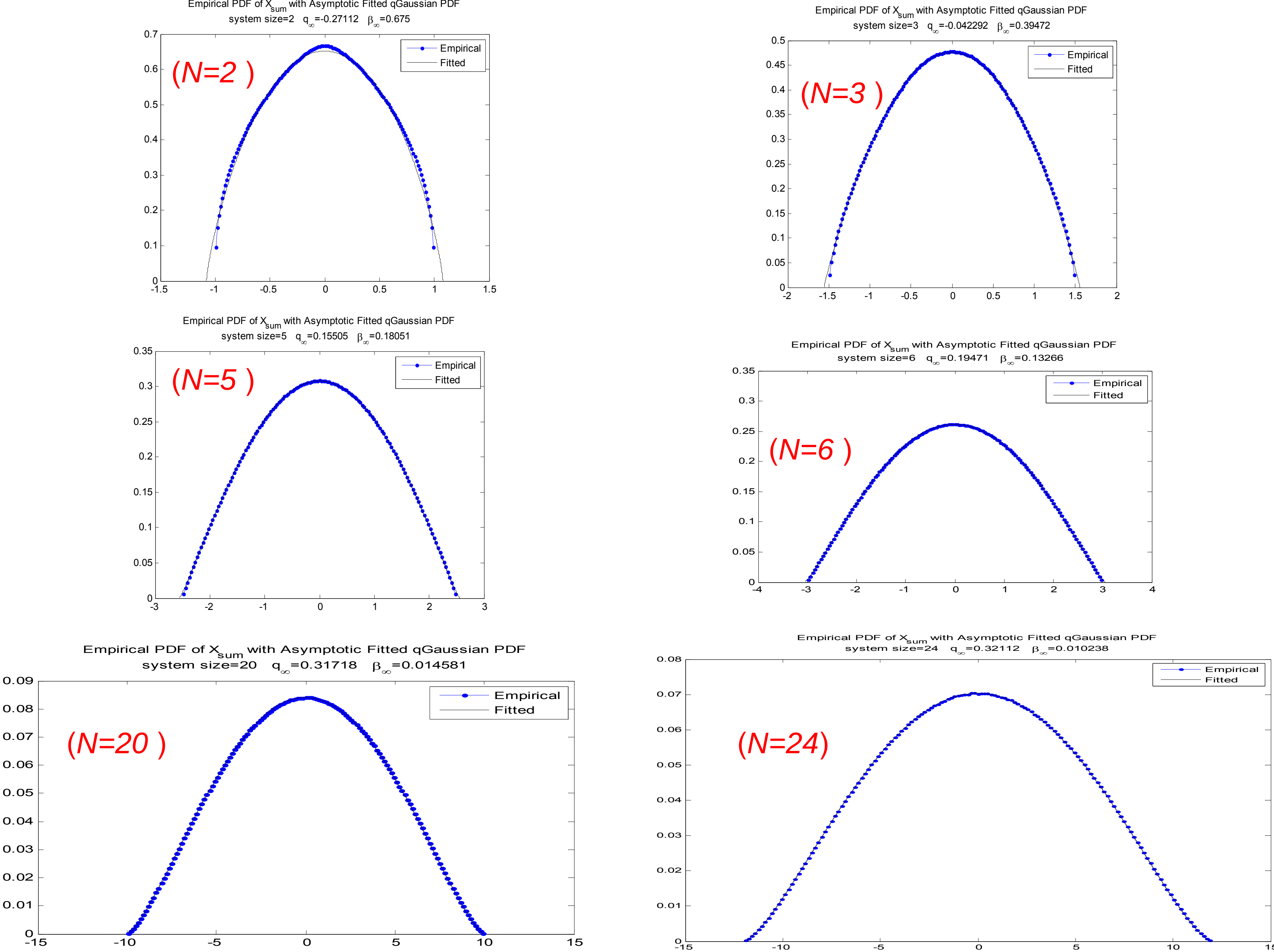


$(N = 2;\ \rho = 0.9)$



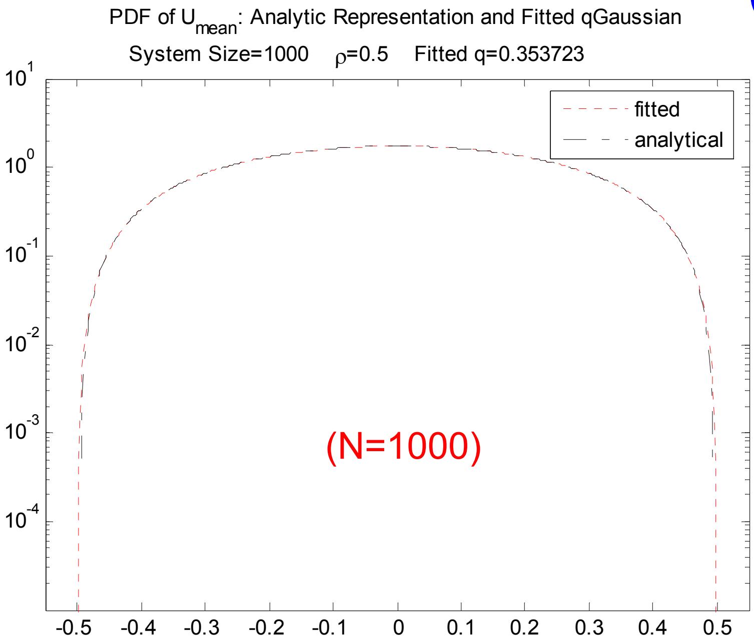


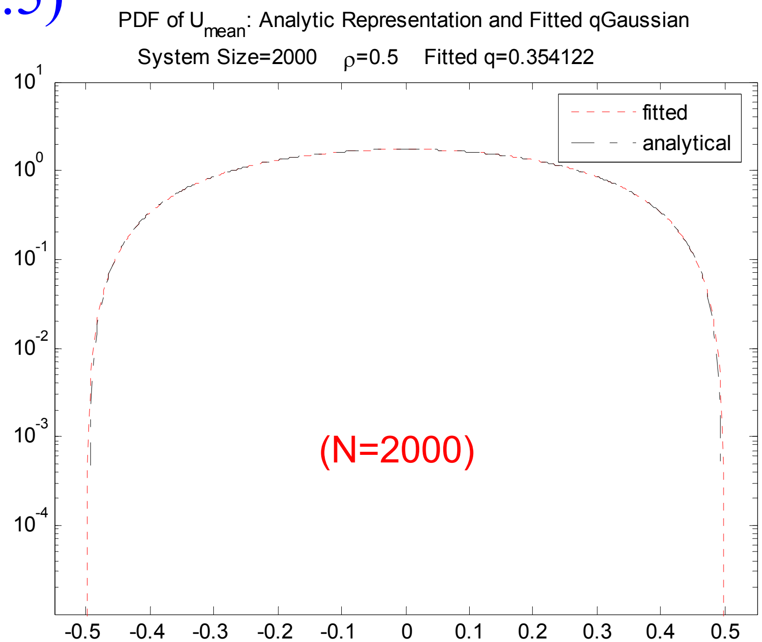
W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

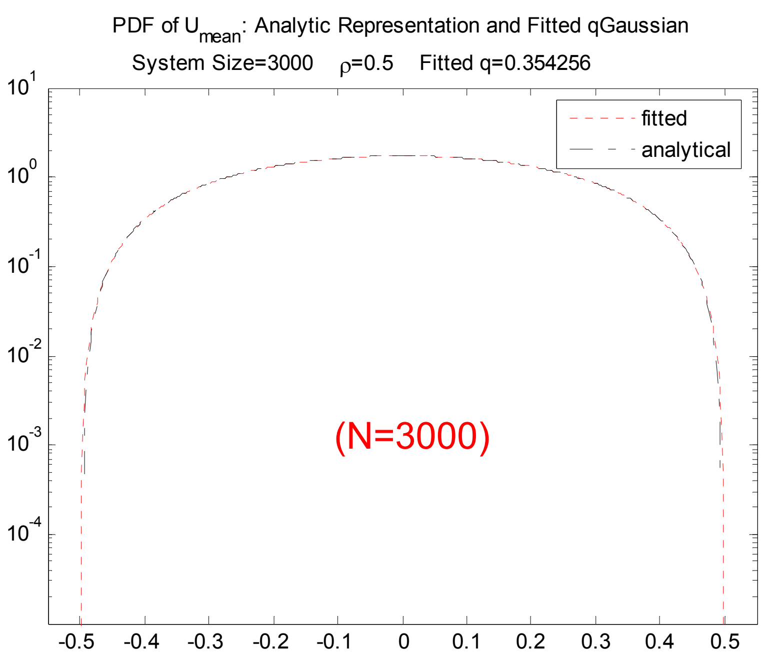


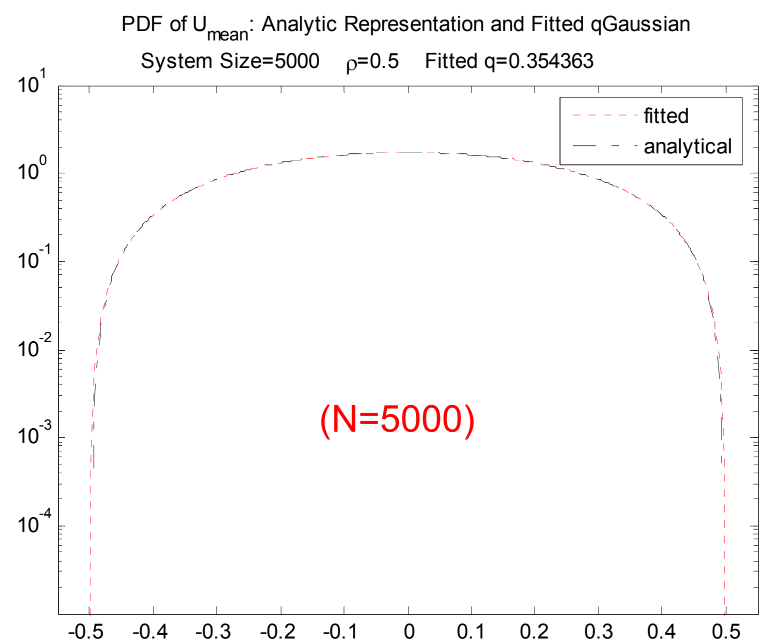
W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

$(\rho = 0.5)$

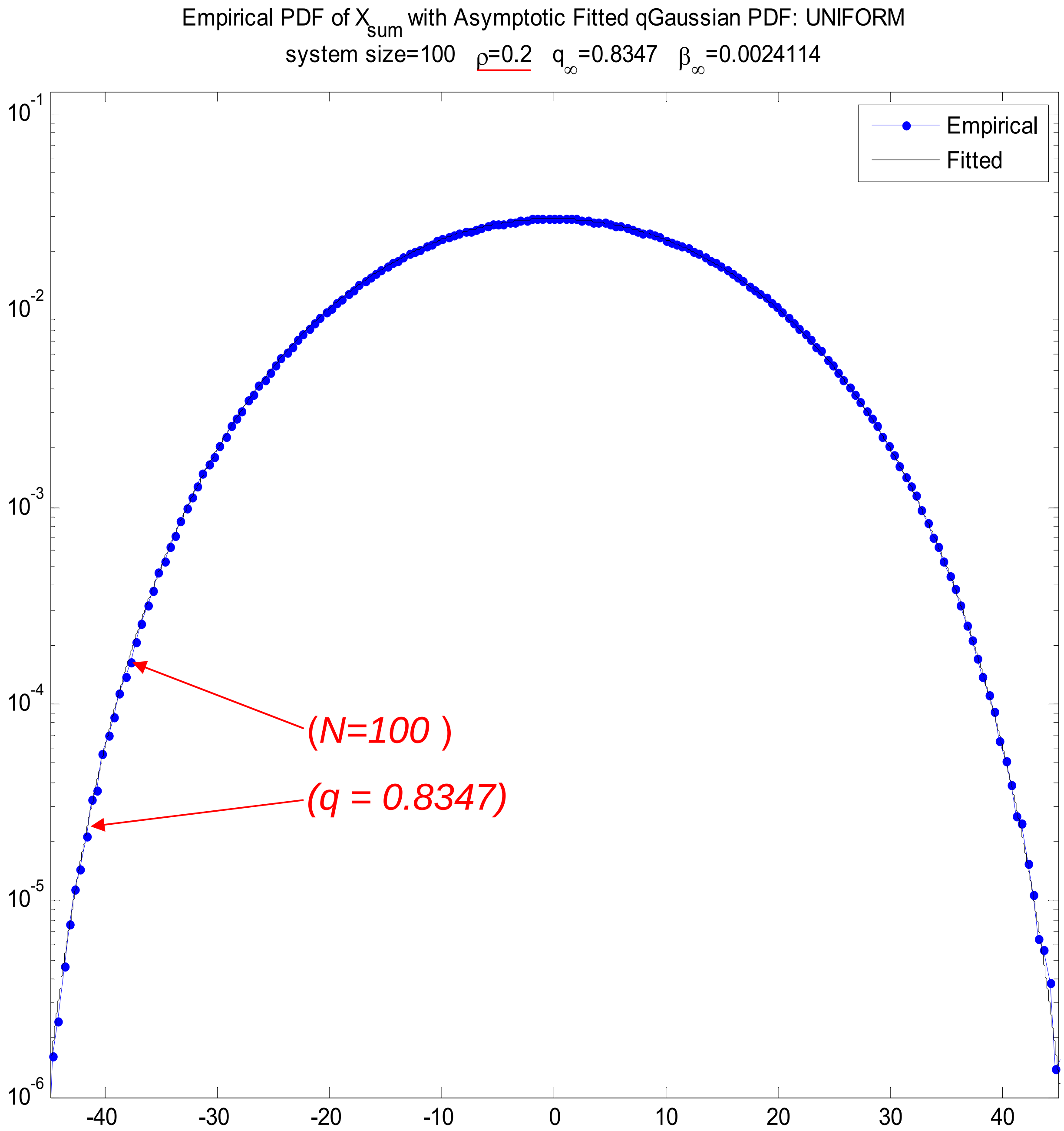








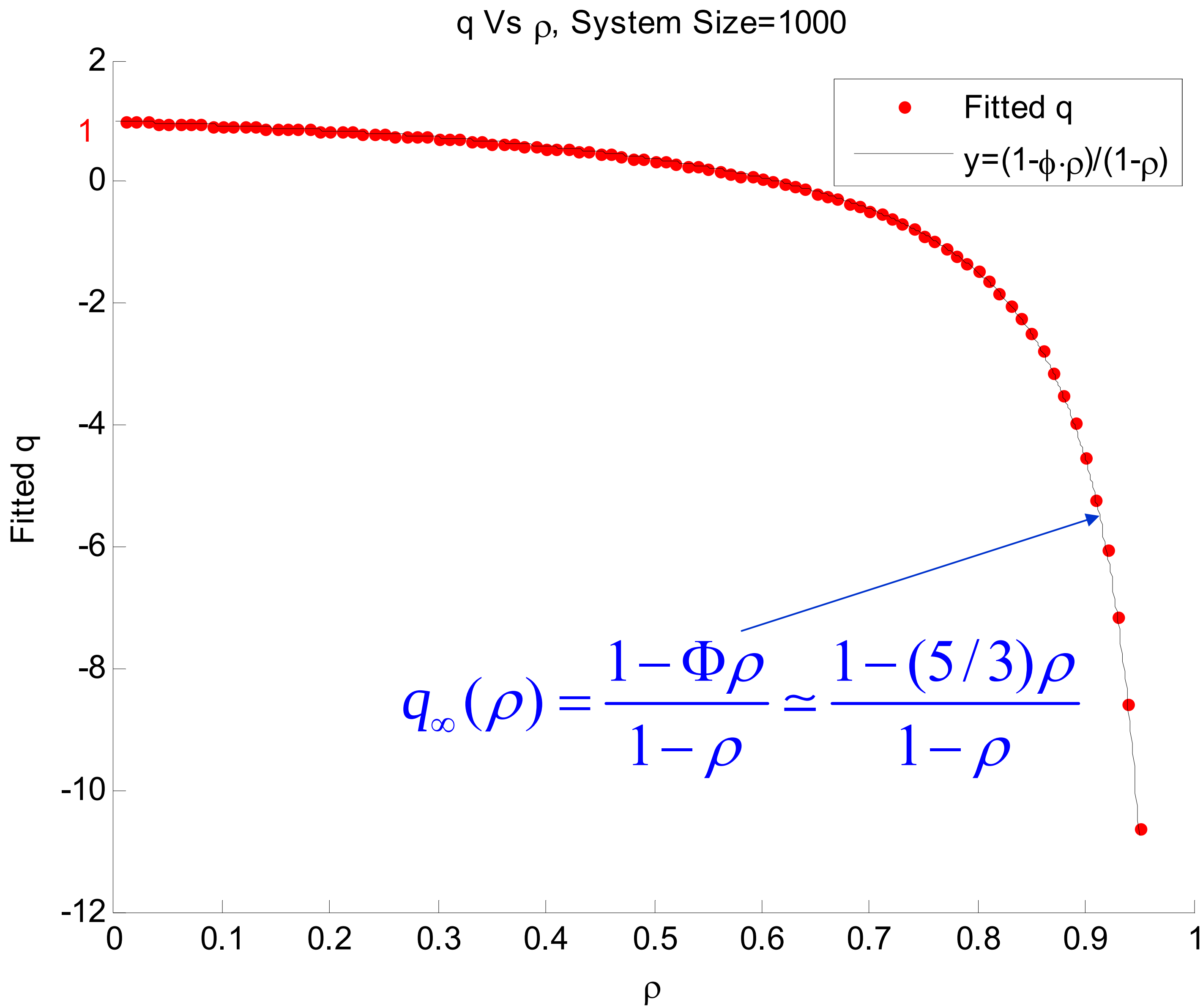
W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)



$$q(\rho, N) = q_{\infty}(\rho) - \frac{A(\rho)}{N^{\delta(\rho)}}$$

$$e.g., \begin{cases} q_{\infty}(0.5) \simeq 0.3545 \\ A(0.5) \simeq 0.5338 \\ \delta(0.5) \simeq 1.9535 \end{cases}$$

W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)



Fitted q Value as a function of correlation coefficient, ρ . The ansatz appearing in the legend is the golden ratio, $\varphi = \left(1+\sqrt{5}\right)/2$.

W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

INFLUENCE OF THE RANGE OF CORRELATIONS DECAYING FAR FROM THE DIAGONAL OF THE COVARIANCE MATRIX:

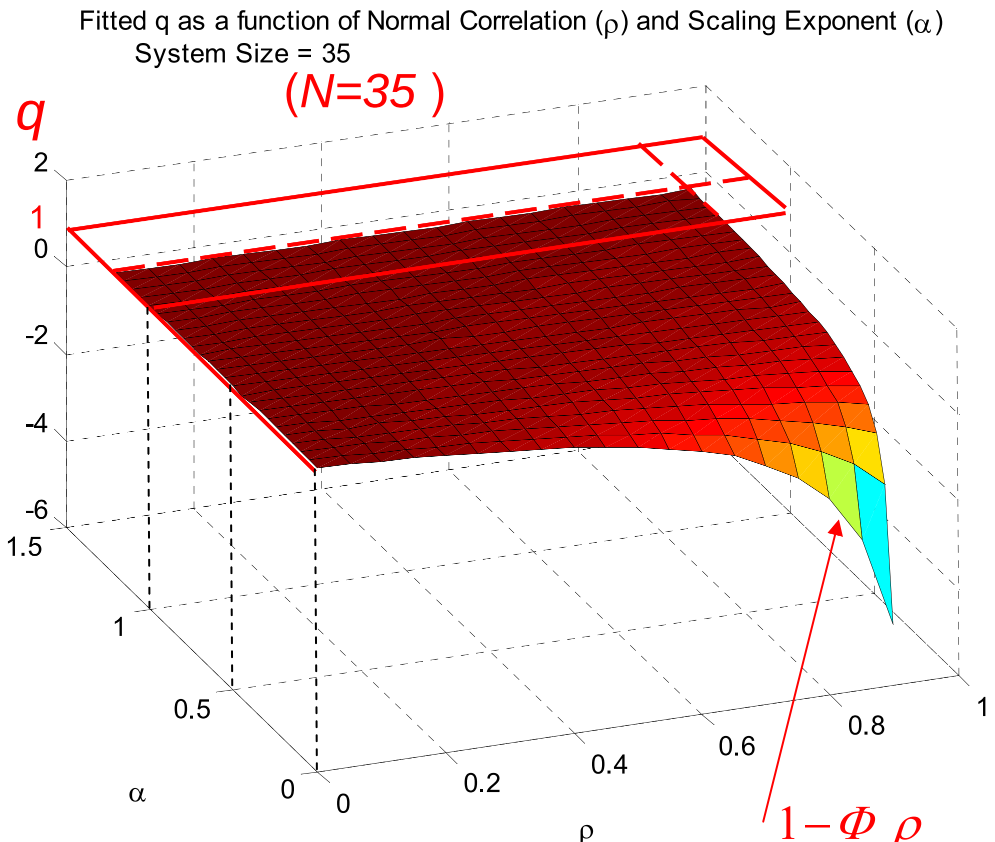
Multivariate Gaussian $N \times N$ covariance matrix given by

$$\begin{pmatrix} 1 & \rho(2) & \rho(3) & \ldots & \rho(N-1) & \rho(N) \\ \rho(2) & 1 & \rho(2) & \ldots & \rho(N-2) & \rho(N-1) \\ \rho(3) & \rho(2) & 1 & \ldots & \rho(N-3) & \rho(N-2) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \rho(N-1) & \rho(N-2) & \ldots & \rho(2) & 1 & \rho(2) \\ \rho(N) & \rho(N-1) & \ldots & \rho(3) & \rho(2) & 1 \end{pmatrix}$$

with

$$\rho(r) = \frac{\rho}{r^{\alpha}}$$

$$(-1 \le \rho \le 1;\ \alpha \ge 0;\ r = 2,\ 3, \ldots,\ N)$$



$$q \simeq \frac{1 - \Phi\ \rho}{1 - \rho} \simeq \frac{1 - (5/3)\rho}{1 - \rho}$$

W. Thistleton, J.A. Marsh, K. Nelson and C. T. (2006)

LOGICAL CONSEQUENCES OF THE q-CENTRAL LIMIT THEOREM, OR JUST A COINCIDENCE? – AN OPEN QUESTION

1- The dynamical attractor at the edge of chaos $[a_c(z)]$ of the one-dimensional dissipative map $x_{t+1} = 1 - a_c(z)\,|\,x_t\,|^z$ is, $\forall z$, well fitted by a q-Gaussian with $q \simeq 1.75$

[U. Tirnakli, C. Beck and C. T., Phys Rev E / RC (in press), cond-mat/0701622]

2- The distribution associated with the fluctuating magnetic field within the solar wind is, as observed in the data from the spacecraft Voyager 1, well fitted by a q-Gaussian with $q = 1.75 \pm 0.06$

[L.F. Burlaga and A. F.-Vinas, Physica A 356, 375 (2005)]

3- The histogram of Brazilian stock market index changes is, for a considerable range of time delays, well fitted by a q-Gaussian with $q \simeq 1.75$

[A.A.G. Cortines and R. Riera, Physica A 377, 181 (2007)]

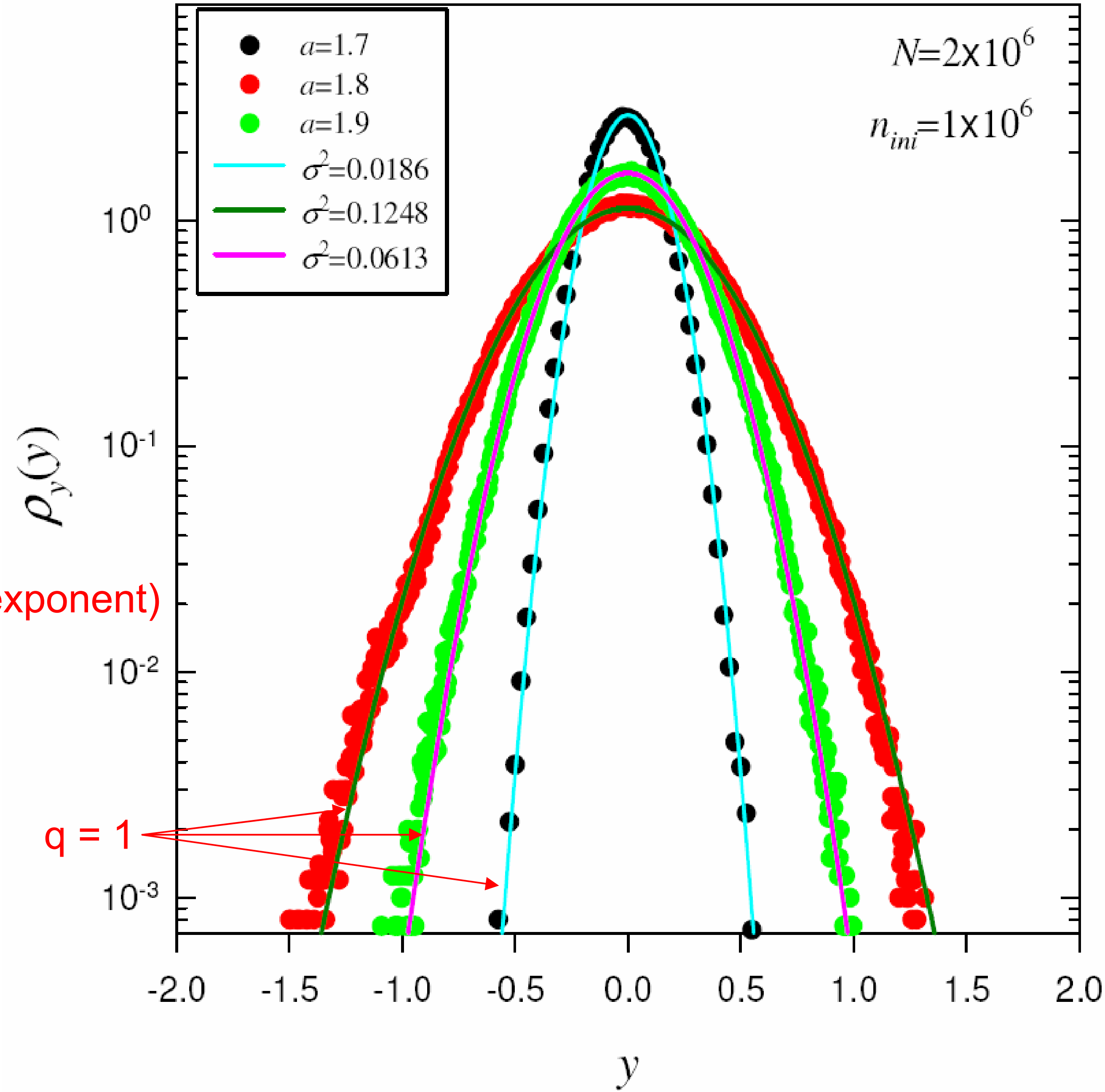
4- The probability distribution of energy differences of subsequent earthquakes in the World Catalog and in Northern California is well fitted by a q-Gaussian with $q \simeq 1.75 \pm 0.15$

[F. Caruso, A. Pluchino, V. Latora, S. Vinciguerra and A. Rapisarda, cond-mat/0606118]

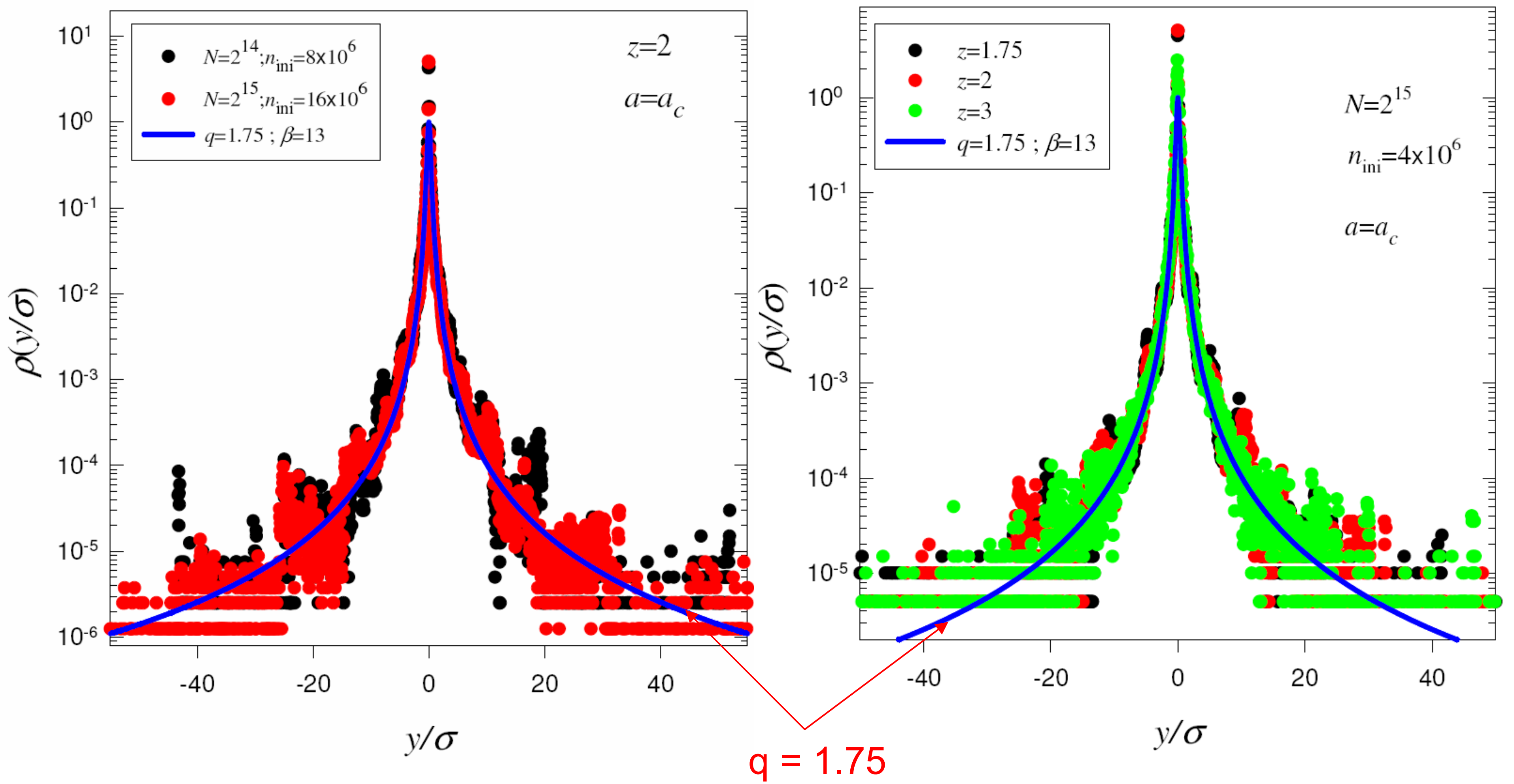
1- The dynamical attractor of the sum at the edge of chaos

$$x_{t+1} = 1 - a\,|\,x_t\,|^{\,z}$$
$$(z = 2)$$

(a = 1.7, 1.8, 1.9,
hence **positive** Lyapunov exponent)



U. Tirnakli, C. Beck and C. T., Phys Rev E / RC (2007), cond-mat/0701622 (a = $a_c(z)$, hence **vanishing** Lyapunov exponent)



U. Tirnakli, C. Beck and C. T., Phys Rev E / RC (2007), cond-mat/0701622

2- The distribution associated with the fluctuating magnetic field

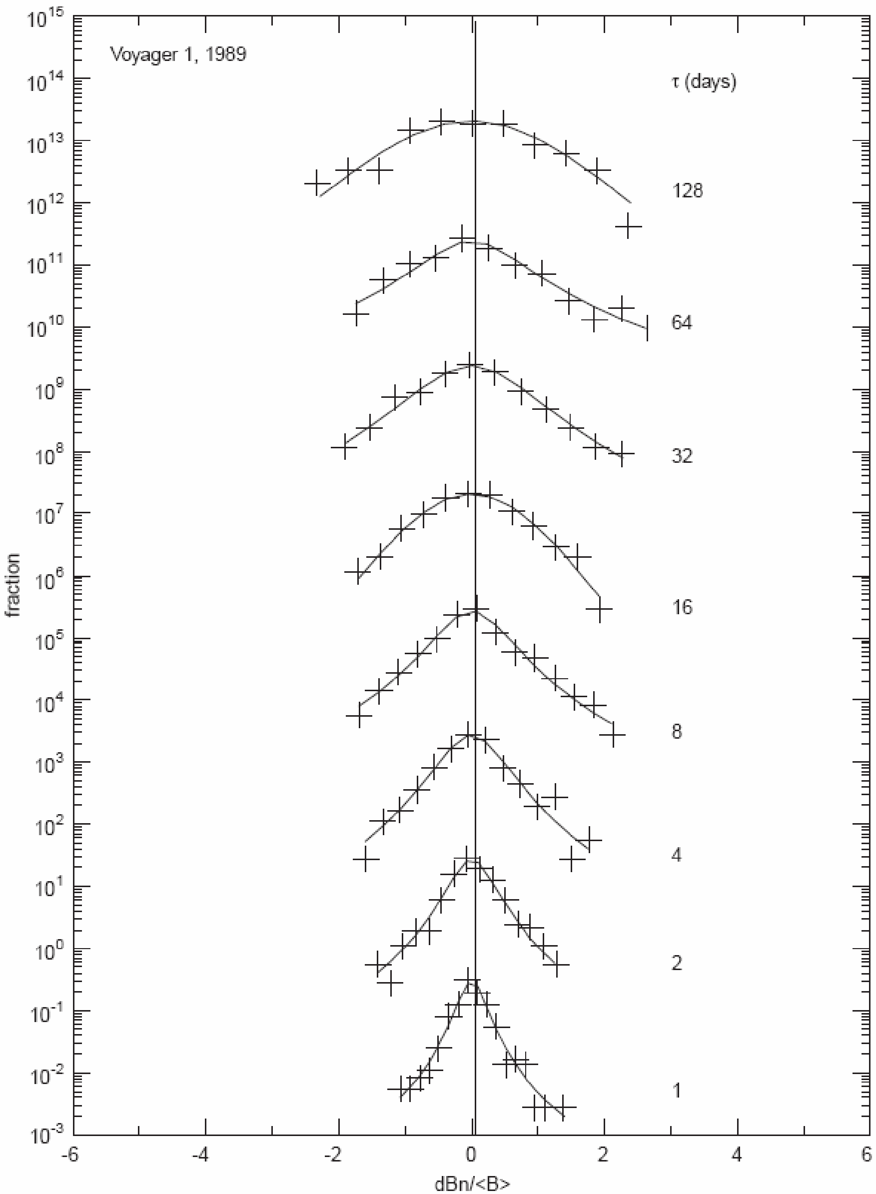


Fig. 2. The symbols are PDFs of relative changes in the magnetic field strength measured by V1 during 1989 on scales from 1 to 128 days. The curves are fits of the data to the *q*-exponential distribution function.

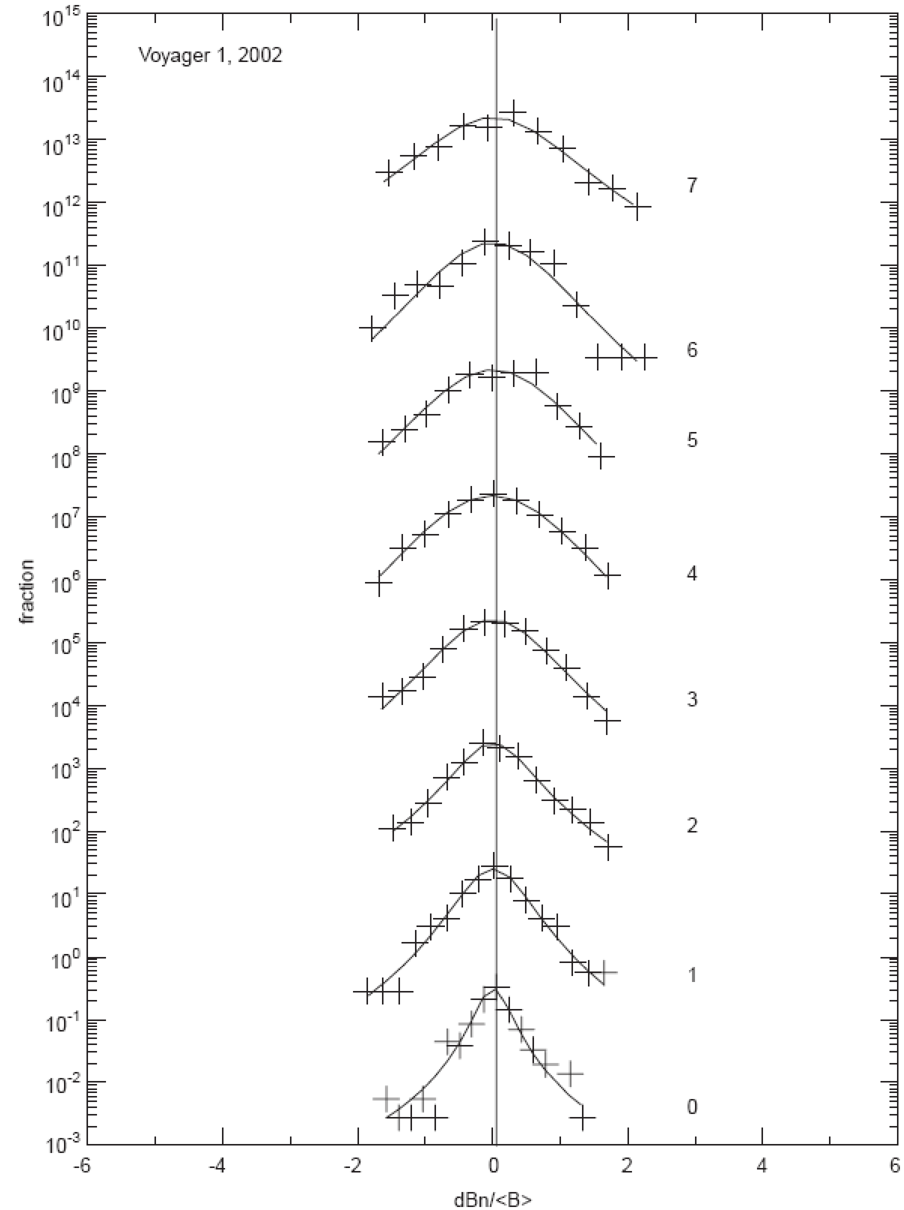
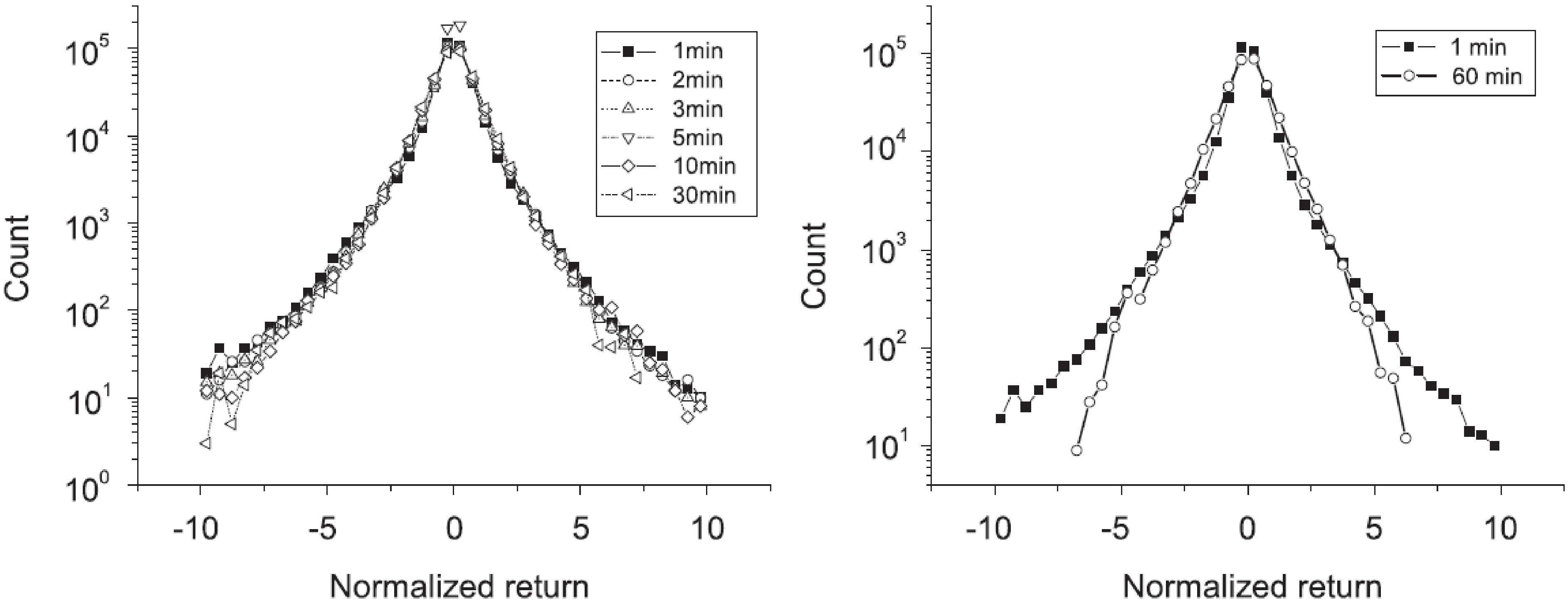


Fig. 3. The symbols are PDFs of relative changes in the magnetic field strength measured by V1 during 2002 on scales from 1 to 128 days. The curves are fits of the data to the *q*-exponential distribution function.

L.F. Burlaga and A.F. Vinas, Physica A **356**, 375 (2005)

3- The histogram of Brazilian stock market index changes

Bovespa index (November 2002 - June 2004)



A.A.G. Cortines and R. Riera, Physica A **377**, 181 (2007)

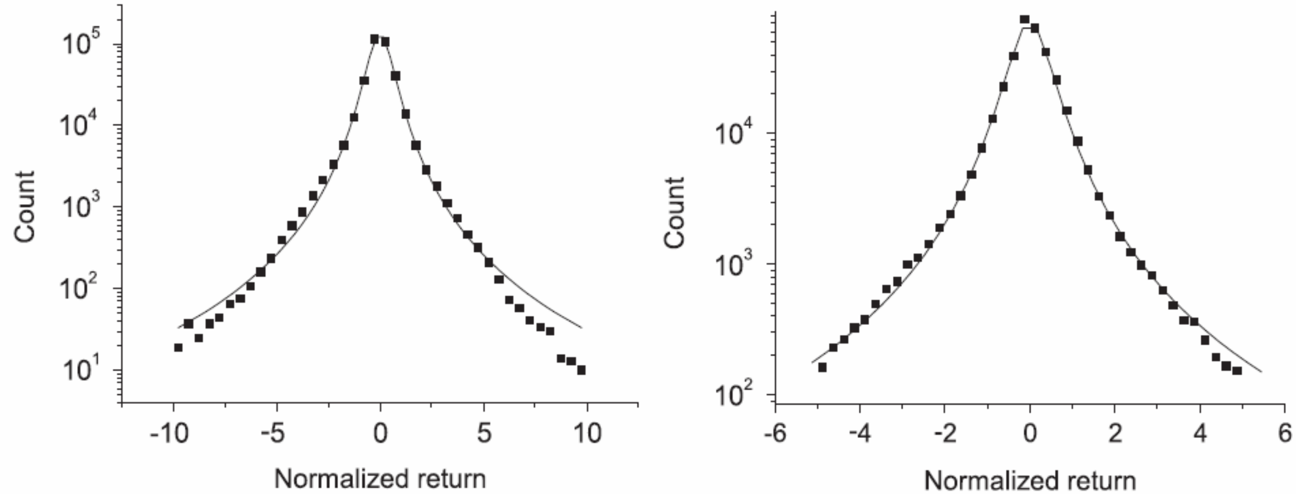


Fig. 5. Probability distribution function of normalized returns for time interval $\tau = 1$ min for BOVESPA index data (black squares) and best fit of the data with Eq. (3): (a) range $x_1^N \in [-10, +10]$, best fit $q = 1.64$ and $\beta = 3.36$ and (b) range $x_1^N \in [-5, +5]$, best fit $q^* = 1.75$ and $\beta = 4.47$.

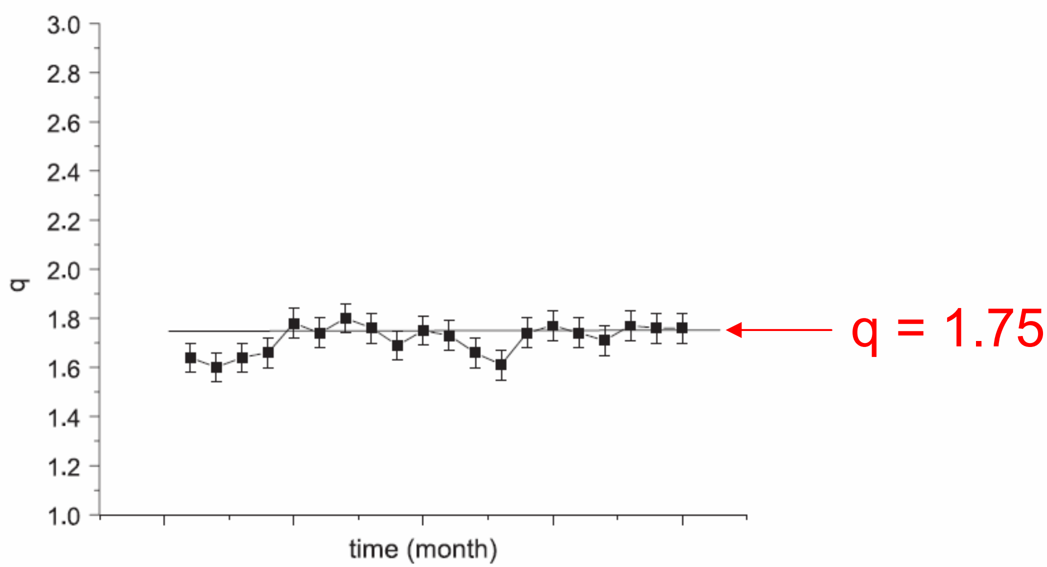
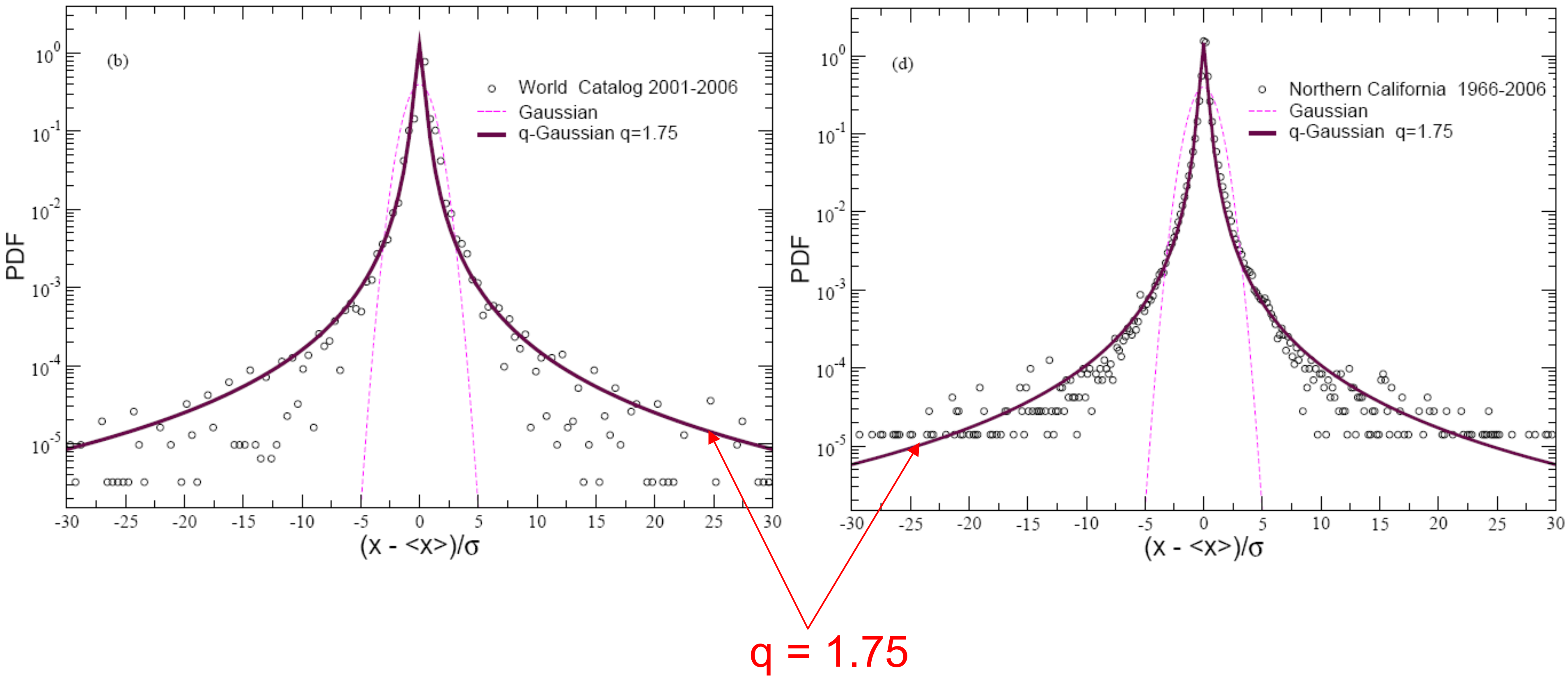


Fig. 6. Time series of optimal parameter q for the bulk of the $\tau = 1$ min returns collected in each month of the data set (see text). The horizontal line represents the optimal parameter $q^* = 1.75$ for the whole period of the analyzed data.

A.A.G. Cortines and R. Riera, Physica A **377**, 181 (2007)

4- The probability distribution of energy differences of subsequent earthquakes



F. Caruso, A. Pluchino, V.Latora, S. Vinciguerra and A. Rapisarda, cond-mat/0606118

THERMODYNAMICAL CONSEQUENCES OF LONG-RANGE INTERACTIONS IN MANY-BODY CLASSICAL SYSTEMS

CLASSICAL SYSTEM:

$$H = K + V = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i \neq j} V(r_{ij})$$

where $V(r)$ has no singularity at the origin,

or an integrable singularity, and, for $r \to \infty$,

$$V(r) \sim \frac{A}{r^{\alpha}} \quad (A > 0;\ \alpha \geq 0)$$

The characteristic potential energy would be typically given by

$$\frac{U_{pot}(N)}{N} \propto -A \int_1^{\infty} dr\ r^{d-1}\ r^{-\alpha} \begin{cases} < \infty & if \quad \alpha / d > 1 \\ \to \infty & if \quad 0 \leq \alpha / d \leq 1 \end{cases}$$

Consequently, a more appropriate estimation is given by

$$\frac{U_{pot}(N)}{N} \propto -A \int_1^{N^{1/d}} dr\ r^{d-1}\ r^{-\alpha} = -\frac{A}{d} N^*$$

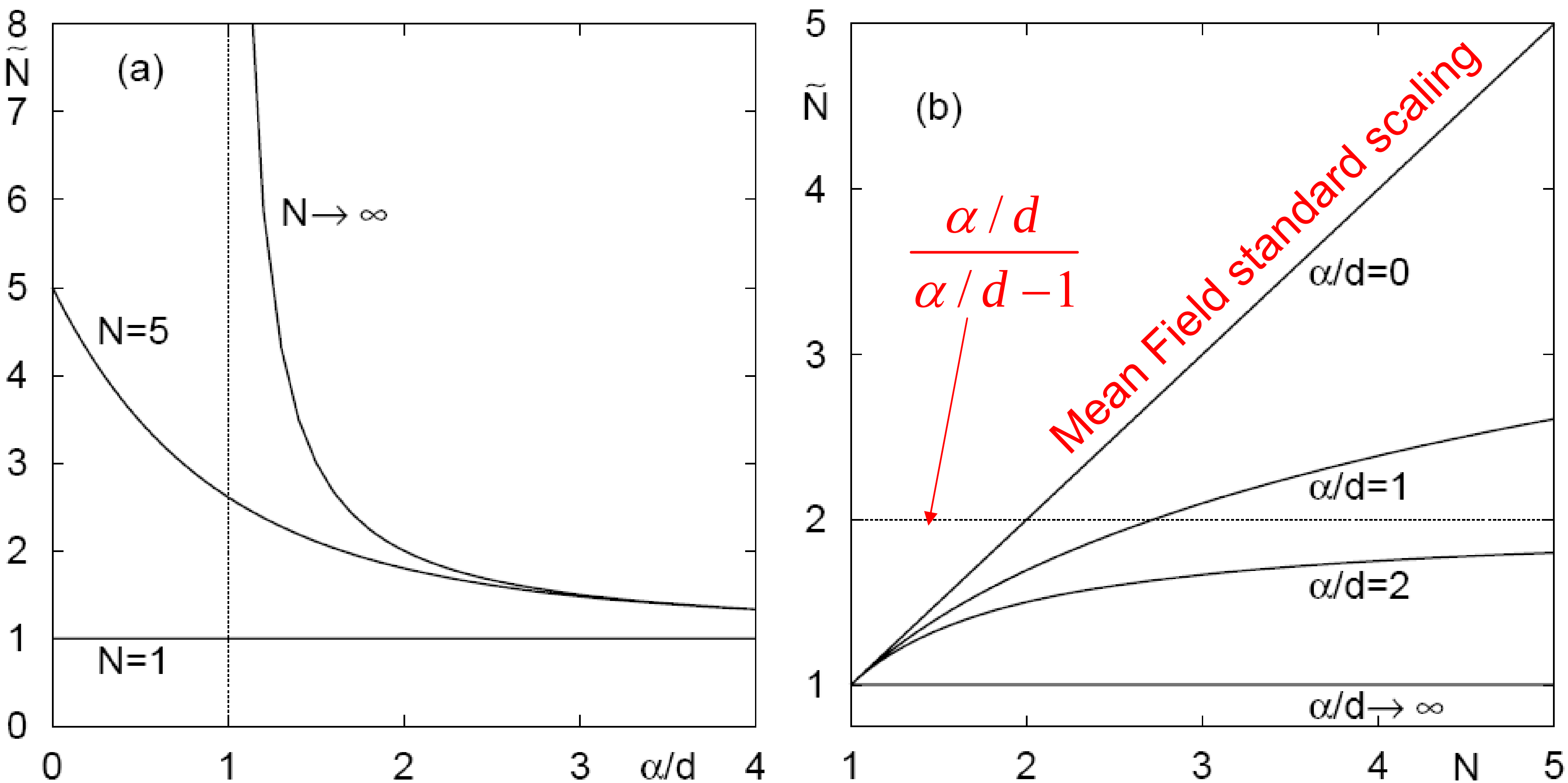
with

$$N^* \equiv \frac{N^{1-\alpha/d} - 1}{1 - \alpha / d} = \ln_{\alpha/d} N \ \sim \begin{cases} \dfrac{1}{\alpha / d - 1} & if \quad \alpha / d > 1 \\ \ln N & if \quad \alpha / d = 1 \\ \dfrac{N^{1-\alpha/d}}{1 - \alpha / d} & if \quad 0 \leq \alpha / d < 1 \end{cases}$$

short-range interaction ($\alpha / d > 1$)

long-range interaction ($\alpha / d = 1$ and $0 \leq \alpha / d < 1$)

$$\tilde{N} \equiv N^* + 1 = \frac{N^{1-\alpha/d} - (\alpha/d)}{1-\alpha/d}$$



$\alpha/d>1$ leads to the traditional *intensive - extensive* classification of thermodynamical variables

SHORT-RANGE INTERACTIONS (*i.e.,* $\alpha / d > 1$ *for clasical systems*):

$$G(N,T,p,H) = U(N,T,p,H) - T\ S(N,T,p,H) + p\ V(N,T,p,H) - H\ M(N,T,p,H)$$

hence

$$\lim_{N\to\infty} \frac{G(N,T,p,H)}{N} = \lim_{N\to\infty} \frac{U(N,T,p,H)}{N} - T \lim_{N\to\infty} \frac{S(N,T,p,H)}{N} + p \lim_{N\to\infty} \frac{V(N,T,p,H)}{N} - H \lim_{N\to\infty} \frac{M(N,T,p,H)}{N}$$

i.e.,

$$g(T,p,H) = u(T,p,H) - T\ s(T,p,H) + p\ v(T,p,H) - H\ m(T,p,H)$$

LONG-RANGE INTERACTIONS (*i.e.,* $0 \le \alpha / d \le 1$ *for clasical systems*):

$$G(N,T,p,H) = U(N,T,p,H) - T\ S(N,T,p,H) + p\ V(N,T,p,H) - H\ M(N,T,p,H)$$

hence

$$\lim_{N\to\infty} \frac{G(N,T,p,H)}{NN^*} = \lim_{N\to\infty} \frac{U(N,T,p,H)}{NN^*} - \lim_{N\to\infty} \frac{T}{N^*} \frac{S(N,T,p,H)}{N} + \lim_{N\to\infty} \frac{p}{N^*} \frac{V(N,T,p,H)}{N} - \lim_{N\to\infty} \frac{H}{N^*} \frac{M(N,T,p,H)}{N}$$

i.e.,

$$g(T^*,p^*,H^*) = u(T^*,p^*,H^*) - T^*\ s(T^*,p^*,H^*) + p^*\ v(T^*,p^*,H^*) - H^*\ m(T^*,p^*,H^*)$$

SHORT-RANGE INTERACTIONS ($\alpha / d > 1$ *for clasical systems*):

Intensive variables : $T,\ p,\ H$ (they do not scale with size N)

Extensive variables: $N,\ G,\ U,\ S,\ V,\ M$ (they scale with N)

i.e., the equations of states are expressed in the variables

$(N\ g,\ N\ u,\ N\ s,\ N\ v,\ N\ m)$ versus $(T,\ p,\ H)$

BOTH SHORT- AND LONG-RANGE INTERACTIONS

($\alpha / d \geq 0$ *for clasical systems*):

Pseudo-intensive variables : $T,\ p,\ H$ (they scale with N^*)

Pseudo-extensive variables: $G,\ U$ (they scale with NN^*)

Extensive variables: $N,\ S,\ V,\ M$ (they scale with N)

i.e., the equations of states are expressed in the variables

$(NN^* g,\ NN^* u,\ N\ s,\ N\ v,\ N\ m)$ versus $\left(\frac{T}{N^*},\ \frac{p}{N^*},\ \frac{H}{N^*}\right)$

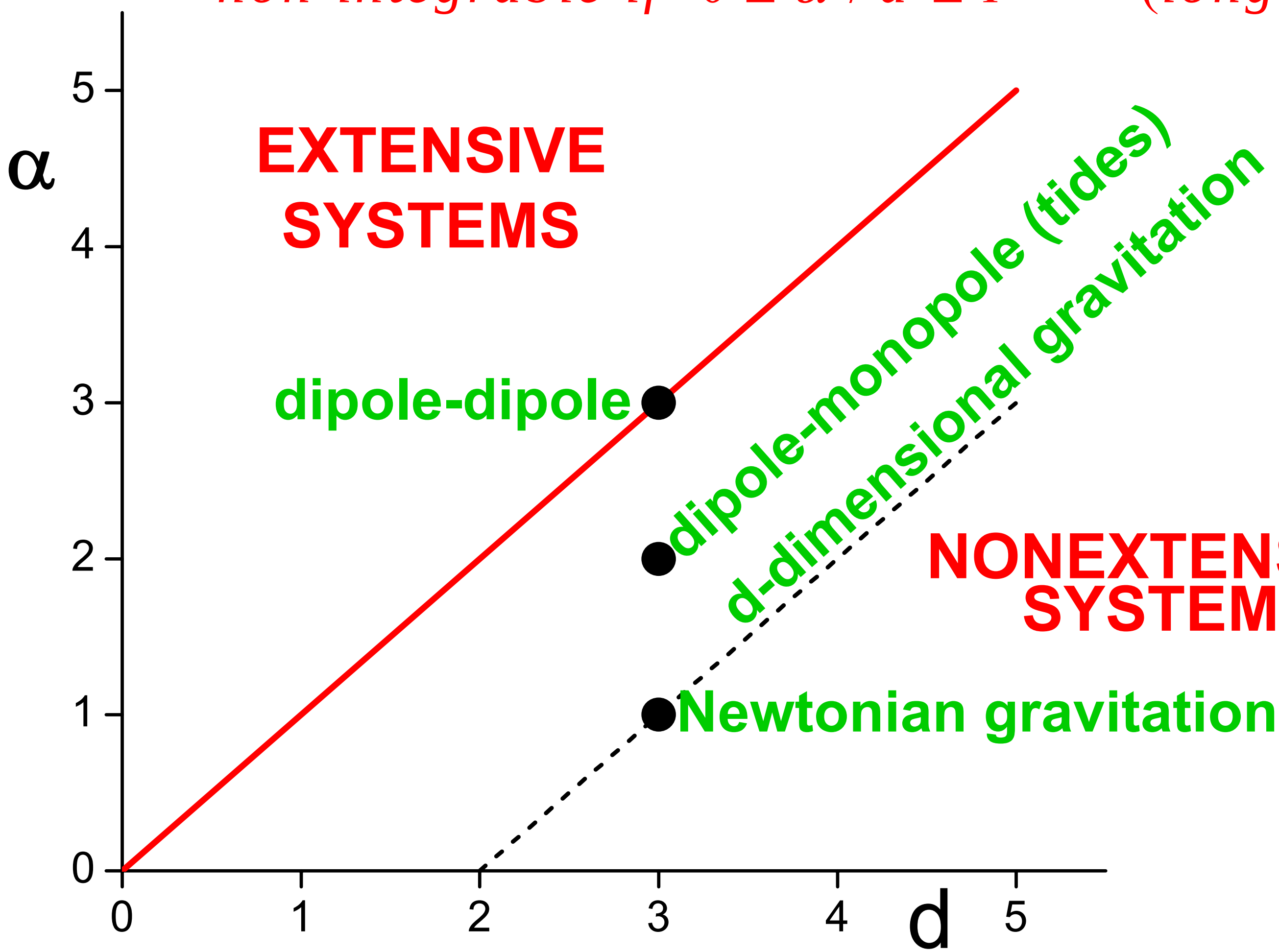
- THE THERMODYNAMICAL ENTROPY IS EXTENSIVE IN ALL CASES!
- THE VARIABLES DEFINING "THERMAL EQUILIBRIUM" $(T_A = T_B,\ p_A = p_B,\ H_A = H_B)$ DO NOT NECESSARILY COINCIDE WITH THOSE DEFINING FINITE EQUATIONS OF STATES!

$$V(\vec{r}) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty)$$

$$(A > 0, \quad \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*



ILLUSTRATIONS OF THE FINITENESS OF THE EQUATIONS OF STATES

Ferrofluid-like model:

P.Jund, S.G. Kim and C. T., Phys Rev B **52**, 50 (1995)

Lennard-Jones-like fluids:

R. Grigera, Phys Lett A **217**, 47 (1996)

Magnetic systems:

L.C. Sampaio, M.P. de Albuquerque and F.S. de Menezes, Phys Rev B **55**, 5611 (1997)

C. Anteneodo and C. T., Phys Rev Lett **80**, 5313 (1998)

R.F.S. Andrade and S.T.R. Pinho, Phys Rev E **71**, 026126 (2005)

Percolation:

H.H.A. Rego, L.S. Lucena, L.R. da Silva and C. T., Physica A **266**, 42 (1999)

U.L. Fulco, L.R. da Silva, F.D. Nobre, H.H.A. Rego and L.S. Lucena, Phys. Lett. A **312**, 331 (2003)

EXTENSIVITY OF THE NONADDITIVE ENTROPY S_q

and

N>>1 ATTRACTORS IN THE SENSE OF THE CENTRAL LIMIT THEOREM

N DISTINGUISHABLE BINARY RANDOM VARIABLES

Attractor / Entropy	BG $q_{ent} = 1 \ (S_{BG}(N) \propto N)$	NONEXTENSIVE $q_{ent} < 1 \ (S_{q_{ent}}(N) \propto N)$	
Gaussian attractor ($i.e., \ q_{att} = 1$)	independent variables	?	CLT
q_{att} − *Gaussian attractor* ($i.e., \ q_{att} \neq 1$)	Leibnitz triangle ($q_{att} \to \infty$) MTG model $\left(q_{att} = 2 - \frac{1}{q_{corr}} \right)$	?	UTS UT
other attractors ($e.g., \ (q, \alpha) - stable$ *distributions*)	TGS1 model (stretched exponential) MFMT2 ($\alpha > 0$) model	TGS2, TGS3 MFMT1 ($\alpha \neq 0$) MFMT2 ($\alpha < 0$) MFMT3	LG UTGS

TGS → C. T., M. Gell-Mann and Y. Sato (2005)

MTG → L.G. Moyano, C. T. and M. Gell-Mann (2006)

MFMT → J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. (2006)

CLT → Central Limit Theorem

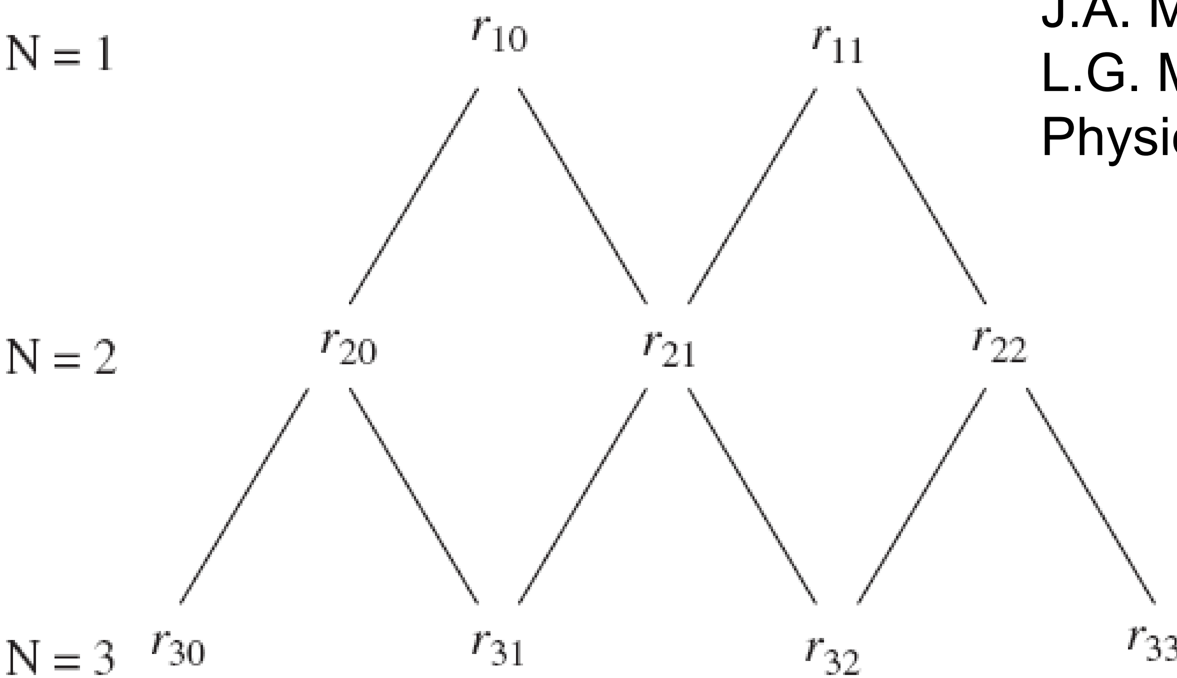
LG → Levy-GnedenkoTheorem

UTS → S. Umarov, C. T. and S. Steinberg (2006)

UT → S. Umarov and C. T. (2007)

UTGS → S. Umarov, C. T., M. Gell-Mann and S. Steinberg (2006)

J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A **372**, 183 (2006)



$$r_{\mathrm{N,n}} + r_{\mathrm{N,n+1}} = r_{\mathrm{N-1,n}}$$ (Leibnitz rule)

$$\sum_{\mathrm{n=0}}^{N} \frac{N!}{(N-n)!n!} r_{\mathrm{N,n}} = 1$$

$$S_{\mathrm{q}}(N) \equiv \frac{1}{q-1}\left(1 - \sum_{\mathrm{i=1}}^{2^N} p_{\mathrm{i}}^q\right) = \frac{1}{q-1}\left(1 - \sum_{\mathrm{n=0}}^{N} \frac{N!}{(N-n)!n!} (r_{\mathrm{N,n}})^q\right)$$

INDEPENDENT VARIABLES

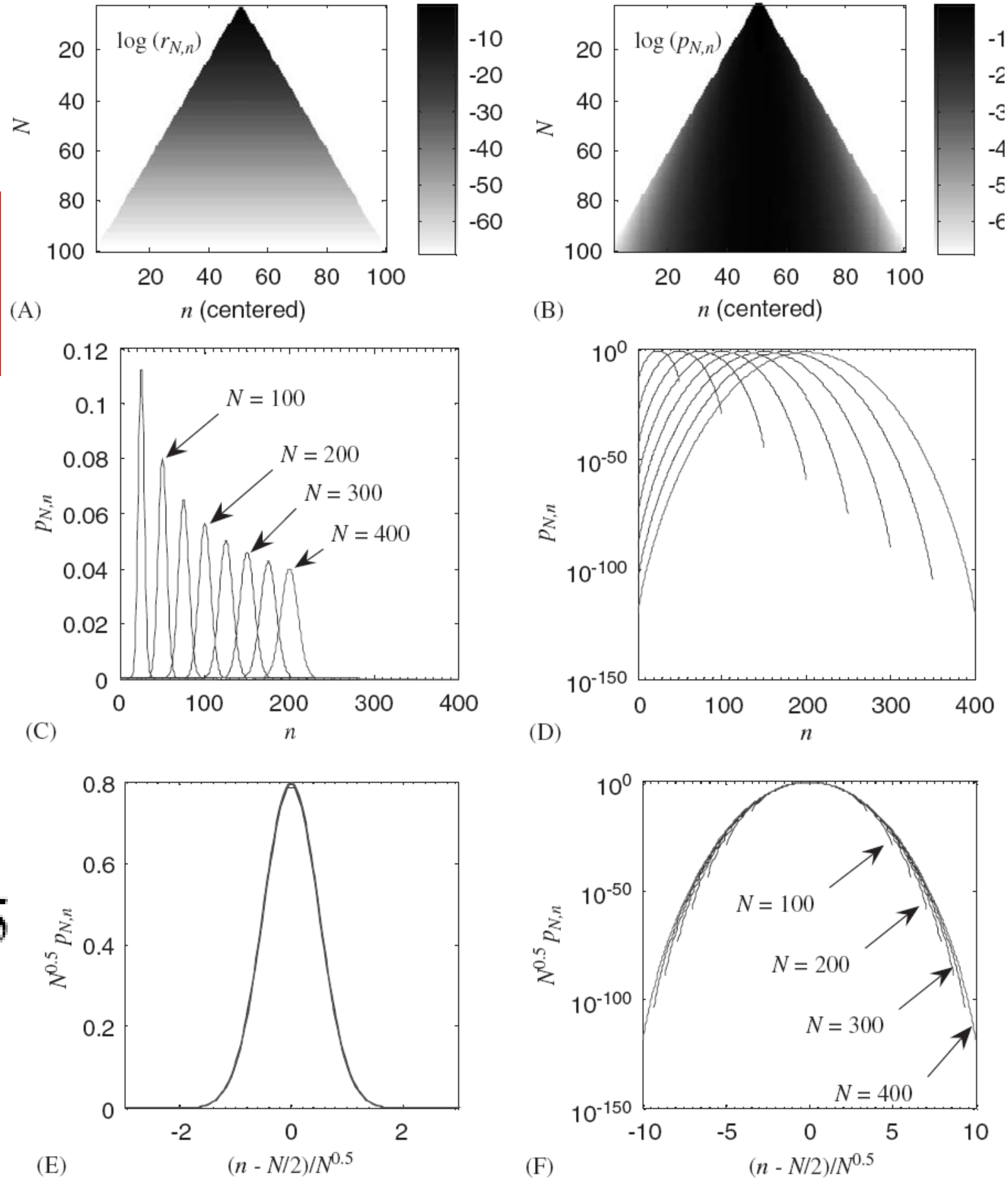
$$p_{N,n} \equiv r_{N,n} \frac{N!}{(N-n)!n!}$$

$$p_{N,n} \rightarrow p_{N,n} N^{\gamma}$$

$$n \rightarrow \frac{(n - n_{center})}{N^{\gamma}}$$

$$r_{N,0} = r_{1,0}^{N} \equiv p^{N}$$

$$p = 0.5 \quad \gamma = 0.5$$



J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A **372**, 183 (2006)

MTG MODEL

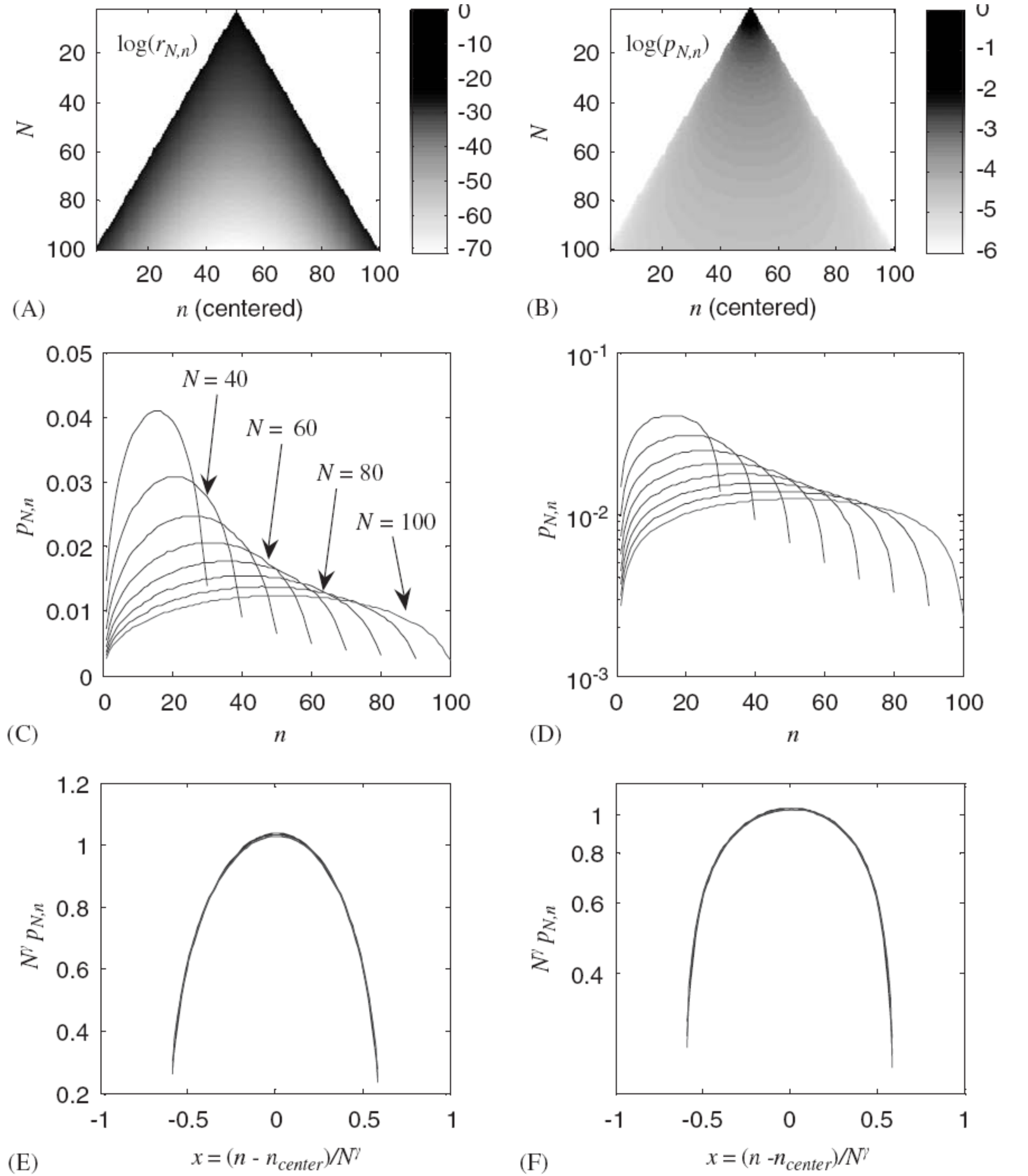
$$r_{\mathrm{N},0} = \left[\left(\frac{1}{p}\right)\otimes_{q_{\mathrm{corr}}}\left(\frac{1}{p}\right)\otimes_{q_{\mathrm{corr}}}\cdots\otimes_{q_{\mathrm{corr}}}\left(\frac{1}{p}\right)\right]^{-1}$$
$$= \left[Np^{q_{\mathrm{corr}}-1} - (N-1)\right]^{\frac{1}{q_{\mathrm{corr}}-1}}$$

$$p = 0.5$$

$$q_{\mathrm{corr}} = 0.30$$

$$\gamma = 0.96$$

$$q_{att} = 2 - \frac{1}{q_{corr}}$$



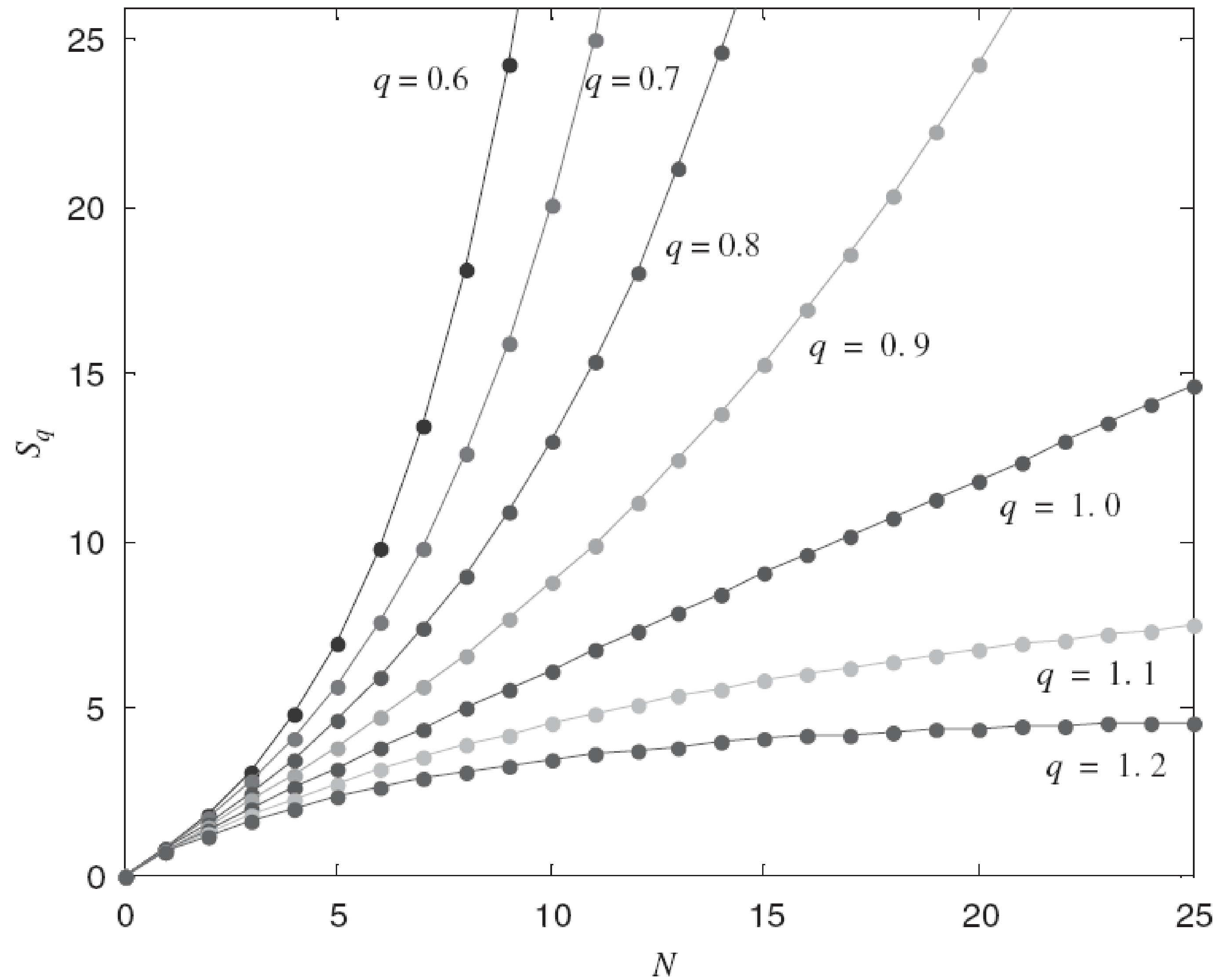
J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

MTG MODEL

$$p = 0.5$$

$$q_{corr} = 0.25$$

$$q_{ent} = 1$$



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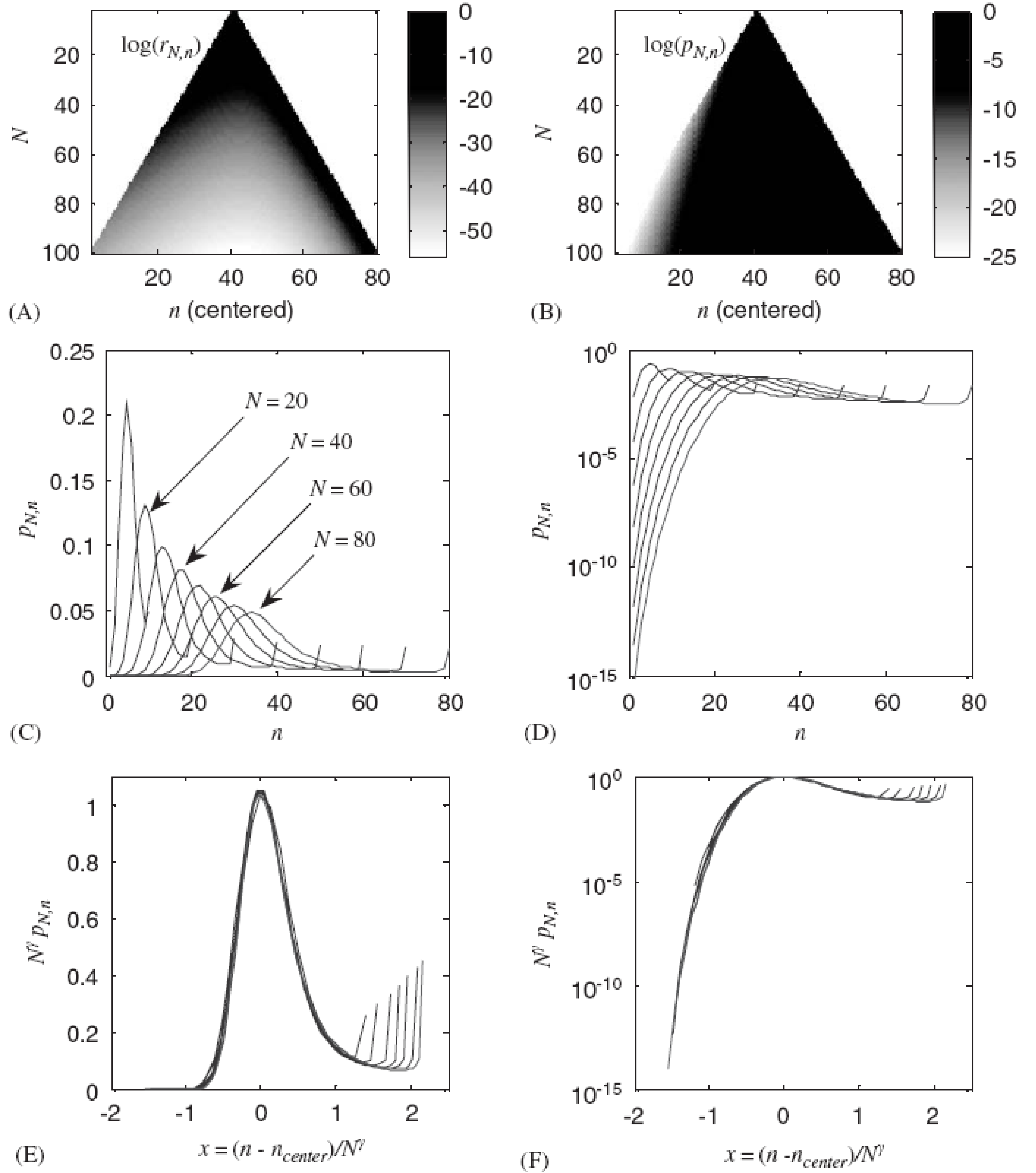
TGS1 MODEL (STRETCHED EXPONENTIAL)

$$r_{N,0} = r_{1,0}^{N^\alpha} = p^{N^\alpha}$$

$$p = 0.5$$

$$\alpha = 0.9$$



$$\gamma = 0.70$$

$$q_{ent} = 1$$

J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

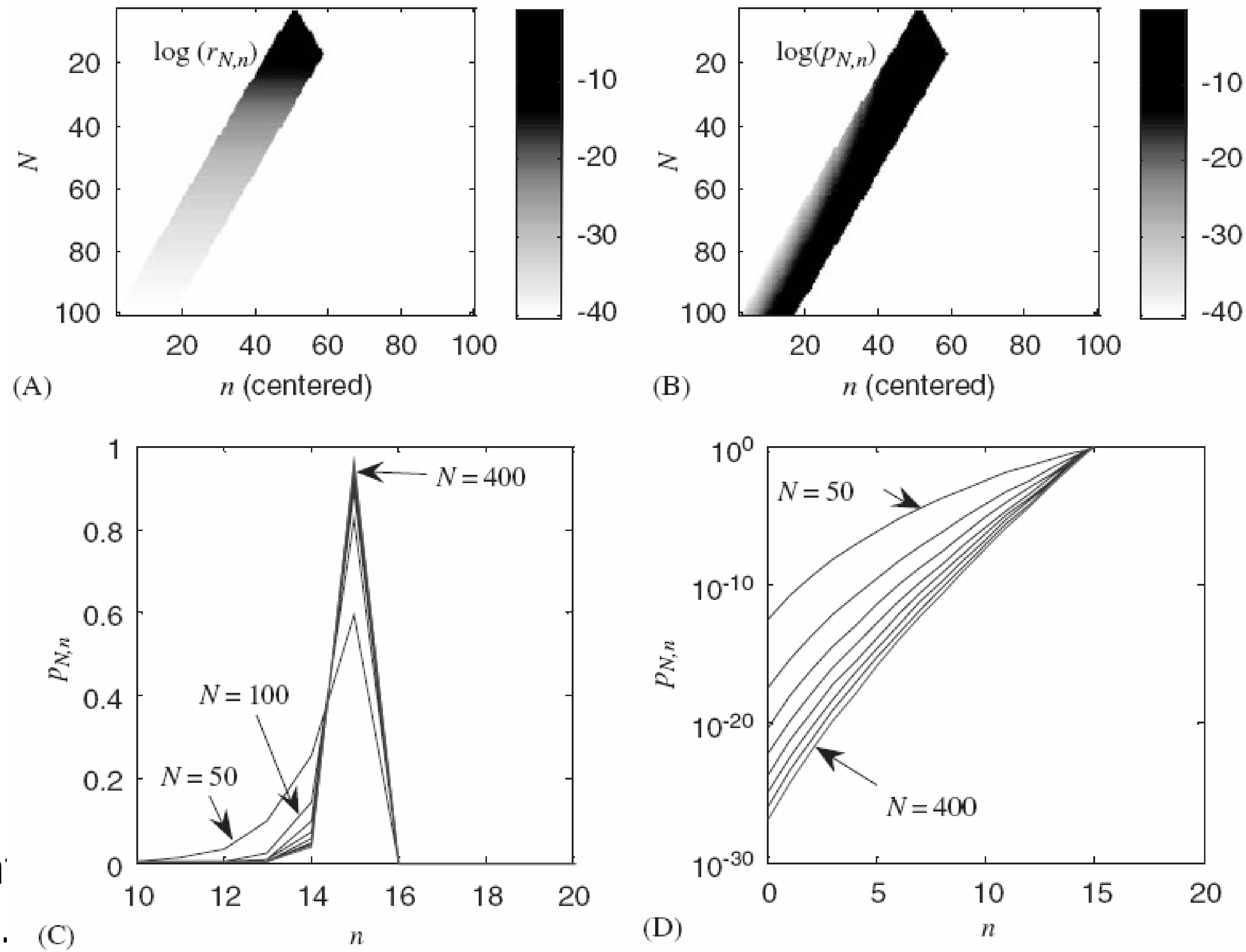
TGS2 MODEL

$$r_{N,n} = \begin{cases} \frac{1}{W^{\text{eff}}} = \left(\sum_{k=0}^{\min(d,N)} \frac{N!}{(N-k)!k!} \right)^{-1} & n \le d \\ 0 & \text{otherwise} \end{cases}$$

(Leibnitz rule is not valid)

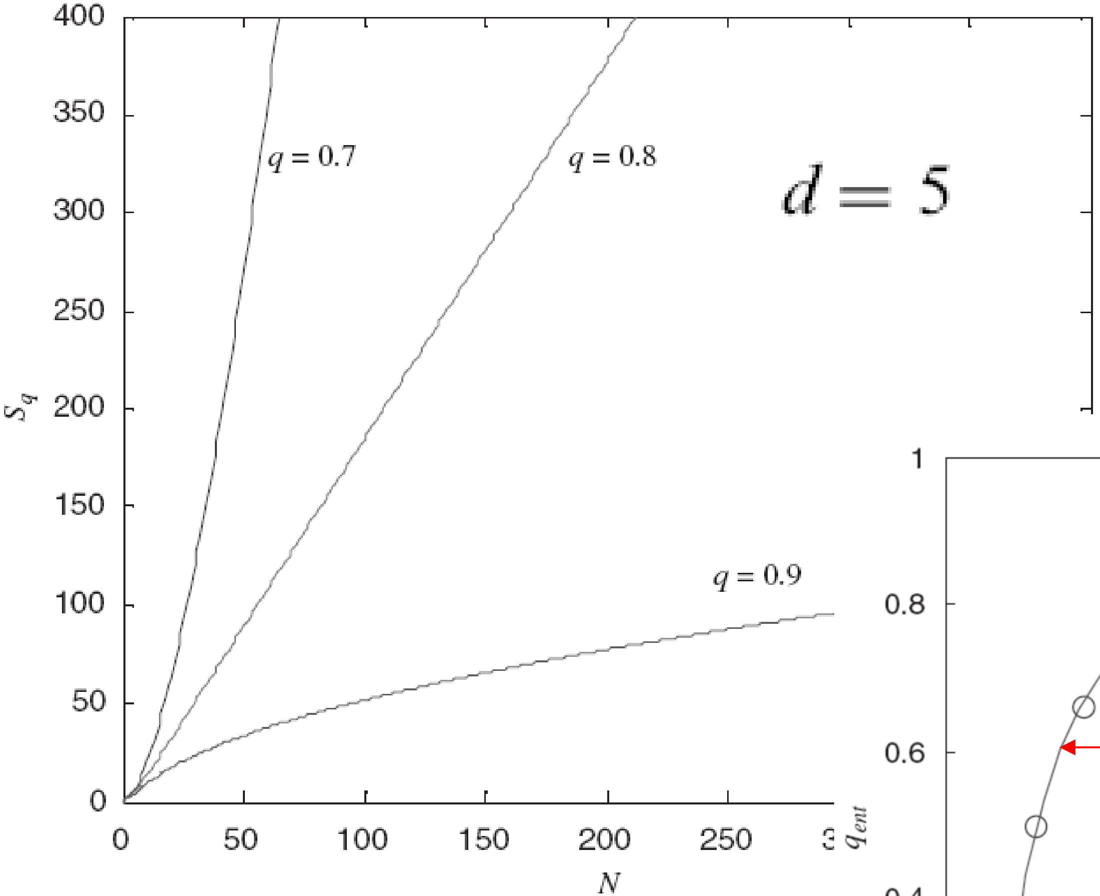
$$d = 15$$

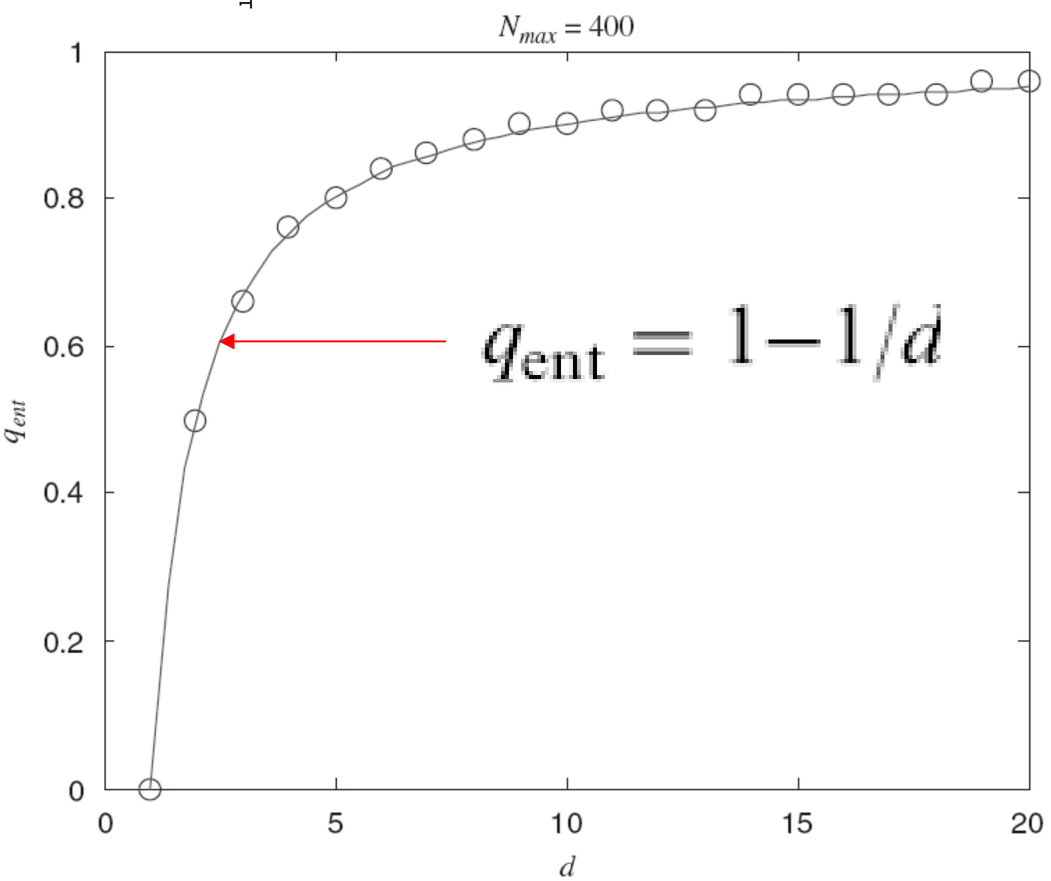
$$\gamma = 0$$



J.A. Marsh, M.A. Fuen
L.G. Moyano and C. T.
Physica A 372, **183** (2006)

TGS2 MODEL





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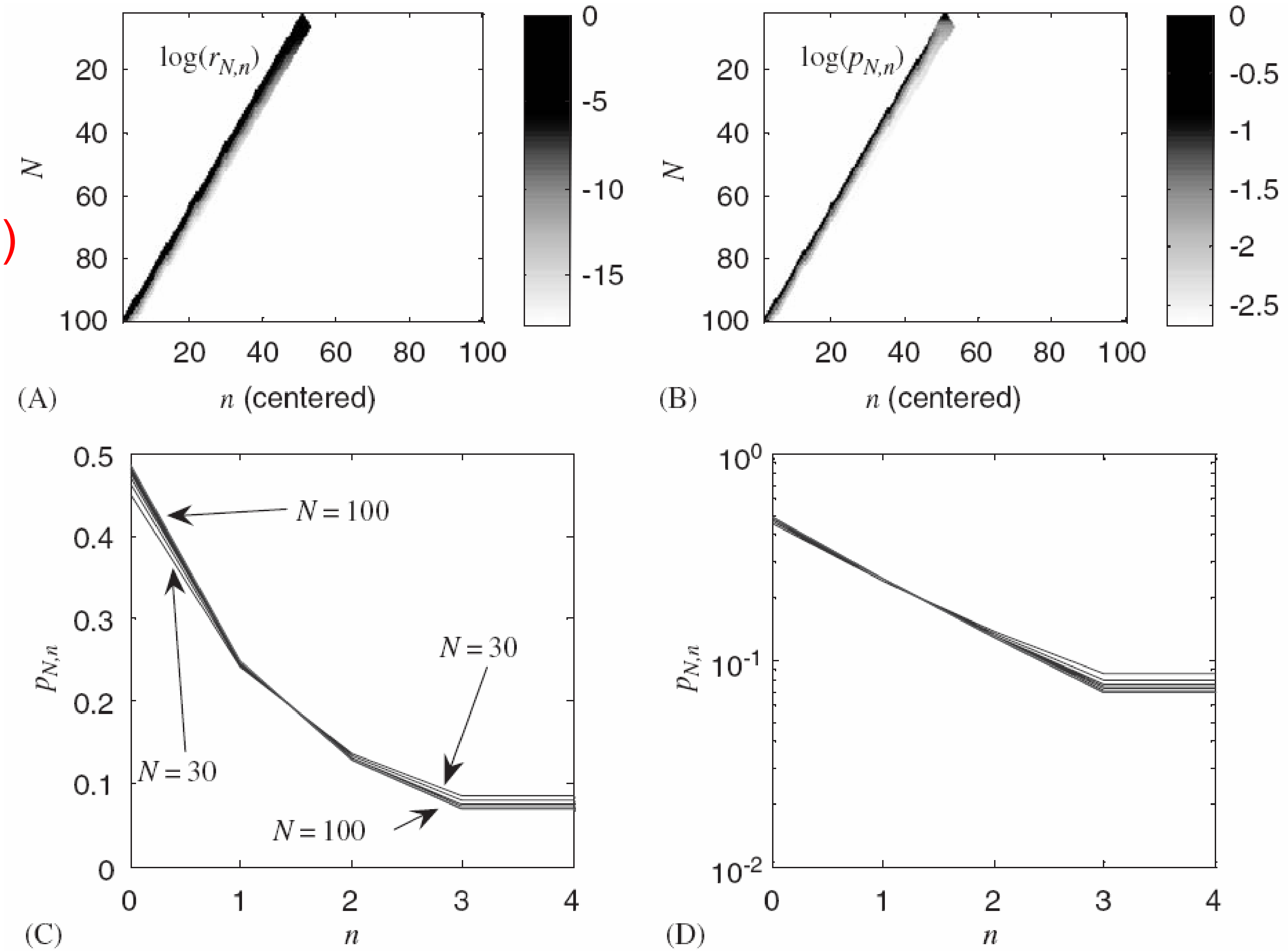
L.G. Moyano and C. T.
Physica A 372, **183** (2006)

TGS3 MODEL

(Leibnitz rule is asymptotically valid)

$d = 4$

$\gamma = 0$



J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

$$q_{\text{ent}} = 1 - 1/d$$

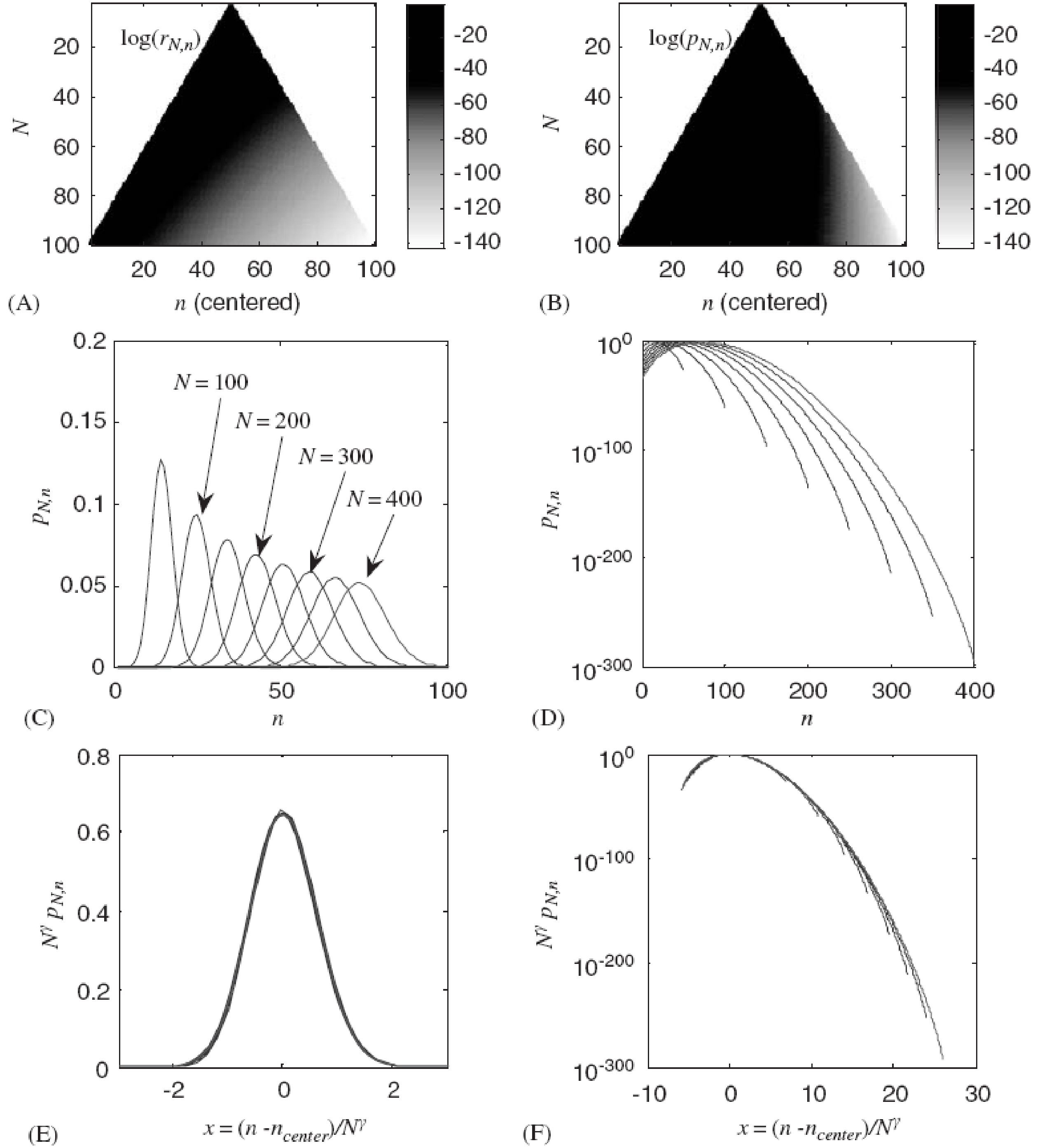
MFMT1 MODEL

$$r_{N,n} = \frac{N^{\alpha n}}{\sum_{k=0}^{N} \frac{N!}{(N-k)!k!} N^{\alpha k}}$$

(Leibnitz rule is not valid)

$$\alpha = -0.2$$

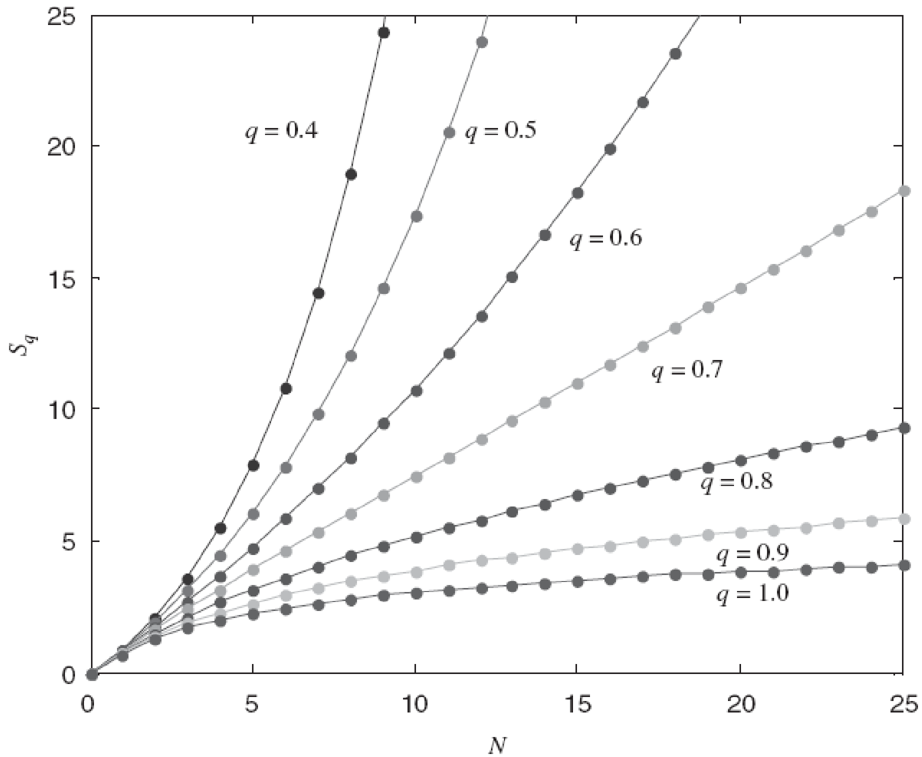
$$\gamma = 0.42$$



J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

MFMT1 MODEL

$$r_{N,n} = \frac{N^{\alpha n}}{\sum_{k=0}^{N} \frac{N!}{(N-k)!k!} N^{\alpha k}}$$



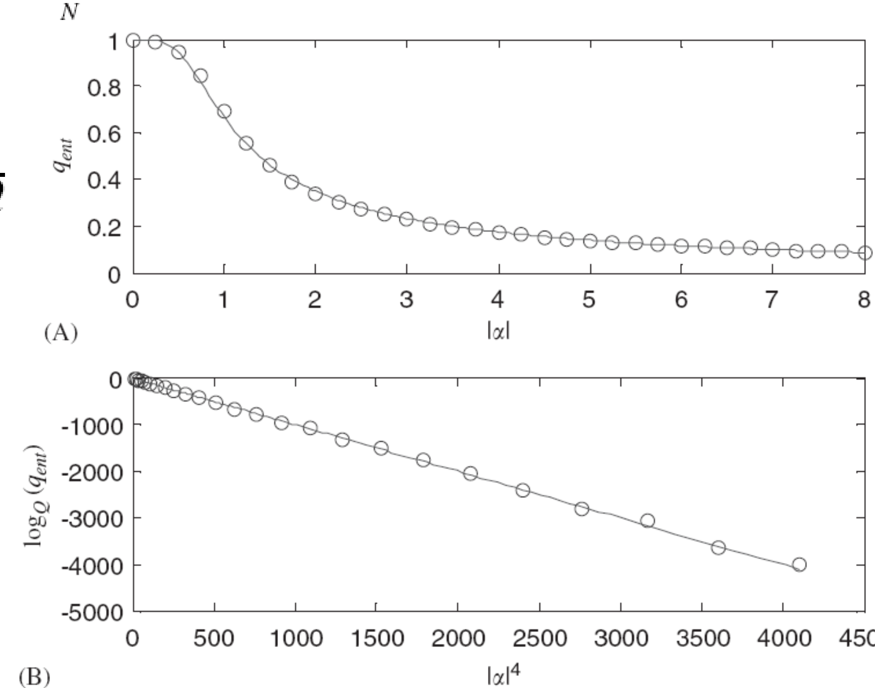
$$\alpha = -1$$

$$q_{\rm ent} \approx 0.7$$

(Leibnitz rule is not valid

$$q_{\rm ent} = e_Q^{-\alpha^4} = \left(1 - (1-Q)\alpha^4\right)^{\frac{1}{1-Q}}$$

$(Q \simeq 5.0)$



J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

MFMT2 MODEL

$$r_{\mathrm{N,n}} = \frac{N^{\alpha n'}}{Z} \quad \text{where}$$

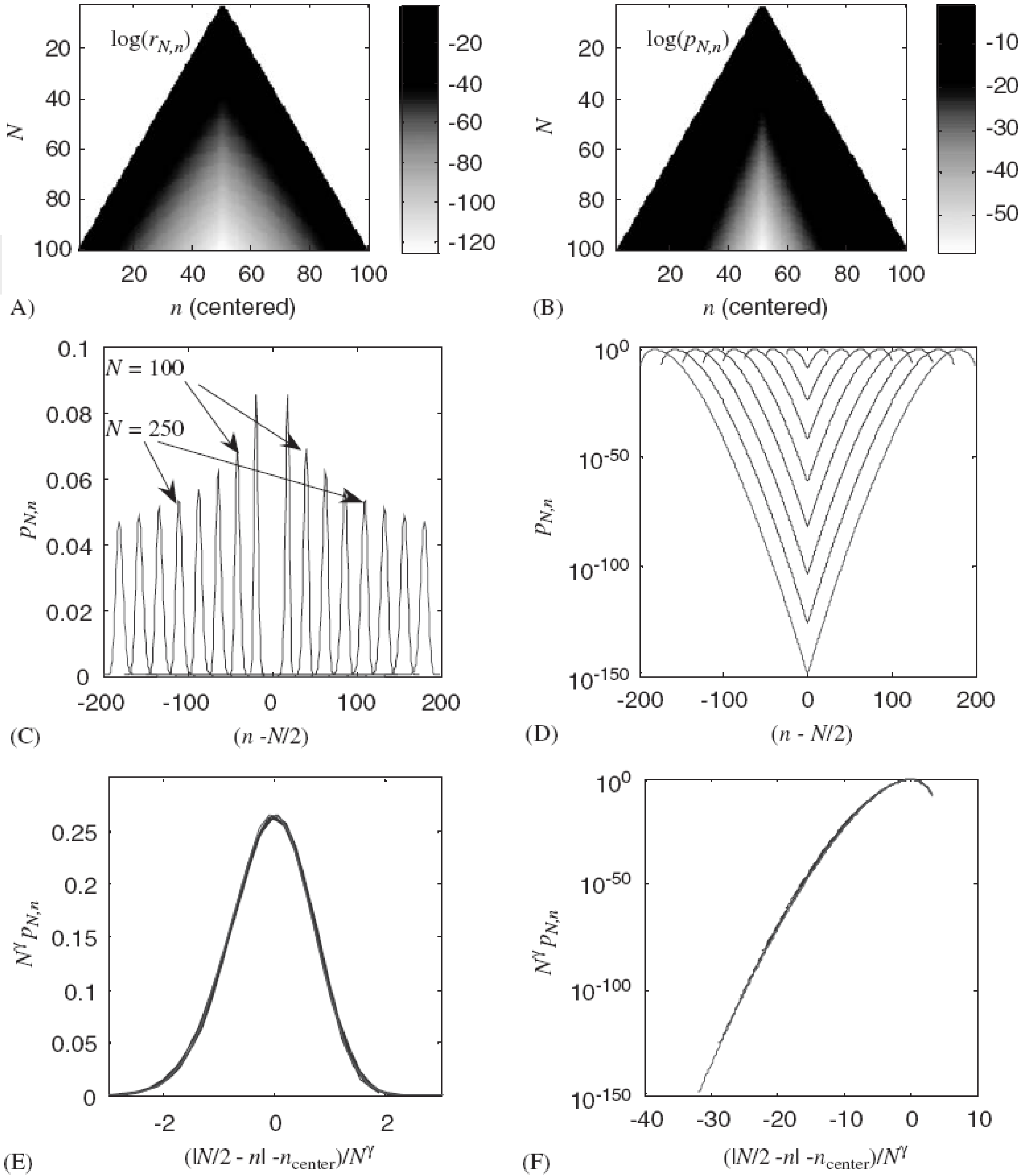
$$n' = \begin{cases} n & n \leq \lfloor N/2 \rfloor \\ N - n & \text{otherwise} \end{cases}$$

$$n = 0, \ldots, N$$

(Leibnitz rule is not valid)

$$\alpha = -0.5$$

$$\gamma = 0.28$$



J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

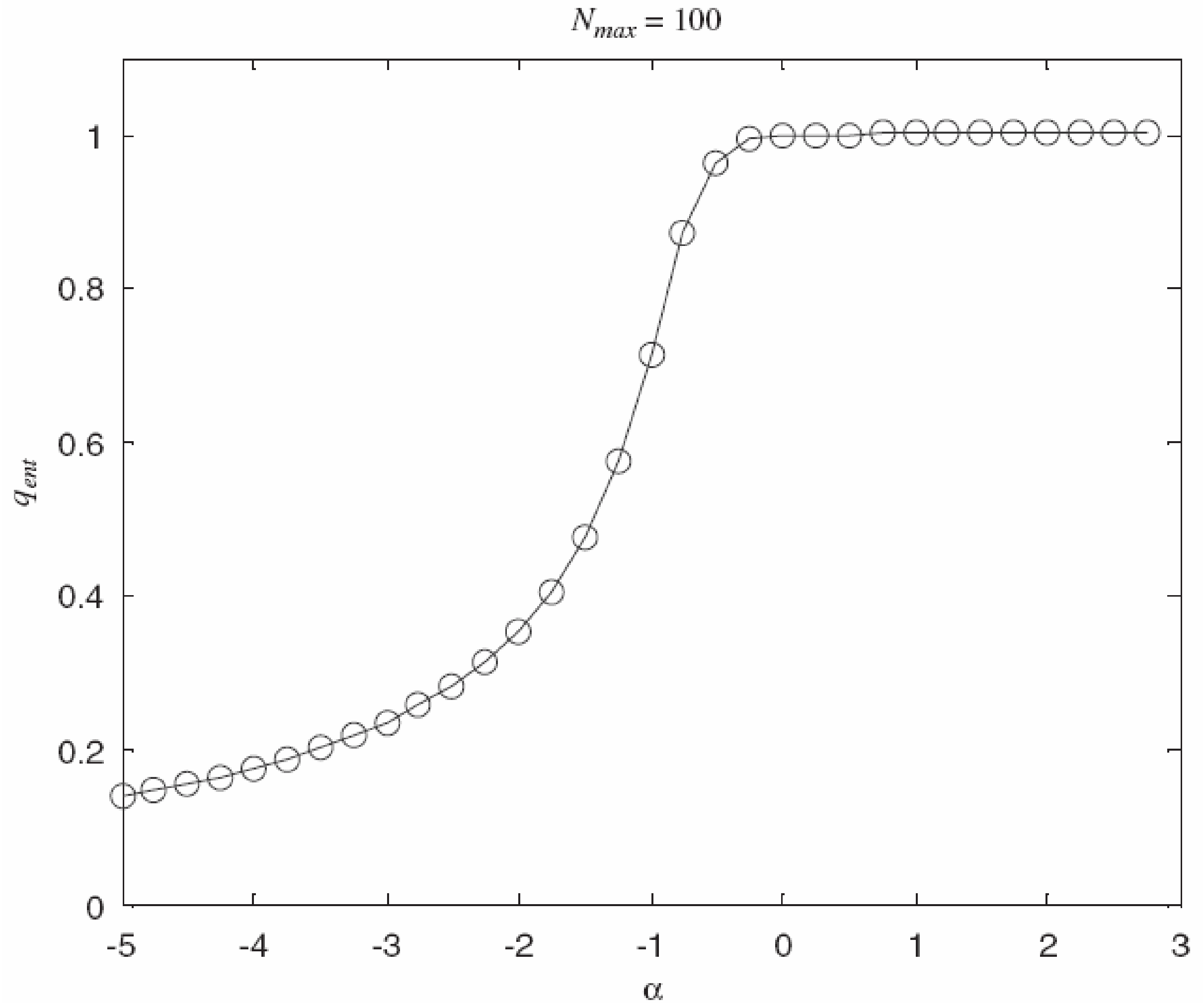
MFMT2 MODEL

$$r_{\mathrm{N,n}} = \frac{N^{\alpha n'}}{Z} \quad \text{where}$$

$$n' = \begin{cases} n & n \leq \lfloor N/2 \rfloor \\ N-n & \text{otherwise} \end{cases}$$

$$n = 0, \ldots, N$$

(Leibnitz rule is not valid)

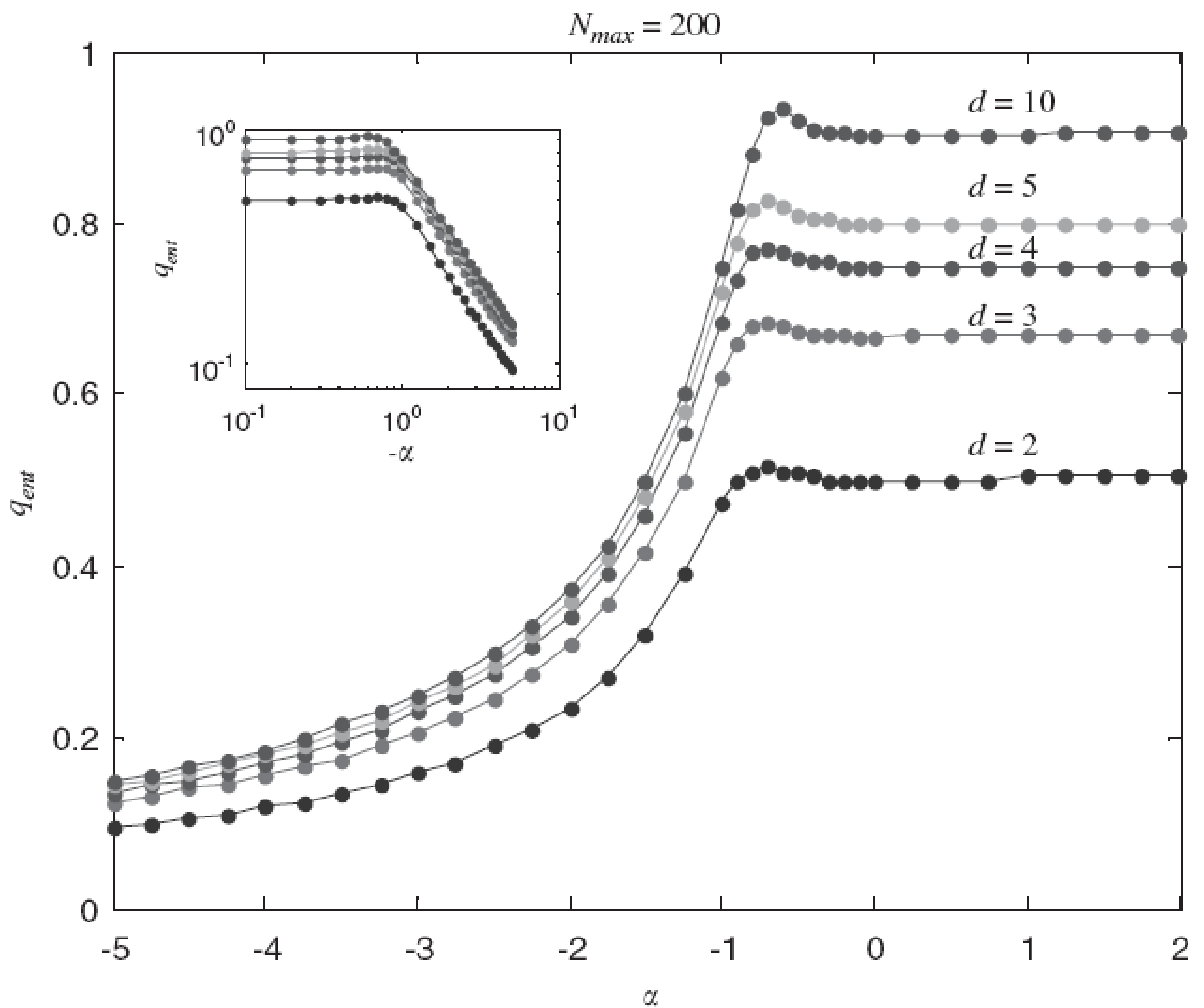


J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

MFMT3 MODEL

$$r_{N,n} = \begin{cases} \frac{N^{\alpha n}}{Z} & n \leq d \\ 0 & \text{otherwise} \end{cases}$$

(Leibnitz rule is not valid)



$$q_{ent} \begin{cases} = 1 - \frac{1}{d} & if \quad \alpha \geq 0 \\ \sim \frac{1}{|\alpha|} & if \quad \alpha \to -\infty \end{cases}$$

J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T. Physica A 372, **183** (2006)

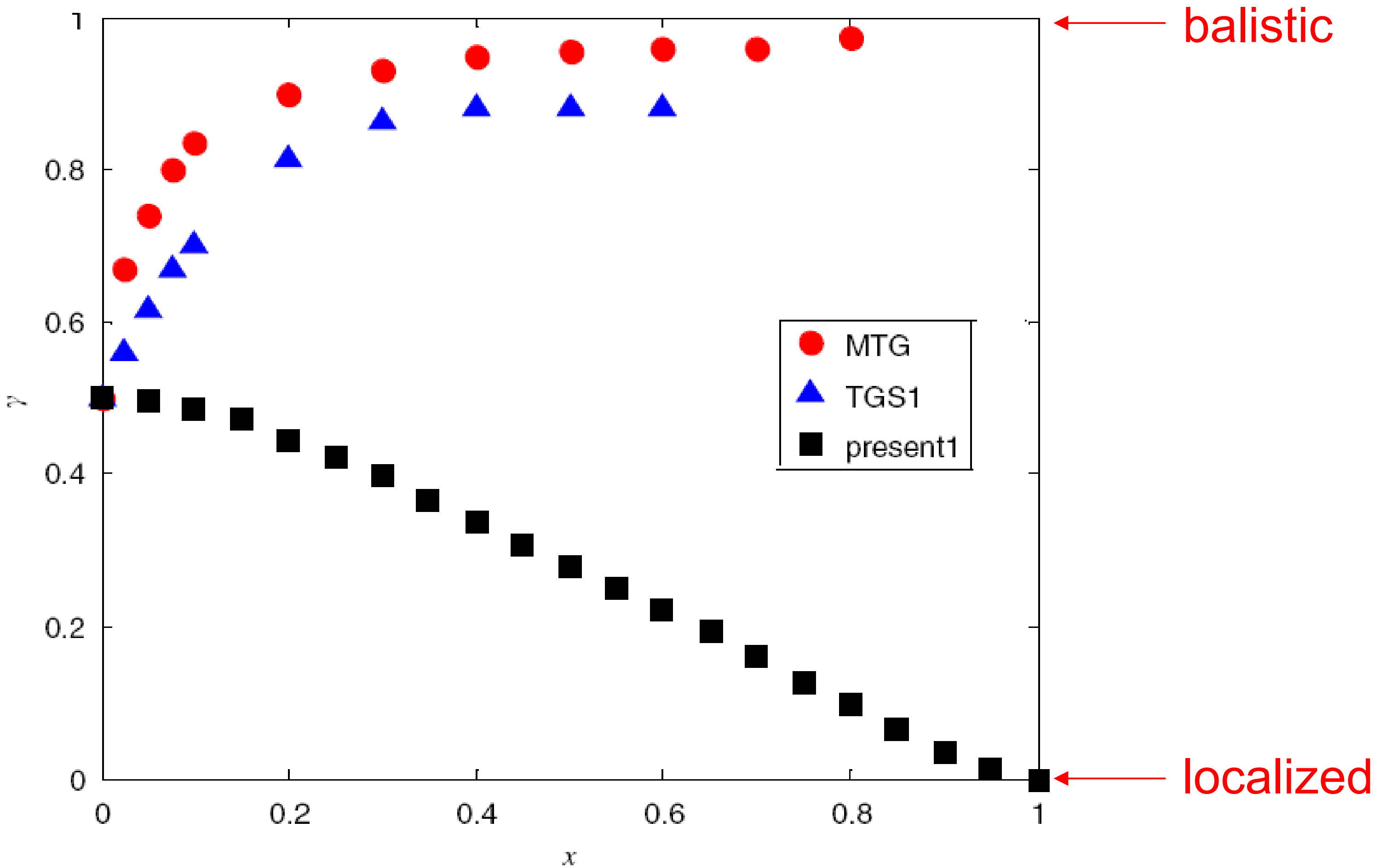


Fig. 20. Diffusion exponent γ vs. model parameters x, for the models MTG, TGS1, and "present1". For all three models, $x = 0$ corresponds to the independent case, with normal diffusion exponent $\gamma = 0.5$. In the model MTG $x = 1 - q_{\mathrm{corr}}$, and in TGS1 $x = 1 - \alpha$ each of these two models exhibiting *superdiffusion*. In the model "present1", $x = |\alpha|$ and the model exhibits *subdiffusion*. The values $\gamma = 0, 1$ respectively correspond to localization and ballistic superdiffusion.

J.A. Marsh, M.A. Fuentes, L.G. Moyano and C. T., Physica A 372, **183** (2006)

CONNECTION WITH

(ASYMPTOTICALLY) SCALE-FREE NETWORKS

AN ANALYTICALLY SOLVABLE MODEL:

R. Albert and A.-L. Barabasi, Phys. Rev. Lett. **85**, 5234 (2000)

At each time step,

m *new links are added with probability* p,

or m *existing links are rewired with probability* r,

or a new node with m *links is added with probability* $1 - p - r$

$$\textit{degree distribution} \equiv p(k) \propto e_q^{-k/\kappa}$$

with $$q = \frac{2m(2-r)+1-p-r}{m(3-2r)+1-p-r}$$

$$\left[\textit{written in the form} \quad p(k) \propto \frac{1}{(k+k_0)^{\gamma}} \quad \textit{by the authors} \right]$$

GEOGRAPHIC PREFERENTIAL ATTACHMENT GROWING NETWORK: THE NATAL MODEL

D.J.B. Soares, C. T. , A.M. Mariz and L.R. Silva, Europhys Lett **70**, 70 (2005)

(1) Locate site $i=1$ at the origin of say a plane

(2) Then locate the next site with

$$P_G \propto 1/r^{2+\alpha_G} \quad (\alpha_G \geq 0)$$

$(r \equiv \textit{distance to the baricenter of the pre-existing cluster})$

(3) Then link it to only one of the previous sites using

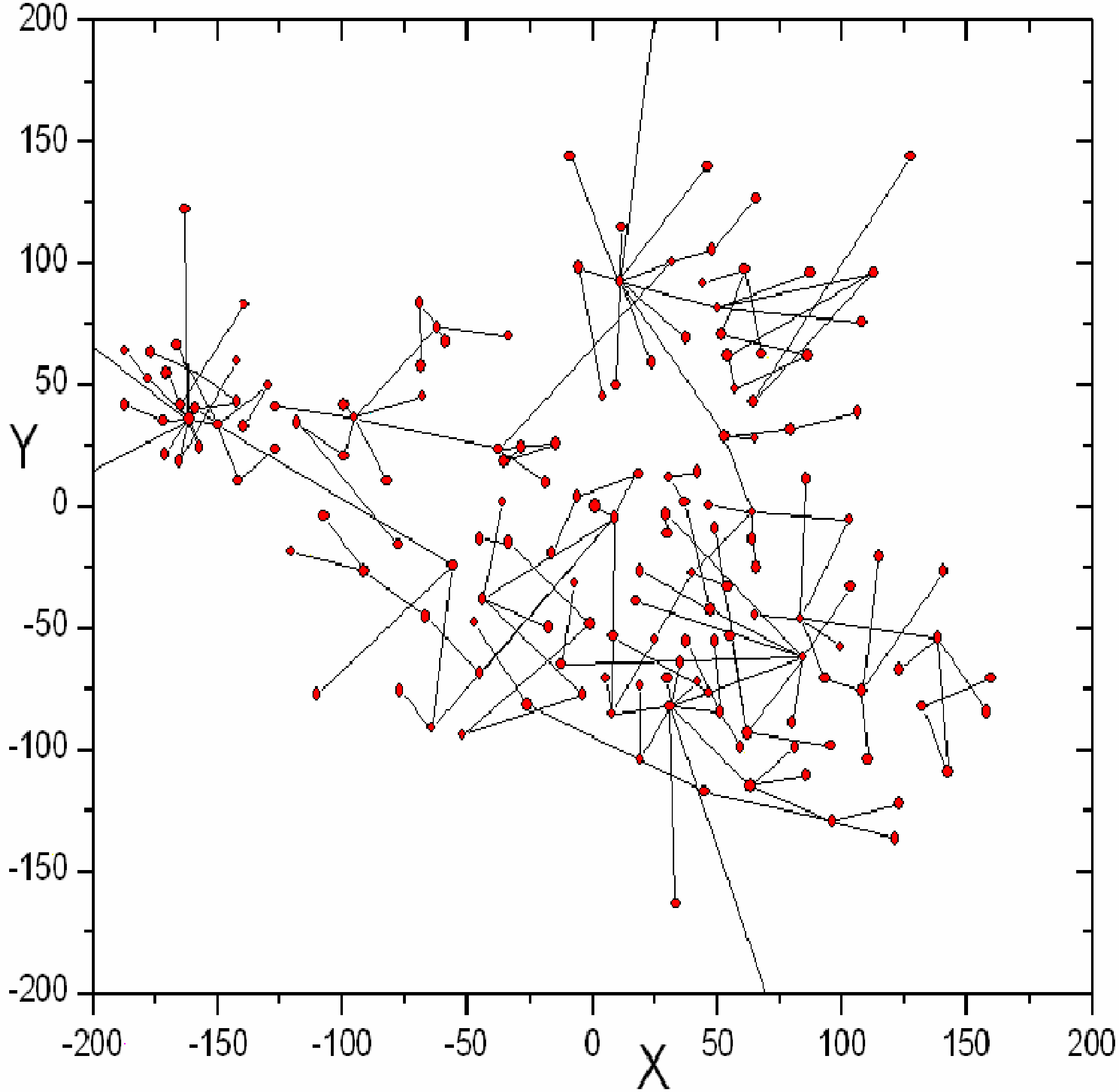
$$p_A \propto k_i / r_i^{\alpha_A} \quad (\alpha_A \geq 0)$$

$(k_i \equiv \textit{links already attached to site } i)$

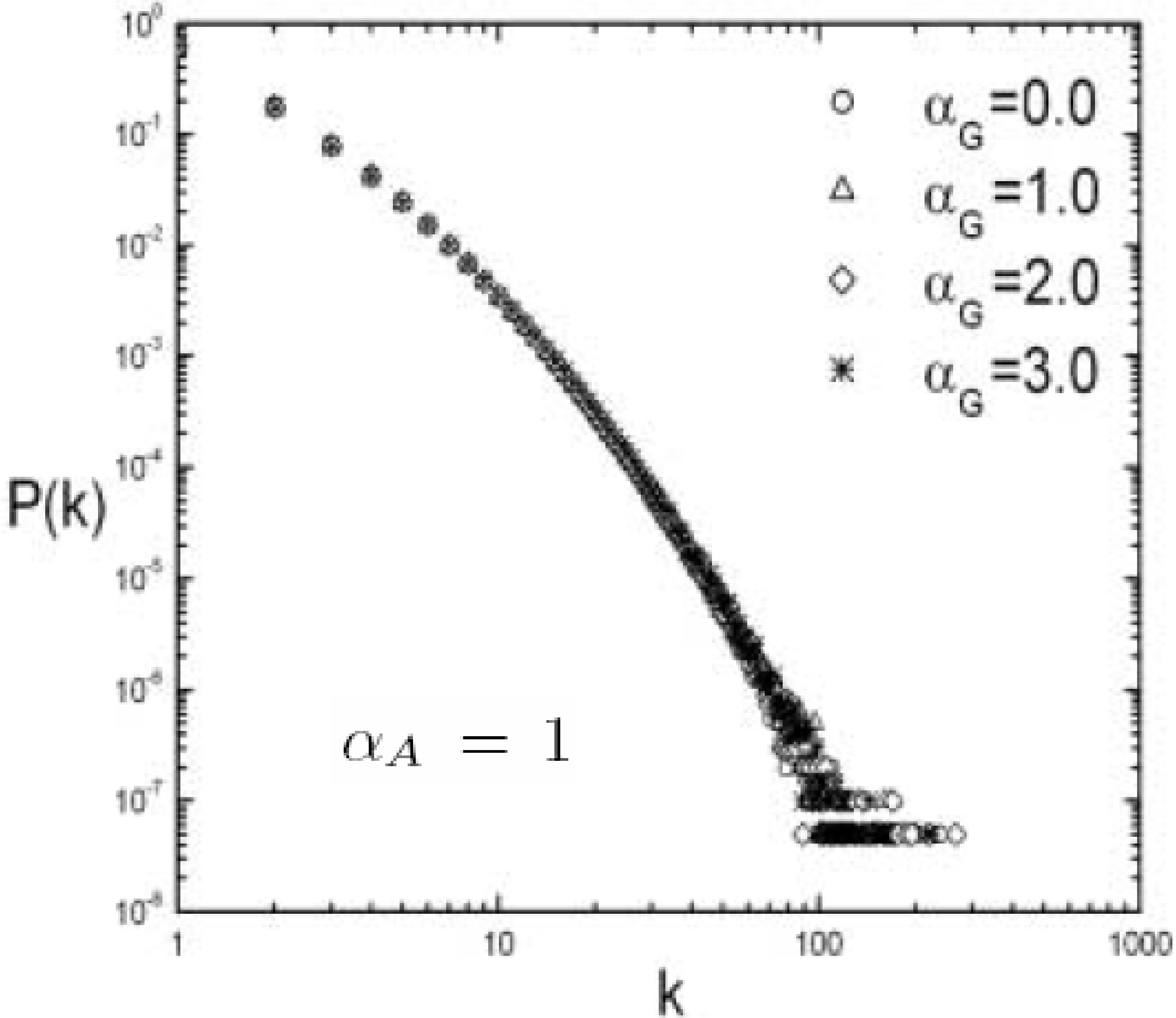
$(r_i \equiv \textit{distance to site } i)$

4) Repeat

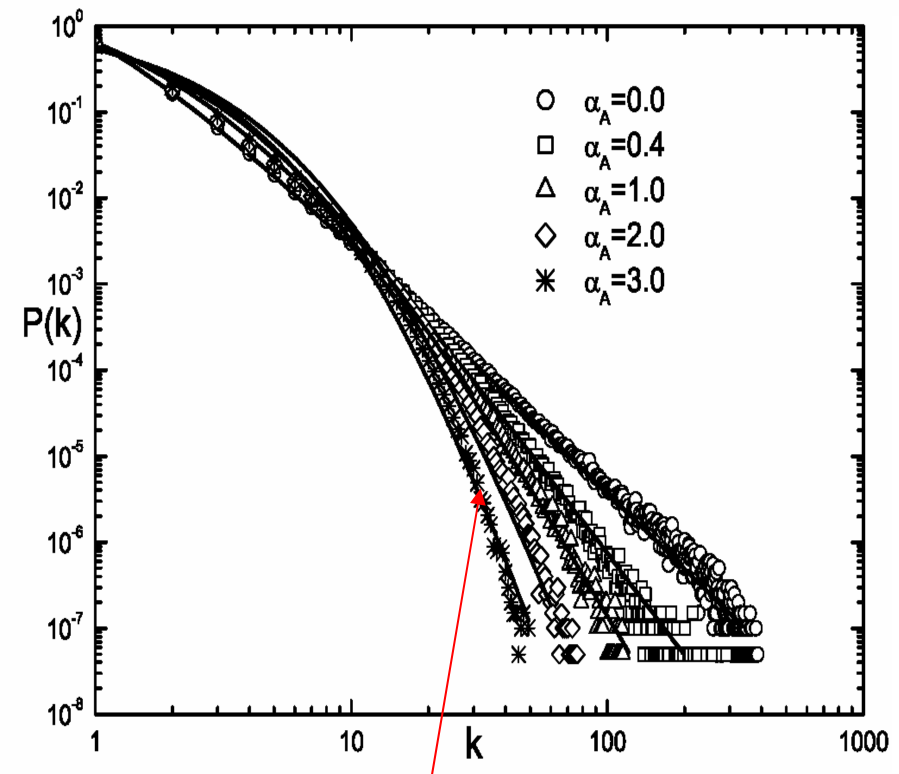
$$(\alpha_G = 1; \alpha_A = 1; N = 250)$$

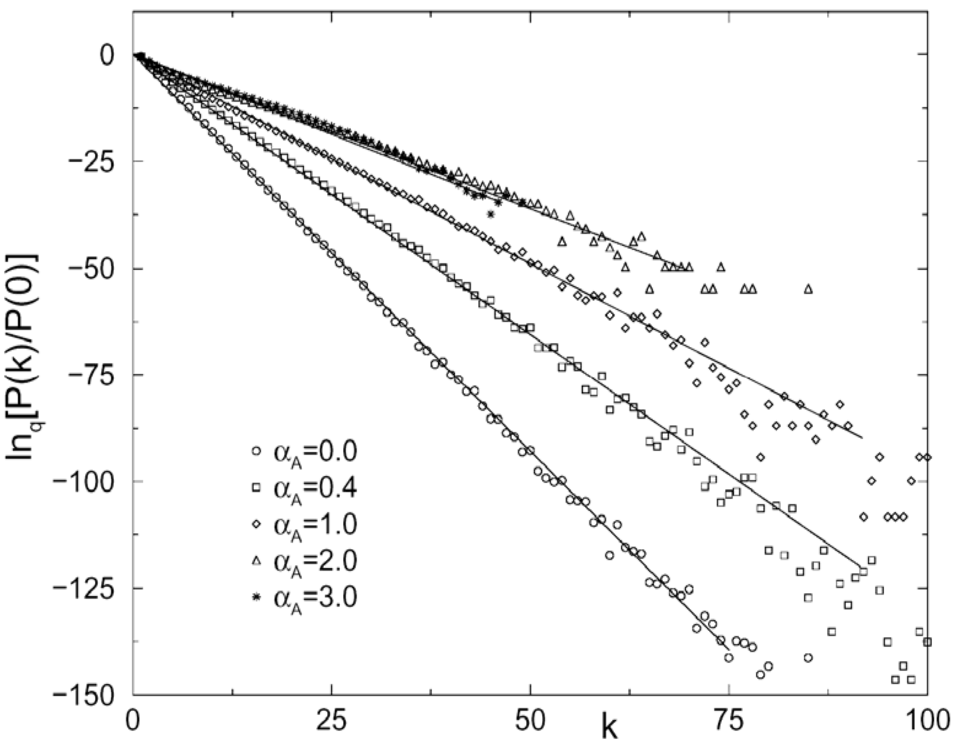


D.J.B. Soares, C. T. , A.M. Mariz and L.R. Silva, Europhys Lett **70**, 70 (2005)



2000 realizations of N =10000 networks

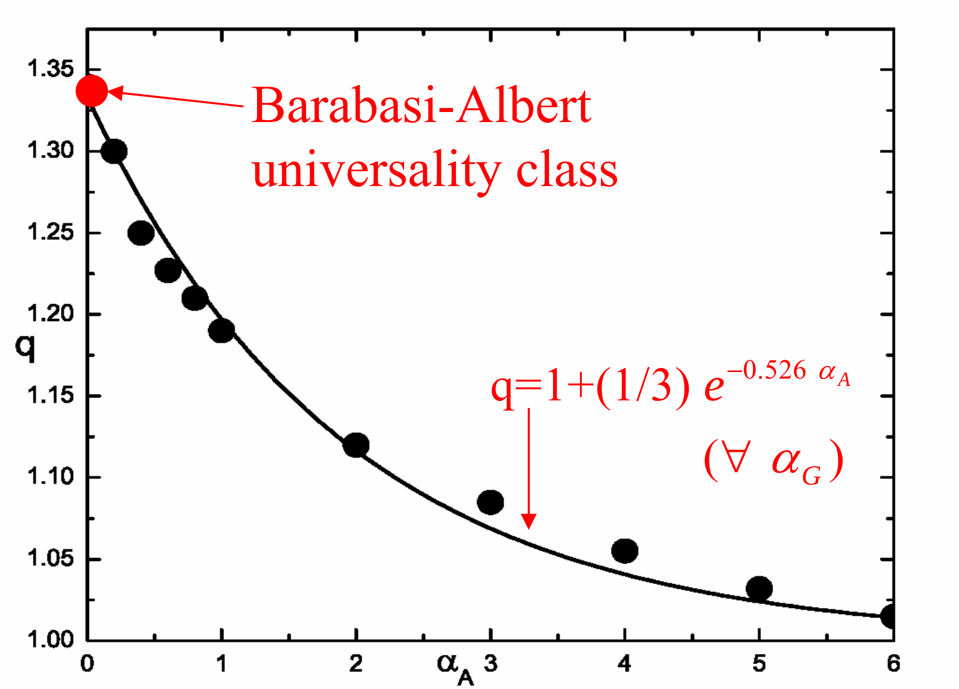


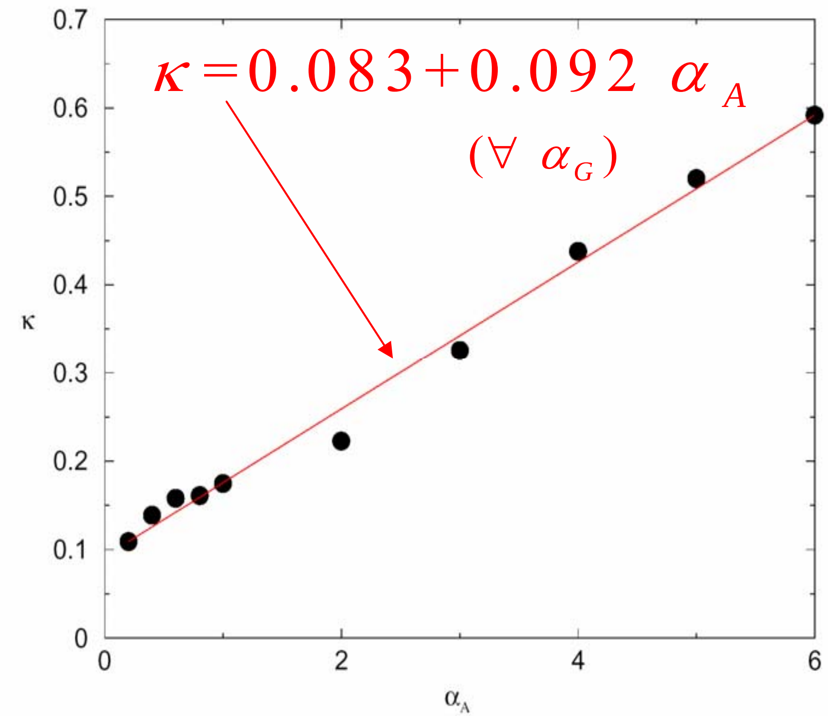


$$P(k)/P(0) = e_q^{-k/\kappa}$$

$$\equiv 1/[1+(q-1)k/\kappa]^{1/(q-1)}$$

D.J.B. Soares, C. T. , A.M. Mariz and L.R. Silva
Europhys Lett **70**, 70 (2005)





D.J.B. Soares, C. T. , A.M. Mariz and L.R. Silva
Europhys Lett **70**, 70 (2005)

GAS-LIKE (NODE COLLAPSING) NETWORK:

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

Number *N* of nodes fixed (*chemostat*); i=1, 2, …, *N*

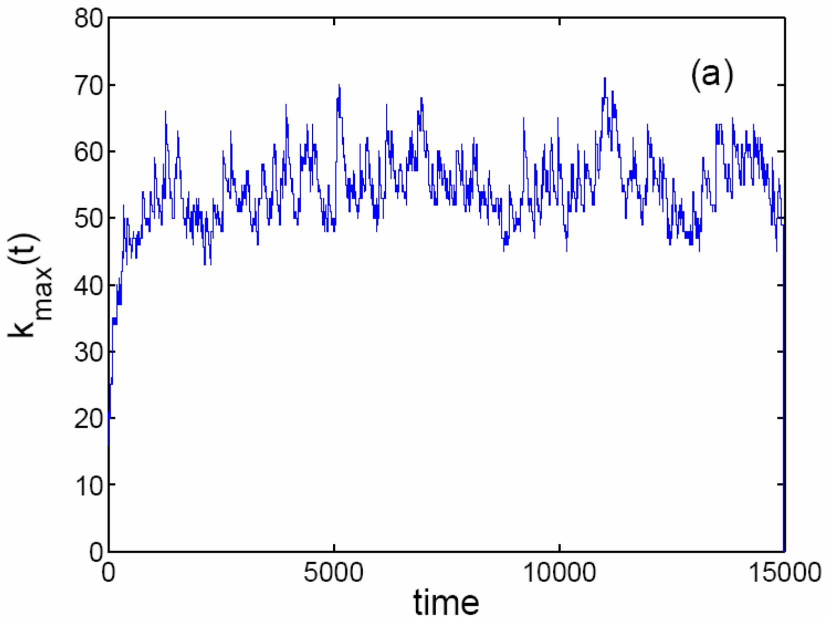
$$\textit{Merging probability}\;\; p_{ij} \propto \frac{1}{d_{ij}{}^{\alpha}} \quad (\alpha \geq 0)$$

$$d_{ij} \equiv \textit{shortest path (chemical distance) connecting nodes } i \textit{ and } j \textit{ on the network}$$

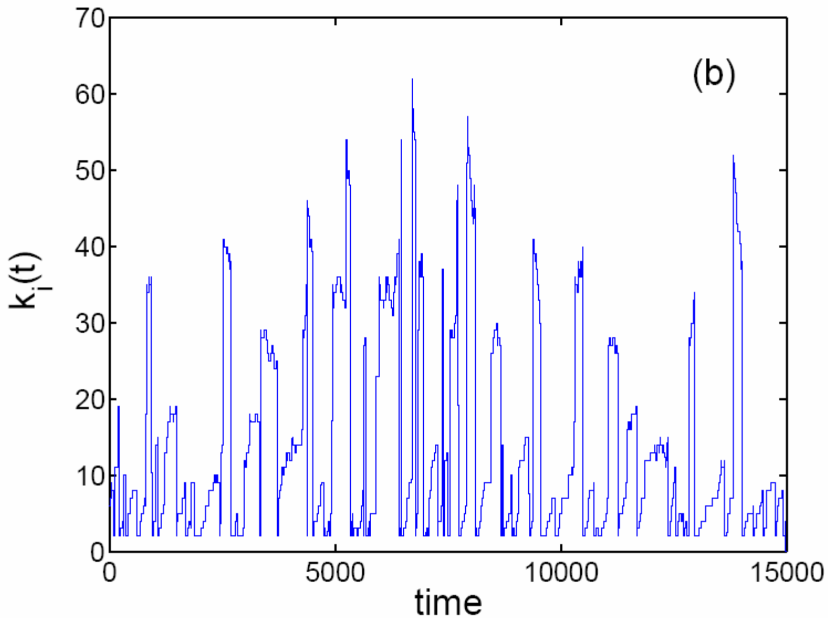
$\alpha = 0$ *and* $\alpha \to \infty$ *recover the random and the neighbor schemes respectively*

(Kim, Trusina, Minnhagen and Sneppen, *Eur. Phys. J. B* 43 (2005) 369)

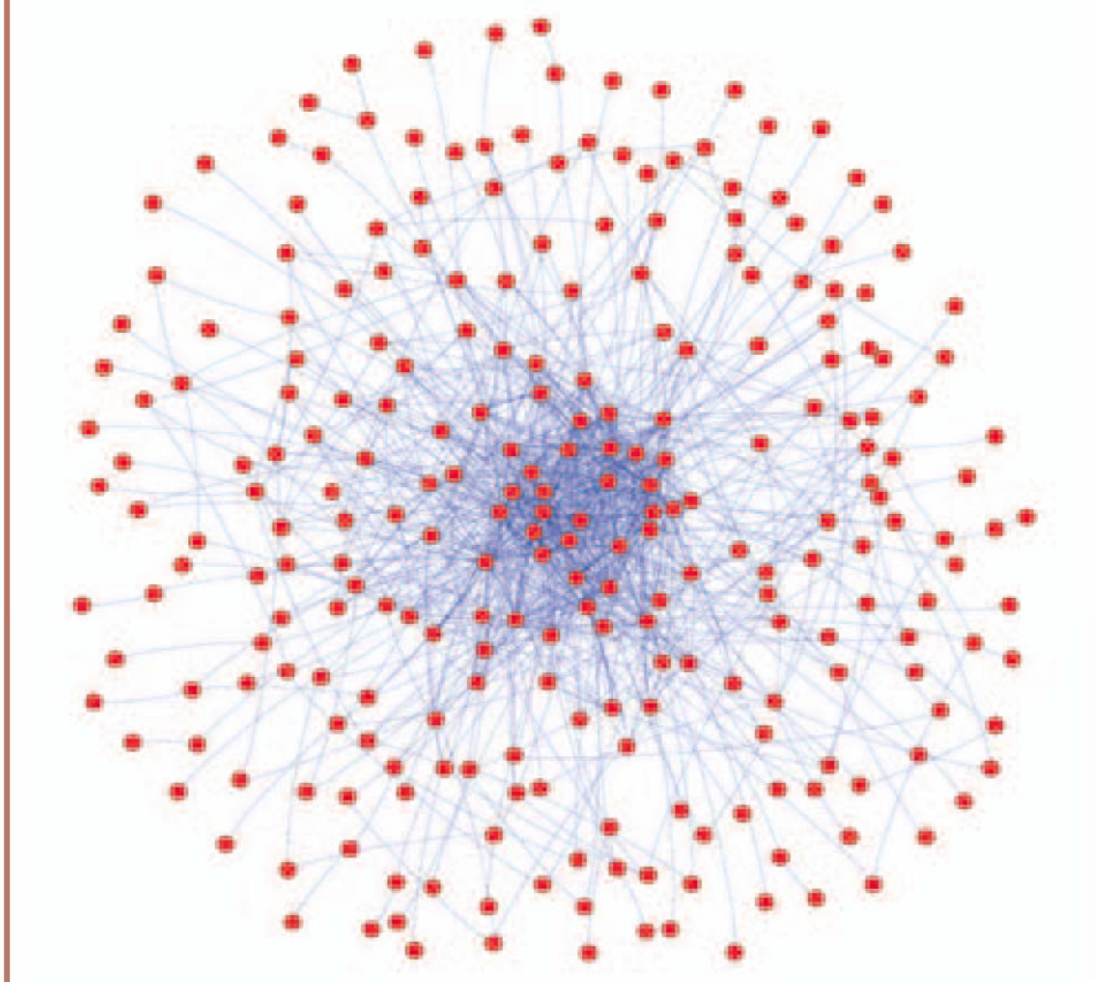
$(N = 2^7;\ \alpha = 0;\ r = 2)$



Degree of the most connected node

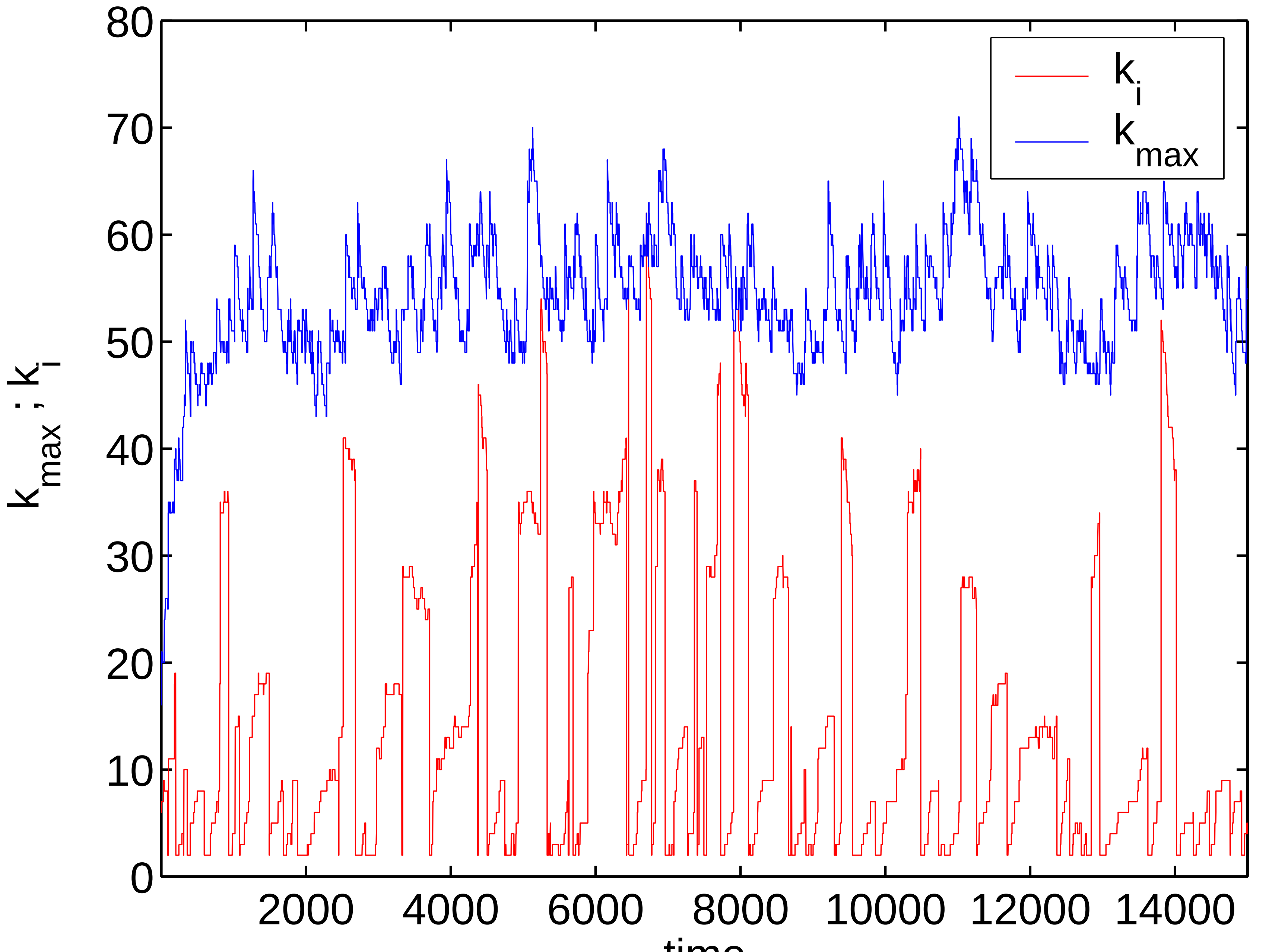


Degree of a randomly chosen node



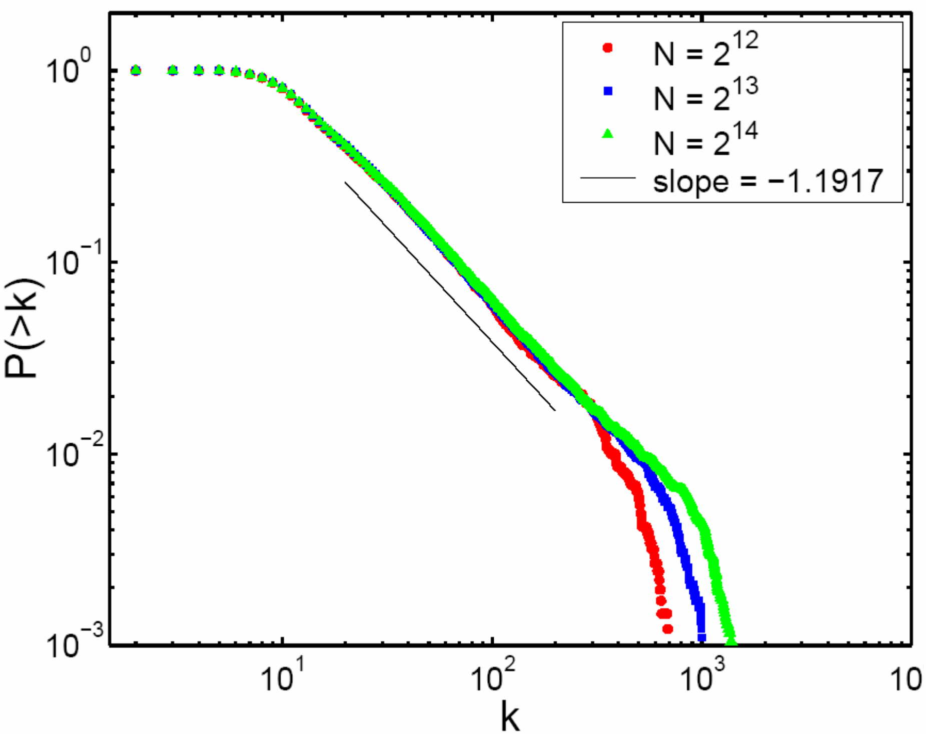
▲ **Fig. 1:** Snapshot of a non-growing dynamic network with q-exponential degree distribution for $N = 256$ nodes and a linking rate of $\bar{r} = 1$, for details see [8, 9]. The shown network is small to make connection patterns visible.

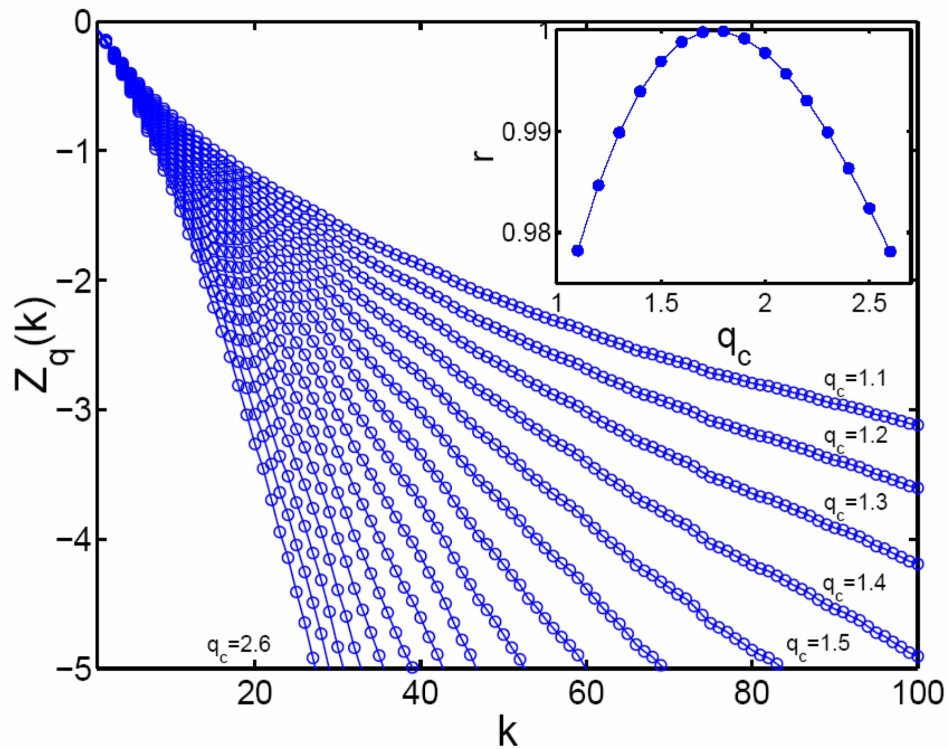
S. Thurner, Europhys News **36**, 218 (2005)



0
10
20
30
40
50
60
70
80
2000
4000
6000
8000
10000
12000
14000
time
k_{max} ; k_i
k_i
k_{max}

$$(\alpha \to \infty\ ;\ \ < r >= 8)$$



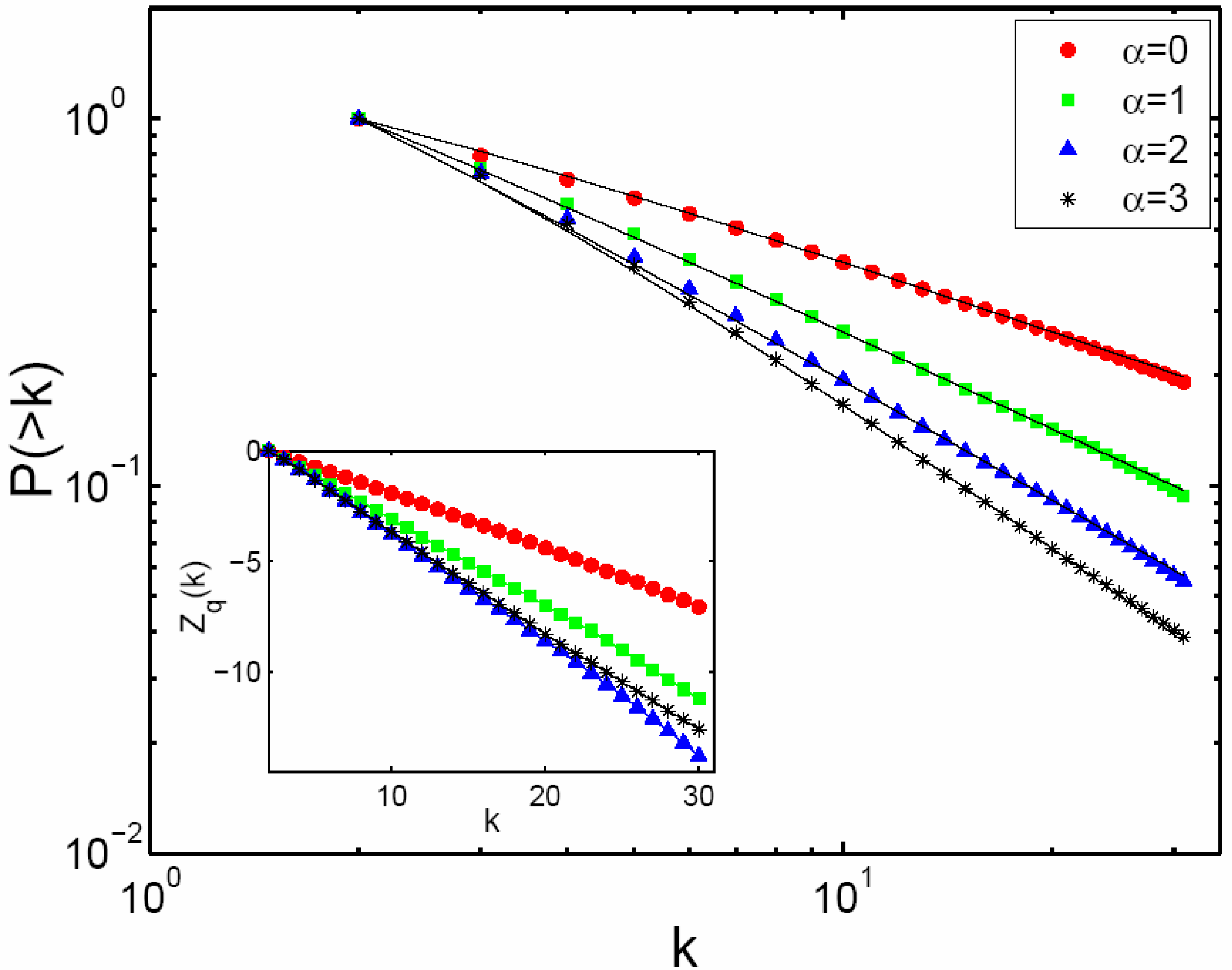


$$Z_q(k) \equiv \ln_q\left[P(>k)\right] \equiv \frac{\left[P(>k)\right]^{1-q} - 1}{1-q}$$

$$(optimal\ \ q_c = 1.84)$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

$(N = 2^9;\ r = 2)$

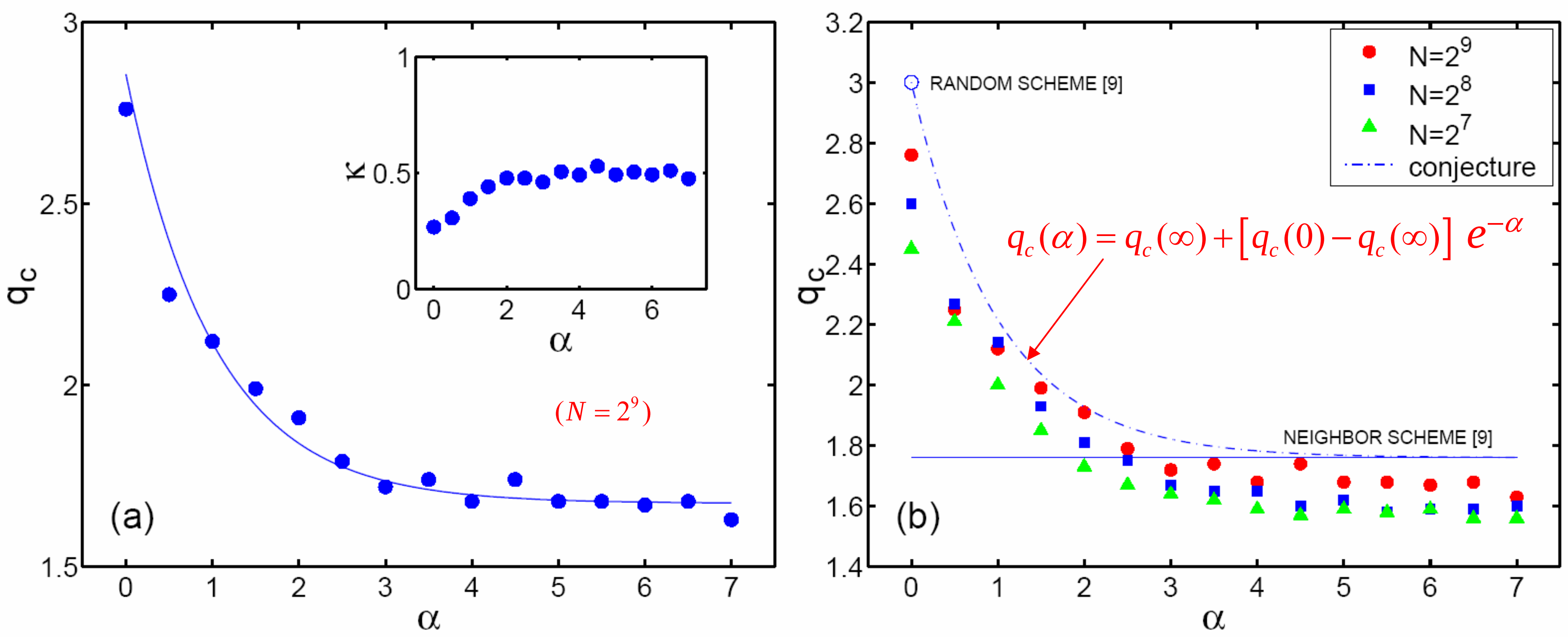


$$P(\geq k) = e_{q_c}^{-(k-2)/\kappa} \quad (k = 2,\ 3,\ 4, \ldots)$$

$$linear\ correlation \in [0.999901, 0.999976]$$

S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

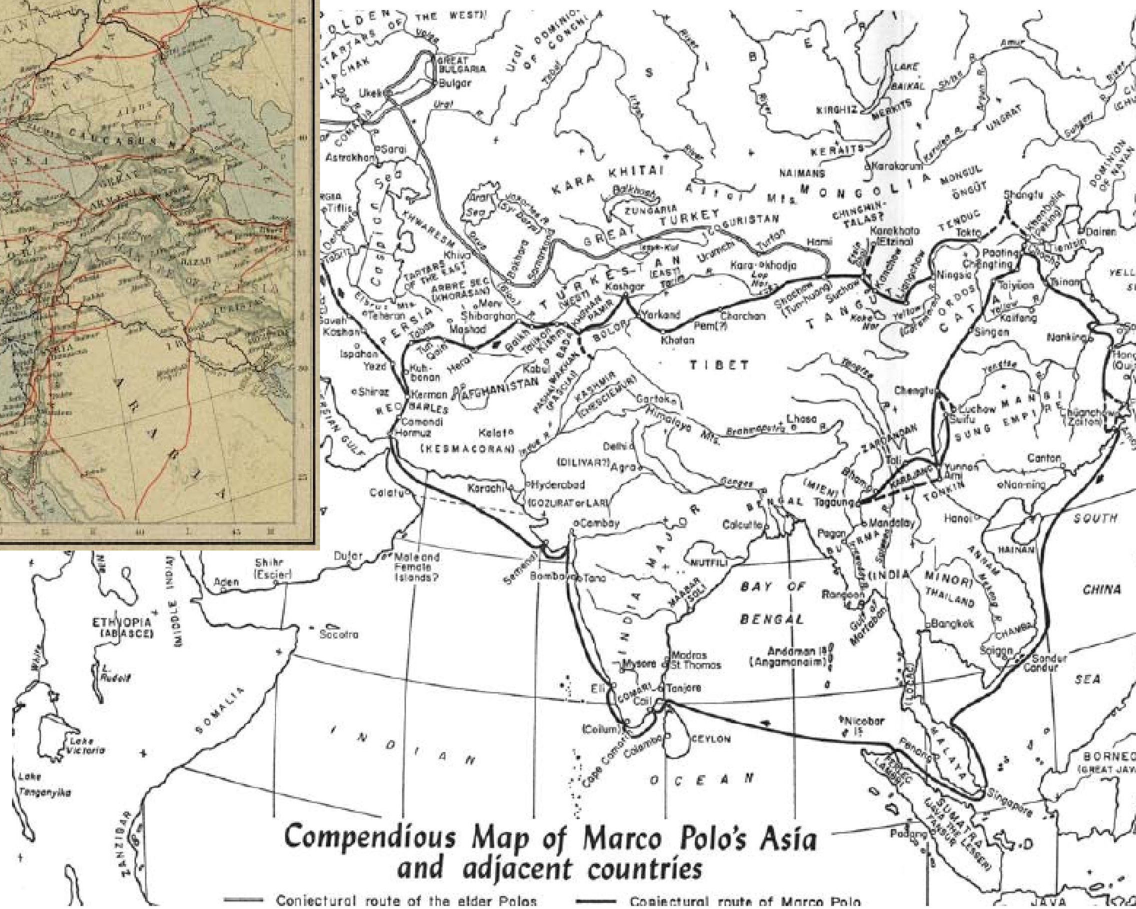
$(r = 2)$



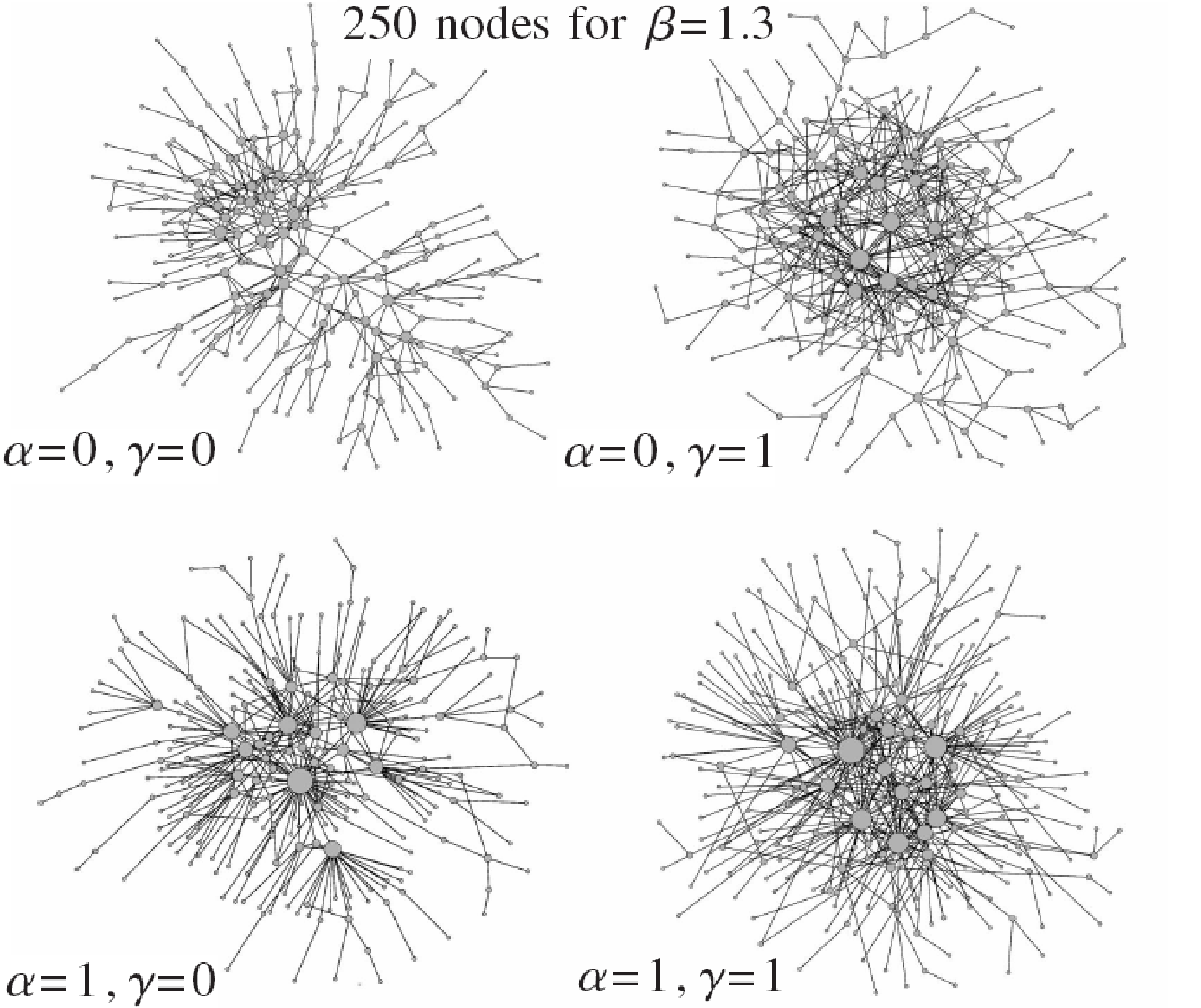
S. Thurner and C. T., Europhys Lett **72**, 197 (2005)

GENERATIVE MODEL FOR FEEDBACK NETWORKS:

D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)

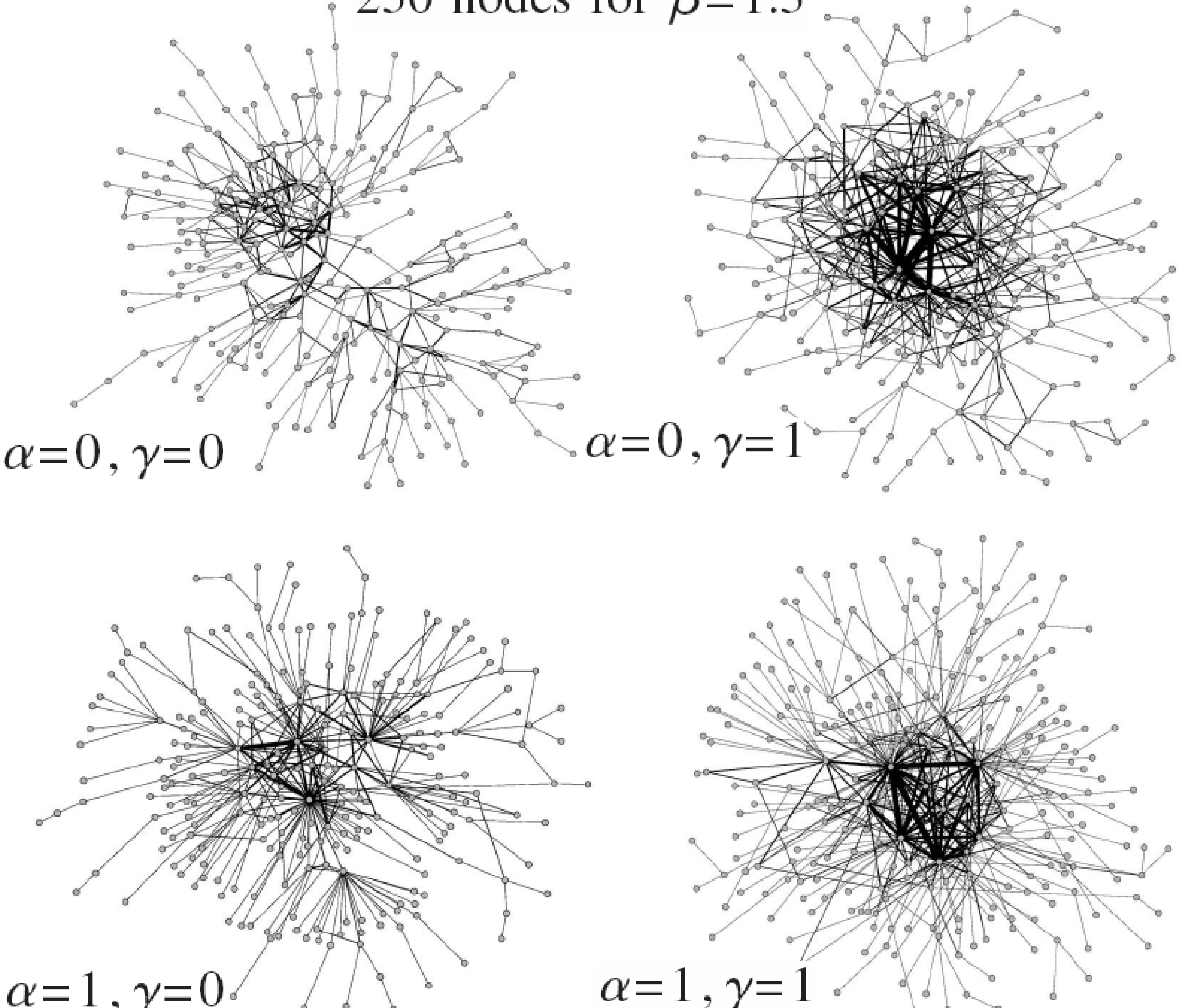


250 nodes for $\beta=1.3$



D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)

250 nodes for $\beta=1.3$



D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)

MODEL:

$$P_\alpha(i) = \frac{[\deg(i)]^\alpha}{\sum_{m=1}^{N} [\deg(m)]^\alpha}$$

attachment parameter $\alpha \geq 0$

$$P_\beta(d) = \frac{d^{-\beta}}{\sum_{m=1}^{\infty} m^{-\beta}}$$

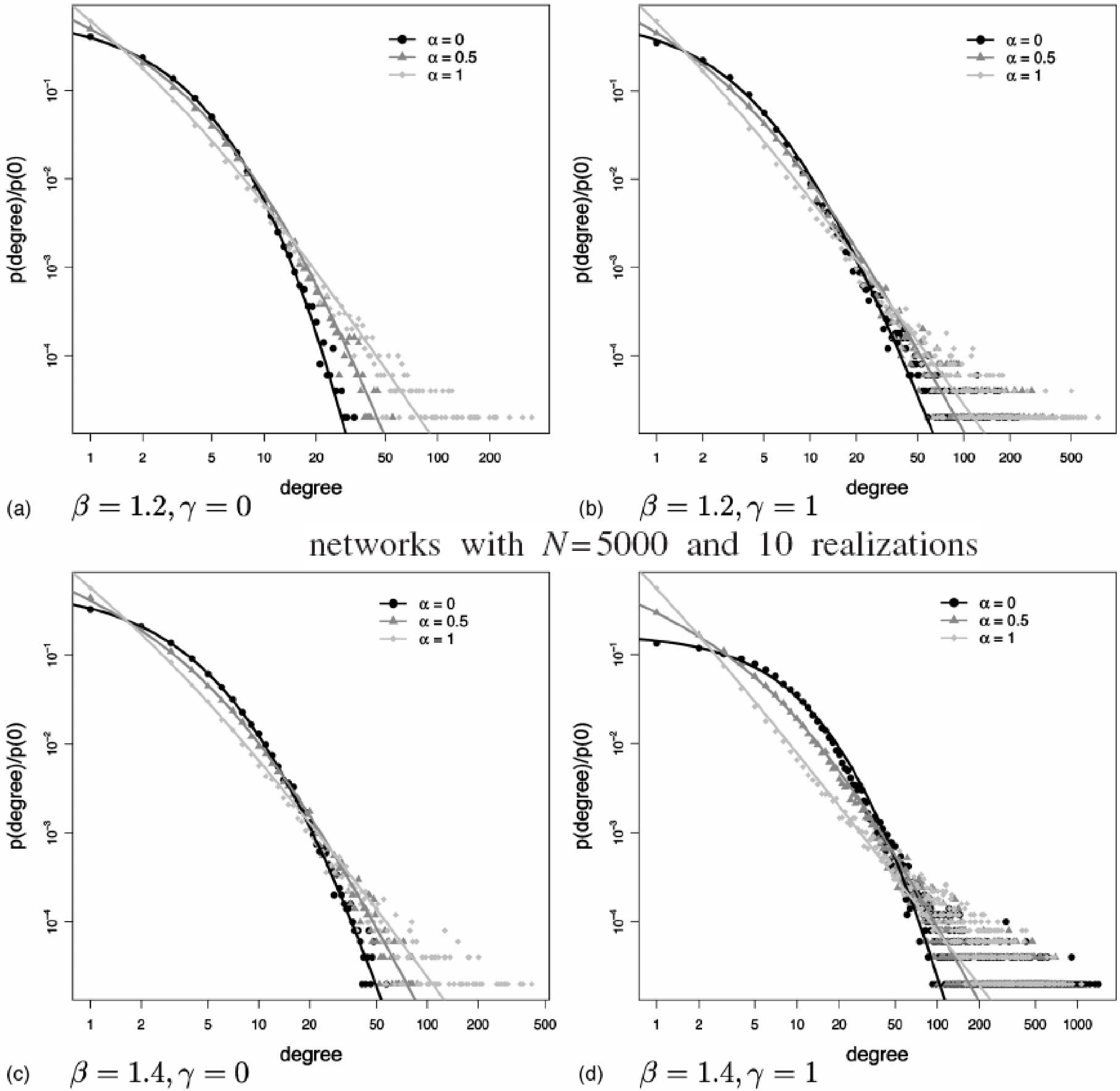
distance decay parameter $\beta > 1;\ d = 1,\ 2,\ 3\ldots$

$$P_\gamma(l) = \frac{[1 + u(l)^\gamma]}{\sum_{m=1}^{M} [1 + u(m)^\gamma]}$$

routing parameter $\gamma > 0$

$u(l) \equiv$ *number of neighbors of node* l *that have not yet been visited*

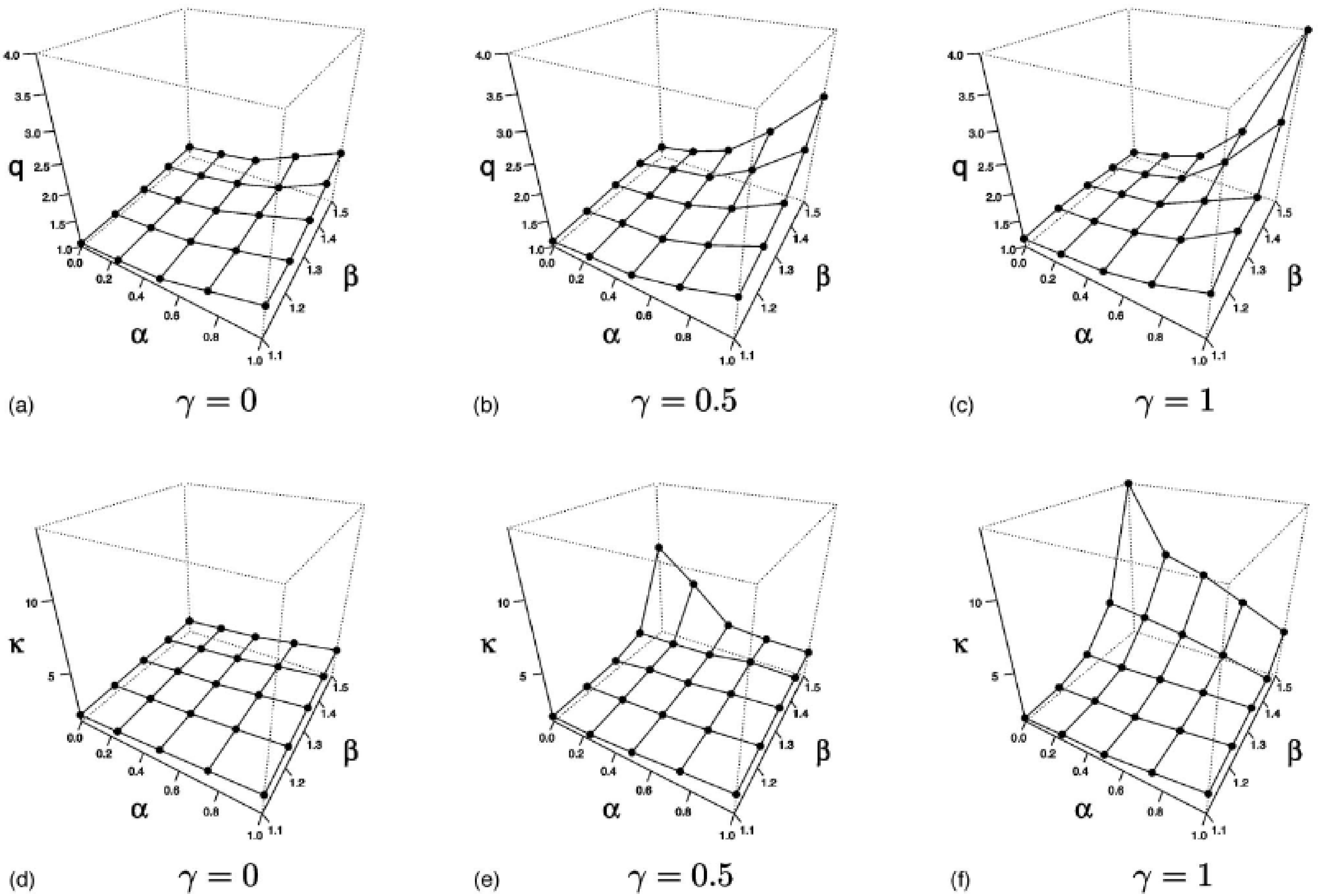
D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)



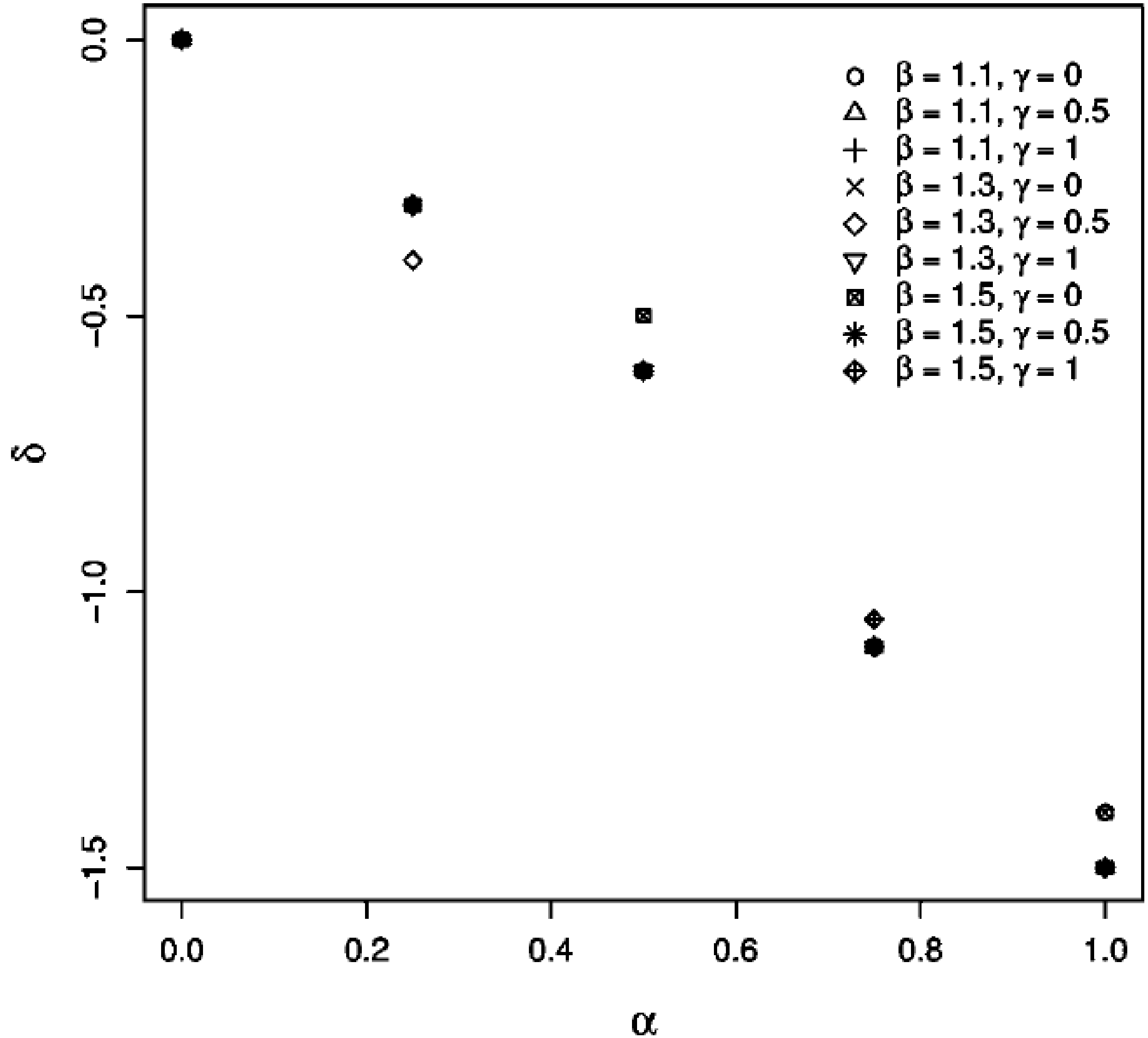
networks with N=5000 and 10 realizations

$$p(k) = p_0 k^{\delta} e_q^{-k/\kappa}$$

D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)



D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)



The quality of the fittings has been shown to be satisfactory through the nonparametric statistical Kolmogorov-Smirnov and the Wilkoxon rank sum tests

D.R. White, N. Kejzar, C. T., D. Farmer and S. White, Phys Rev E 73, 016119 (2006)

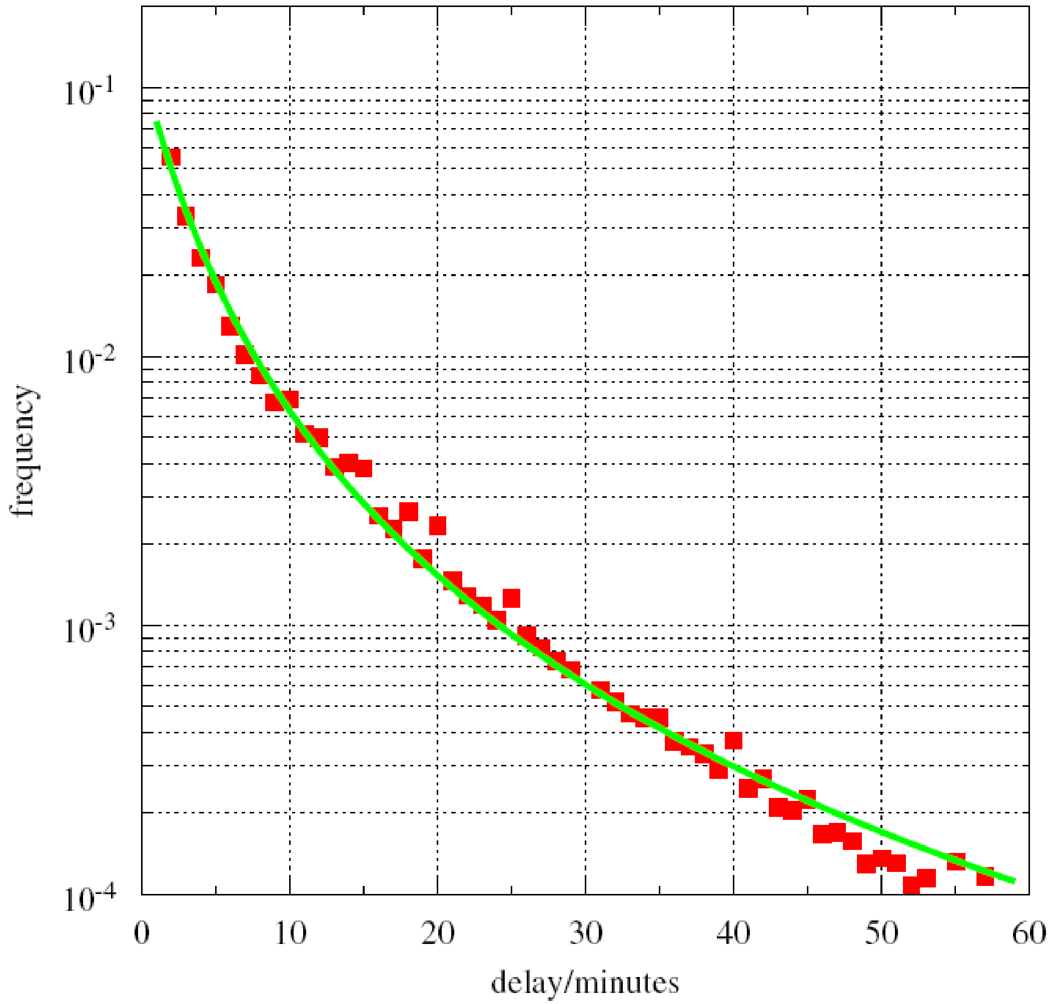
HOW COME THE DEGREE DISTRIBUTION COINCIDES WITH THAT MAXIMIZING S*q* ?

If we associate with each bond an "energy" ε,
we may associate with each node $(i = 1,\ 2, \ldots,\ N)$ *the energy* $k_i \varepsilon / 2$.

Therefore the degree distribution coincides with the energy distribution!

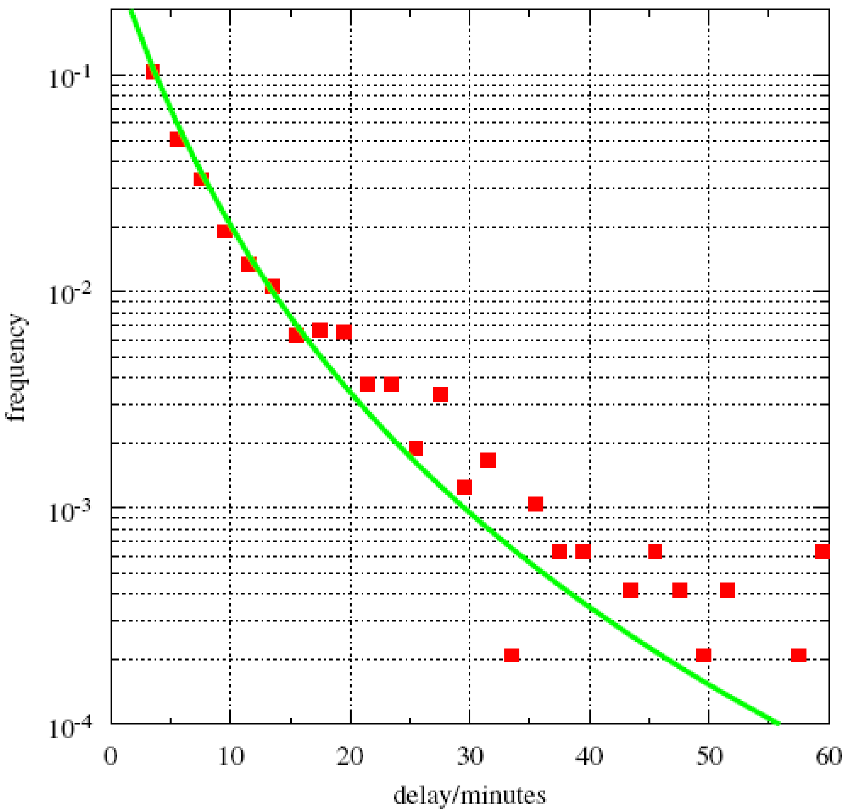
TRAIN DELAYS ON THE BRITISH RAILWAY NETWORK:

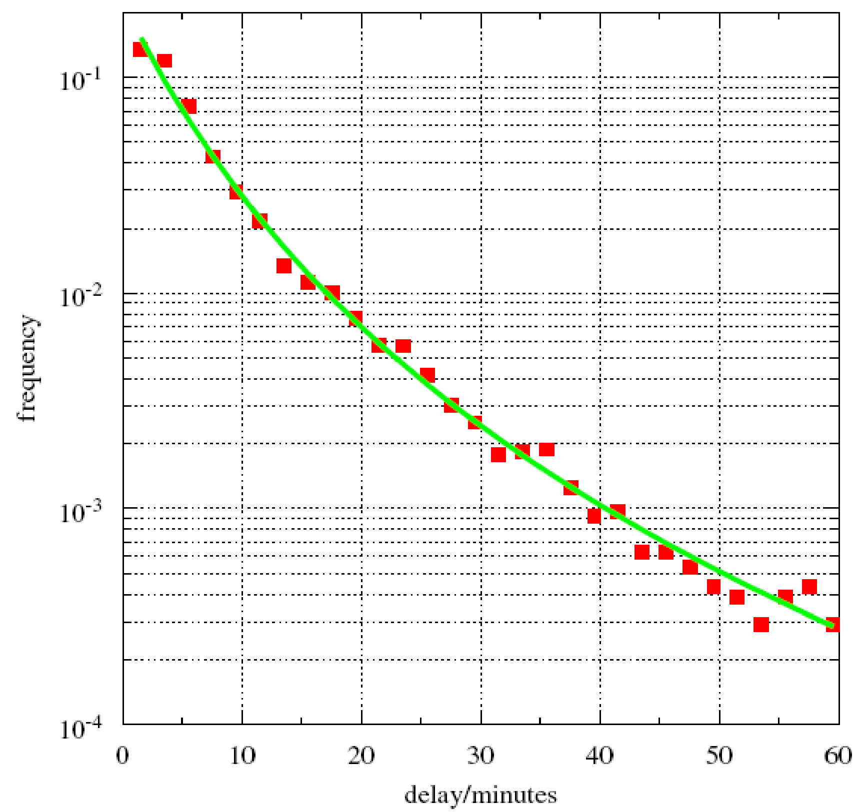
K. Briggs and C. Beck, Physica A 378, 498 (2007)



All train data and best-fit q-exponential: $q = 1.355 \pm 8.8 \times 10^{-5}$, $b = 0.524 \pm 2.5 \times 10^{-8}$.

$$e_{q,b,c}(t) = c(1 + b(q-1)t)^{1/(1-q)}$$





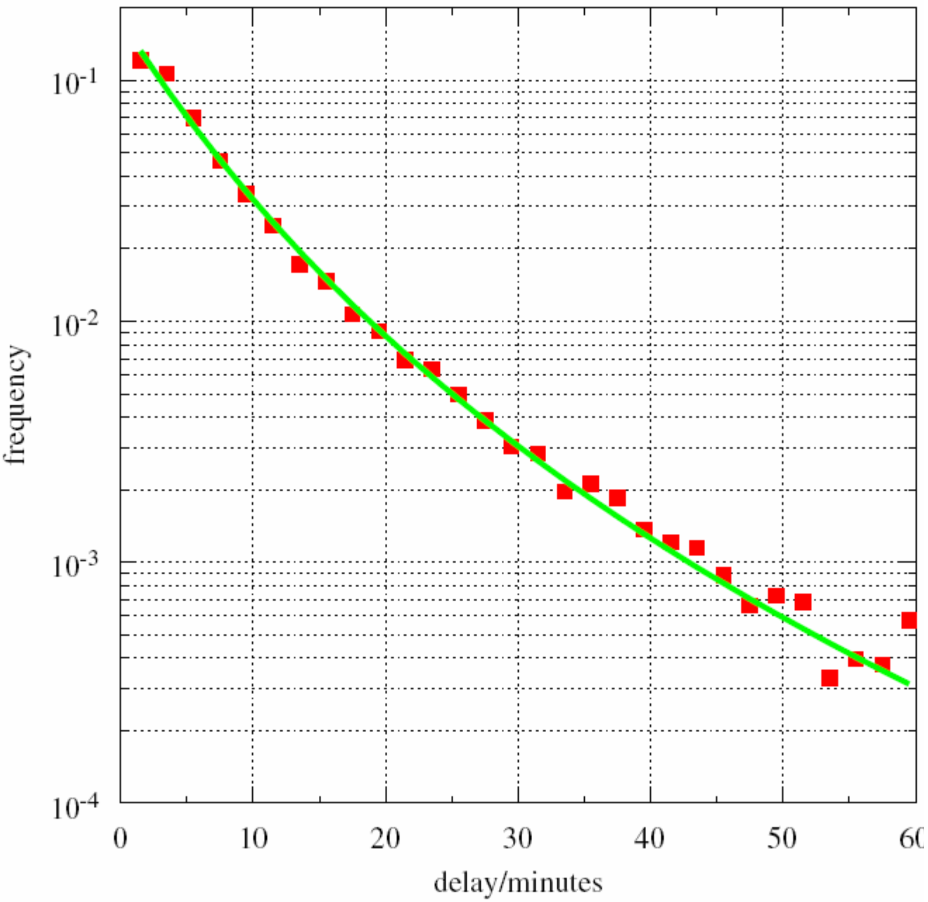
Bath Spa to London Paddington

$$q = 1.215 \pm 0.015$$

Swindon to London Paddington

$$q = 1.230 \pm 0.0086$$

K. Briggs and C. Beck, Physica A 378, 498 (2007)



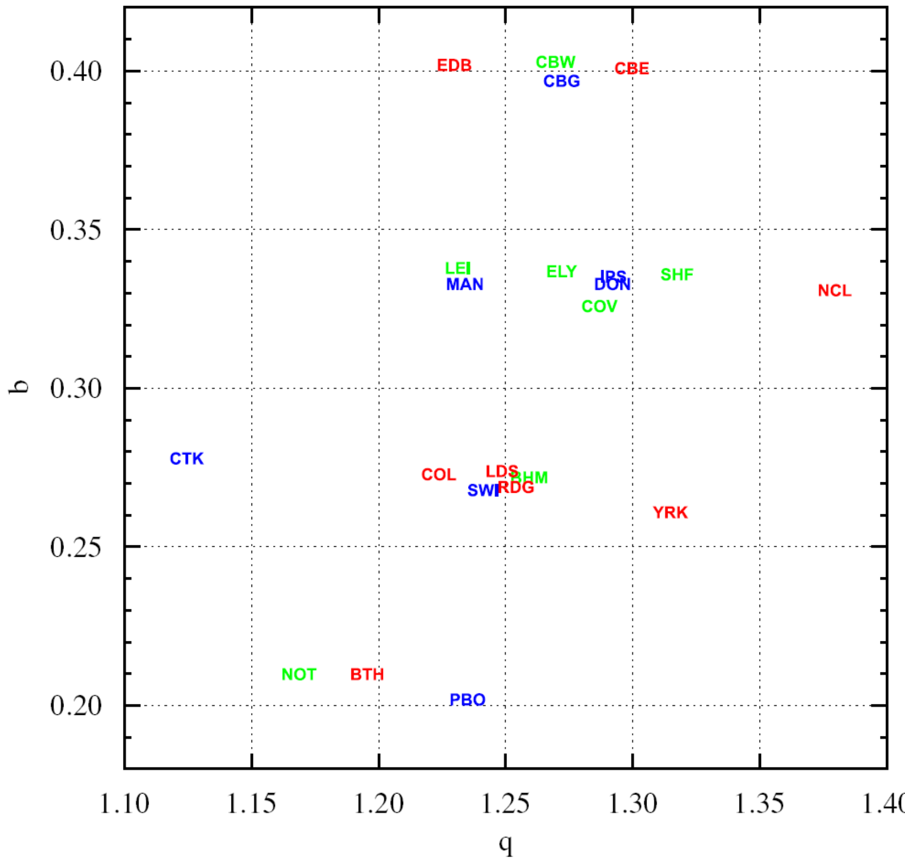


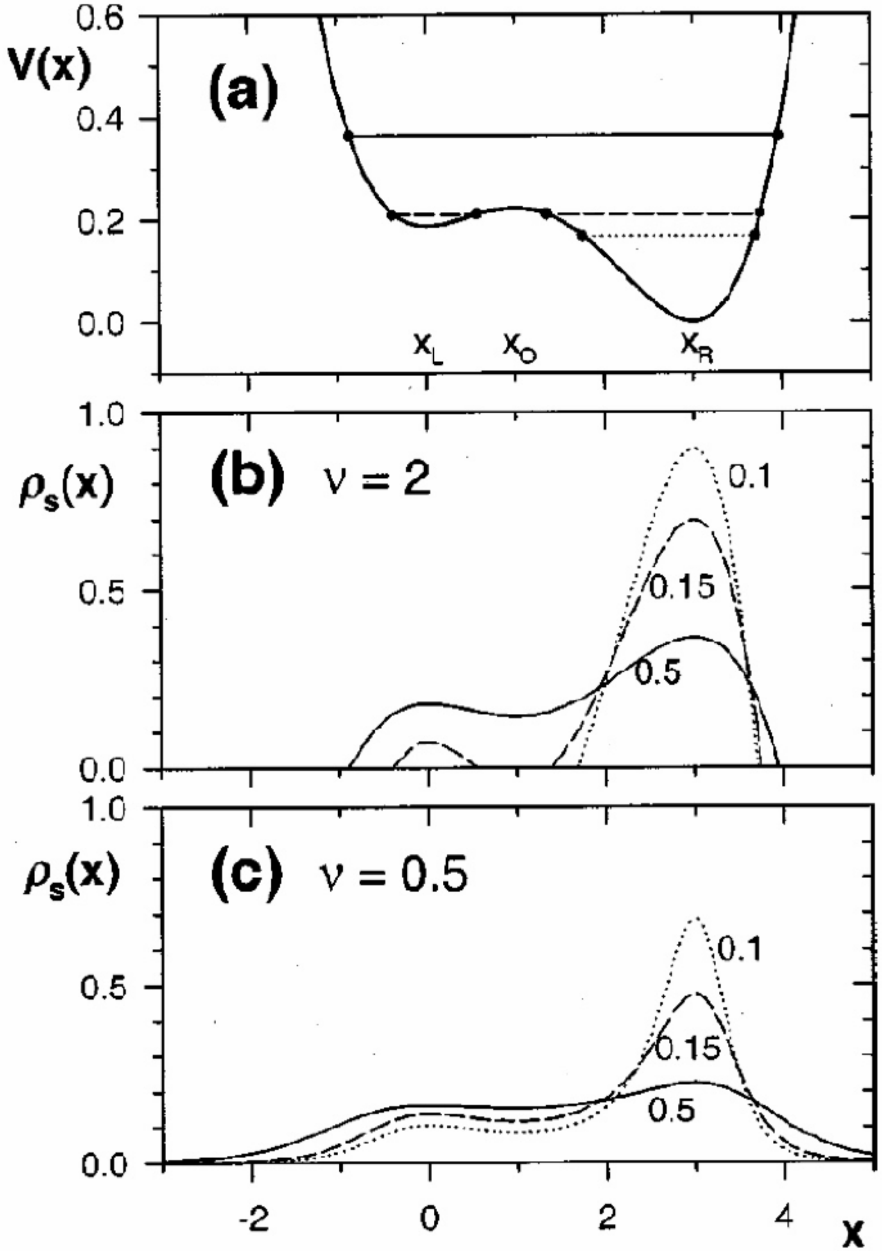
Fig. 5. The estimated parameters q and b for 23 stations.

Reading to London Paddington

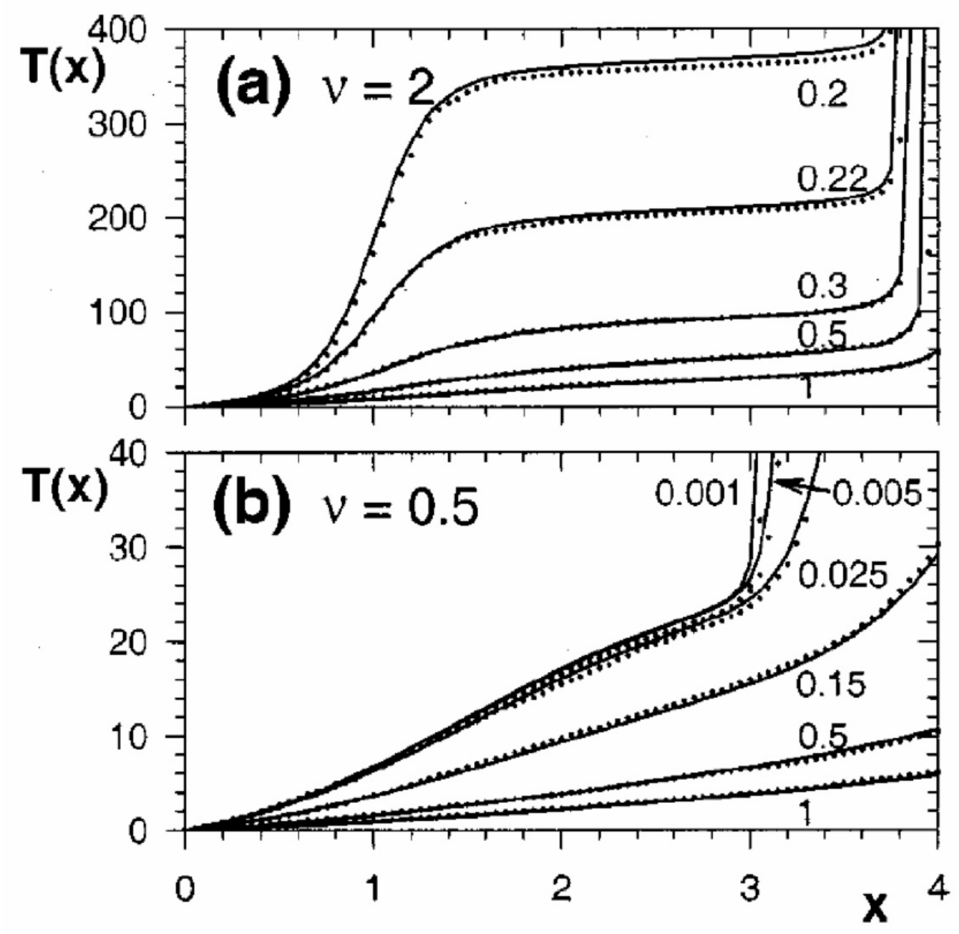
$$q = 1.183 \pm 0.0063$$

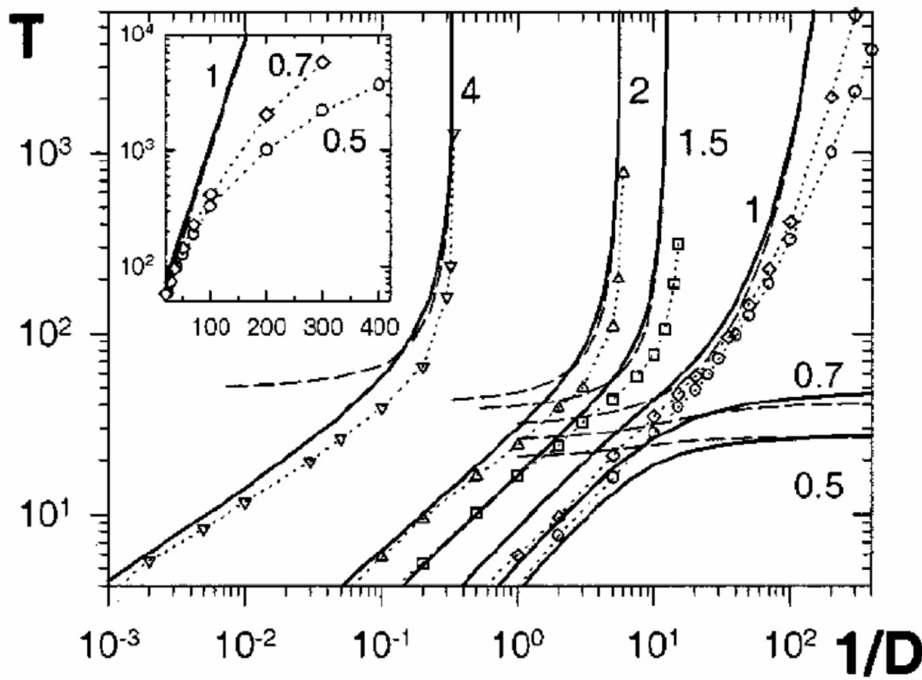
K. Briggs and C. Beck, Physica A 378, 498 (2007)

ANOMALOUS DIFFUSION, ESCAPE TIME AND GENERALIZED ARRHENIUS LAW



E.K. Lenzi, C. Anteneodo and L. Borland, Phys Rev E **63**, 051109 (2001)





E.K. Lenzi, C. Anteneodo and L. Borland, Phys Rev E **63**, 051109 (2001)

THEORETICAL PREDICTIONS

AND

EXPERIMENTAL – OBSERVATIONAL – COMPUTATIONAL VERIFICATIONS

PREDICTION OF A SCALING RELATION:

The solution of

$$\frac{\partial\, p(x,t)}{\partial t} = D\,\frac{\partial^2 [p(x,t)]^{2-q}}{\partial x^2} \qquad [p(x,0)=\delta(0)] \qquad (q<3)$$

is given by

$$p(x,t) \propto \left[1+(1-q)\,x^2 / \,(\Gamma t)^{2/(3-q)}\right]^{1/(1-q)} \equiv e_q^{-x^2/\,(\Gamma t)^{2/(3-q)}} \qquad (\Gamma \propto D)$$

hence

x^2 *scales like* t^{γ} $\quad$ (*e.g.*, $\langle x^2 \rangle \propto t^{\gamma}$)

with

$$\gamma = \frac{2}{3-q}$$

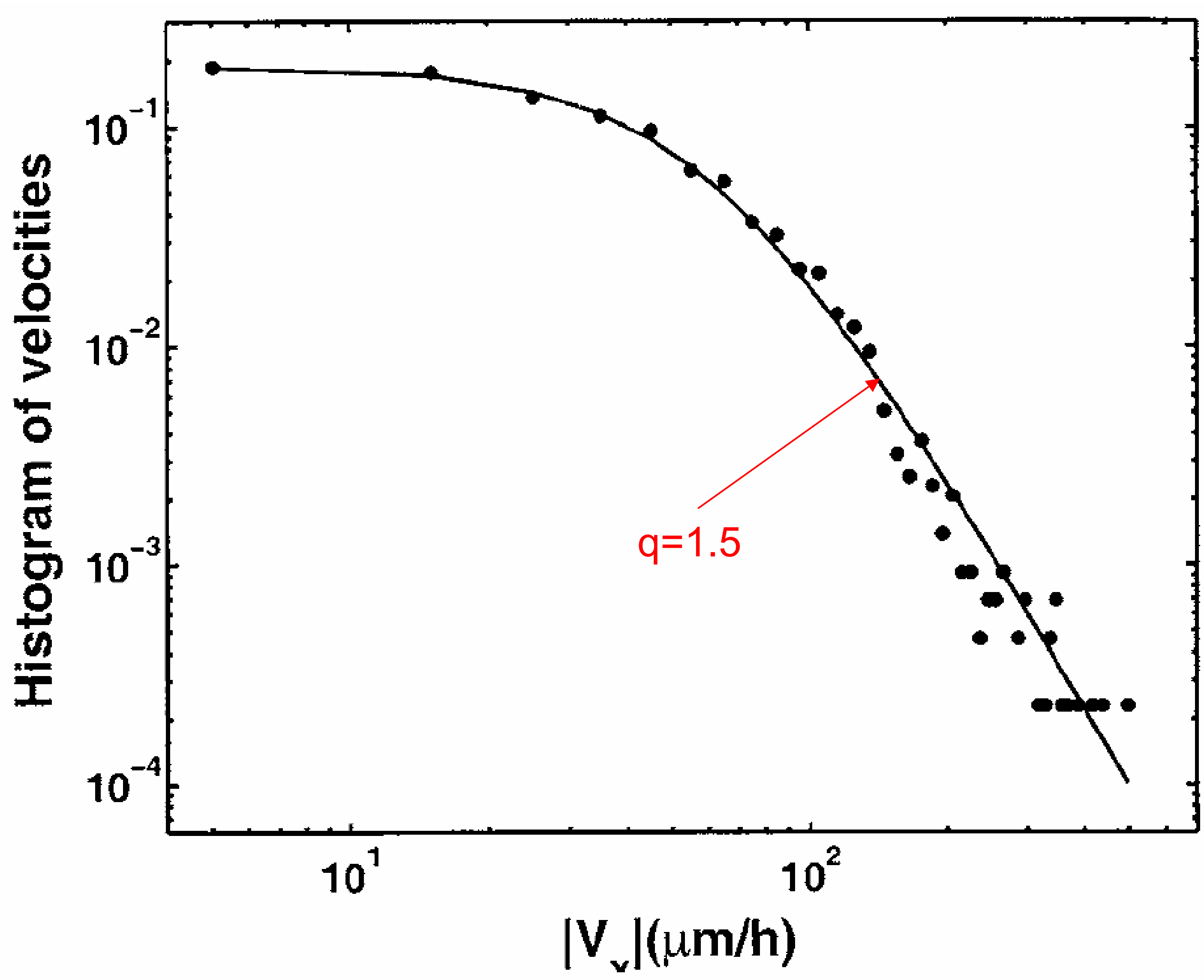
$(e.g.,\ q=1 \Rightarrow \gamma=1\ ,\ i.e.,\ normal\ diffusion)$

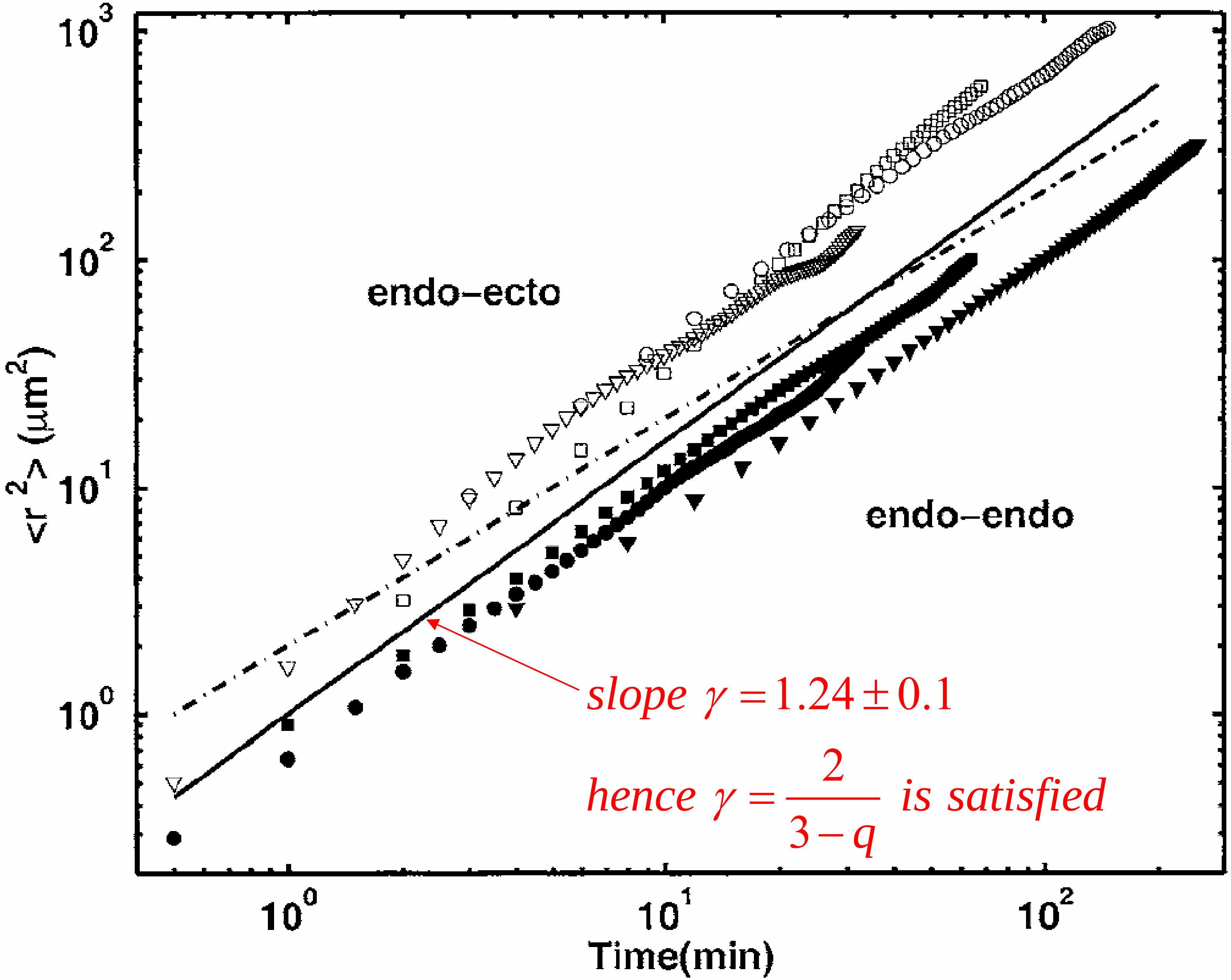
C.T. and D.J. Bukman, Phys Rev E **54**, R2197 (1996)

(see also AR Plastino and A Plastino, Physica A **222**, 347 (1995))

Hydra viridissima:

A. Upadhyaya, J.-P. Rieu, J.A. Glazier and Y. Sawada
Physica A **293**, 549 (2001)

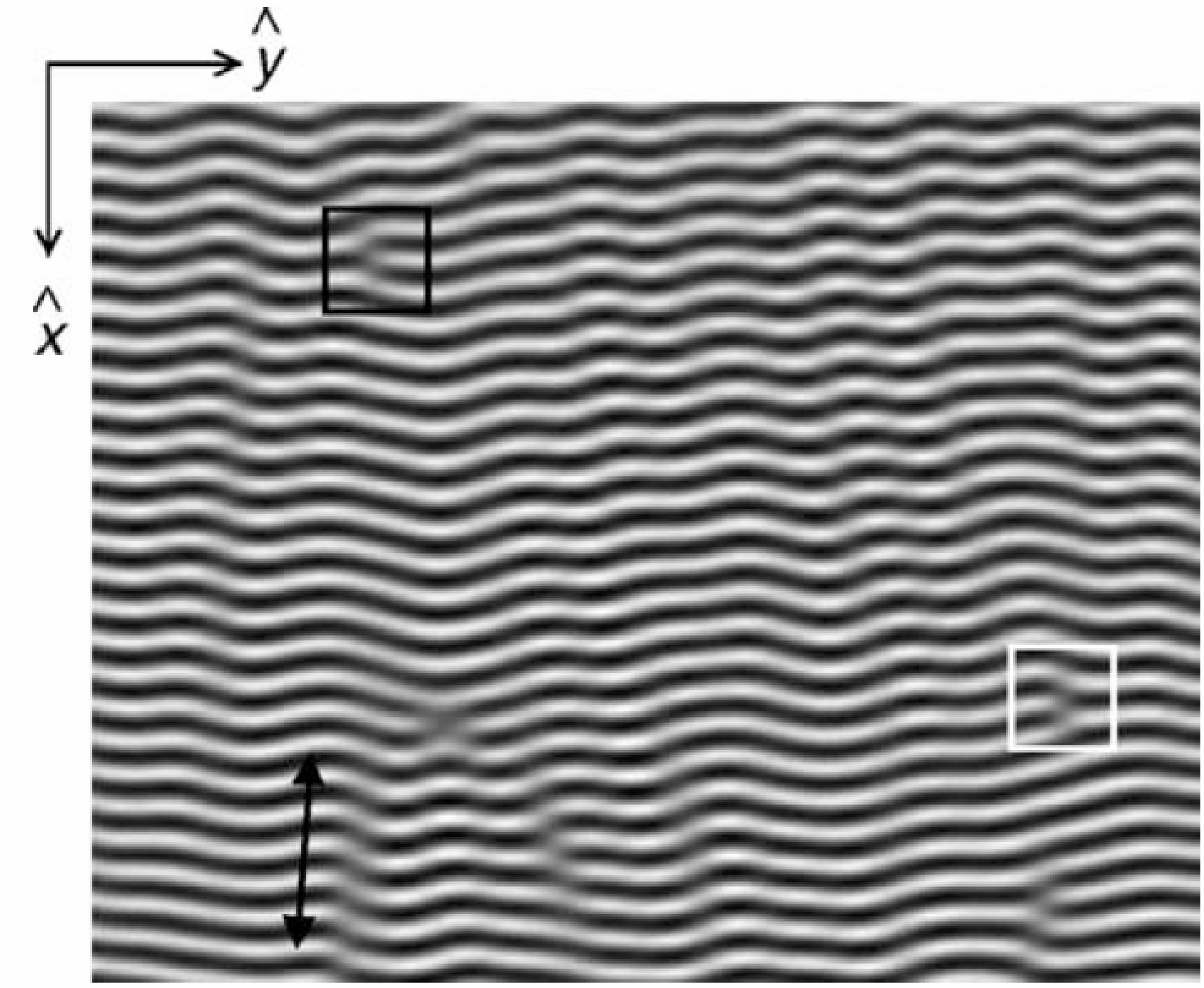


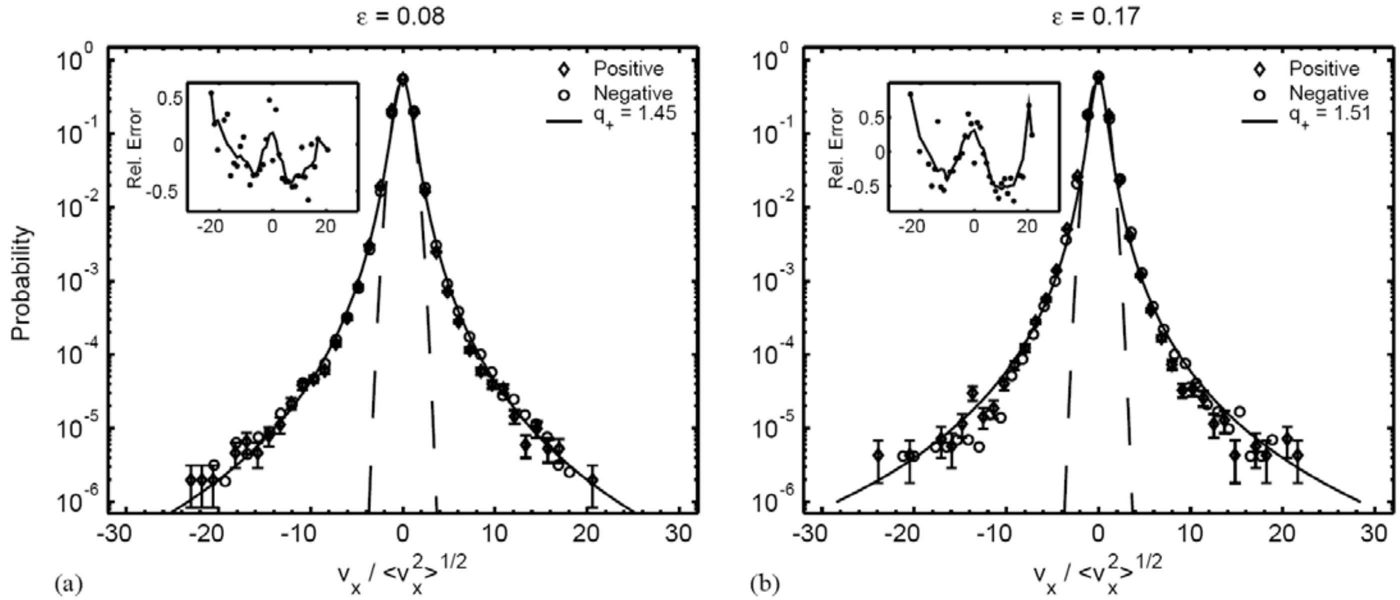


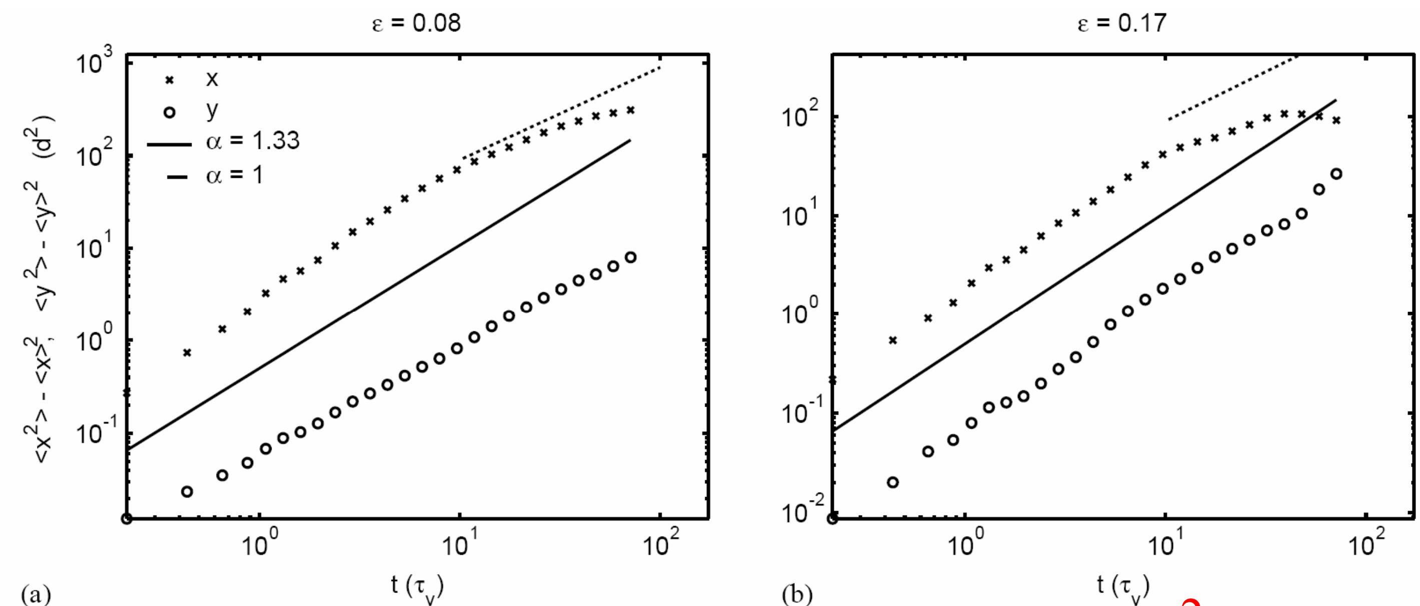
endo−ecto
endo−endo
<r 2> (μm2)
Time(min)
slope γ = 1.24 ± 0.1
hence γ = 2/(3 − q) is satisfied

Defect turbulence:

K.E. Daniels, C. Beck and E. Bodenschatz, Physica D **193**, 208 (2004)





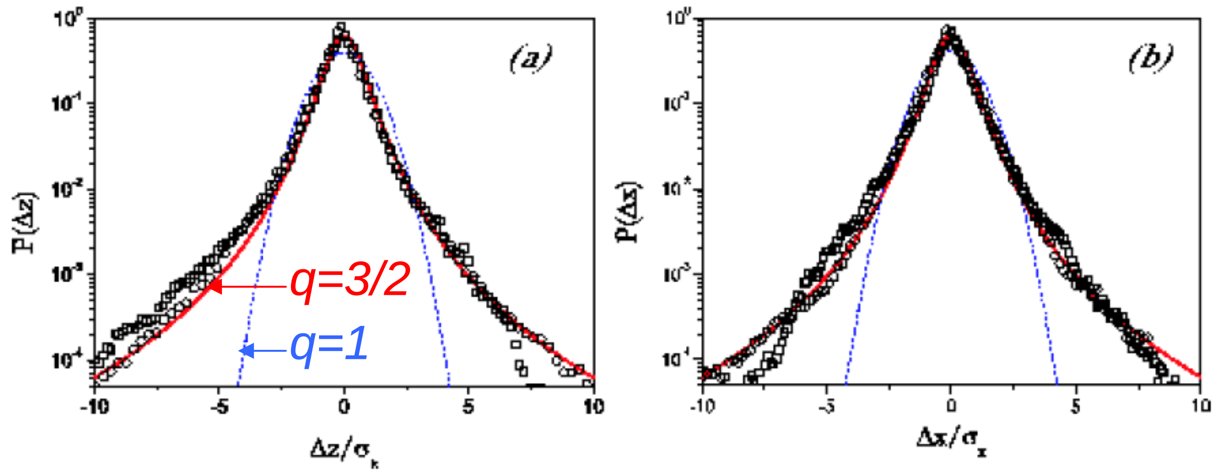


$q \approx 1.5$ *and* $\gamma \approx 4/3$ *are consistent with* $\gamma = \dfrac{2}{3-q}$

K.E. Daniels, C. Beck and E. Bodenschatz, Physica D **193**, 208 (2004)

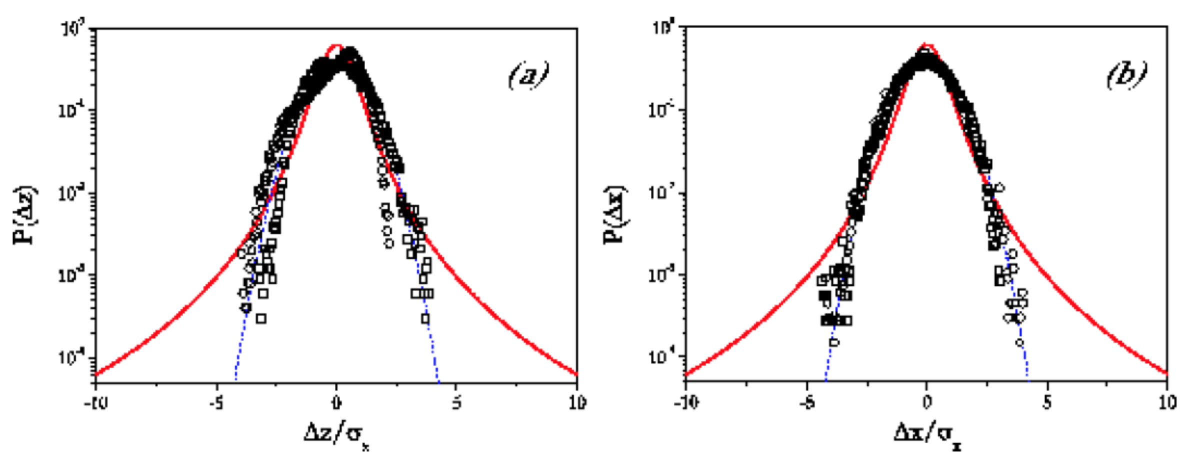
Silo drainage:

R. Arevalo, A. Garcimartin and D. Maza, cond-mat/0607365 (2006)



(intermediate regime)

Fig. 2 – Normalized PDFs for (a) vertical and (b) horizontal displacements. The symbols indicate the orifice aperture in the silo: circles for 3.8 d and squares for 11 d. The dotted line is a gaussian and the continuous line is Eq. (5).

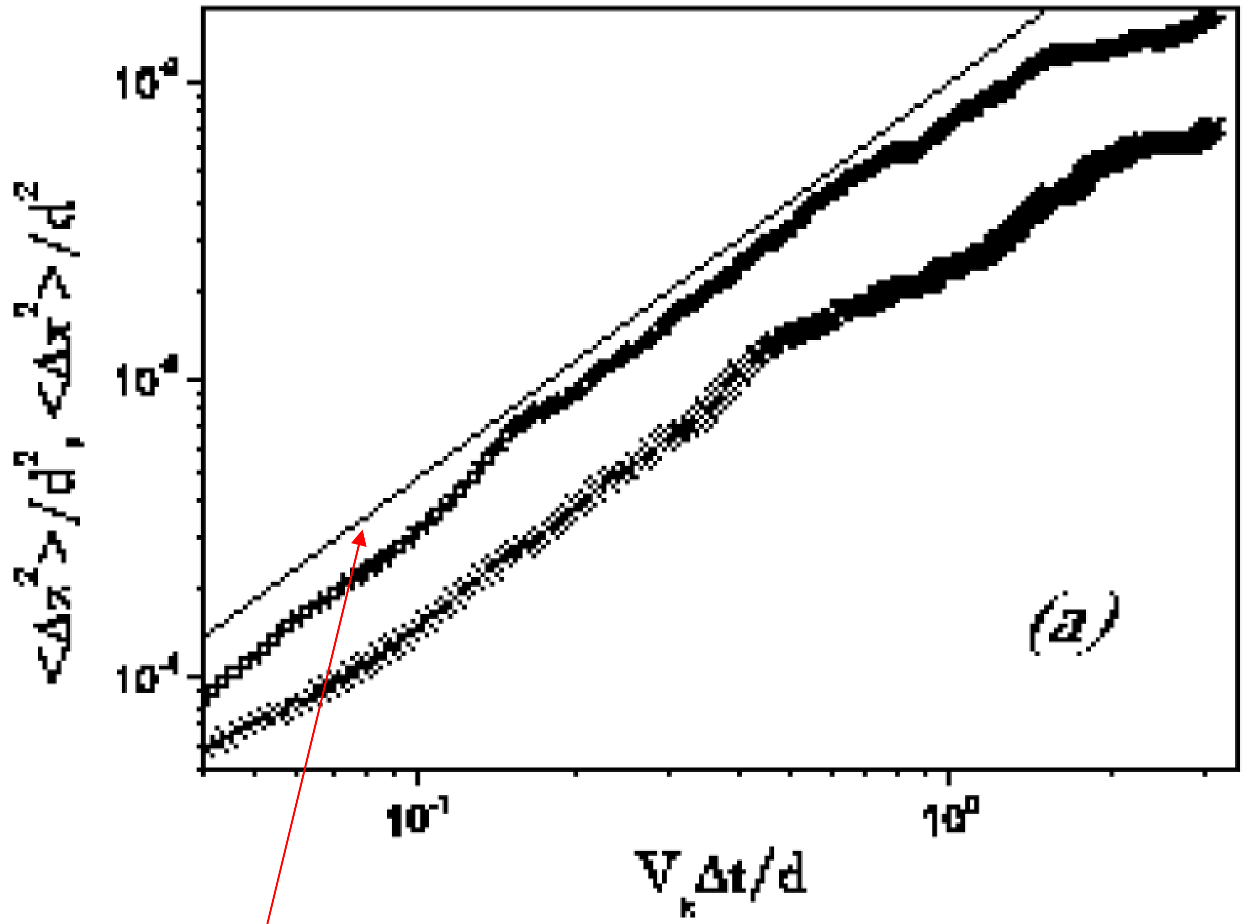


(fully developed regime)

Fig. 4 – Normalized PDFs for (a) vertical and (b) horizontal displacements for the fully developed discharge regime. Both cases fit well to a gaussian profile. Nevertheless, some skewness, similar to the experimental results (particularly in the case of the 11 d orifice) can be observed for the vertical direction. It is probably related to the difficulty of adequately defining the mean value of the vertical velocity.

R. Arevalo, A. Garcimartin and D. Maza, cond-mat/0607365 (2006)

(outlet size 3.8 d)



$$\textit{slope}\ \ \gamma = 4/3$$

$$\textit{hence}\ \ \gamma = \frac{2}{3-q}\ \ \textit{is satisfied}$$

d-DIMENSIONAL CLASSICAL INERTIAL *XY* FERROMAGNET:

(We illustrate with the *XY* (i.e., *n=2*) model; the argument holds however true for any *n>1* and any *d*-dimensional Bravais lattice)

$$H = K + V = \frac{1}{2I}\sum_{i=1}^{N} L_i^2 + \frac{J}{\mathfrak{A}}\sum_{i,j}\frac{1-\cos(\vartheta_i - \vartheta_j)}{r_{ij}^{\alpha}} \qquad (I>0,\; J>0)$$

$$\text{with} \;\; \mathfrak{A} \equiv \sum_{j=1}^{N} r_{ij}^{-\alpha} \propto \left\{ \begin{array}{lll} N^{1-\alpha/d} & \text{if} & 0 \le \alpha/d < \\ \ln N & \text{if} & \alpha/d = \\ \text{constant} & \text{if} & \alpha/d > \end{array} \right\}$$

and periodic boundary conditions.

[*The HMF model corresponds to* $\alpha/d = 0$]

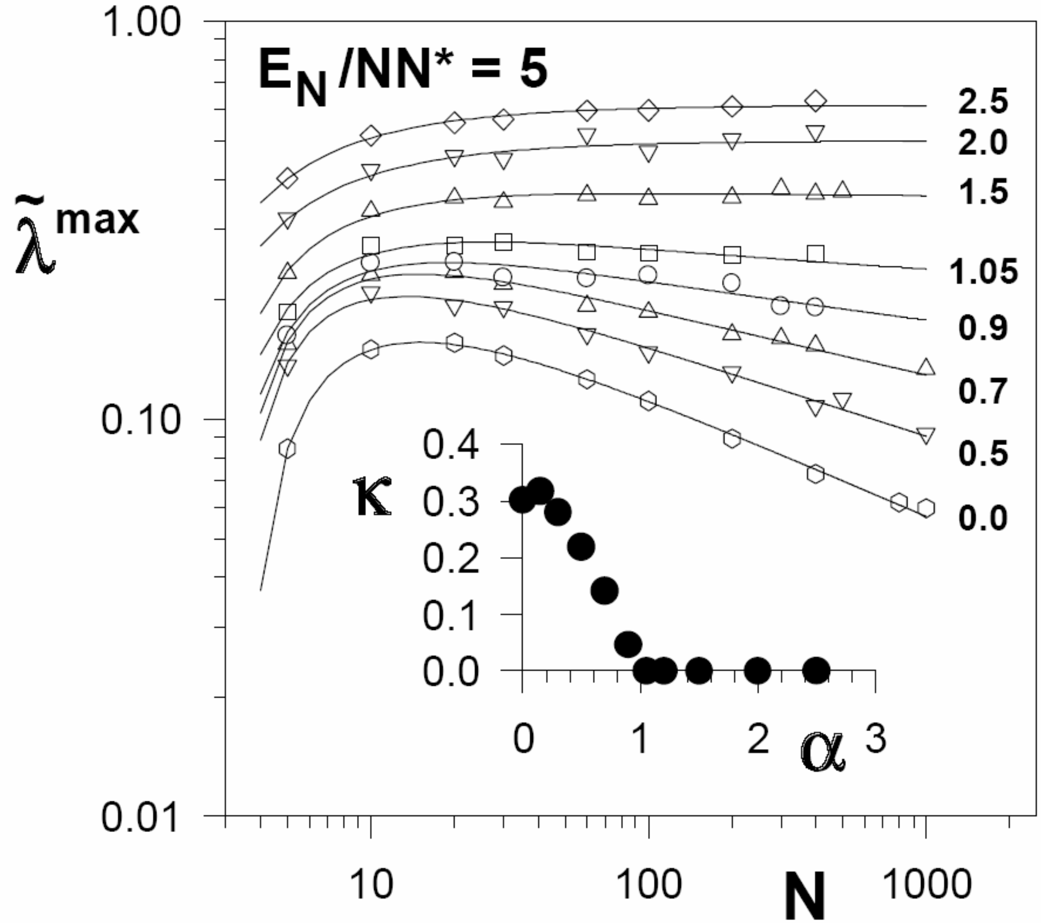
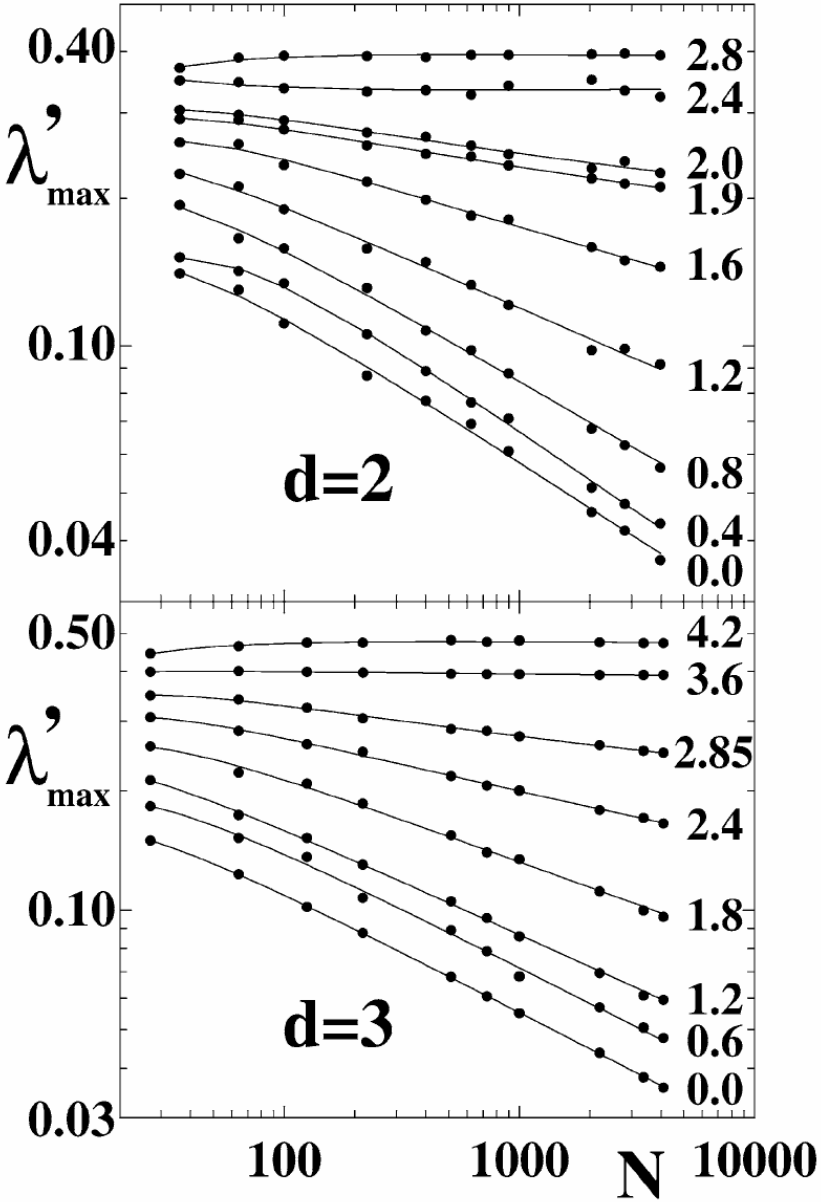
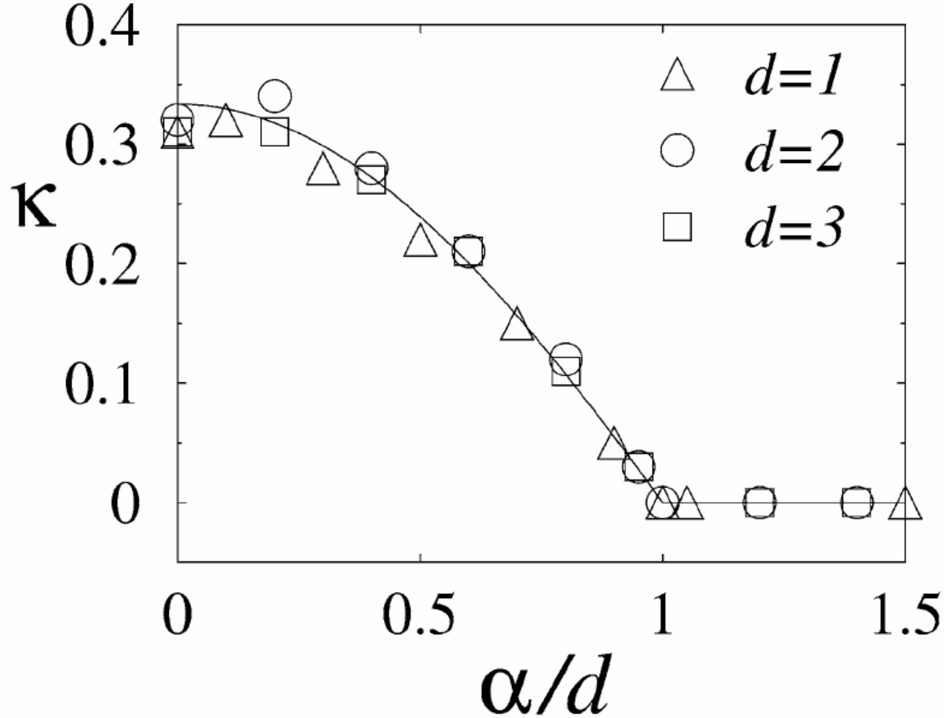
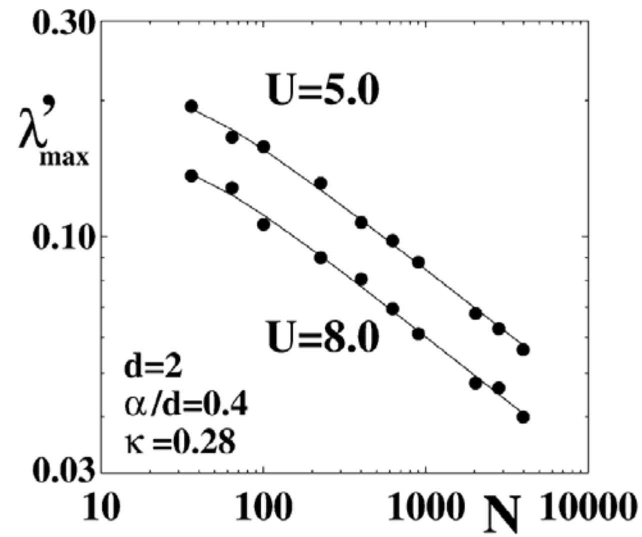


FIG. 3. $\tilde{\lambda}_N^{\max}$ versus N (log-log plot) for typical values of α and $\frac{E_N}{NN^*} = 5$. The full lines are the best fittings with the forms $(a - \frac{b}{N})/(N^*)^c$. Consequently, $\tilde{\lambda}_N^{\max} \propto N^{-\kappa(\alpha)}$ where $\kappa(\alpha) = (1 - \alpha)\,c$ for $0 \le \alpha < 1$ and $\kappa(\alpha) = 0$ for $\alpha > 1$; for $\alpha = 1$, $\tilde{\lambda}_N^{\max}$ is expected to vanish as a power of $1/\ln N$. Inset: κ versus α (related random matrices arguments will be detailed elsewhere).

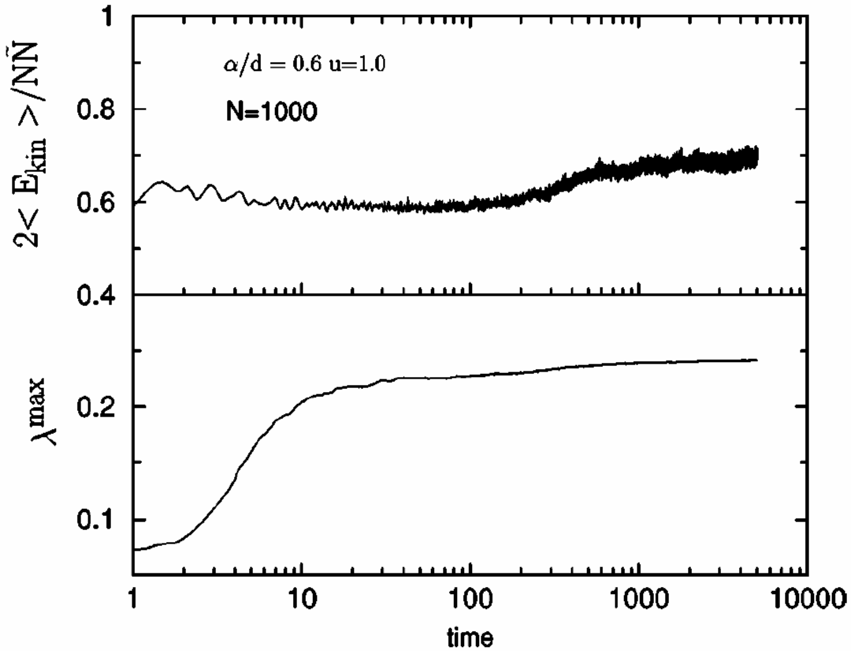
C. Anteneodo and C. T., Phys Rev Lett **80**, 5313 (1998)

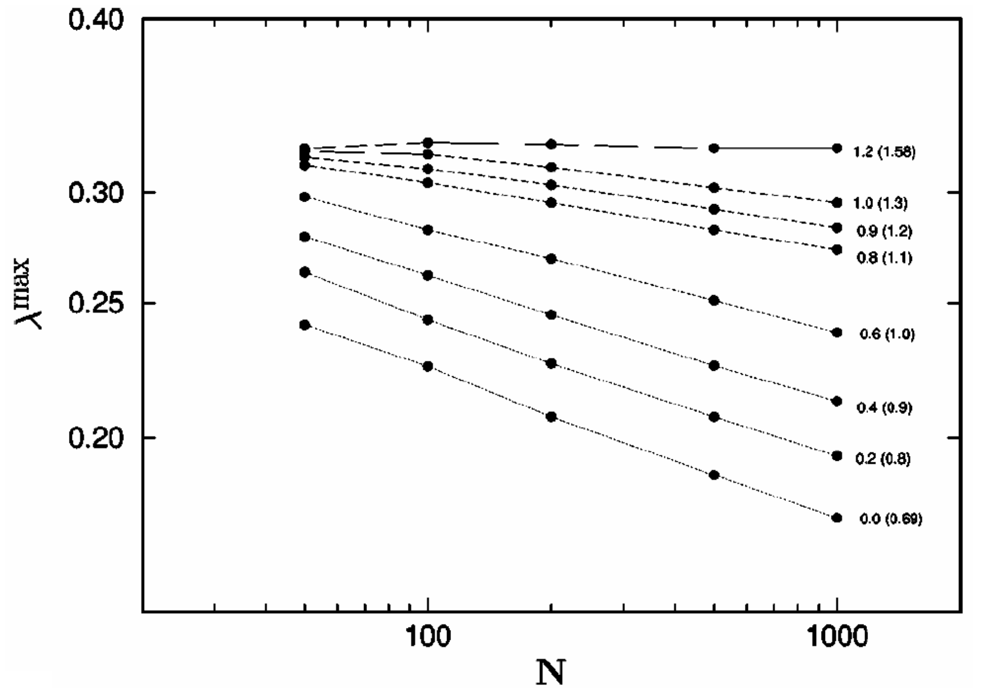


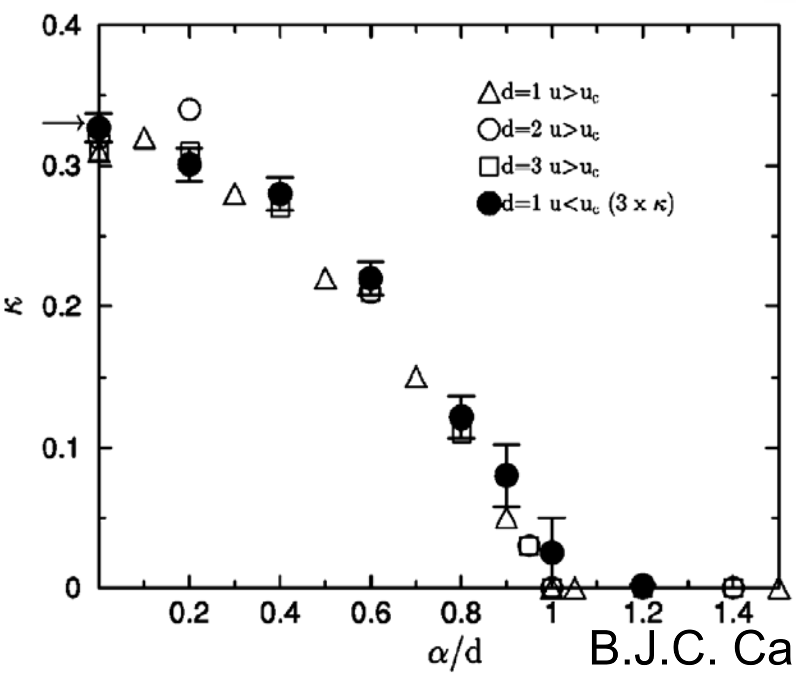




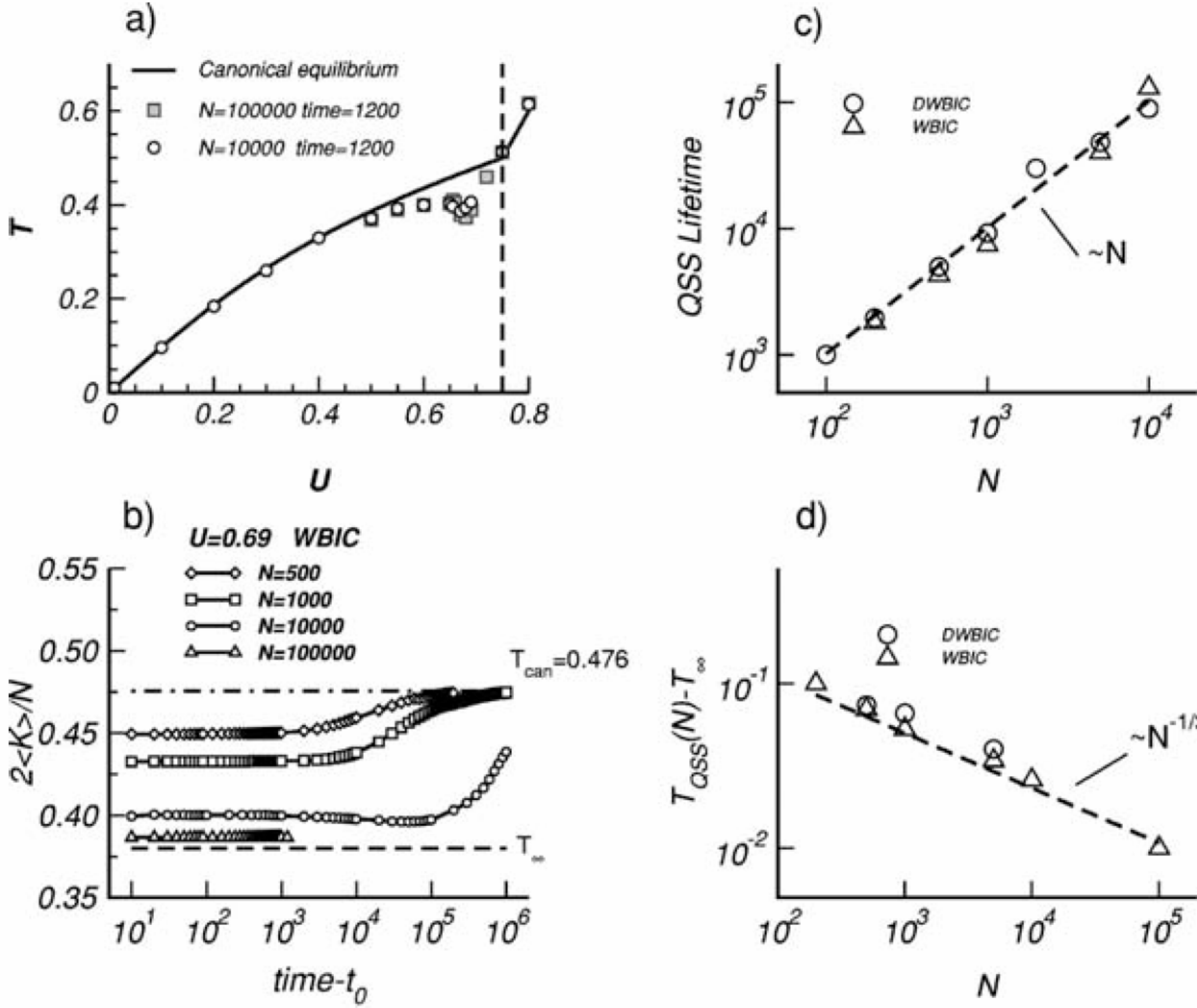
A. Campa, A. Giansanti, D. Moroni and C. T., Phys Lett A **286**, 251 (2001)







B.J.C. Cabral and C. T., Phys Rev E **66**, 065101 (2002)

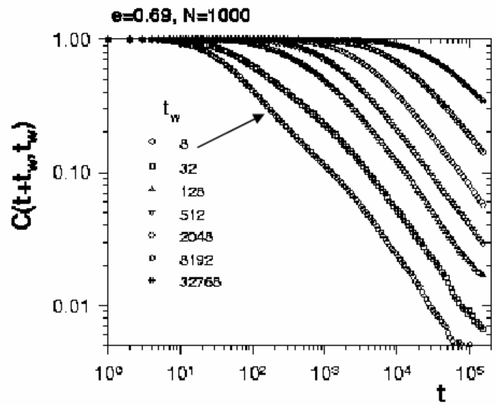


V. Latora, A. Rapisarda and C. T., Phys Rev E **64**, 056134 (2001)

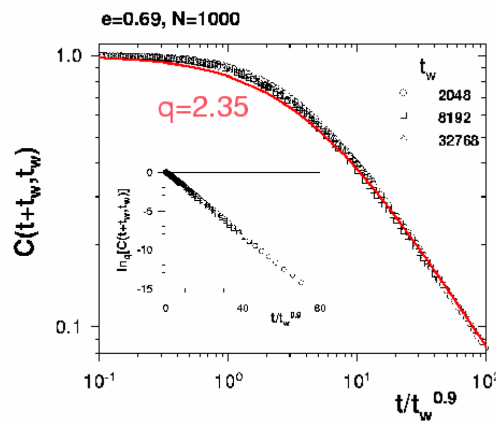
AGING $[\ \vec{x} \equiv (\vec{\theta}, \vec{L})\ ]$

Montemurro, Tamarit and Anteneodo, PRE (2003)

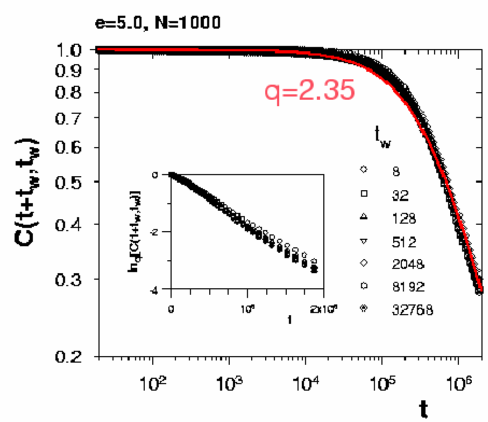
e=0.69



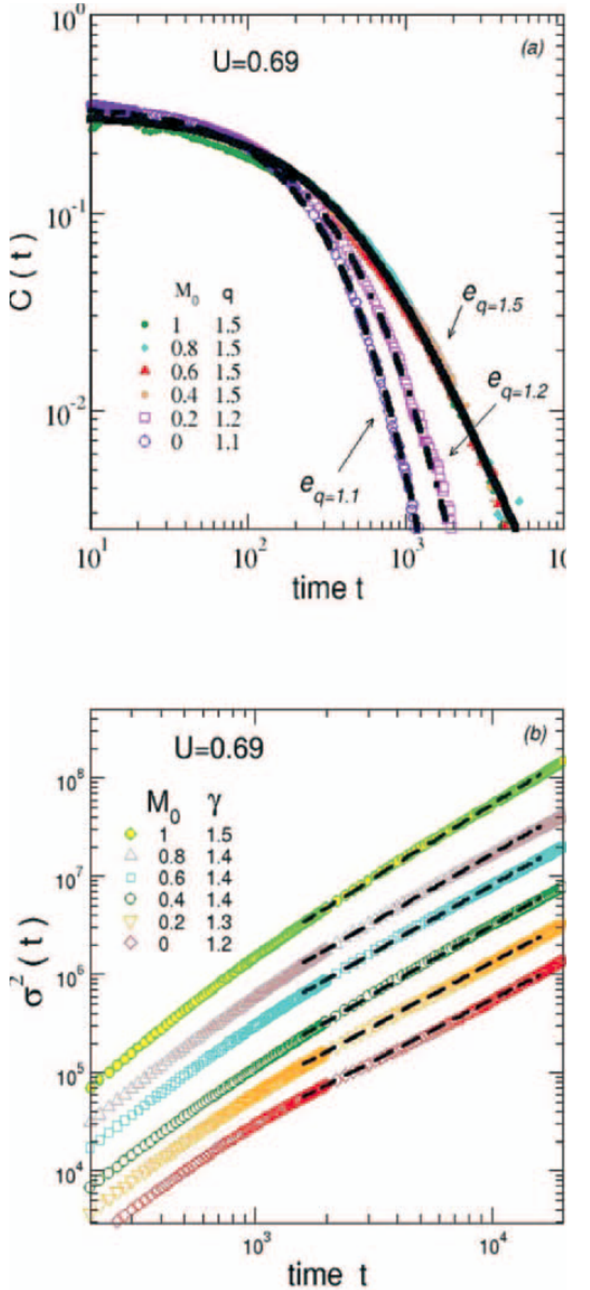
e=0.69



e=5.0

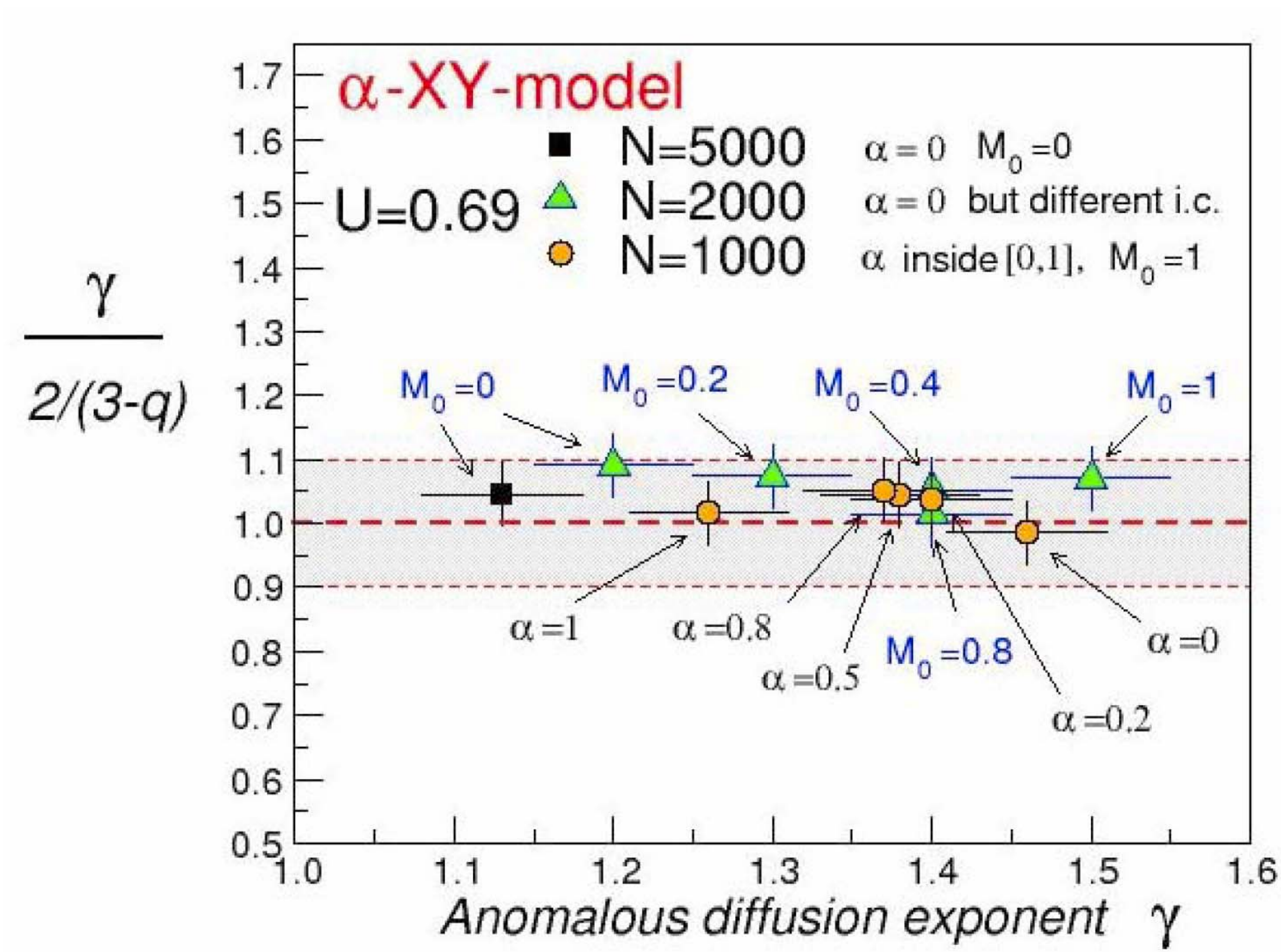


XY FERROMAGNET WITH LONG-RANGE INTERACTIONS:



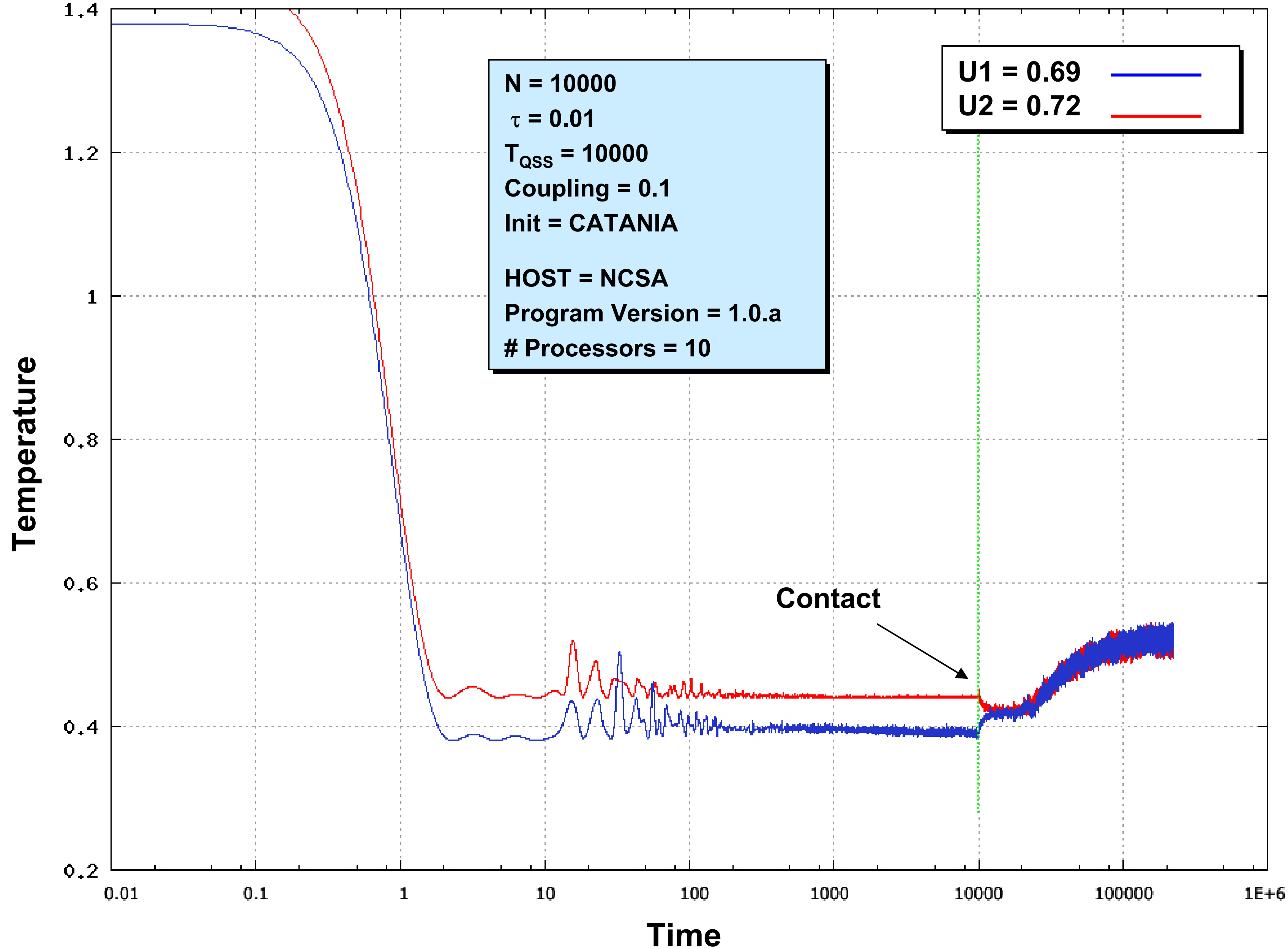
A. Rapisarda and A. Pluchino,
Europhys News **36**, 202
(European Physical Society, Nov/Dec 2005)

XY FERROMAGNET WITH LONG-RANGE INTERACTIONS:



A. Rapisarda and A. Pluchino, Europhys News **36**, 202 (2005)
(European Physical Society)

Charm++ NExtComp Molecular Dynamics – 2 Systems Interactions



BOLTZMANN-GIBBS STATISTICAL MECHANICS

(Maxwell 1860, Boltzmann 1872, Gibbs $\leq$ 1902)

Entropy $$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i$$

Internal energy $$U_{BG} = \sum_{i=1}^{W} p_i E_i$$

Equilibrium distribution $$p_i = e^{-\beta E_i} / Z_{BG} \quad \left(Z_{BG} \equiv \sum_{j=1}^{W} e^{-\beta E_j} \right)$$

Paradigmatic differential equation $$\left. \begin{aligned} \frac{dy}{dx} &= a\,y \\ y(0) &= 1 \end{aligned} \right\} \Rightarrow \quad y = e^{ax}$$

	x	a	$y(x)$
Equilibrium distribution	E_i	$-\beta$	$Z\,p(E_i)$
Sensitivity to initial conditions	t	λ	$\xi \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)} = e^{\lambda t}$
Typical relaxation of observable O	t	$-1/\tau$	$\Omega \equiv \frac{O(t) - O(\infty)}{O(0) - O(\infty)} = e^{-t/\tau}$

S_{BG} → extensive, concave, Lesche-stable, finite entropy production

NONEXTENSIVE STATISTICAL MECHANICS

(C. T. 1988, E.M.F. Curado and C. T. 1991, C. T., R.S. Mendes and A.R. Plastino 1998)

Entropy $$S_q = k\left(1-\sum_{i=1}^{W} p_i^q\right)/(q-1)$$

Internal energy $$U_q = \sum_{i=1}^{W} p_i^q E_i / \sum_{j=1}^{W} p_j^q$$

Stationary state distribution $$p_i = e_q^{-\beta_q (E_i - U_q)} / Z_q \qquad \left(Z_q \equiv \sum_{j=1}^{W} e_q^{-\beta_q (E_j - U_q)} \right)$$

Paradigmatic differential equation $$\left.\begin{array}{l} \dfrac{dy}{dx} = a\, y^{q} \\ y(0)=1 \end{array}\right\} \Rightarrow \quad y = e_q^{ax} \equiv [1 + (1-q)ax]^{\frac{1}{1-q}}$$

	x	a	$y(x)$
Stationary state distribution	E_i	$-\beta_{q_{stat}}$	$Z_{q_{stat}}\, p(E_i)$ (typically $q_{stat} \geq 1$)
Sensitivity to initial conditions	t	$\lambda_{q_{sen}}$	$\xi = e_{q_{sen}}^{\lambda_{q_{sen}} t}$ (typically $q_{sen} \leq 1$)
Typical relaxation of observable O	t	$-1/\tau_{q_{rel}}$	$\Omega = e_{q_{rel}}^{-t/\tau_{q_{rel}}}$ (typically $q_{rel} \geq 1$)

$S_q \rightarrow$ extensive, concave, Lesche-stable, finite entropy production

C. T., Physica A **340**,1 (2004)

Prediction of the *q* - triplet: C. T., Physica A **340**,1 (2004)

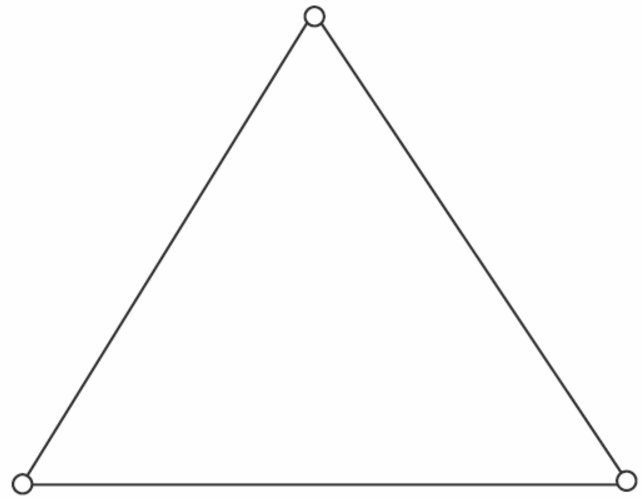
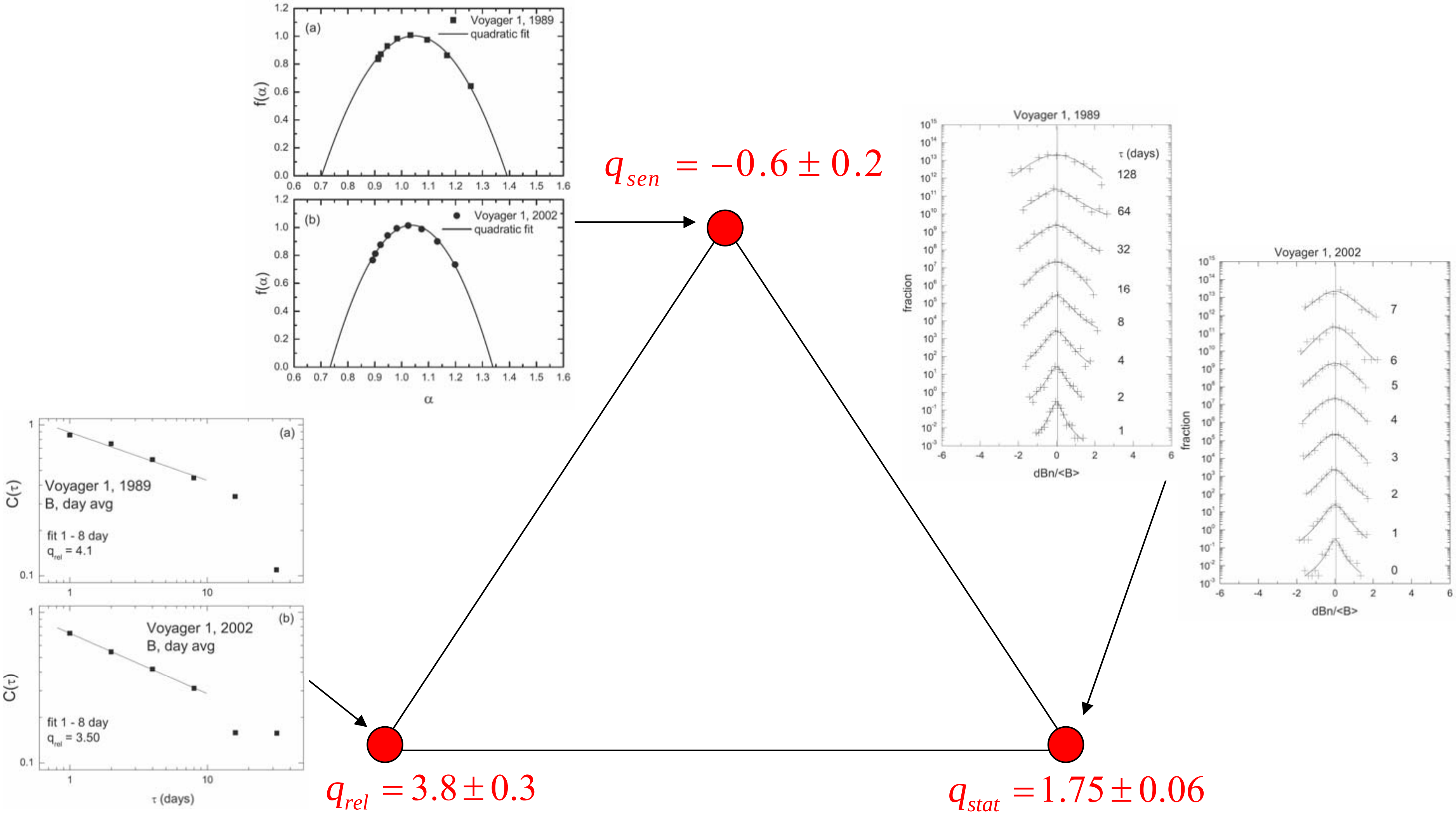


Fig. 2. The triangle of the basic values of q, namely those associated with sensitivity to the initial conditions, relaxation and stationary state. For the most relevant situations we expect $q_{sen} \leqslant 1$, $q_{rel} \geqslant 1$ and $q_{stat} \geqslant 1$. These indices are presumably inter-related since they all descend from the particular dynamical exploration that the system does of its full phase space. For example, for long-range Hamiltonian systems characterized by the decay exponent α and the dimension d, it could be that q_{stat} decreases from a value above unity (e.g., 2 or $\frac{3}{2}$) to unity when α/d increases from zero to unity. For such systems one expects relations like the (particularly simple) $q_{stat} = q_{rel} = 2 - q_{sen}$ or similar ones. In any case, it is clear that, for $\alpha/d > 1$ (i.e., when BG statistics is known to be the correct one), one has $q_{stat} = q_{rel} = q_{sen} = 1$. All the weakly chaotic systems focused on here are expected to have well defined values for q_{sen} and q_{rel}, but only those associated with a Hamiltonian are expected to *also* have a well defined value for q_{stat}.

SOLAR WIND: Magnetic Field Strength

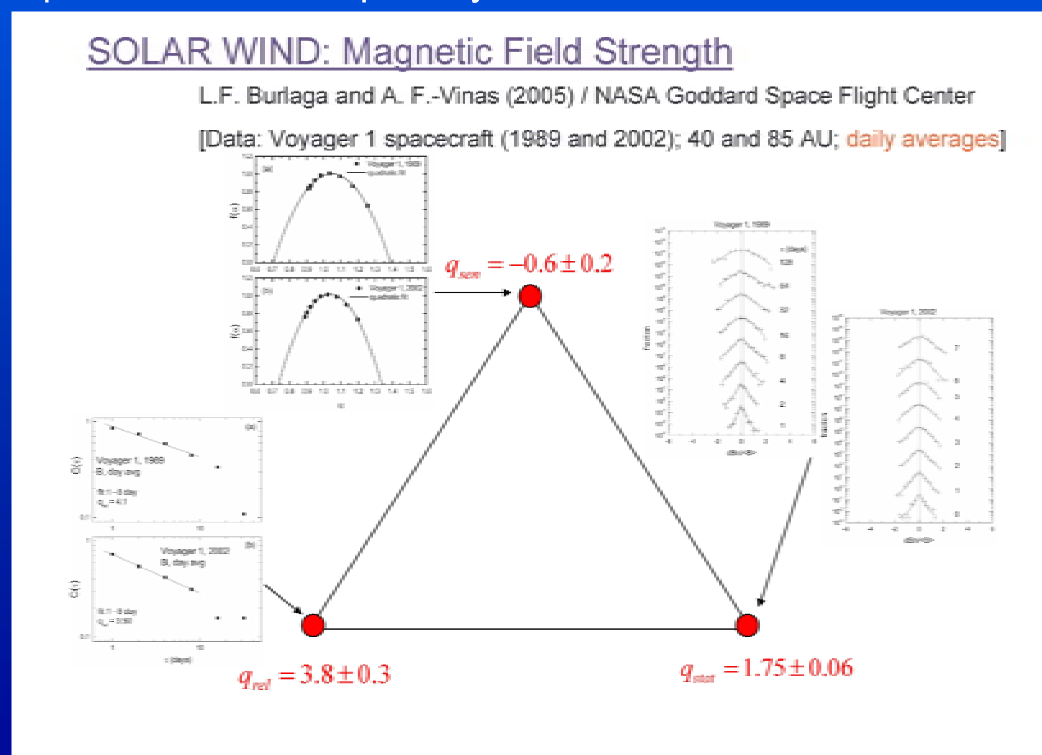
L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]



IHY 2007: VOYAGER 1: Fundamental Physics

The atmosphere of the Sun beyond a few solar radii, known as HELIOSPHERE, is fully ionized plasma expanding at supersonic speeds, carrying solar magnetic fields with it. This solar wind is a driven non-linear non-equilibrium system. The Sun injects matter, momentum, energy, and magnetic fields into the heliosphere in a highly variable way. Voyager 1 observed magnetic field strength variations in the solar wind near 40 AU during 1989 and near 85 AU during 2002. Tsallis' non-extensive statistical mechanics, a generalization of Boltzmann-Gibbs statistical mechanics, allows a physical explanation of these magnetic field strength variations in terms of departure from thermodynamic equilibrium in an unique way:



Playing with additive duality $(q \rightarrow 2-q)$

and with multiplicative duality $(q \rightarrow 1/q)$

(*and using numerical results related to the* $q-$*generalized central limit theorem*)

we conjecture

$$q_{rel} + \frac{1}{q_{sen}} = 2 \qquad \text{and} \qquad q_{stat} + \frac{1}{q_{rel}} = 2$$

hence $$1 - q_{sen} = \frac{1-q_{stat}}{3-2\,q_{stat}}$$

hence only one independent!

Burlaga and Vinas (*NASA*) *most precise value of the* $q-$*triplet is*

$$q_{stat} = 1.75 = 7/4$$

hence $q_{sen} = -\,0.5 = -\,1/2$ (*consistent with* $q_{sen} = -0.6 \pm 0.2$!)

and $q_{rel} = 4$ (*consistent with* $q_{rel} = 3.8 \pm 0.3$!)

C.T., M. Gell-Mann and Y. Sato
Proc Natl Acad Sc USA **102**, 15377 (2005)

Generic pitchfork bifurcations:

$$x_{t+1} = x_t + b\ sign(x_t)\,|\,x_t\,|^z\ (z > 1;\ b > 0)$$

Generic tangent bifurcations:

$$x_{t+1} = x_t + b\,|\,x_t\,|^z\ (z > 1;\ b > 0)$$

The fixed point map is a q-exponential with

$$q = z$$

and the sensitivity to the initial conditions is a q_{sen}-exponential with

$$q_{sen} = 2 - \frac{1}{q}$$

Example: *The* $\varsigma -$ *logistic family of maps*

$$x_{t+1} = 1 - a\,|\,x_t\,|^\varsigma\ (\varsigma > 1;\ 0 \le a \le 2;\ \varsigma > 1)$$

has

$$z = 3\ \ for\ pitchfork\ bifurcations\ (\forall \varsigma),\ hence\ \ q = 3\ \ and\ \ q_{sen} = \frac{5}{3}\ ;$$

$$z = 2\ \ for\ tangent\ bifurcations\ (\forall \varsigma),\ hence\ \ q = 2\ \ and\ \ q_{sen} = \frac{3}{2}\ .$$

A. Robledo, Physica D **193**, 153 (2004)

ELEMENTARY 1D CELLULAR AUTOMATA
(SHORT AND LONG MEMORY)

CA WITH SHORT AND LONG MEMORY:

T. Rohlf and C. T., Physica A (2007), in press

$$\sigma_i(t) = f\left[\sigma_{i-1}(t-1),\ \Theta\left(\Xi_i(t) - \frac{1}{2}\right),\ \sigma_{i+1}(t-1)\right]$$

where

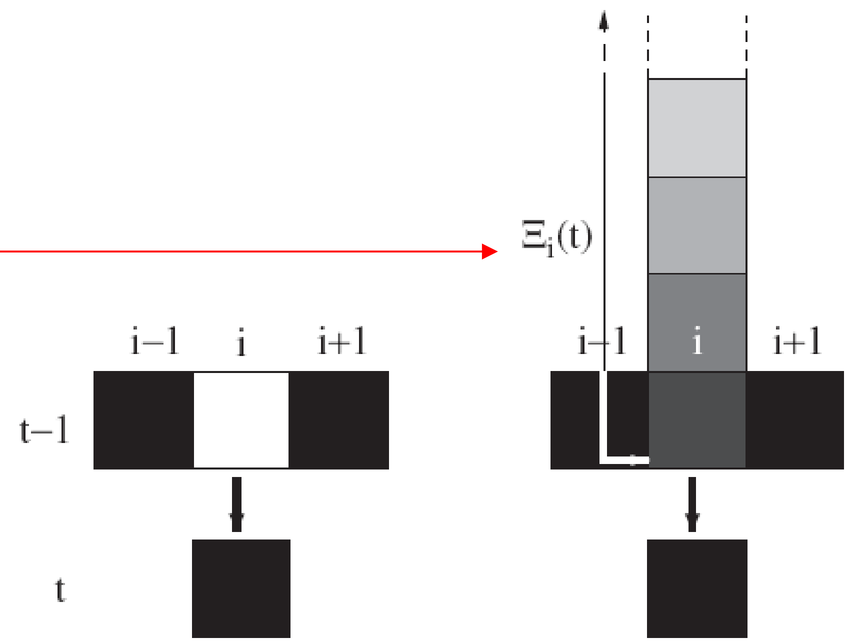
f is one of the Wolfram $2^{2^3} = 256$ two-state 3-neighborhood 1D elementary CA rules

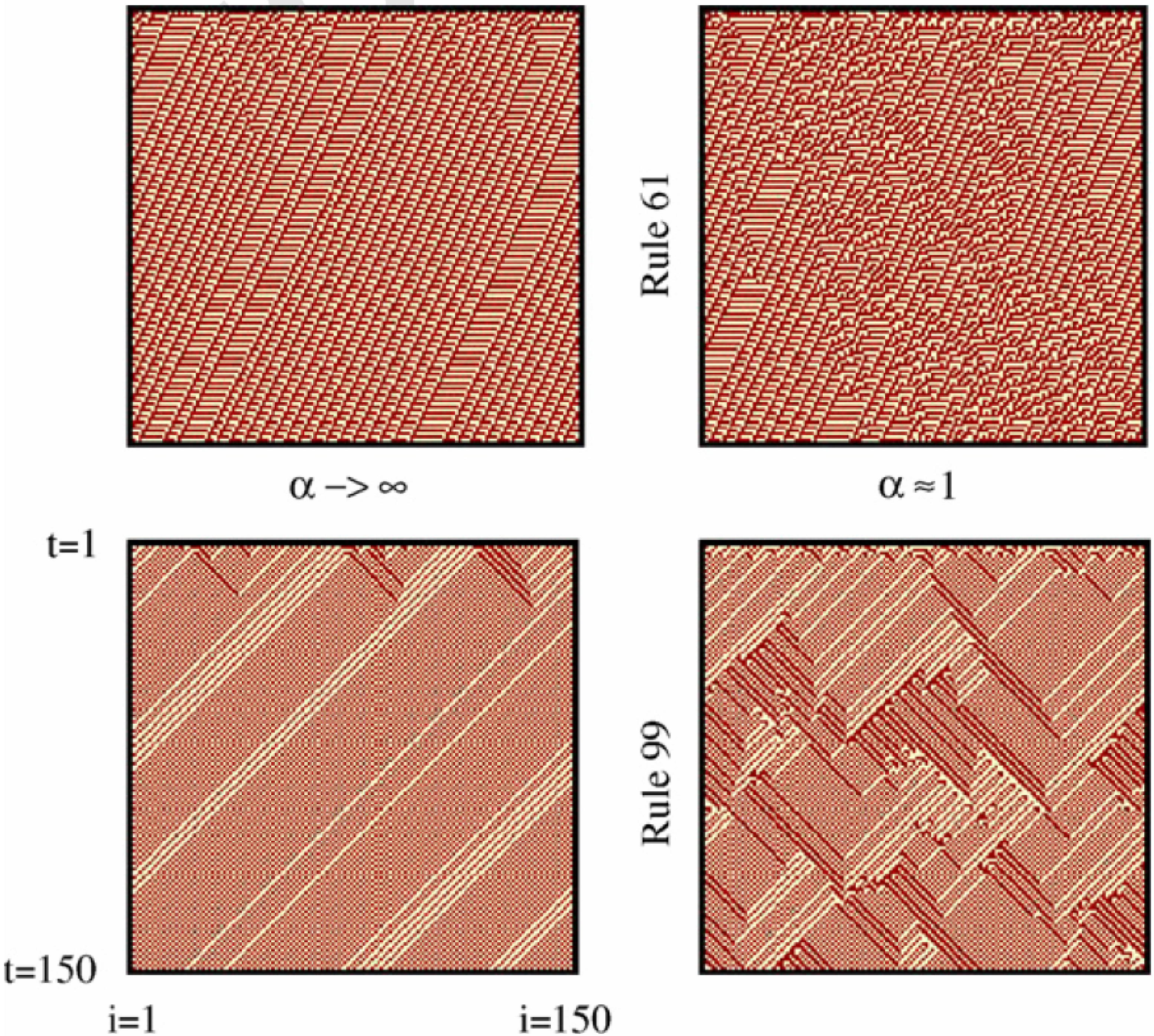
$$\Xi_i(t) \equiv \lim_{T\to\infty} \frac{\sum_{\tau=1}^{T} \frac{\sigma_i(t-\tau)}{\tau^\alpha}}{\sum_{\tau=1}^{T} \frac{1}{\tau^\alpha}} \in [0,1] \qquad (\alpha \geq 0)$$

$\Theta \equiv$ Heaviside step function

$\alpha < \infty$

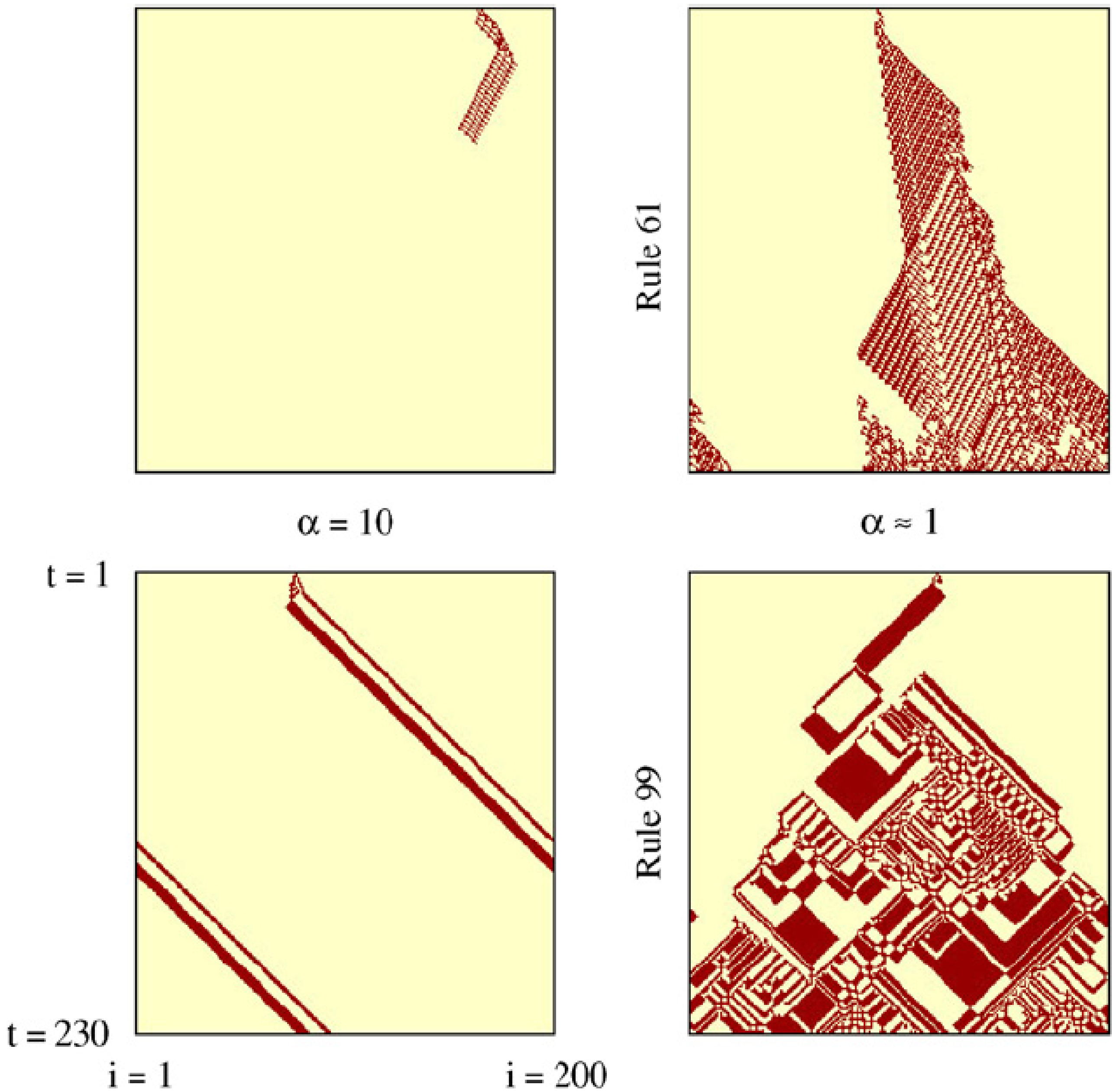
$\alpha \to \infty$



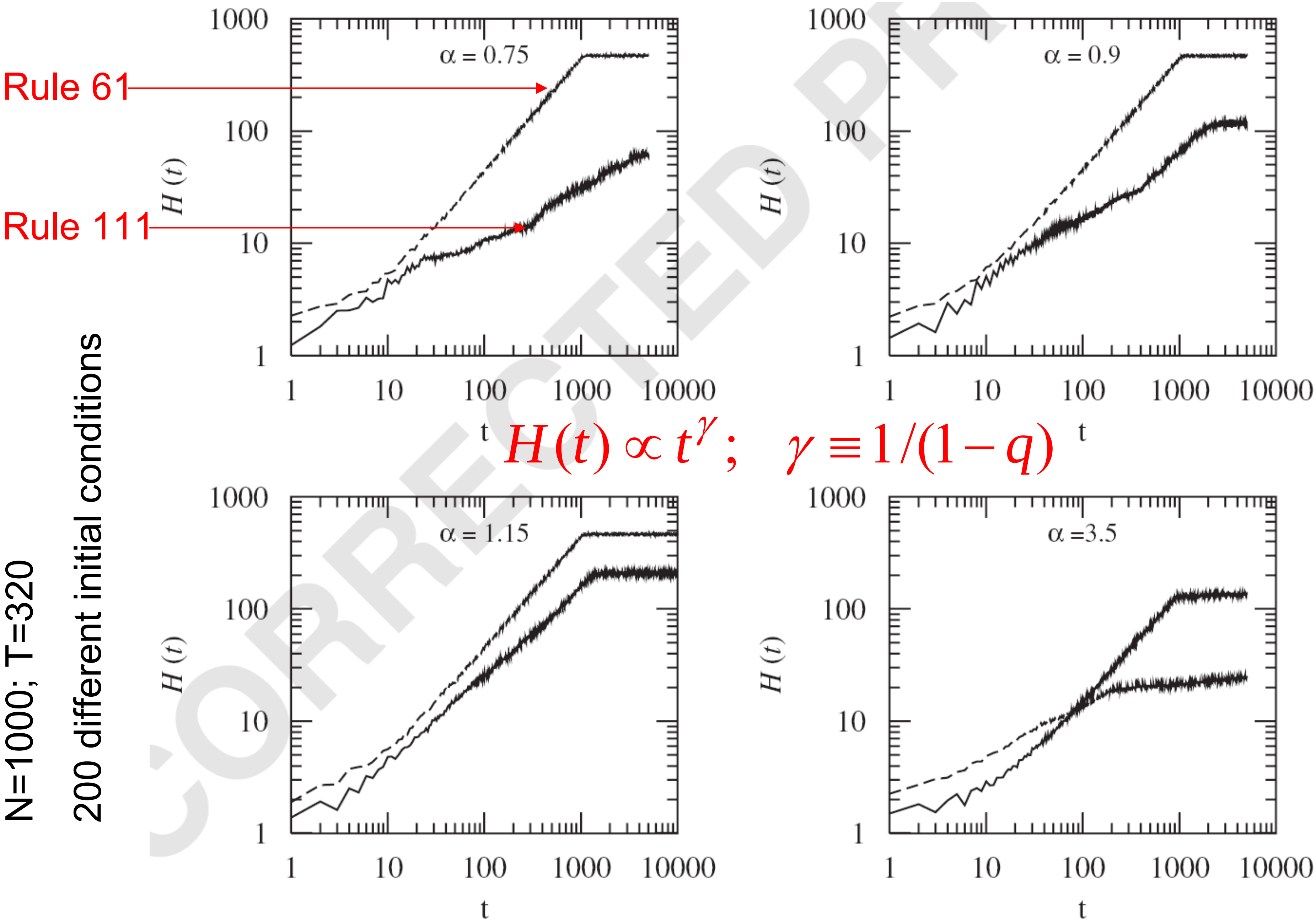


T. Rohlf and C. T., Physica A (2007), in press

Difference patterns with initial configurations differing in one randomly chosen bit



T. Rohlf and C. T., Physica A (2007), in press



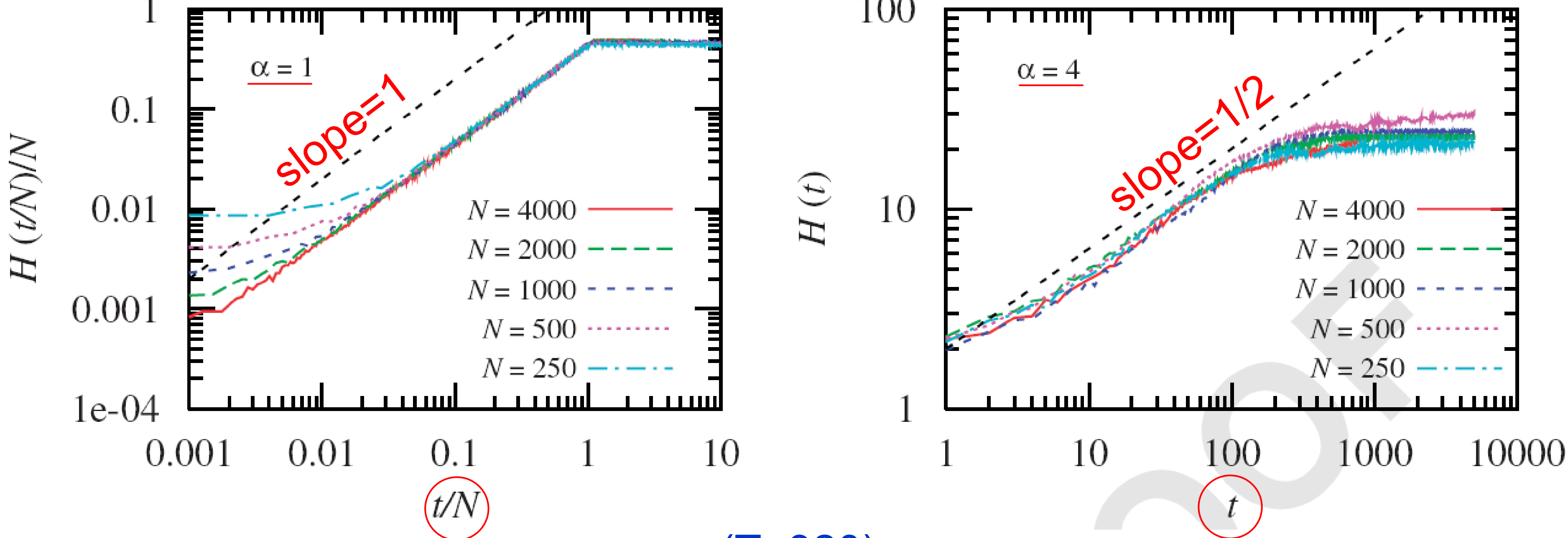
$$H(t) \propto t^{\gamma}; \quad \gamma \equiv 1/(1-q)$$

N=1000; T=320

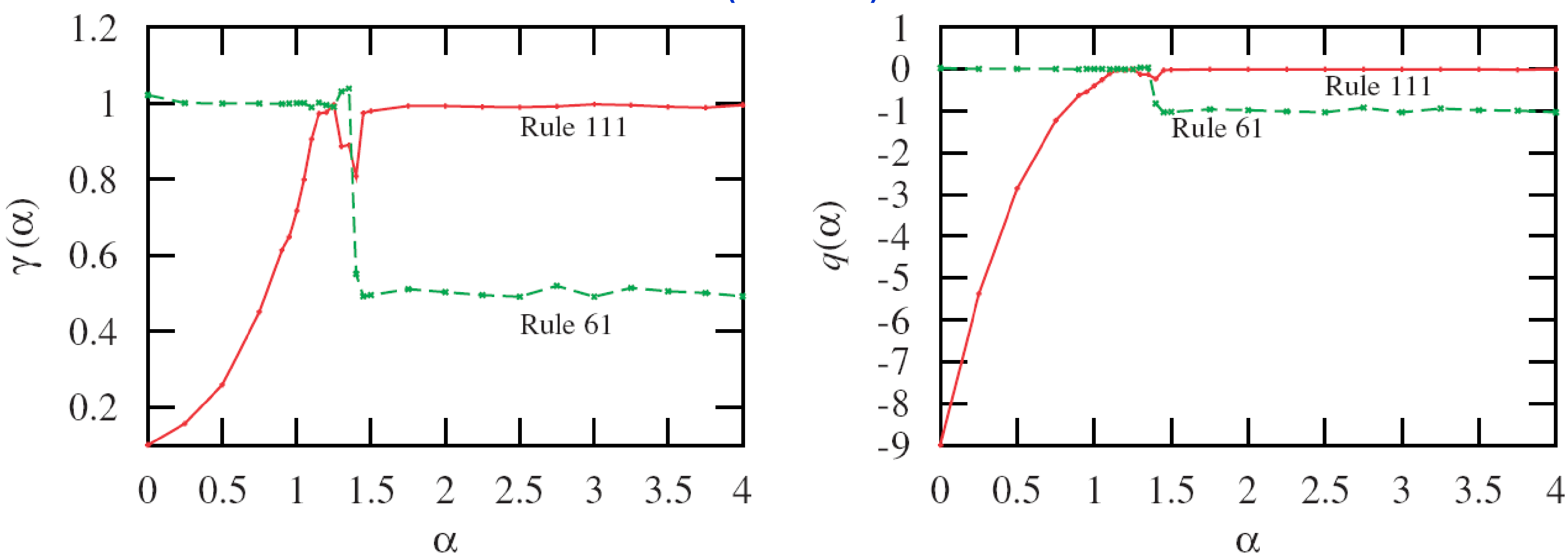
200 different initial conditions

T. Rohlf and C. T., Physica A (2007), in press

(Rule 61; T=320)



(T=320)



A TYPICAL OPEN QUESTION: For 1<< t < N

$$\text{Rule 61} \begin{cases} \alpha < 1.4 \Rightarrow \gamma \simeq 1 \Rightarrow q = \dfrac{\gamma - 1}{\gamma} \simeq 0 & \left[H(t,N) \propto \dfrac{t}{N} \right] \\ \alpha > 1.4 \Rightarrow \gamma \simeq \dfrac{1}{2} \Rightarrow q = \dfrac{\gamma - 1}{\gamma} \simeq -1 & \left[H(t,N) \propto \sqrt{t} \ (\forall N) \right] \end{cases}$$

WHY?

T. Rohlf and C. T., Physica A (2007), in press

HADRONIC JETS FROM ELECTRON-POSITRON ANNIHILATION:

I. Bediaga, E.M.F. Curado and J.M. de Miranda, Physica A **286** (2000) 156

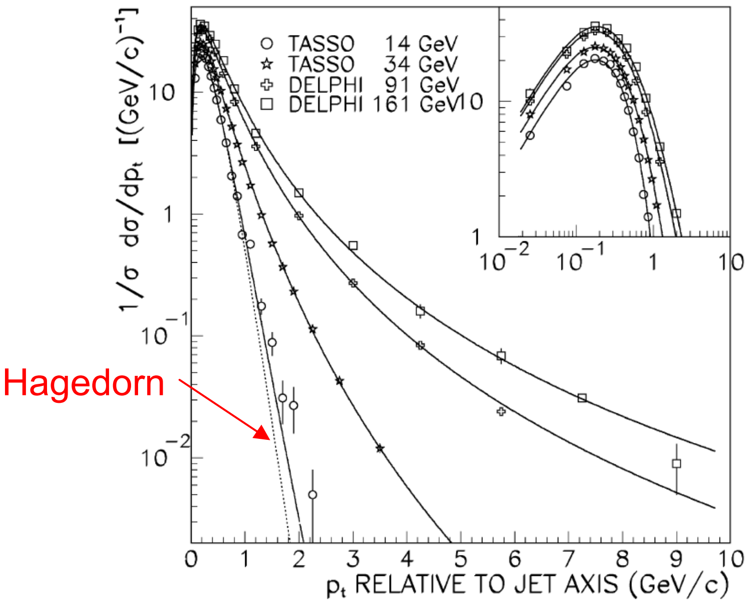
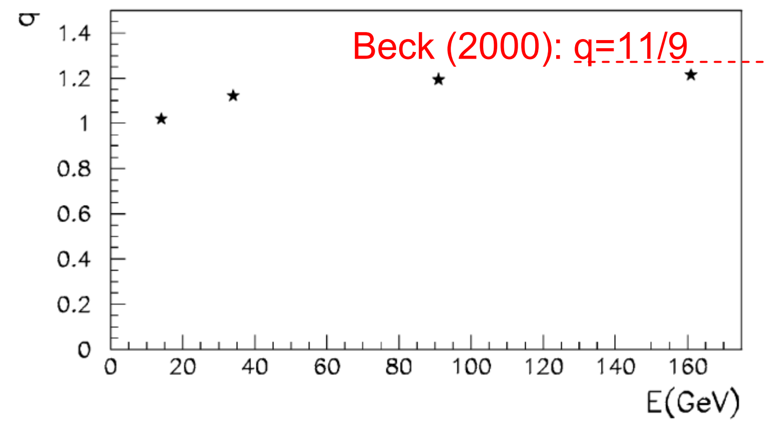
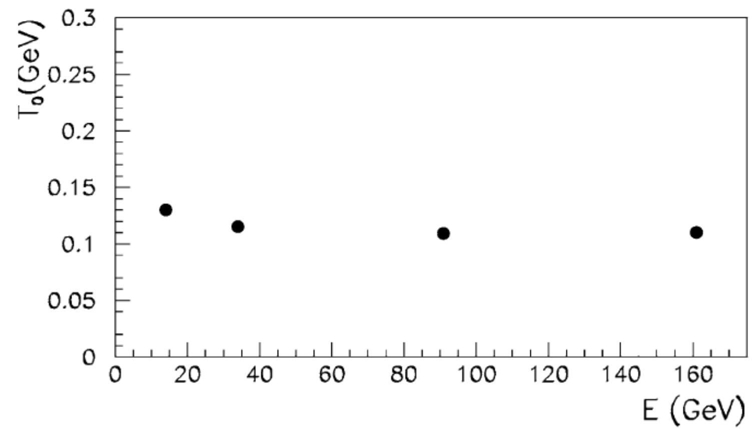


Fig. 1. Transverse momentum distribution. The distribution $(1/\sigma)\,\mathrm{d}\sigma/\mathrm{d}p_t$ of the transverse momentum p_t of charged hadrons with respect to jet axis (defined in these experimental results as the sphericity axis) is sketched for four different experiments, whose center-of-mass energies vary from 14 and 34 GeV (TASSO) up to 91 and 161 GeV (DELPHI). The Hagedorn predicted exponential behavior is shown by the dotted line. We can see that the deviation of the exponential behavior increases when the energy increases. The continuous lines are obtained from our Eq. (3) and agree very well with the experimental data. The inset shows the transverse momentum distribution for small values of p_t.





GENERALIZED SIMULATED ANNEALING AND RELATED ALGORITHMS

q-GENERALIZED SIMULATED ANNEALING (GSA):

C.T. and D.A. Stariolo, Notas de Fisica / CBPF (1994); Physica A **233**, 395 (1996)

Visiting algorithm:

Boltzmann machine $\rightarrow$ *Gaussian*

Generalized machine $\rightarrow$ $q_V - Gaussian$

Acceptance algorithm:

Boltzmann machine $\rightarrow$ *Boltzmann weight*

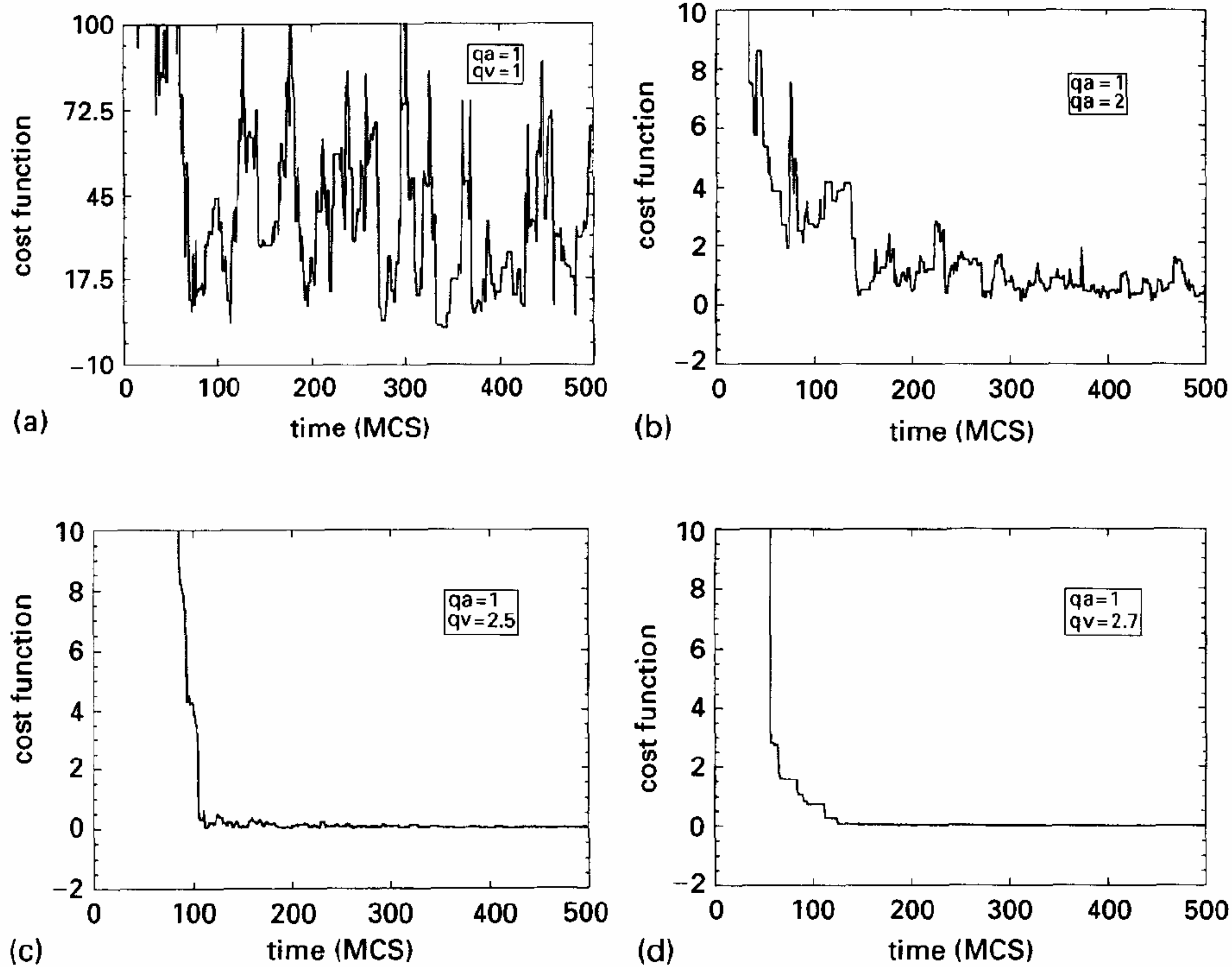
Generalized machine $\rightarrow$ $q_A - exponential\ weight$

Cooling algorithm:

Boltzmann machine $\rightarrow$ $\dfrac{\mathrm{T(t)}}{\mathrm{T(1)}} = \dfrac{\ln 2}{\ln(1+t)}$

Generalized machine $\rightarrow$ $\dfrac{\mathrm{T(t)}}{\mathrm{T(1)}} = \dfrac{2^{q_V - 1} - 1}{(1+t)^{q_V - 1} - 1}$

[*Typical values:* $1 < q_V < 3$ *and* $q_A < 1$]



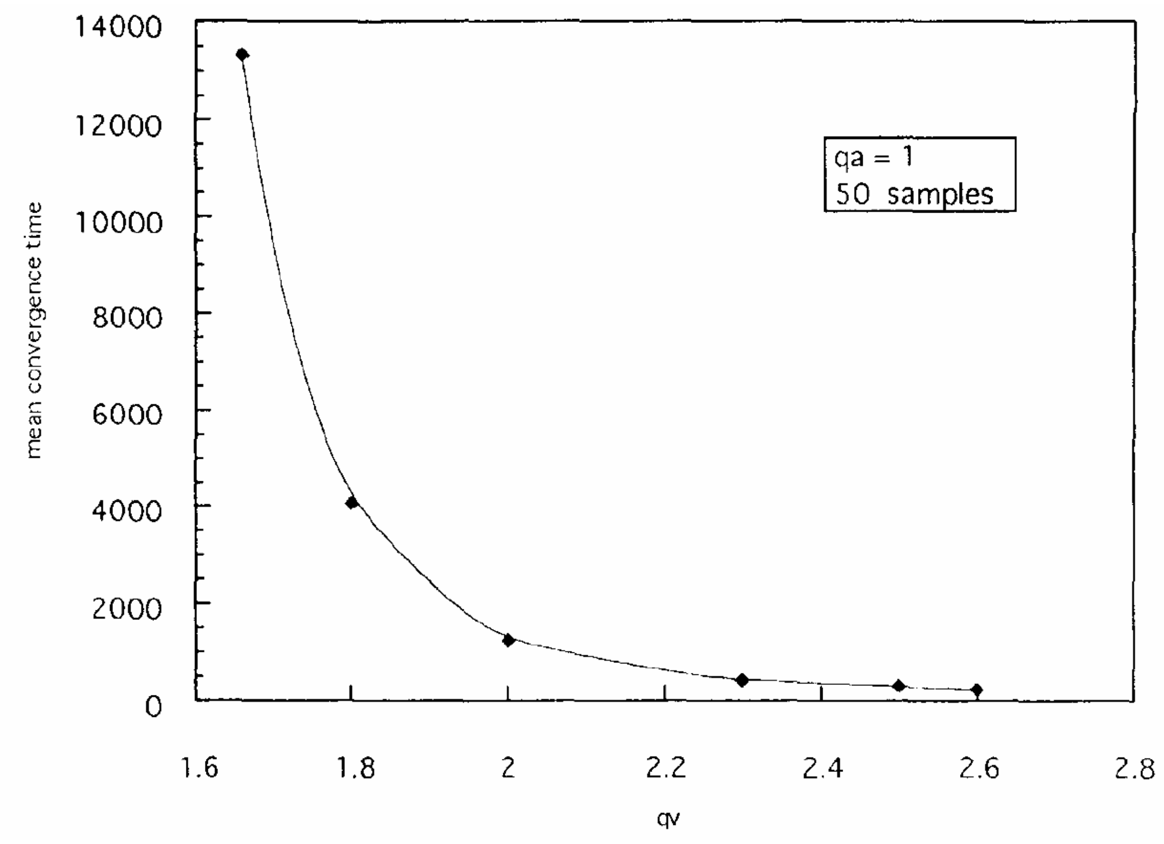
C. T. and D.A. Stariolo, Physica A **233**, 395 (1996)

q-GENERALIZED SIMULATED ANNEALING (GSA):

Illustration: $$E(x_1, x_2, x_3, x_4) = \sum_{i=1}^{4} (x_i^2 - 8)^2 + 5\sum_{i=1}^{4} x_i$$

(15 *local minima and one global minimum*)

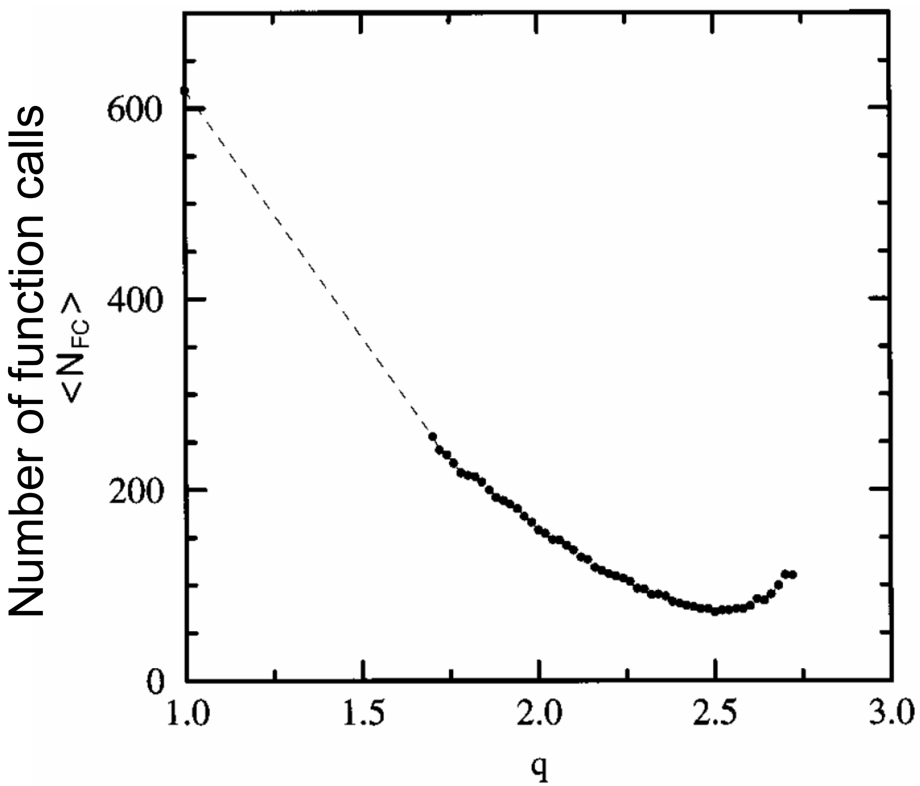
$(q_V = 1 \Rightarrow \textit{mean convergence time} \approx 50000)$



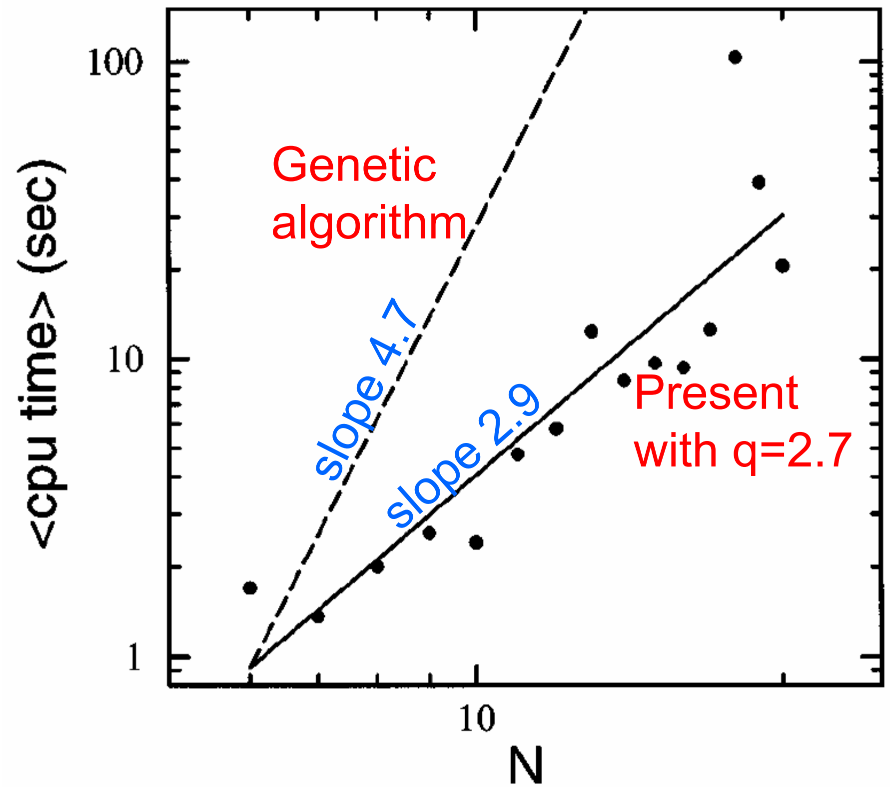
q-GENERALIZED PIVOT METHOD:

P. Serra, A.F. Stanton and S. Kais, Phys Rev E **55**, 1162 (1997)

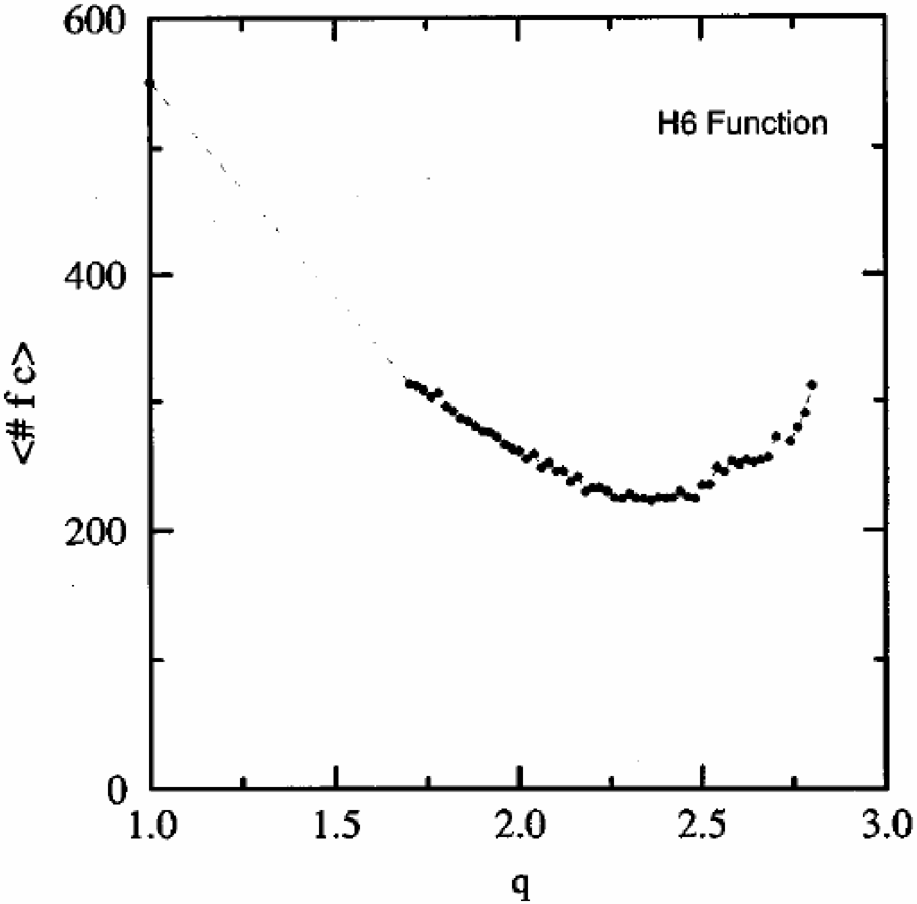
(Branin function)

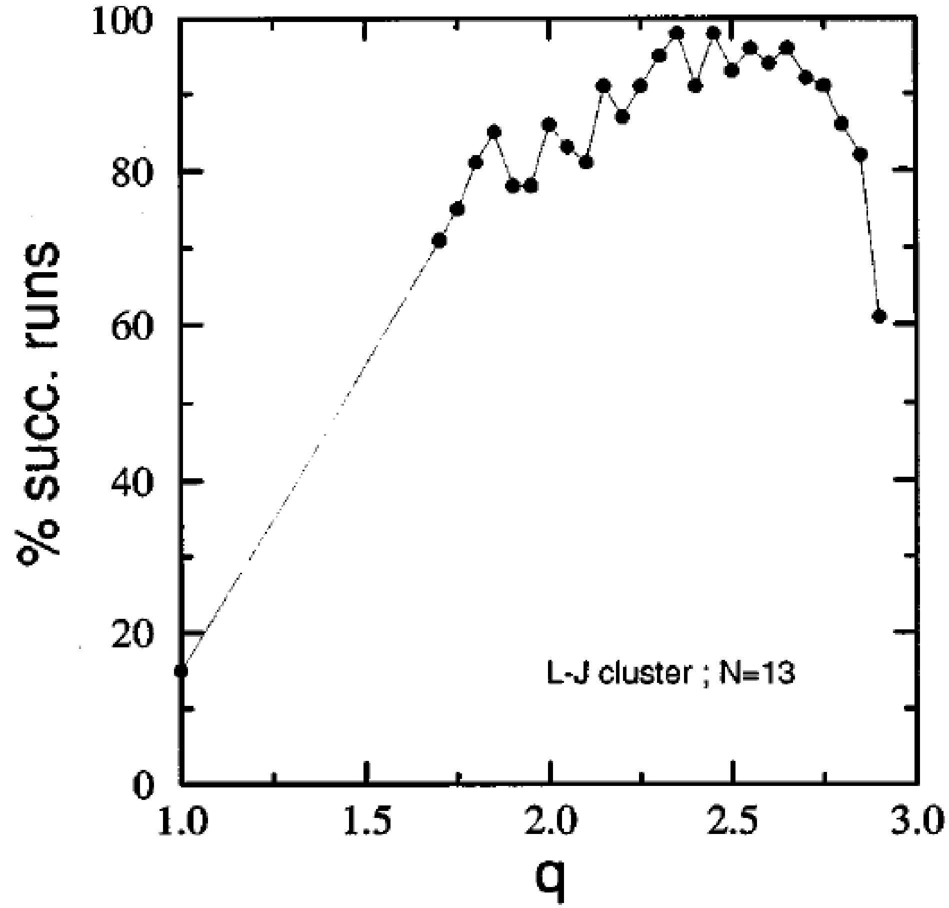


(Lennard-Jones clusters)



Recently: M.A. Moret, P.G. Pascutti, P.M. Bisch, M.S.P. Mundim and K.C. Mundim
Classical and quantum conformational analysis using Generalized Genetic Algorithm
Physica A **363**, 260 (2006)





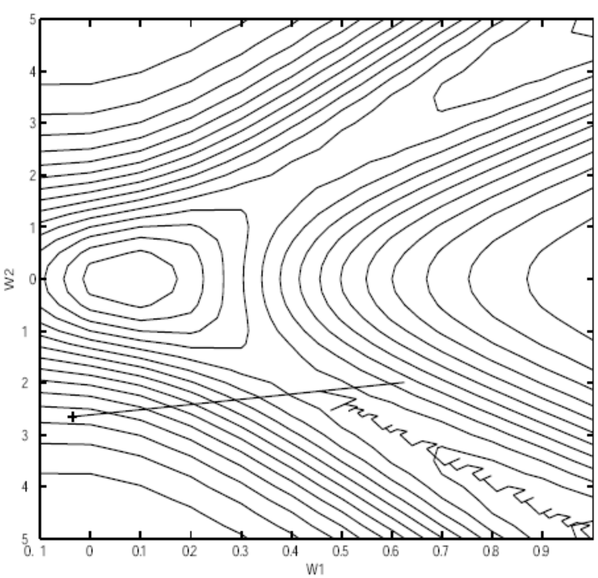
P. Serra, A.F. Stanton, S. Kais and R.E. Bleil
J. Chem. Phys **106**, 7170 (1997)

HYBRID LEARNING OF NEURAL NETWORKS

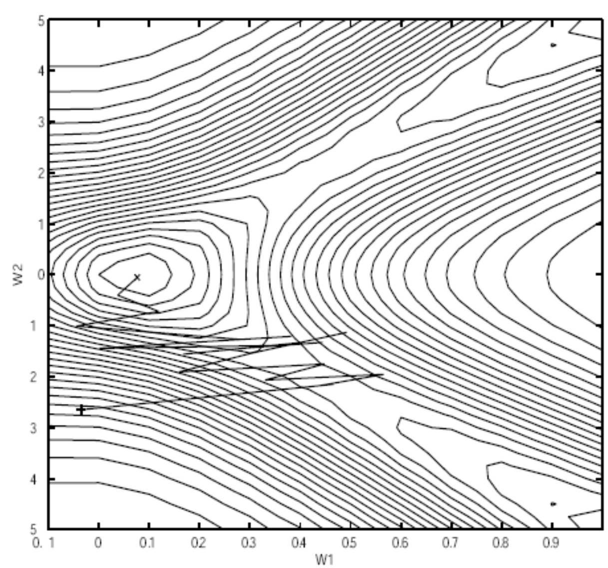
A.D. Anastasiadis and G.D. Magoulas, Physica A **344**, 372 (2004)

A.D. Anastasiadis, Proc. Int. Summer School Complex Systems (June 2005, Santa Fe Institute, NM)

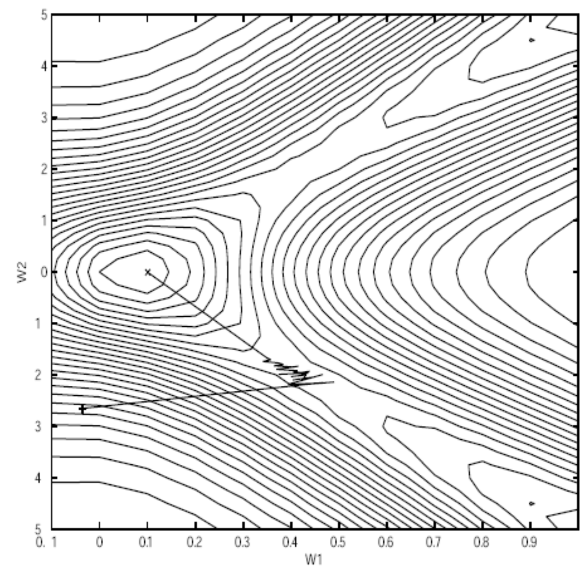
Rprop



HLS
(Hybrid Learning Scheme; q > 1)



CHLS
(Cooling Hybrid Learning Scheme; q > 1)



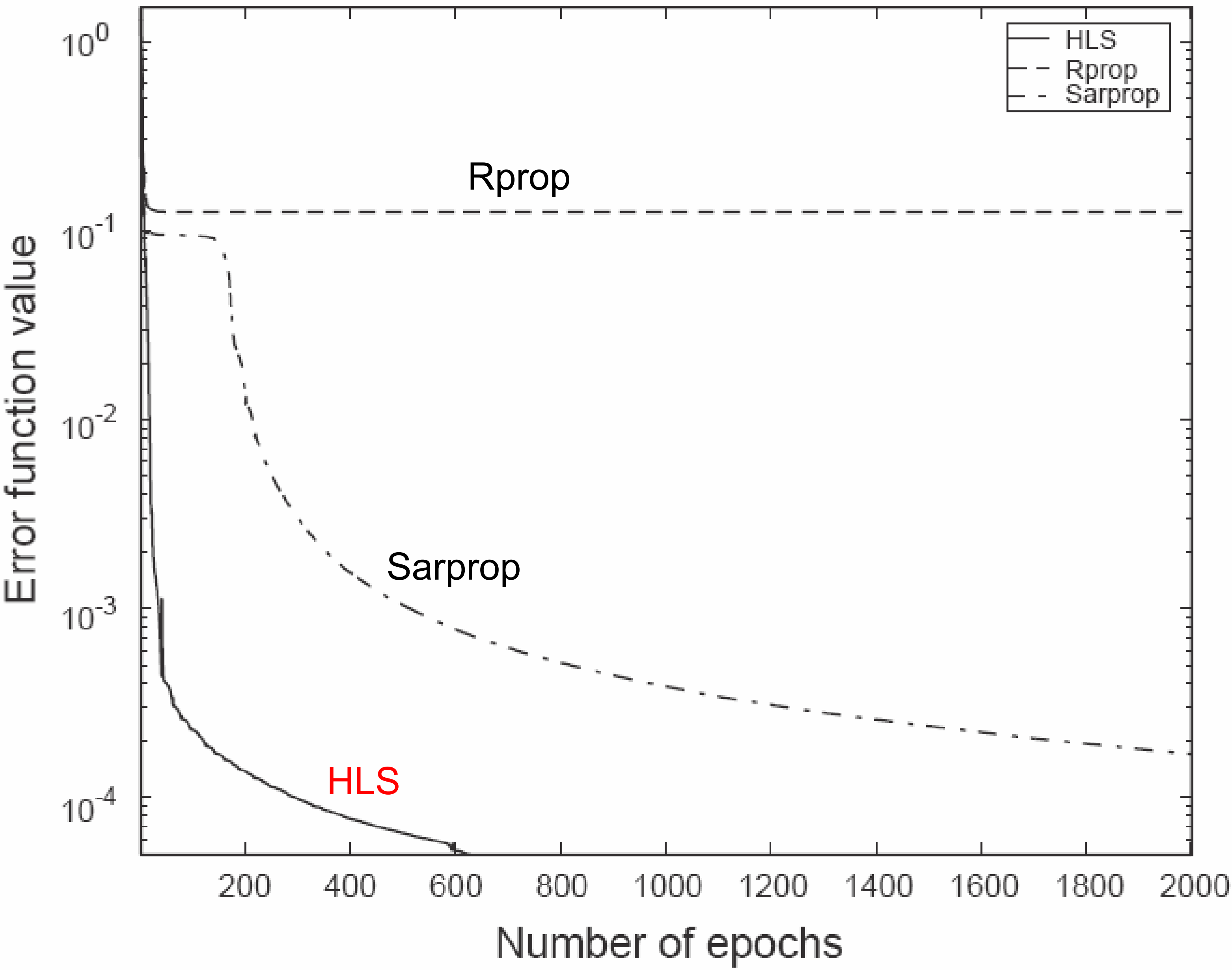


Fig. 2. Typical learning error curve for the Parity-3 problem.

Algorithm	Iris			Cancer		
	IT	*GEN.*	*CONV.*	*IT*	*GEN.*	*CONV.*
Rprop	1400 (+)	98.4 (+)	96	279 (+)	97.2 (−)	94
SARprop	1430 (+)	98.9 (+)	96	282 (+)	97.6 (−)	87
HLS	1377	99.5	100	157	97.5	100

Algorithm	Diabetes			Thyroid		
	IT	*GEN.*	*CONV.*	*IT*	*GEN.*	*CONV.*
Rprop	357 (+)	75.9 (+)	96	793 (+)	98.0 (−)	78
SARprop	325 (+)	75.8 (+)	96	736 (+)	98.1 (−)	92
HLS	223	76.1	100	460	98.2	100

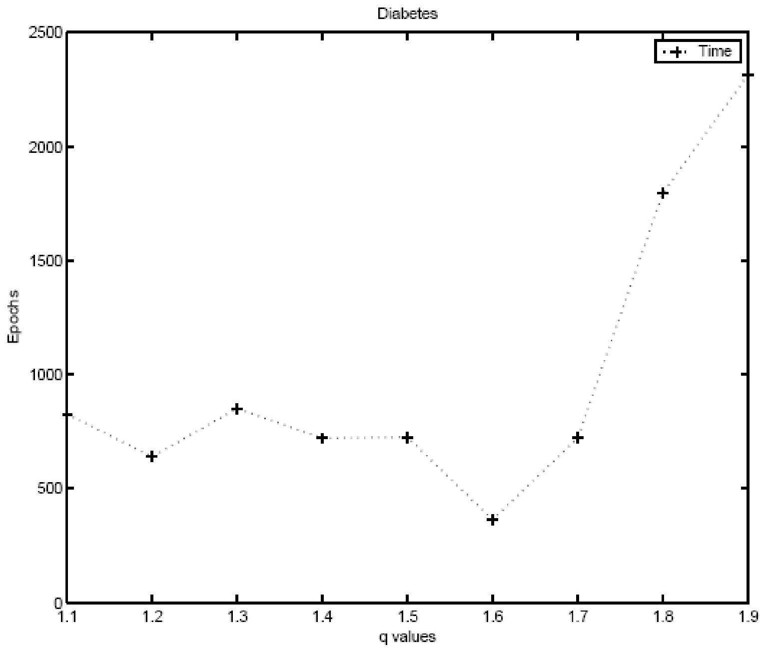
Algorithm	XOR			Parity 4		
	IT	*GEN.*	*CONV.*	*IT*	*GEN.*	*CONV.*
Rprop	1110 (+)	100 (−)	23	1360 (+)	100 (−)	42
SARprop	168 (+)	100 (−)	98	1378 (+)	100 (−)	48
HLS	49	100	100	1270	100	100

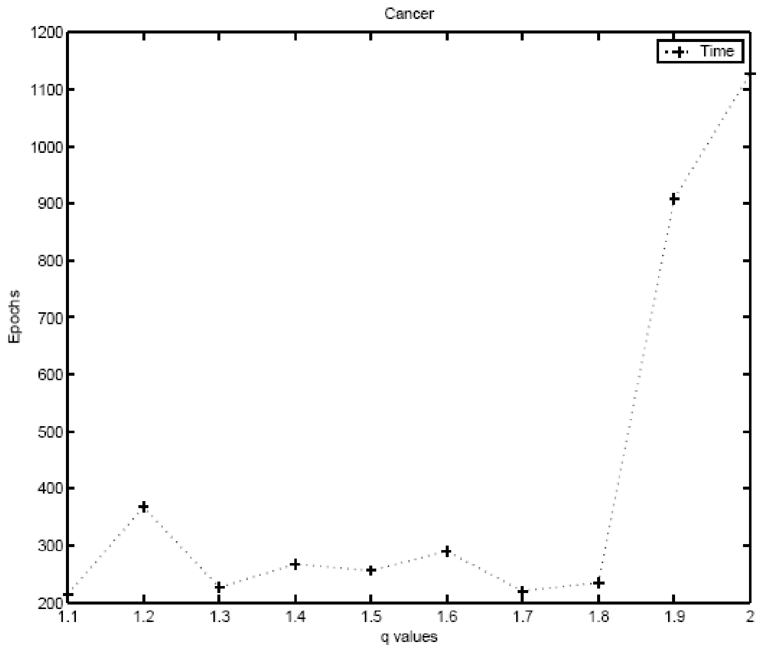
Algorithm	Parity 3			Parity 5		
	IT	*GEN.*	*CONV.*	*IT*	*GEN.*	*CONV.*
Rprop	1105 (+)	100 (−)	22	416 (+)	100 (−)	67
SARprop	882 (+)	100 (−)	78	394 (+)	100 (−)	95
HLS	640	100	100	20	100	100

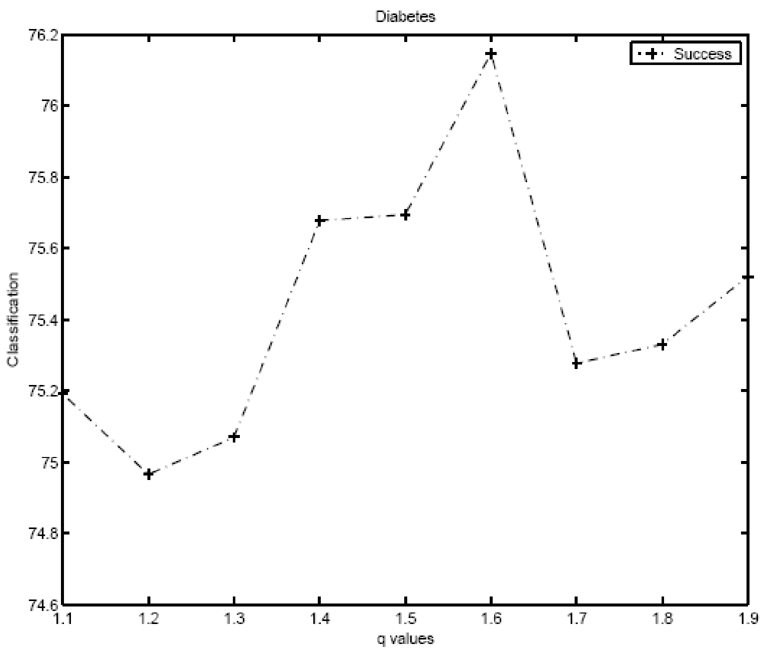
Anastasiadis and Magoulas (2004)

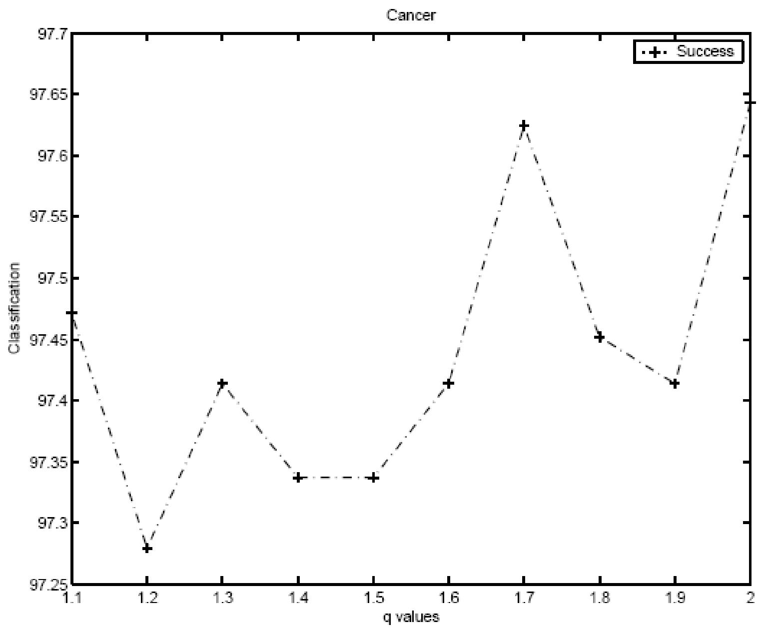
DIABETES

CANCER









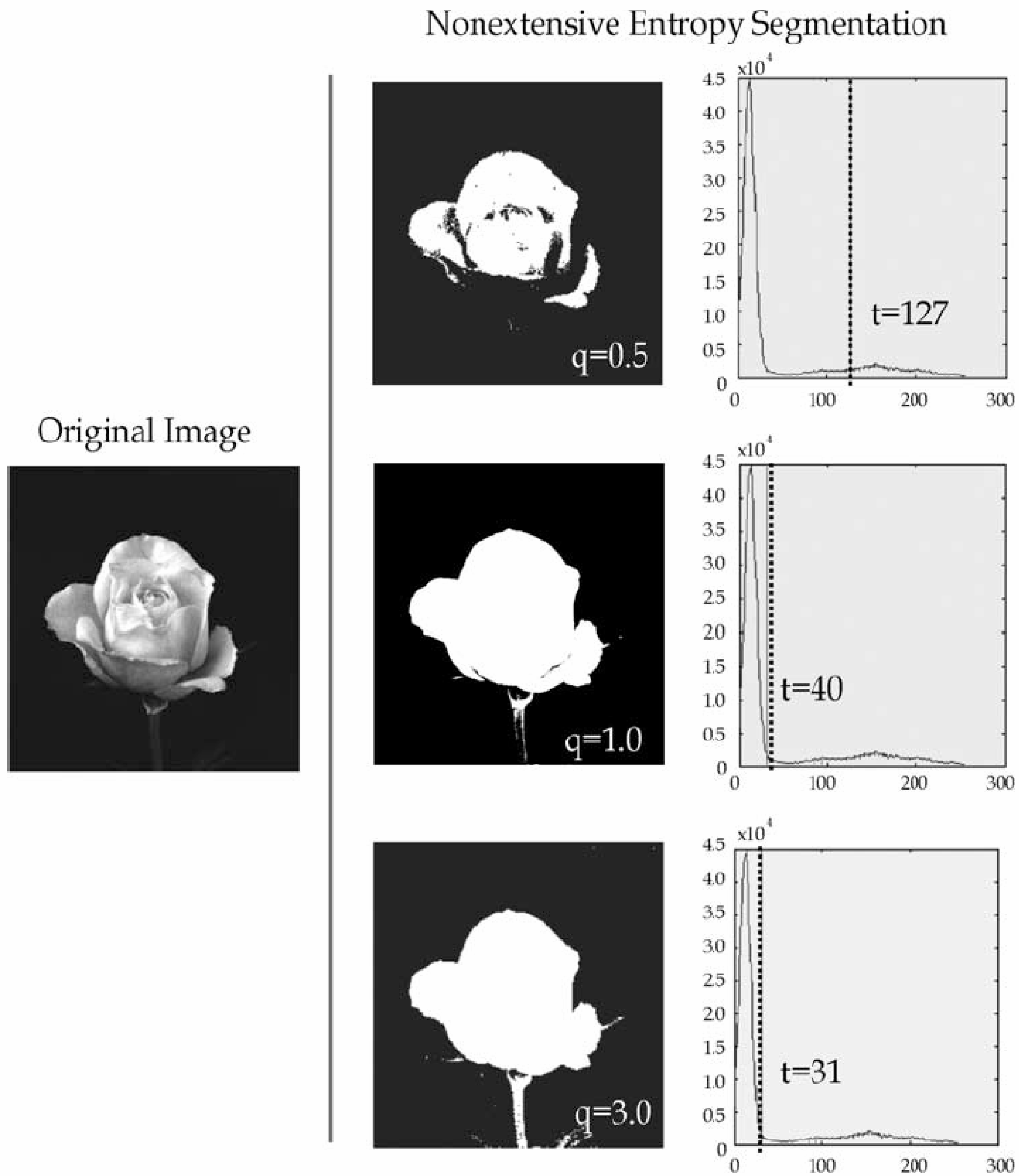


Fig. 4. Influence of parameter q in natural images: $q = 0.5$, $q = 1.0$ (classical entropic segmentation) and $q = 3.0$.

M.P. de Albuquerque, I.A. Esquef, A.R.G. Mello and M.P. de Albuquerque
Pattern Recognition Letters **25**, 1059 (2004)

IMAGE THRESHOLDING:

M.P. de Albuquerque, I.A. Esquef, A.R.G. Mello and M.P. de Albuquerque
Pattern Recognition Letters **25**, 1059 (2004)

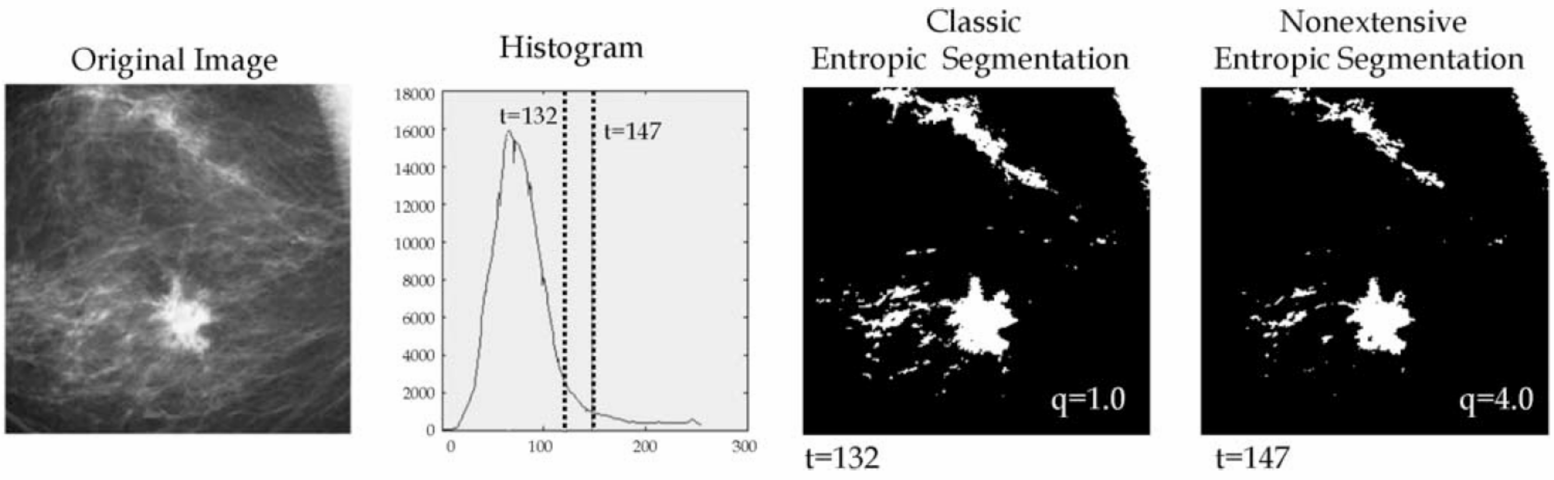


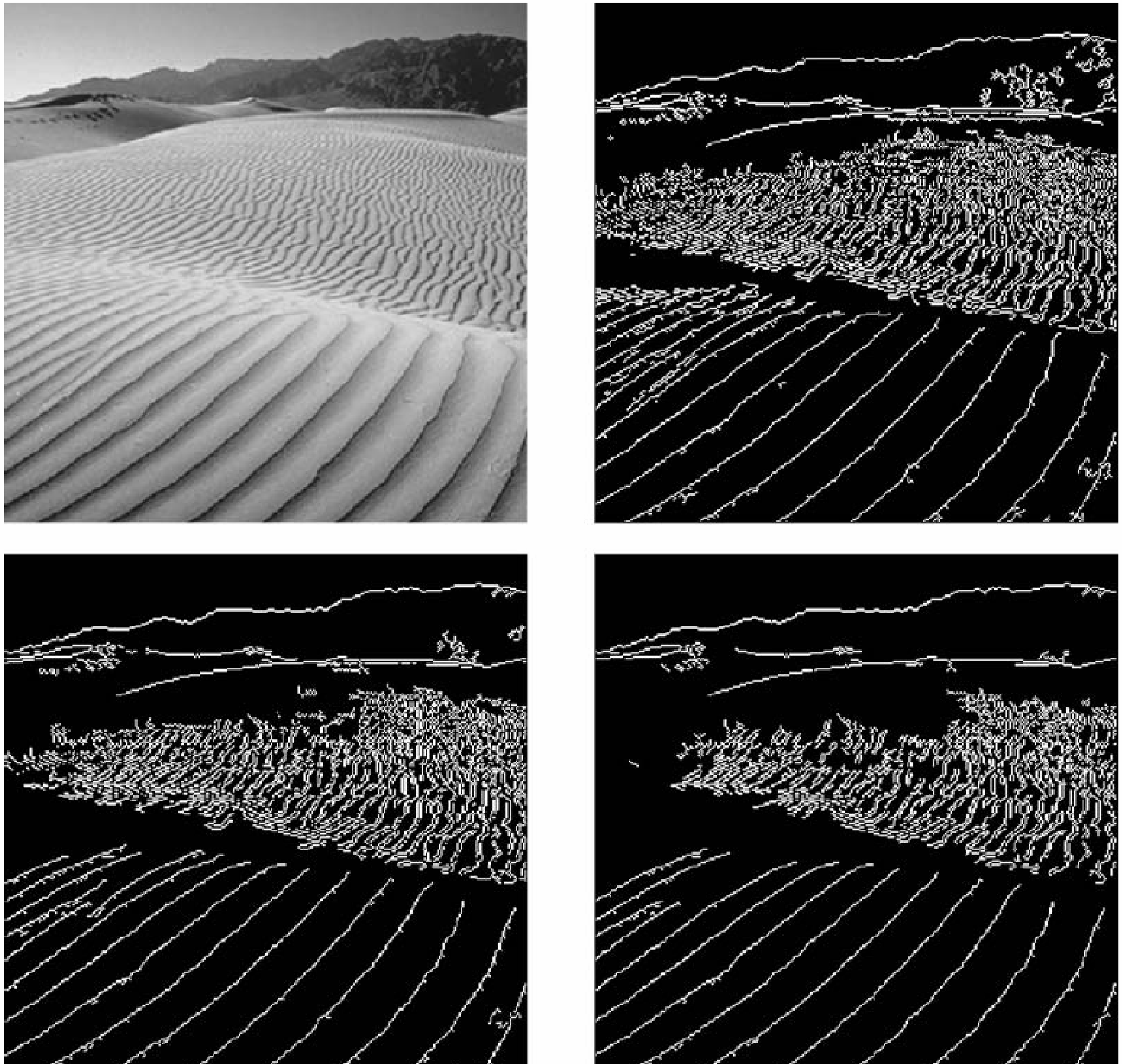
Fig. 3. Example of entropic segmentation for mammography image with an inhomogeneous spatial noise. Two image segmentation results are presented for $q = 1.0$ (classic entropic segmentation) and $q = 4.0$.

IMAGE EDGE DETECTION

[A. Ben Hanza, J. Electronic Imaging **15**, 013011 (2006)]

Original image

q = 1.5

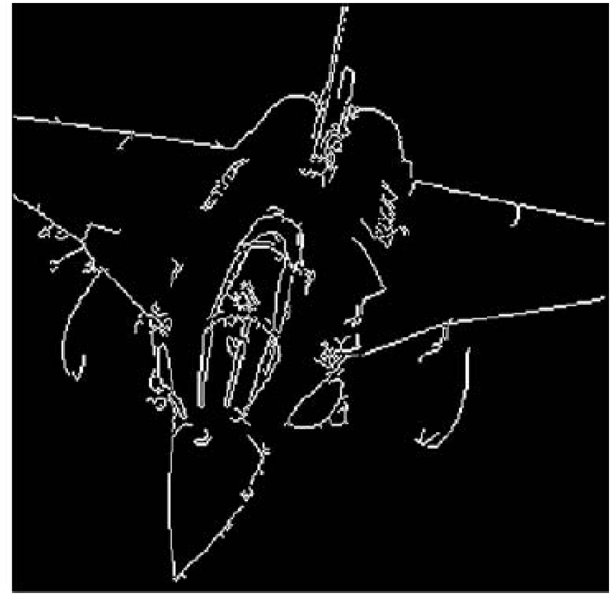


Canny edge detector

q = 1 (Jensen-Shannon)

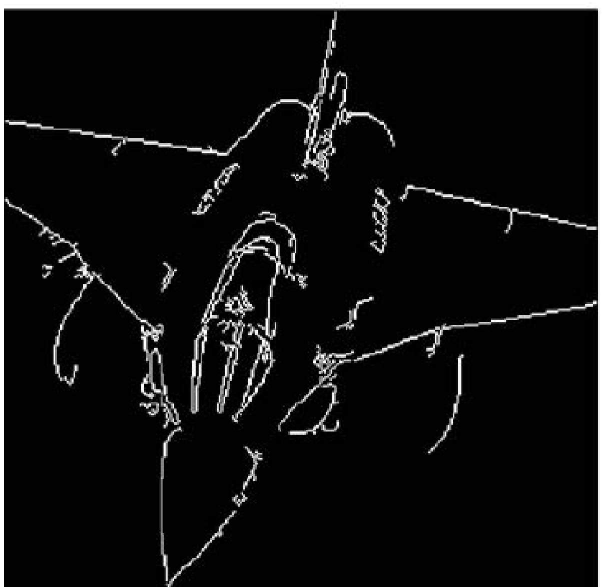
IMAGE EDGE DETECTION [A. Ben Hanza, J. Electronic Imaging **15**, 013011 (2006)]

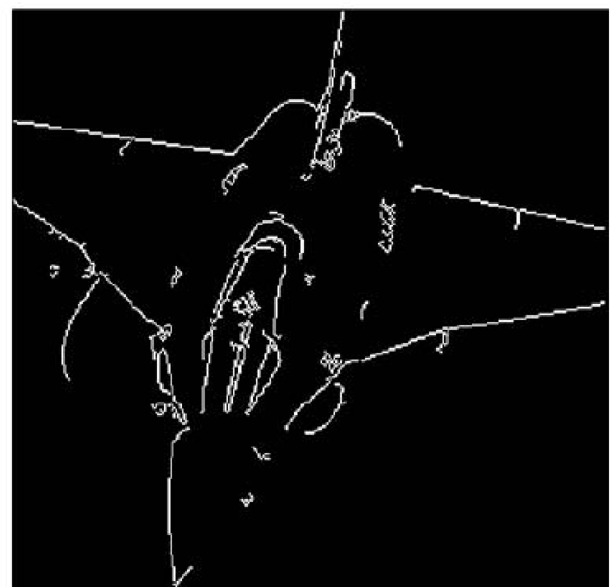
Original image



q = 1.5

Canny edge detector





q = 1

(Jensen-Shannon)

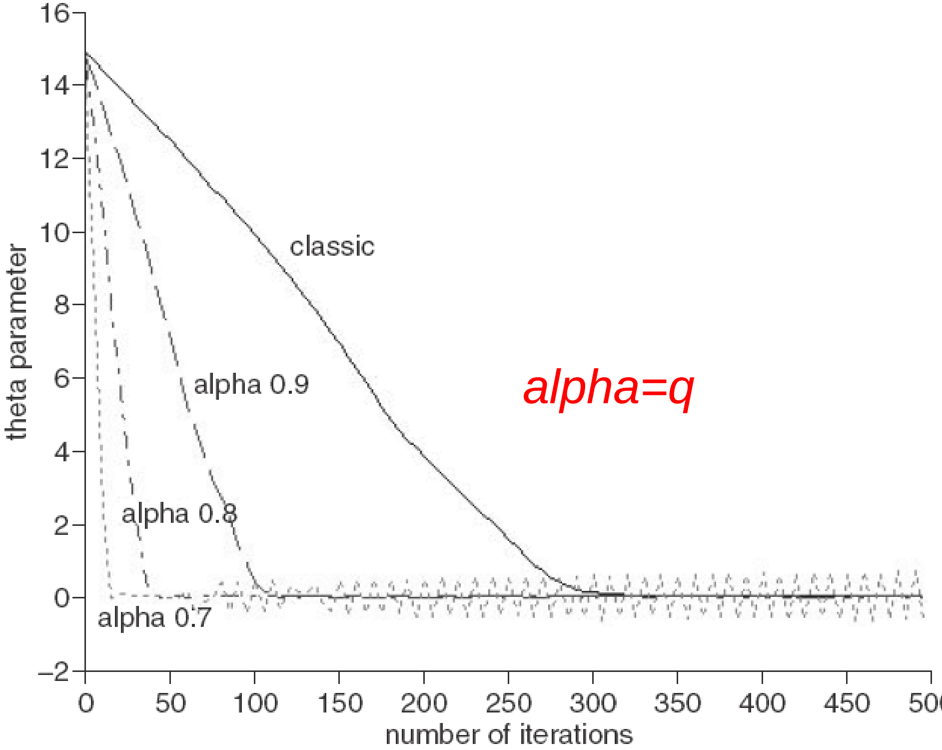
Magnetic Resonance and Computed Tomography Images

Fast and accurate image registration using Tsallis entropy and simultaneous perturbation stochastic approximation

S. Martin, G. Morison, W. Nailon and T. Durrani

The Tsallis measure of mutual information is combined with the simultaneous perturbation stochastic approximation algorithm to register images. It is shown that Tsallis entropy can improve registration accuracy and speed of convergence, compared with Shannon entropy, in the calculation of mutual information. Simulation results show that the new algorithm achieves up to seven times faster convergence and four times more precise registration than using a classic form of entropy.

ELECTRONICS LETTERS 13th May 2004 Vol. 40 No. 10



alpha=q

Image Fusion

ELECTRONICS LETTERS 25th May 2006 Vol. 42 No. 11

Image fusion metric based on mutual information and Tsallis entropy

N. Cvejic, C.N. Canagarajah and D.R. Bull

A novel image fusion performance metric using mutual information is proposed. The metric is based on Tsallis entropy, which is a one-parameter generalisation of Shannon entropy. Experimental results have confirmed that the proposed metric outperforms the standard MI metric by correlating better with the subjective quality of fused images.

INFRARED VISIBLE

Fig. 1 *Top: input IR image (left), input visible image (middle) and fused image using averaging (right); Bottom: fused image using ratio pyramid (left), LT (middle) and DT-CWT (right)*

INFRARED VISIBLE

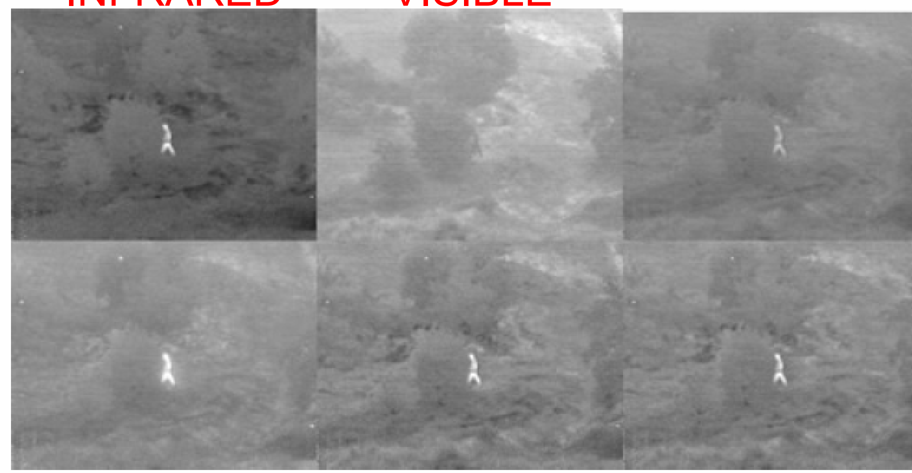


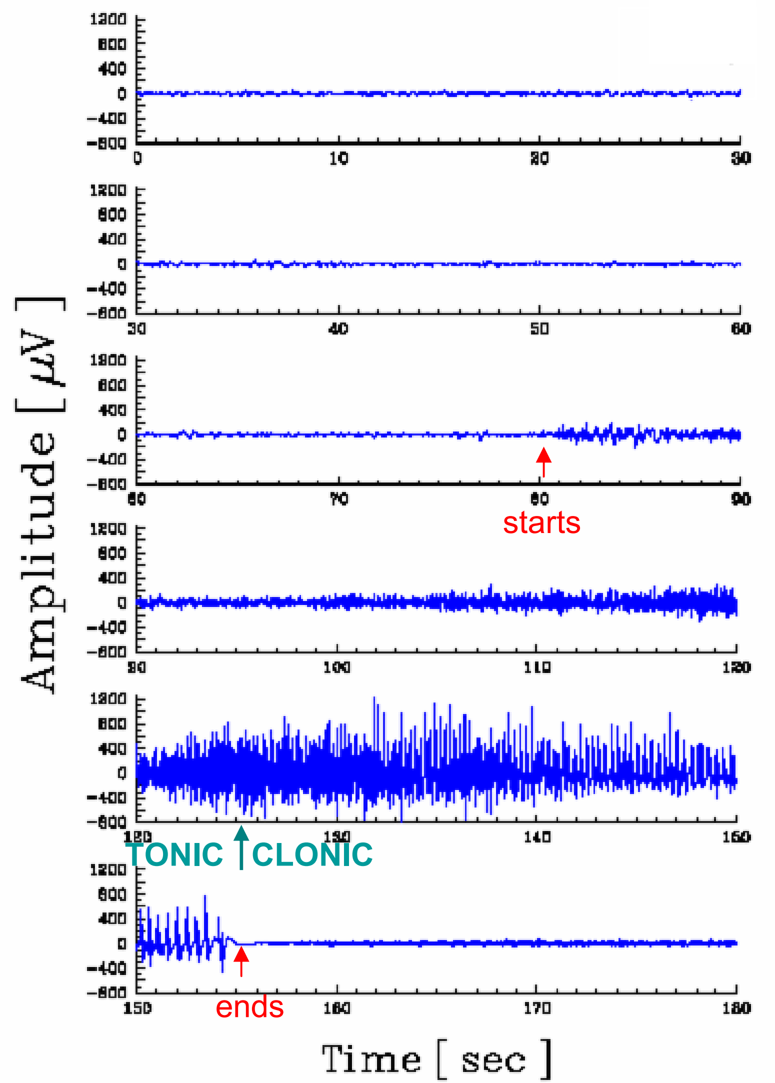
Fig. 2 *Top: input IR image (left), input visible image (middle) and fused image using averaging (right); Bottom: fused image using ratio pyramid (left), LT (middle) and DT-CWT (right)*

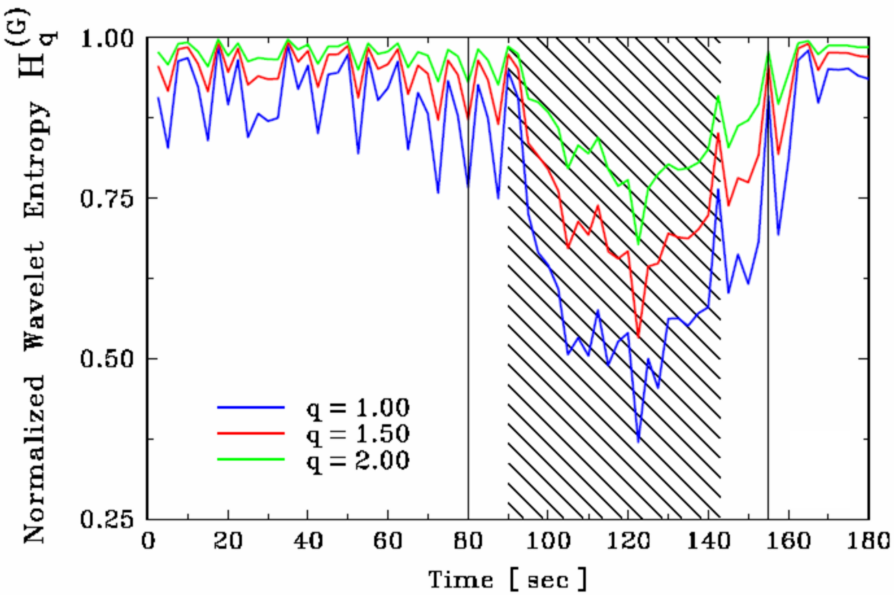
Table 1: Comparison of different objective quality metrics

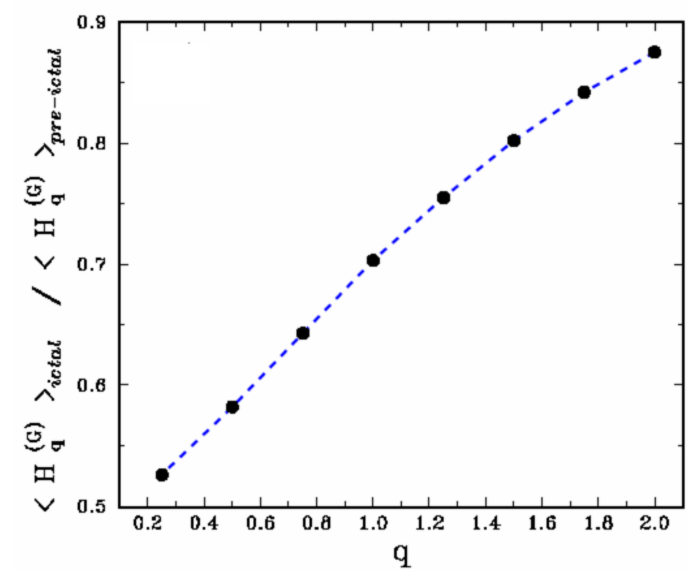
	UN camp (Fig. 1)				Trees image (Fig. 2)			
Method/ metric	MI q=1	Proposed q=1.85	Petrovic	Piella	MI	Proposed	Petrovic	Piella
Average	**0.819**	8.394	0.349	0.865	**0.848**	11.048	0.445	0.909
Ratio	0.752	6.985	0.416	0.861	0.689	10.375	0.470	0.918
Laplace	0.759	8.687	0.506	0.913	0.732	13.138	**0.554**	0.926
DT-CWT	0.740	**9.073**	**0.509**	**0.921**	0.722	**14.125**	**0.554**	**0.927**

ELECTROENCEPHALOGRAMS (tonic-clonic transition in epilepsy):

(INCLUDING MUSCULAR ACTIVITY CONTRIBUTION)







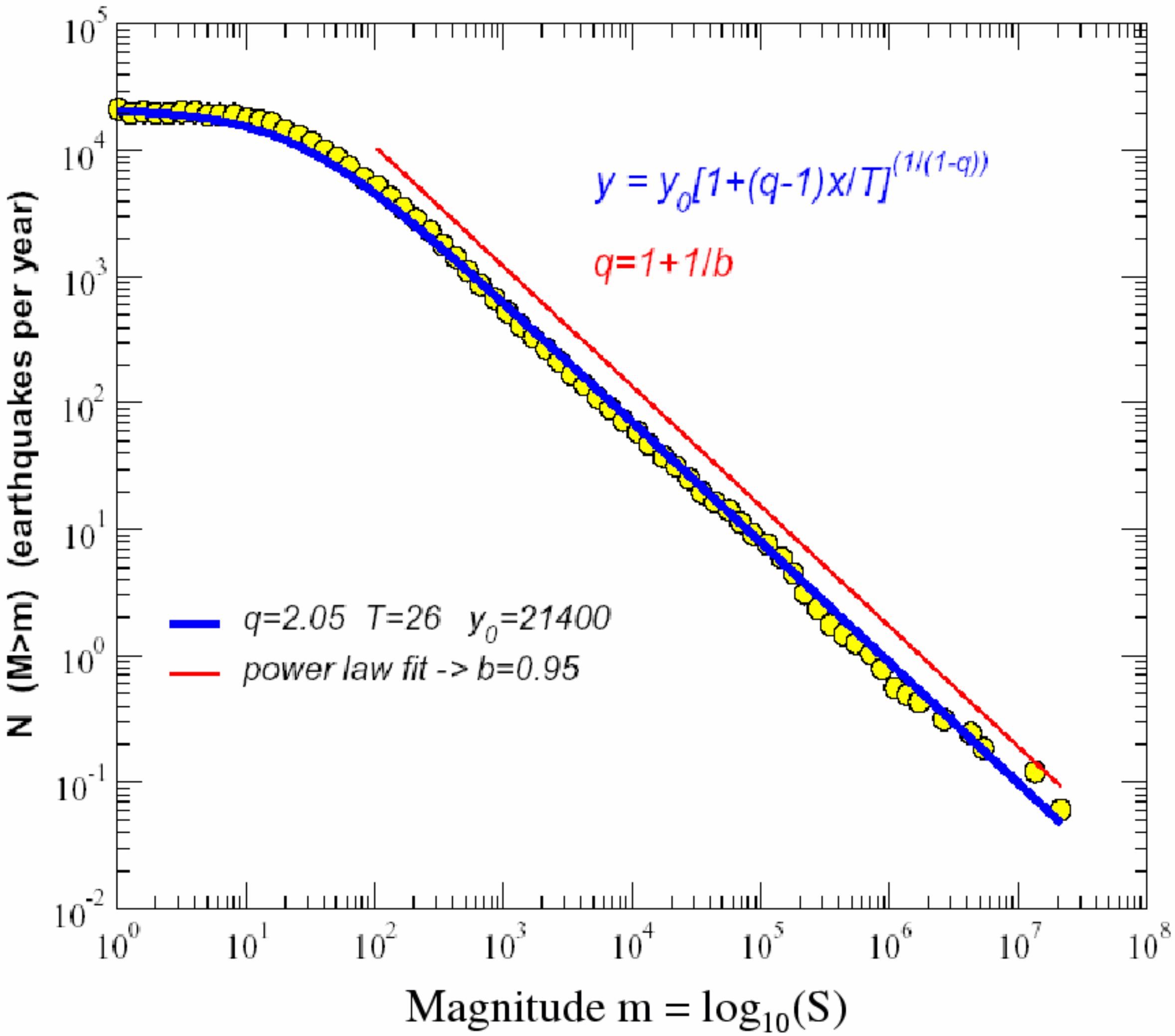
A. Plastino and O.A. Rosso
Europhysics News **36** (6), 224 (2005) [European Physical Society]

EARTHQUAKES

Earthquakes

Data from

P.Bak, K. Christensen, L. Danon and T. Scanlon, Phys Rev Lett **88**, 178501 (2002)

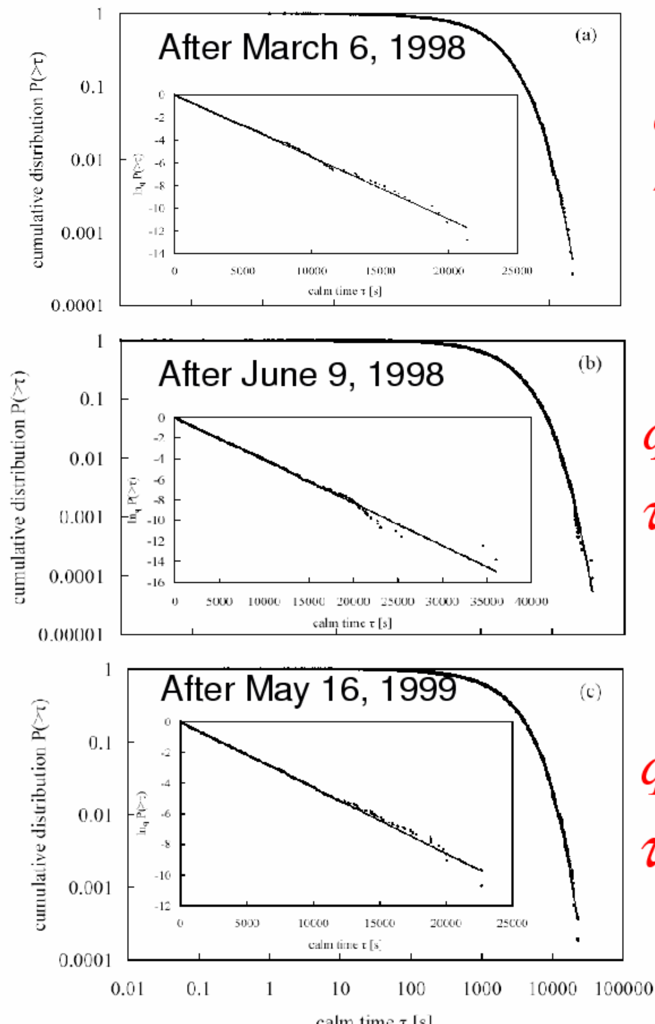


TIME INTERVALS BETWEEN EARTHQUAKES
Southern California data [S. Abe and N. Suzuki (2004)]

Calm periods (stationary states) between major earthquakes,

i.e., excluding the Omori-regime periods (nonstationary states)

$$P(t>\tau)=\frac{1}{[1+(q-1)\tau/\tau_0]^{1/(q-1)}}$$

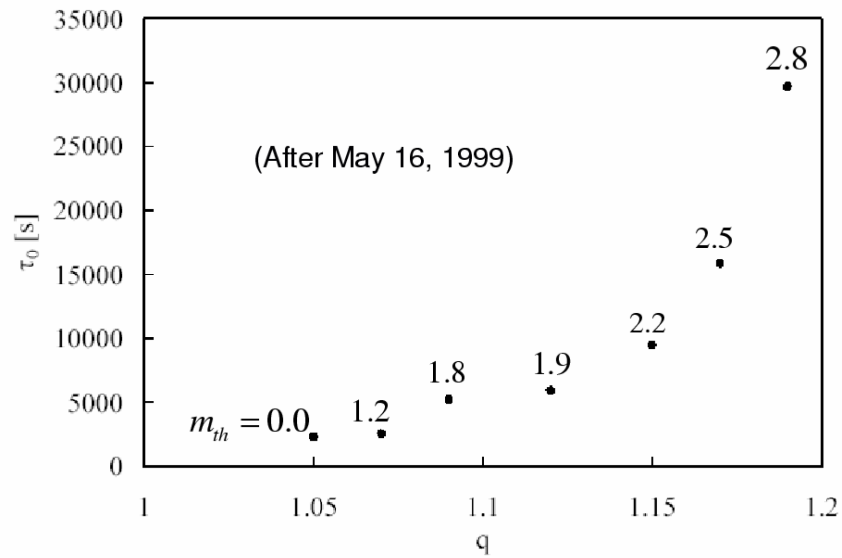


$q = 1.10$
$\tau_0 = 1830$ s

$q = 1.08$
$\tau_0 = 2410$ s

$q = 1.05$
$\tau_0 = 2330$ s

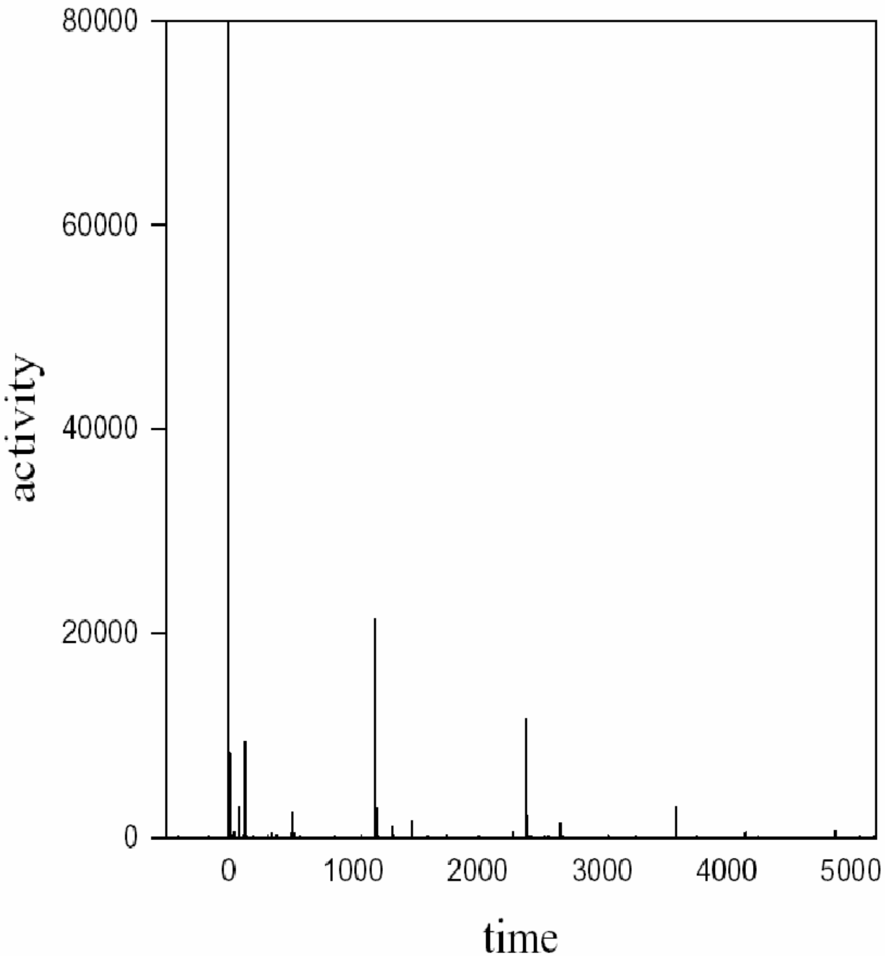
Influence of the threshold m_{th}

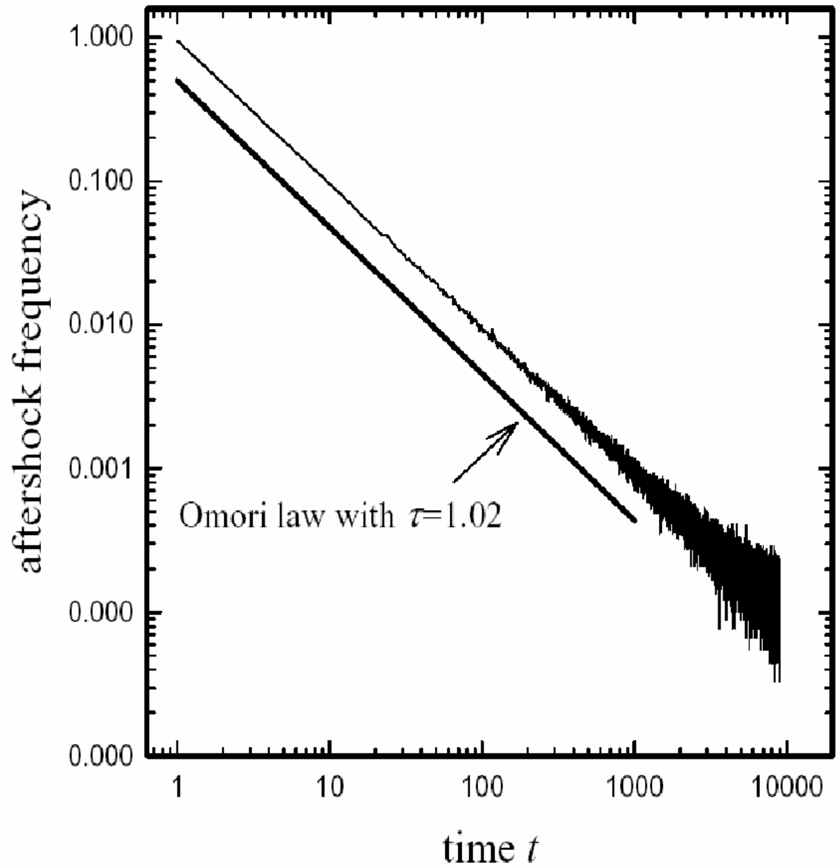


AGING IN THE NEWMAN MODEL FOR COHERENT NOISE:

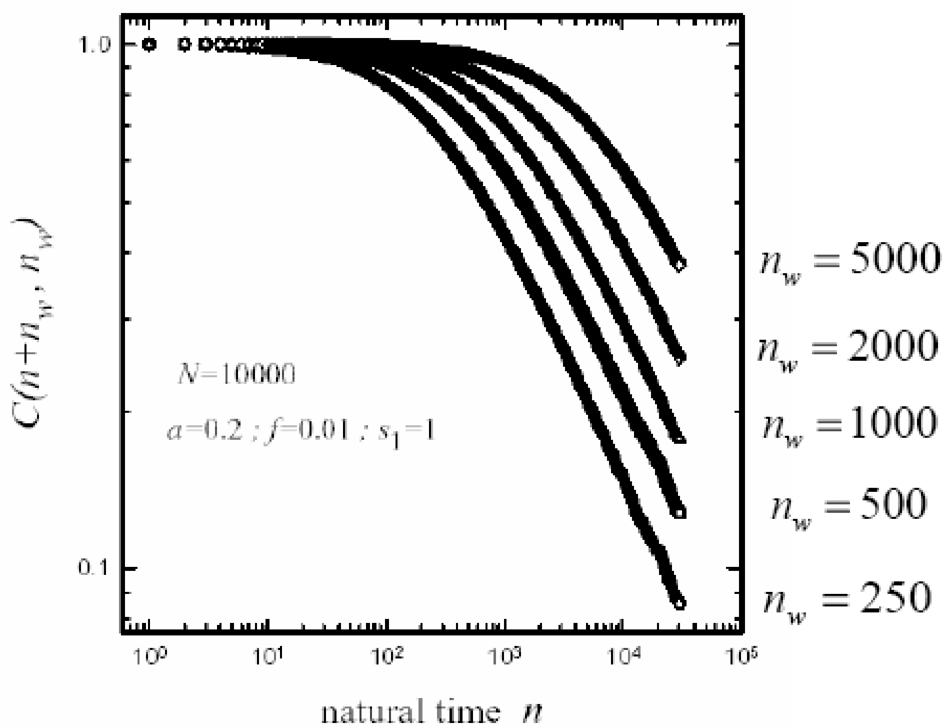
Model:
M.E.J. Newman, Proc. R. Soc. London B **263,** 1605 (1996);
M.E.J. Newman and K. Sneppen, Phys. Rev. E **54,** 6226 (199

Aging:
U. Tirnakli and S. Abe, Phys. Rev. E **70**, 056120 (2004)

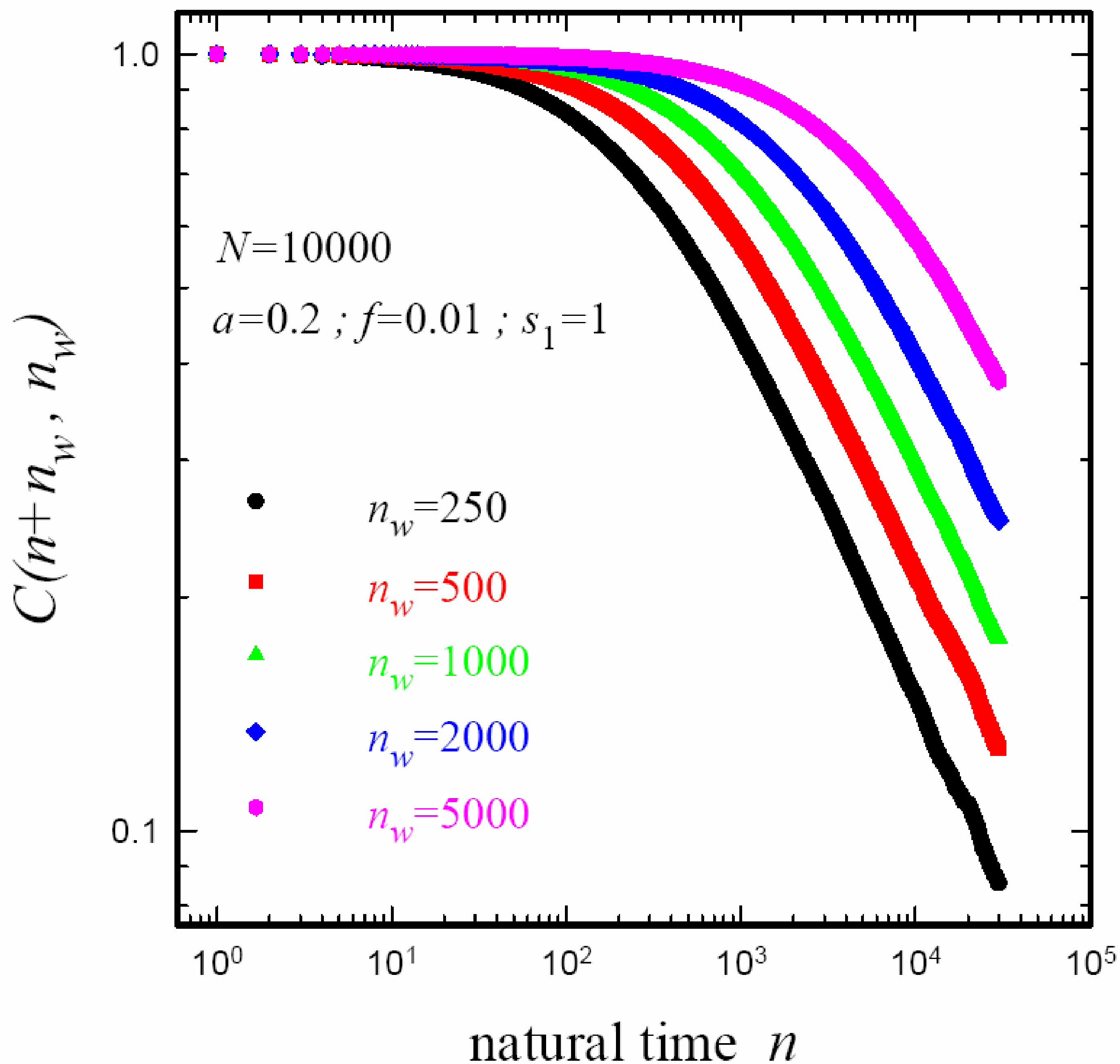




$$C(n+n_w, n_w) \equiv \frac{\langle t_{n+n_w} t_{n_w} \rangle - \langle t_{n+n_w} \rangle \langle t_{n_w} \rangle}{(\sigma^2_{n+n_w} \sigma^2_{n_w})}$$

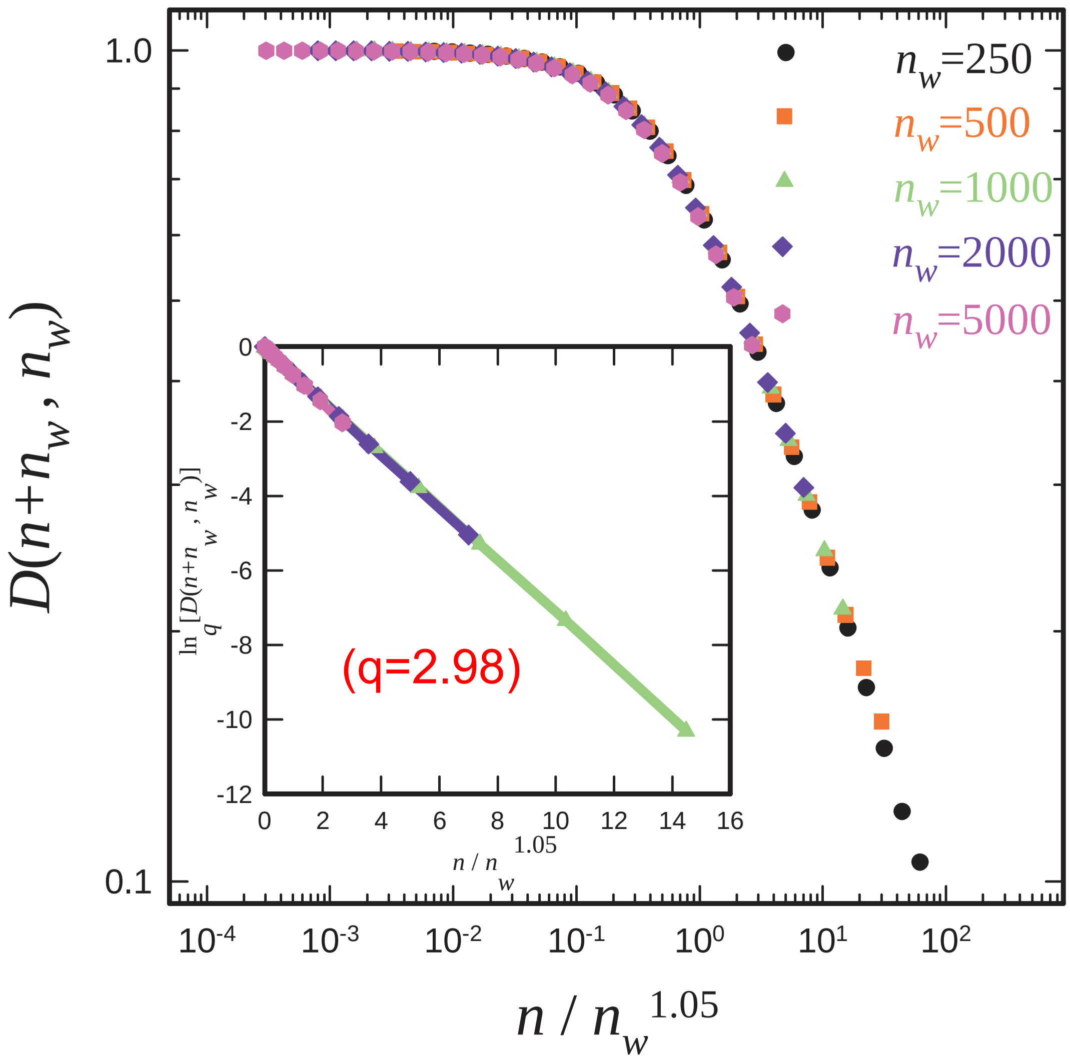


"Natural time" suggested in
P.A. Varotsos, N.V. Sarlis and E.S. Skordas,
Phys Rev E **66**, 011902 (2002); **67**, 021109; **68**, 031106 (2003).

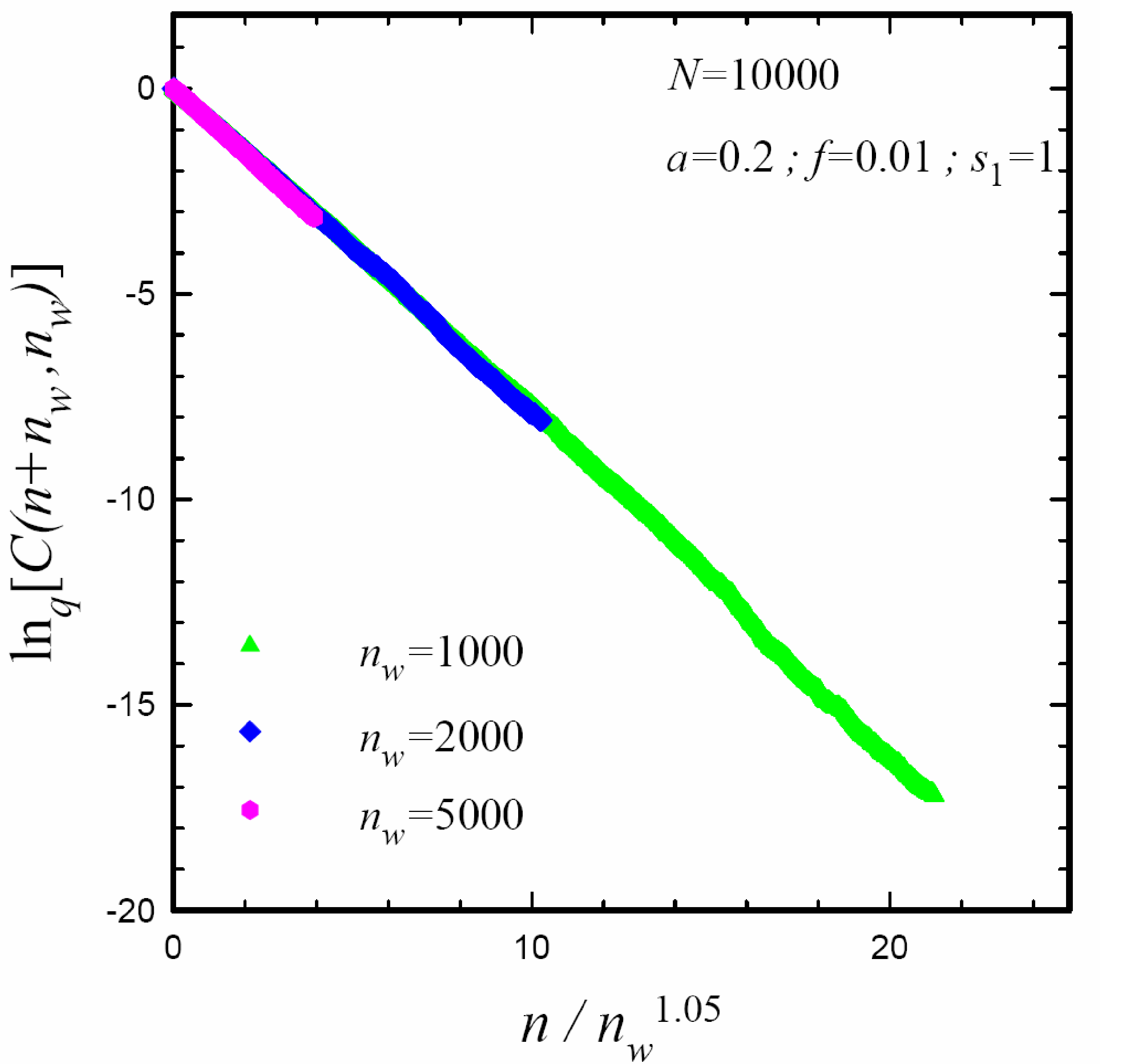


1.0
0.1
$C(n+n_w, n_w)$
N=10000
a=0.2 ; f=0.01 ; s_1=1
n_w=250
n_w=500
n_w=1000
n_w=2000
n_w=5000
10^0
10^1
10^2
10^3
10^4
10^5
natural time n

MODEL FOR EARTHQUAKES (OMORI REGIME):



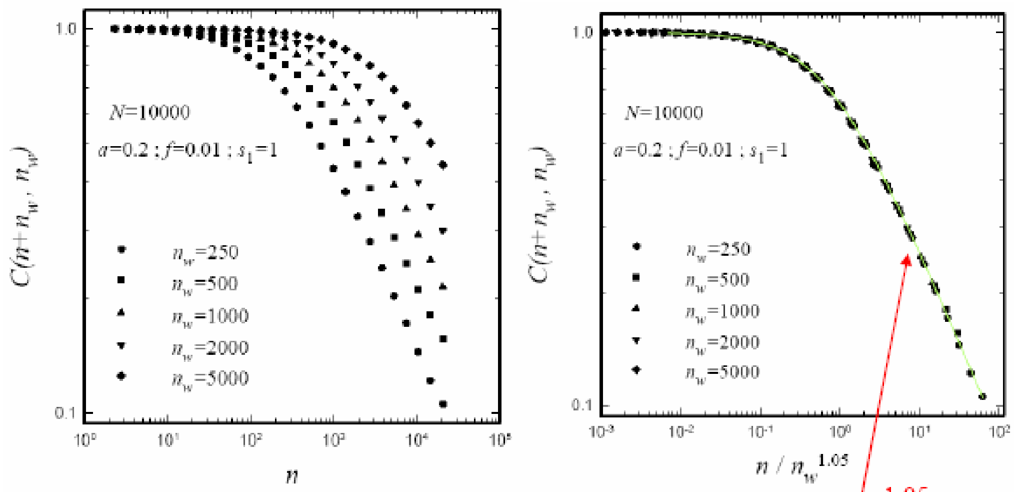
S. Abe, U. Tirnakli and P.A. Varotsos
Europhysics News **36** (6), 206 (2005) [European Physical Society]



N=10000
a=0.2 ; f=0.01 ; s1=1
ln_q[C(n+n_w, n_w)]
n / n_w^1.05
n_w=1000
n_w=2000
n_w=5000
0
-5
-10
-15
-20
0
10
20

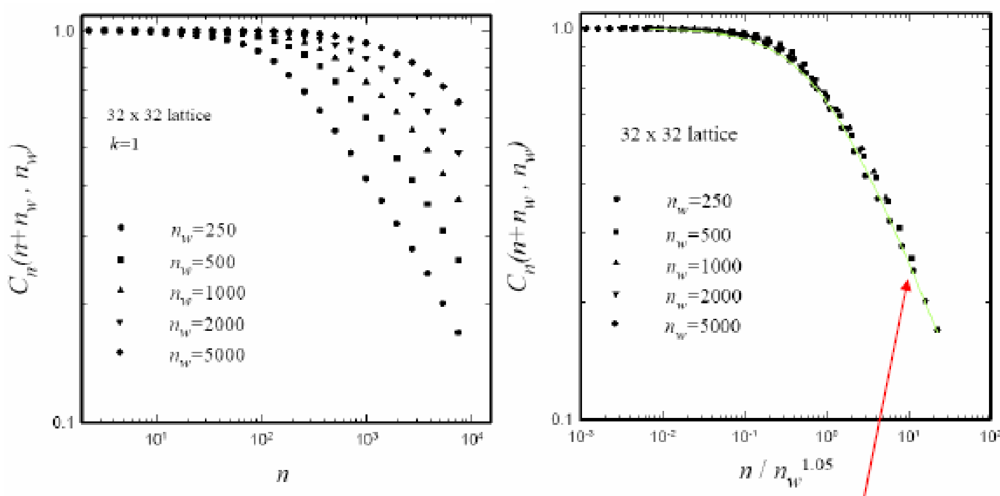
EARTHQUAKES:

NEWMAN MODEL (average over 100,000 realizations)



$e_{2.98}(0.7\ n / n_W^{1.05})$

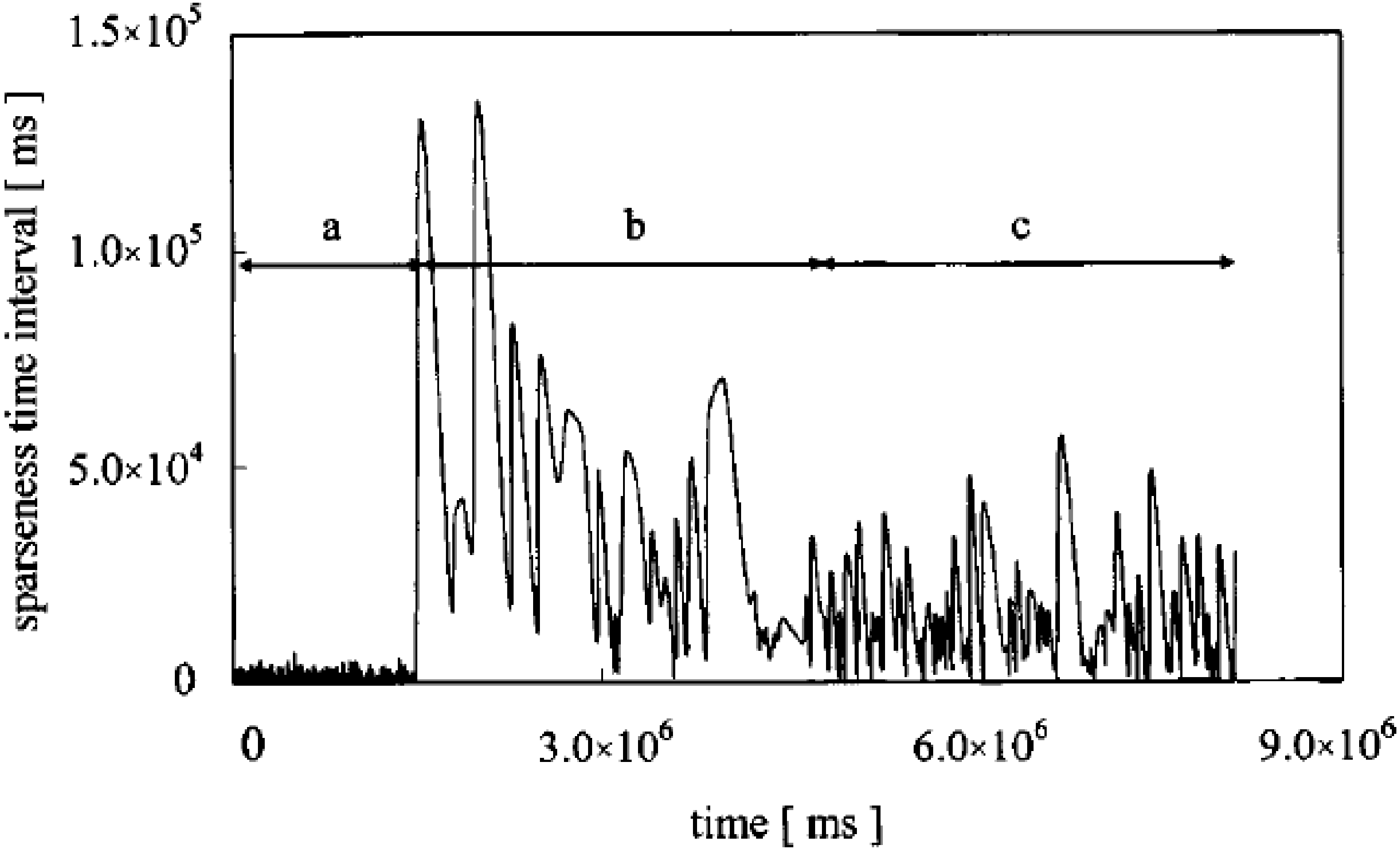
OLAMI-FEDER-CHRISTENSEN MODEL (average over 20,000 realizations)

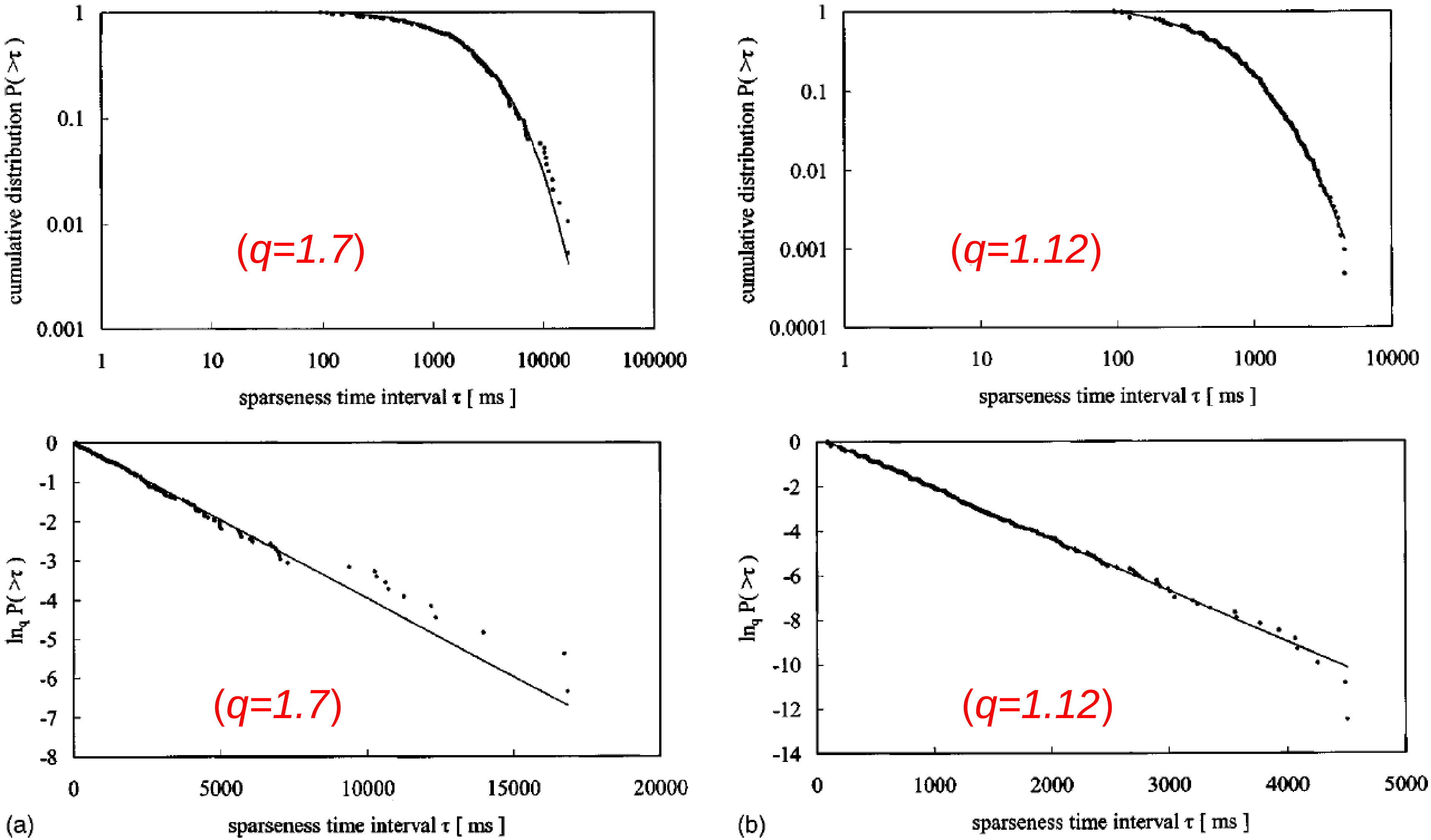


$e_{2.9}(0.6\ n / n_W^{1.05})$

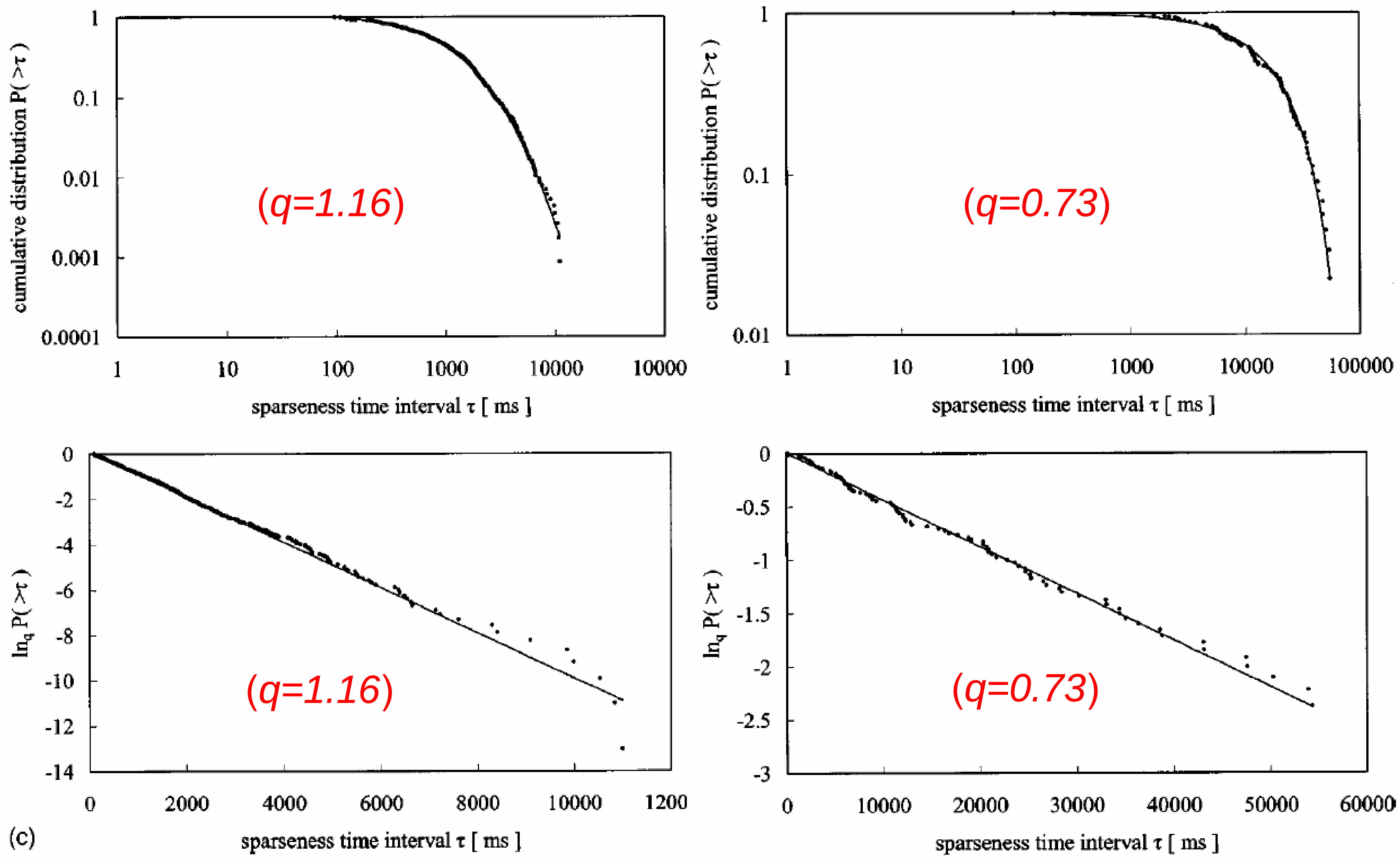
U. Tirnakli, in *Complexity, Metastability and Nonextensivity*,
eds. C. Beck, G. Benedek, A. Rapisarda and C. T. (World Scientific, Singapore, 2005), page 350

INTERNET (PING): S. Abe and Suzuki, Phys Rev E **67**, 016106 (2003)





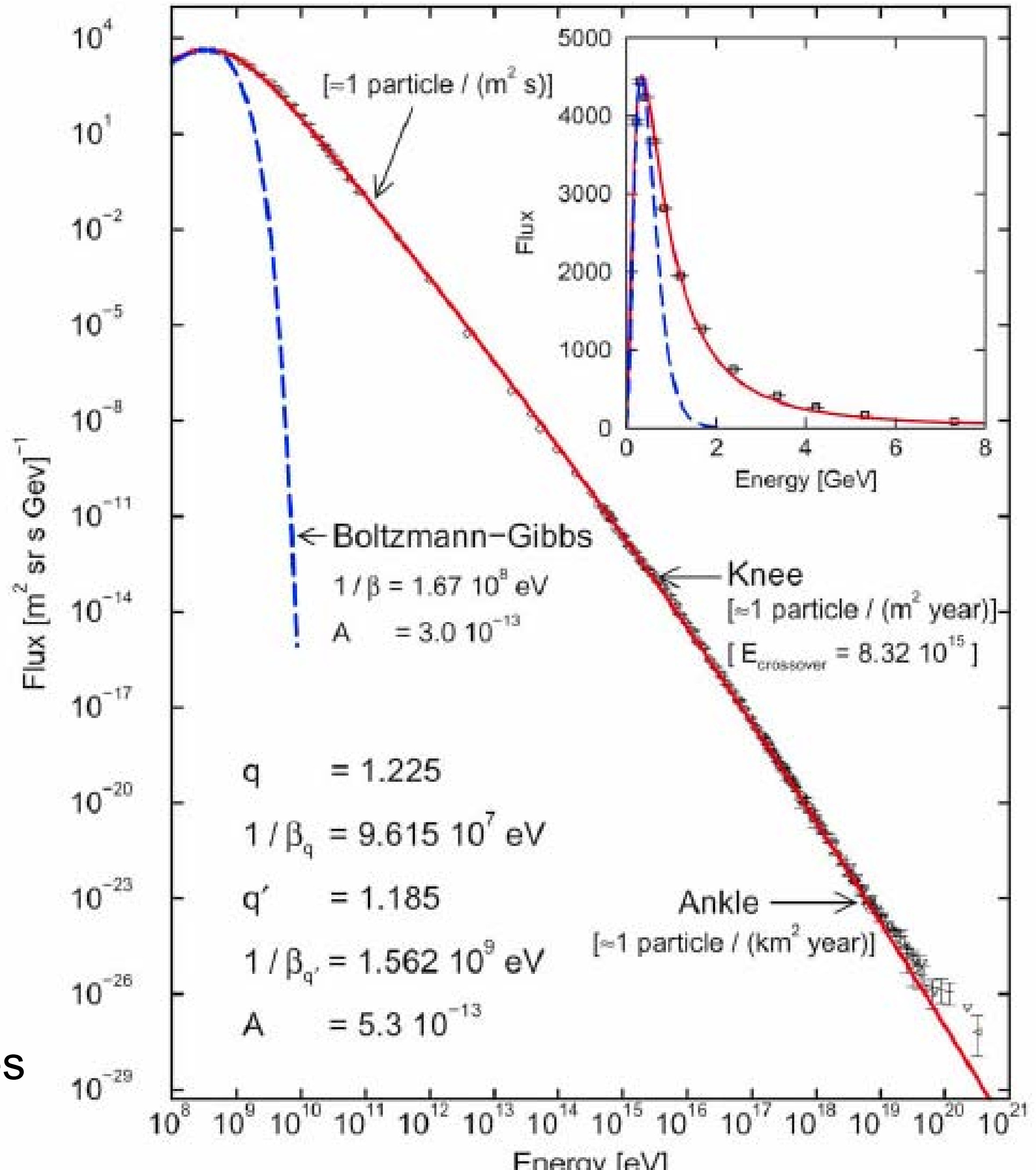
S. Abe and Suzuki, Phys Rev E **67**, 016106 (2003)



S. Abe and Suzuki, Phys Rev E **67**, 016106 (2003)

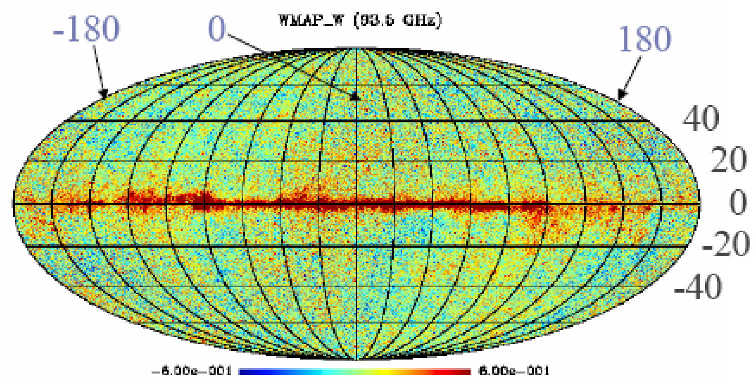
ASTROPHYSICS

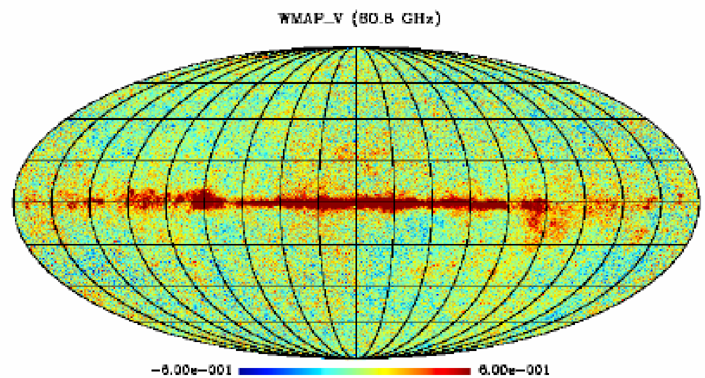
FLUX OF COSMIC RAYS:

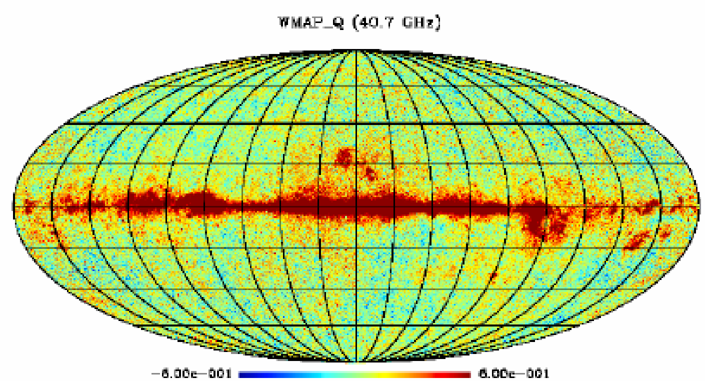


C. T, J. Anjos and E.P. Borges
Phys. Lett. A **310**, 372 (2003)

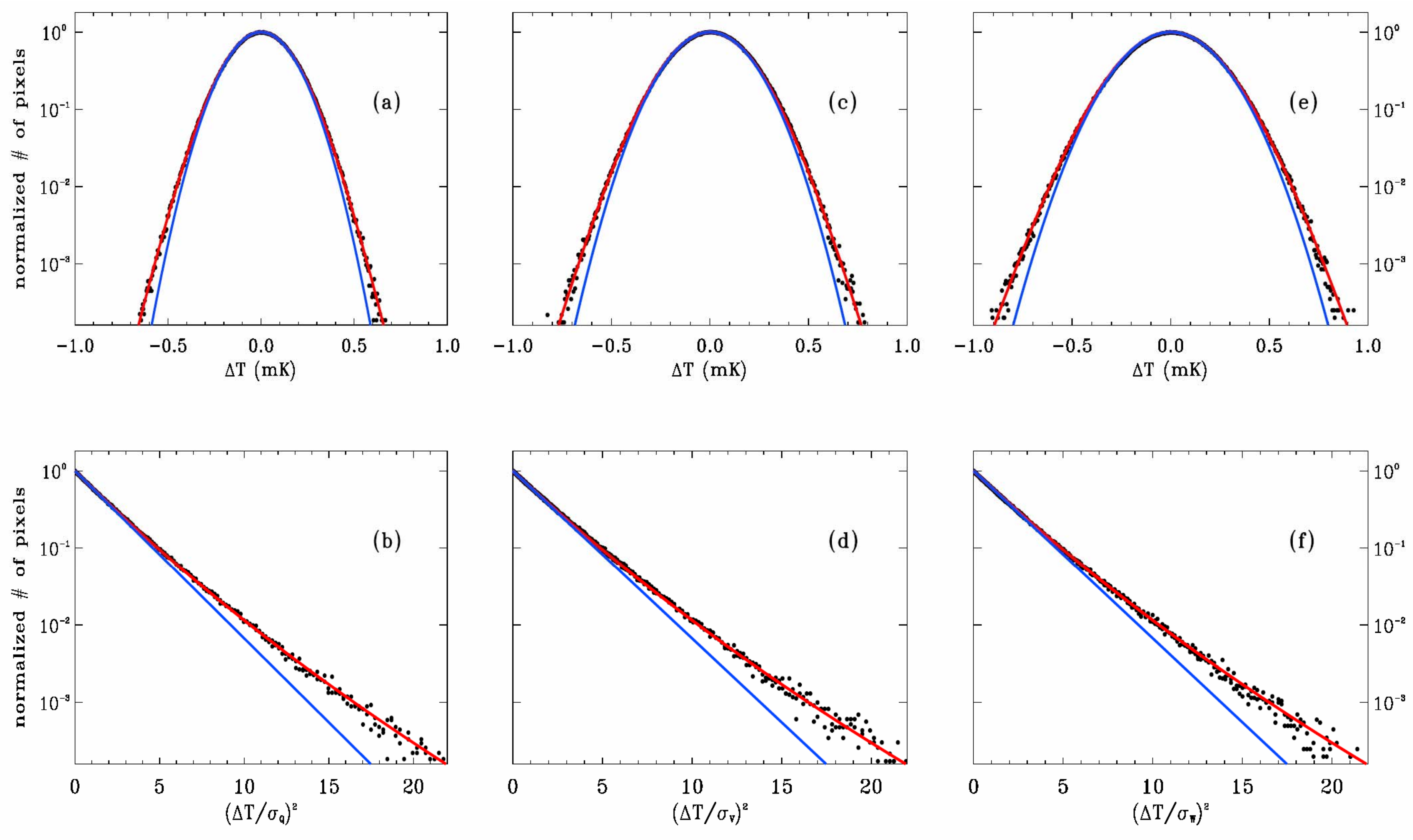
COSMIC MICROWAVE BACKGROUND RADIATION: TEMPERATURE FLUCTUATIONS





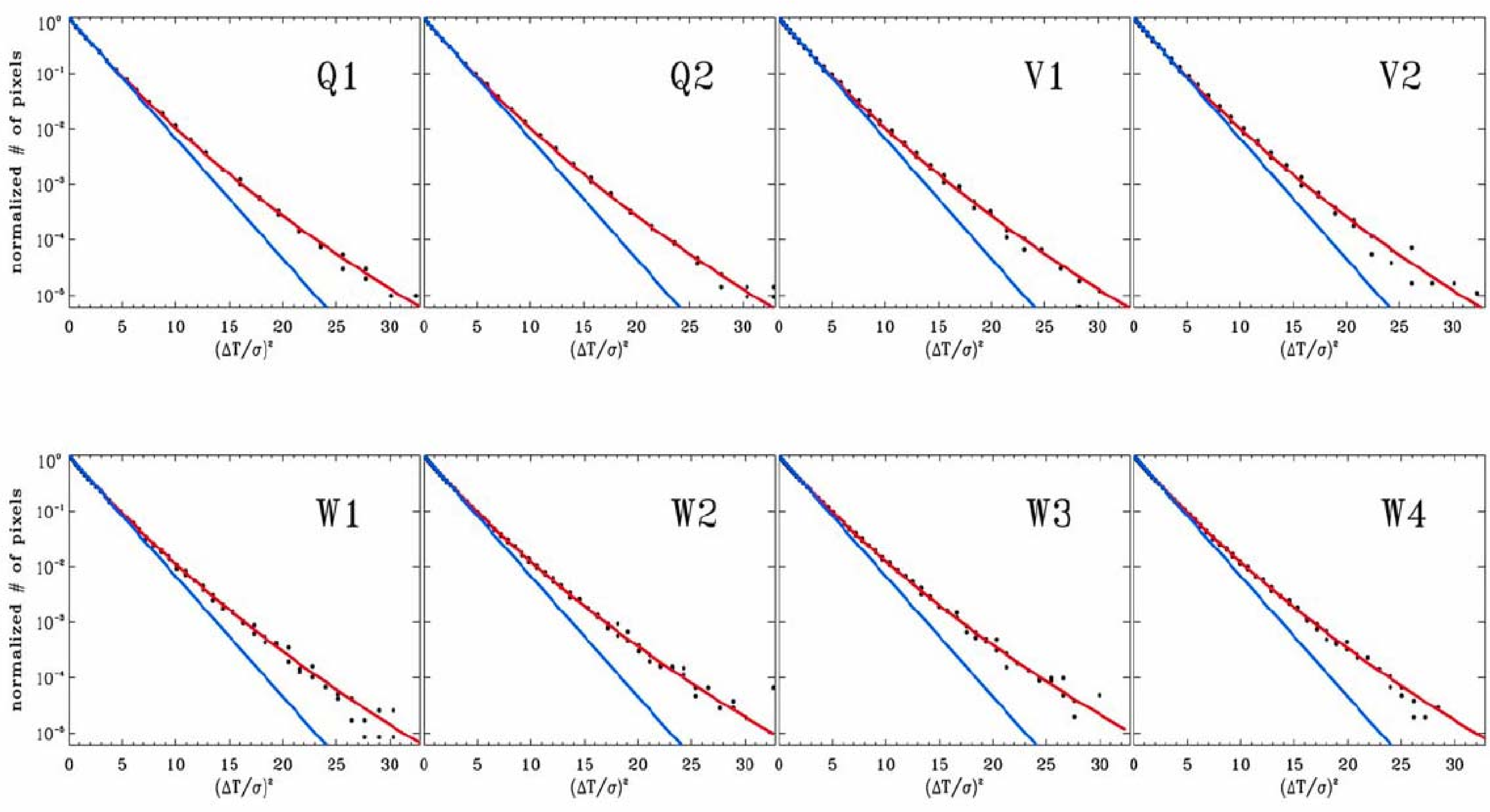


(Band Q: 22.8 GHz) (Band V: 60.8 GHz) (Band W: 93.5 GHz)



(Data after using Kp0 mask) $q = 1.045 \pm 0.005$ (99 % confidence level)

A. Bernui, C. T. and T. Villela, Phys Lett **356**, 426 (2006)

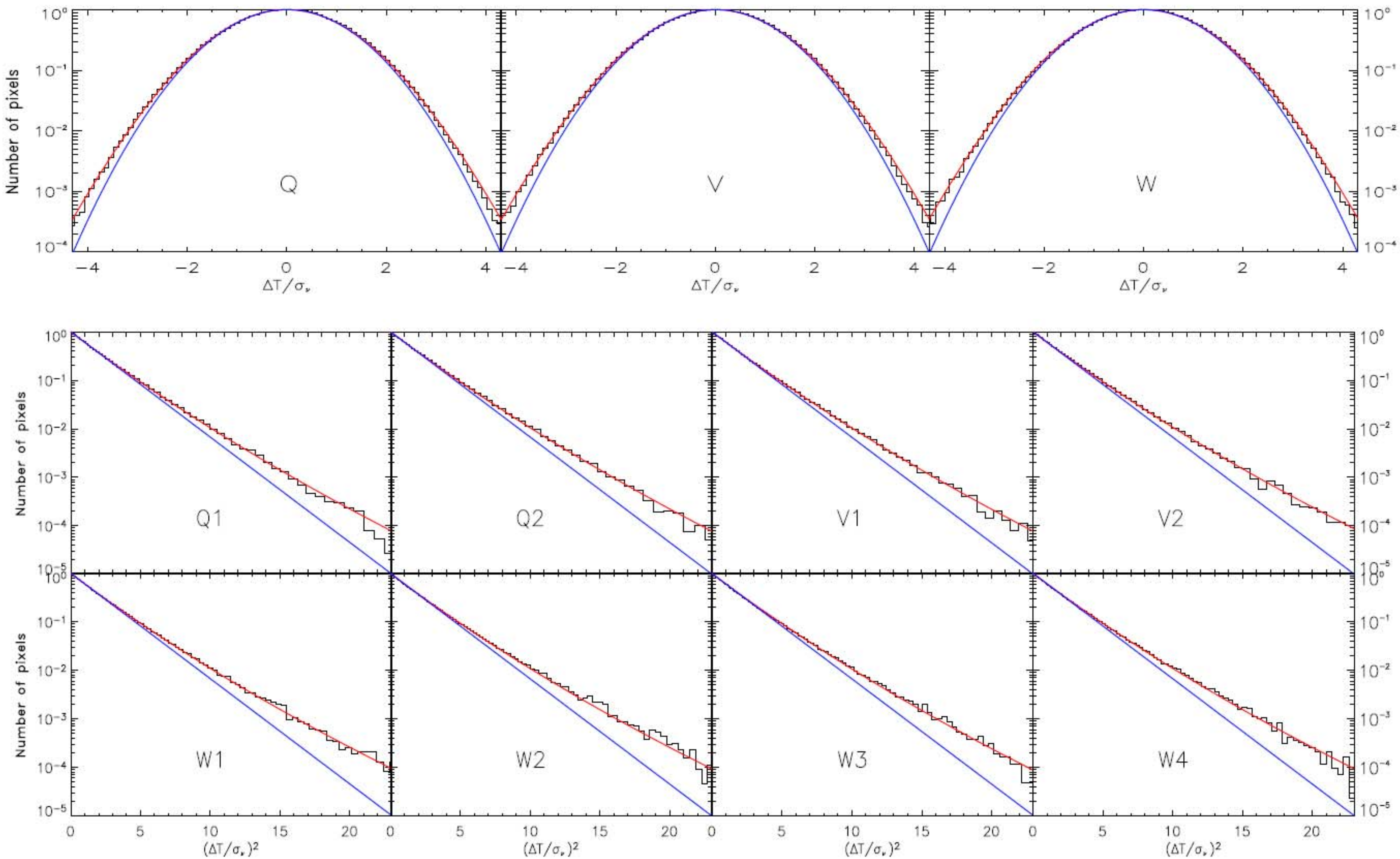


$$q = 1.045 \pm 0.005 \quad (99\ \% \ \text{ confidence level})$$

A. Bernui, C. T. and T. Villela, Phys Lett **356**, 426 (2006)

Using the release of 3-year data:

$$q = 1.04 \pm 0.01$$

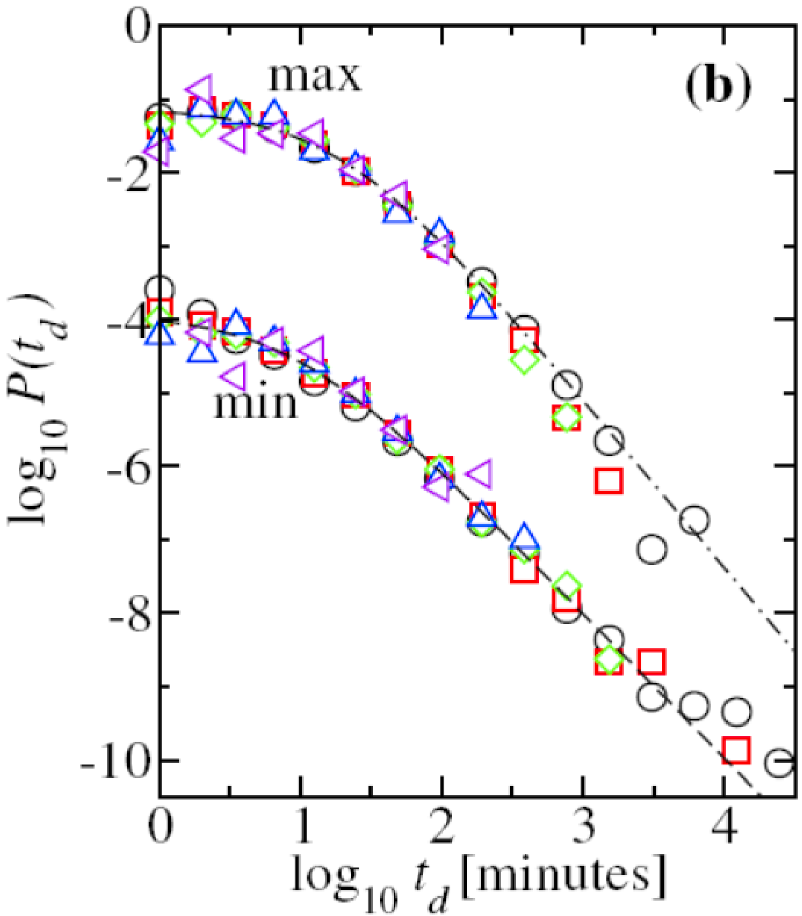


A. Bernui, C. T. and T. Villela, Europhys Lett **78**, 19001 (2007)

SOLAR FLARES:

$q_{max} \simeq 1.43$

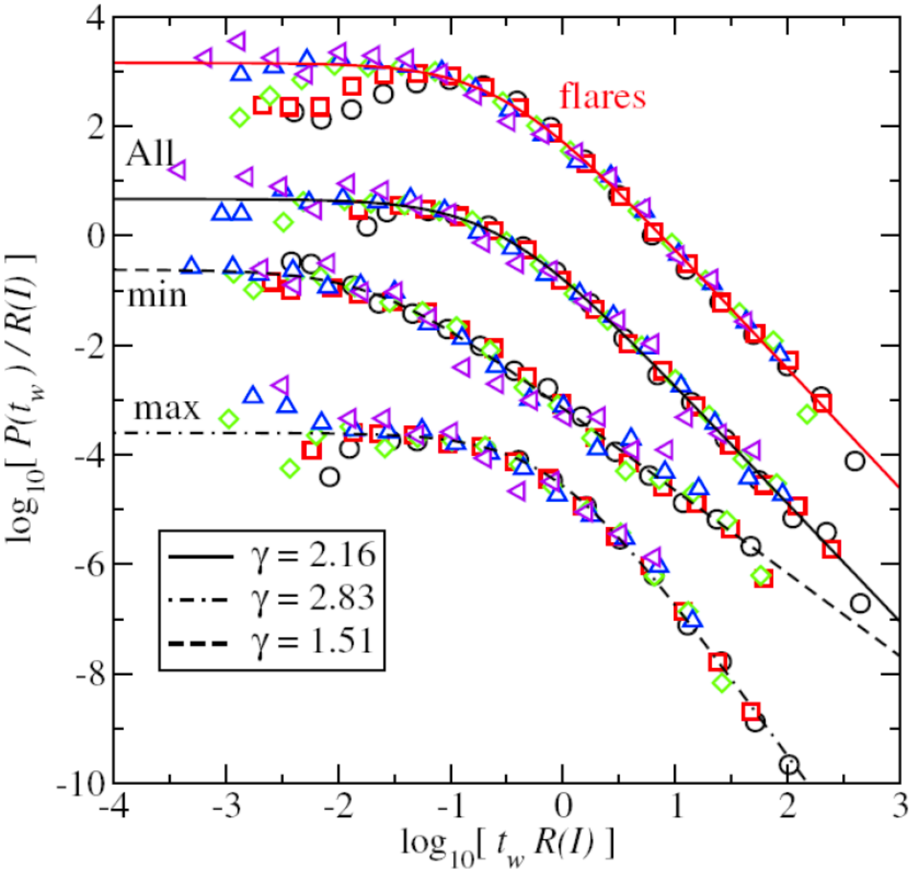
$q_{min} \simeq 1.50$



$q_{All} \simeq 1.46$

$q_{min} \simeq 1.66$

$q_{max} \simeq 1.35$



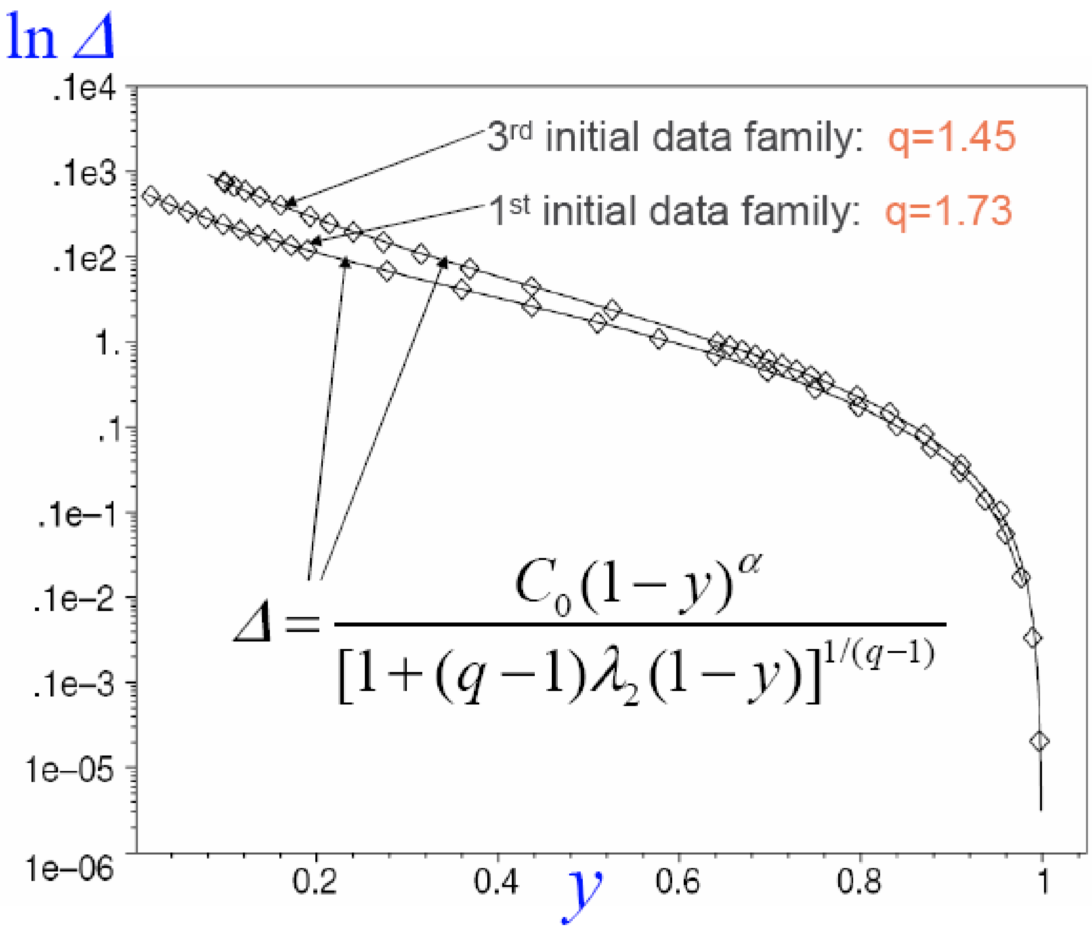
M. Baiesi, M. Paczuski and A. Stella, Phys Rev Lett **96**, 051103 (2006)

GRAVITATIONAL EMISSION FROM A BLACK HOLE

[Oliveira and Damiao Soares, Phys. Rev. D **70**, 084041(2004)]

$$\Delta \equiv \textit{fraction of mass extracted} \equiv \frac{M_{init} - M_{\infty}}{M_{init}}$$

$$y \equiv \textit{dimensionless initial mass} \propto M_{init}$$



$$\Delta = \frac{C_0 (1-y)^{\alpha}}{[1+(q-1)\lambda_2 (1-y)]^{1/(q-1)}}$$

CHEMICAL RE-ASSOCIATION

$$\frac{dy}{dx} = -a_1 y - (a_q - a_1) y^q \quad with \quad y(0) = 1$$

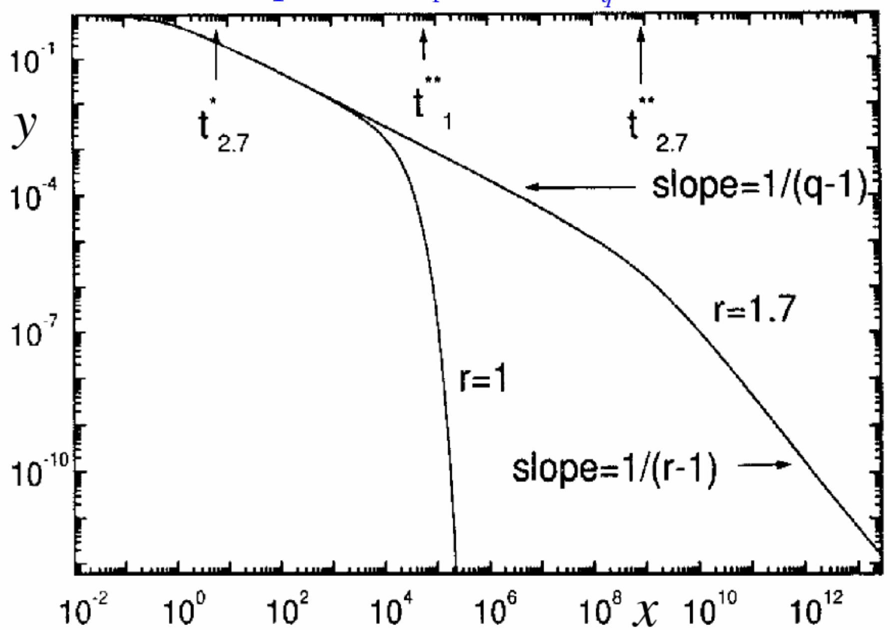
$$\Rightarrow y = \frac{1}{\left[1 - \frac{a_q}{a_1} + \frac{a_q}{a_1} e^{(q-1)a_1 x}\right]^{\frac{1}{q-1}}}$$

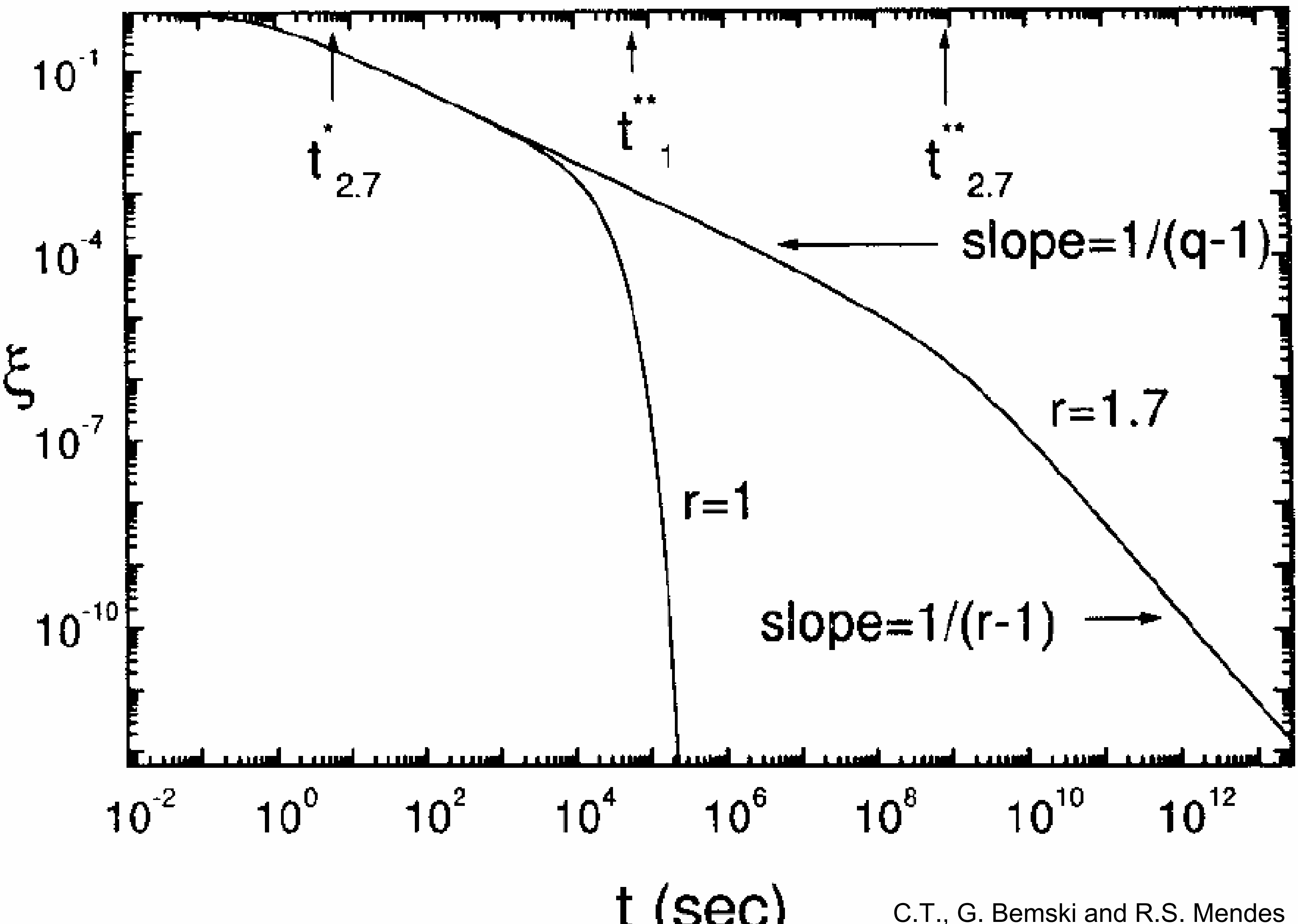
$$\frac{dy}{dx} = -a_r y^r - (a_q - a_r) y^q \quad with \quad y(0) = 1 \qquad (r \le q)$$

$$\Rightarrow x = f(y\ ; a_r, a_q, r, q)$$

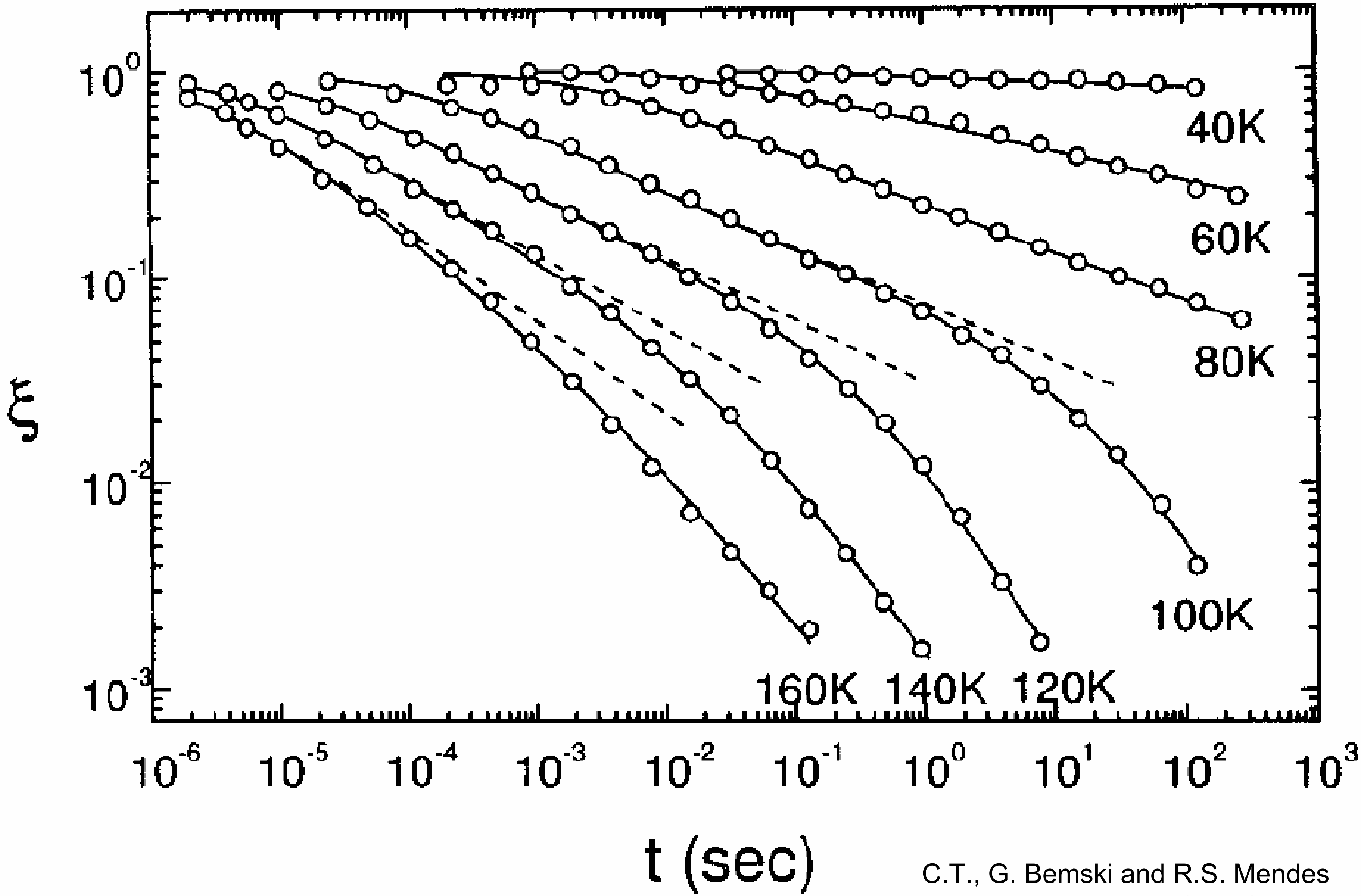
where f involves hypergeometric functions

$(q = 2.7;\ a_1 = 10^{-5};\ a_q = 1)$

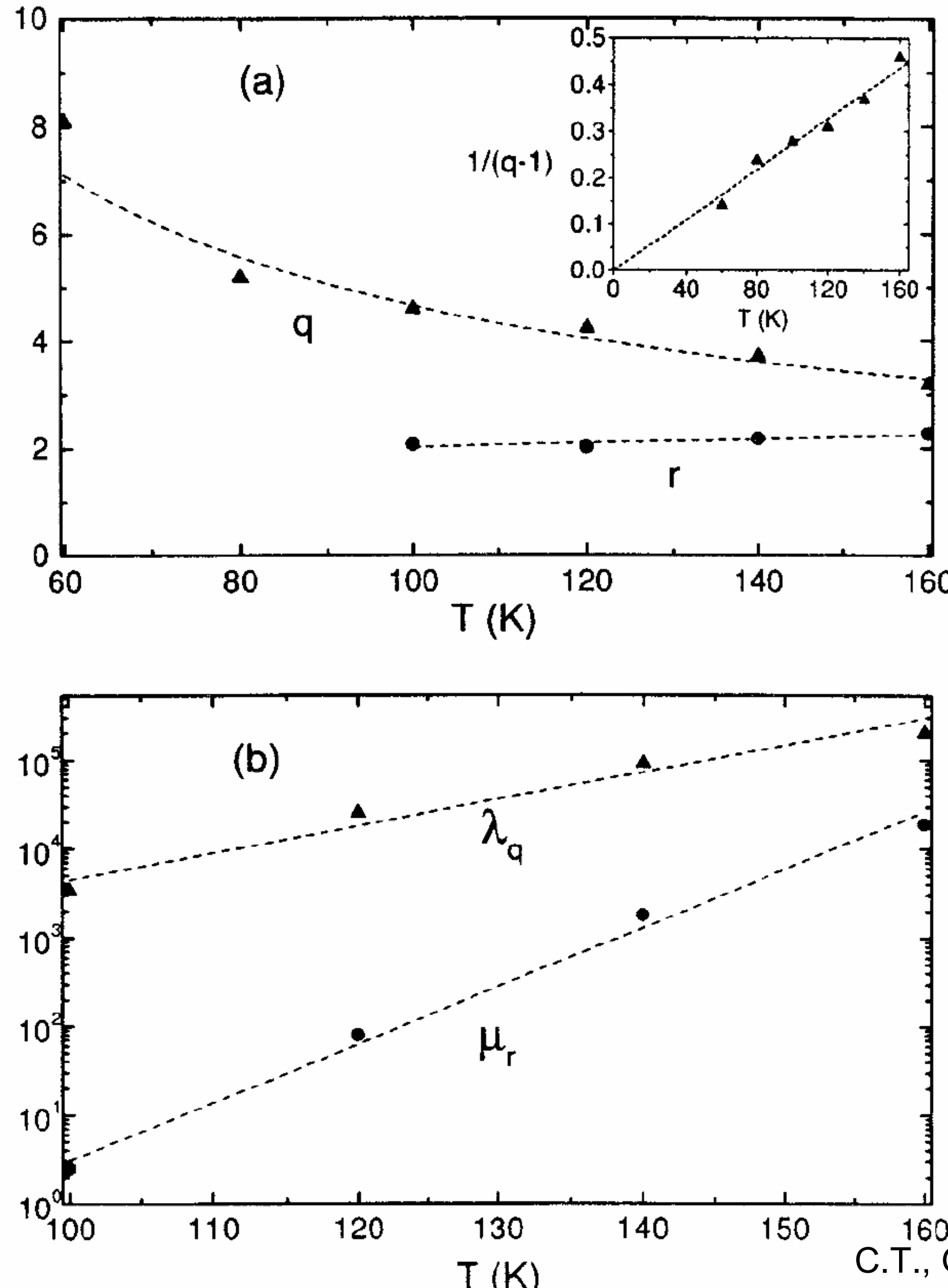




C.T., G. Bemski and R.S. Mendes
Phys. Lett. A **257**, 93 (1999)

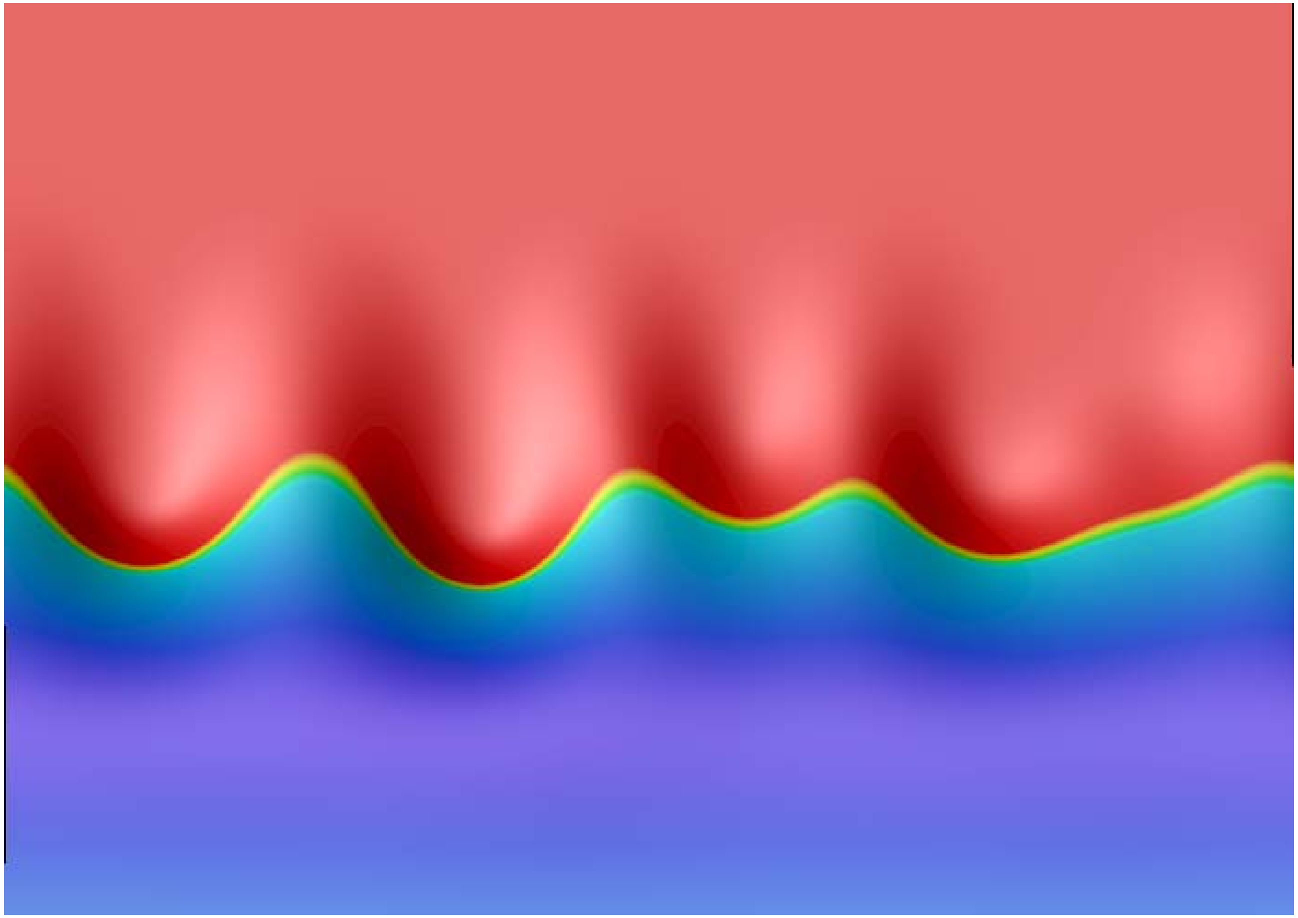


C.T., G. Bemski and R.S. Mendes
Phys. Lett. A **257**, 93 (1999)

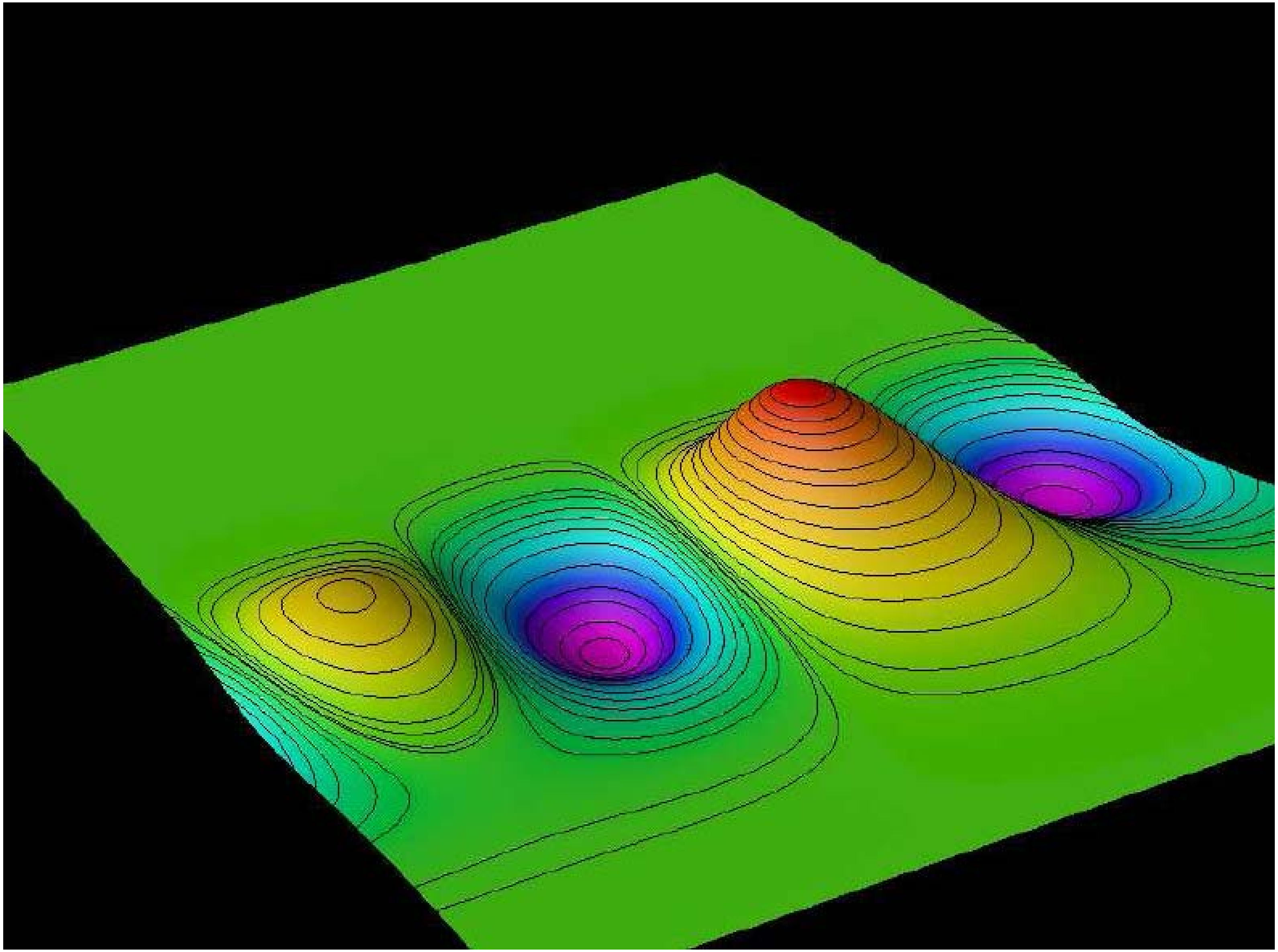


C.T., G. Bemski and R.S. Mendes
Phys. Lett. A **257**, 93 (1999)

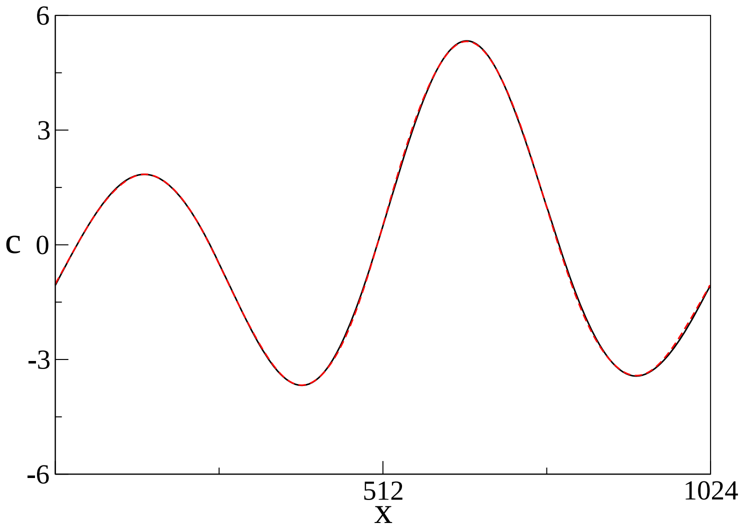
FINGERING

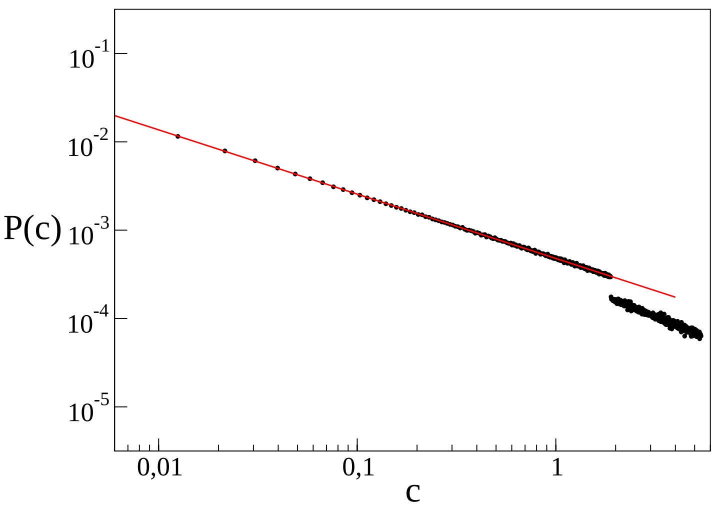


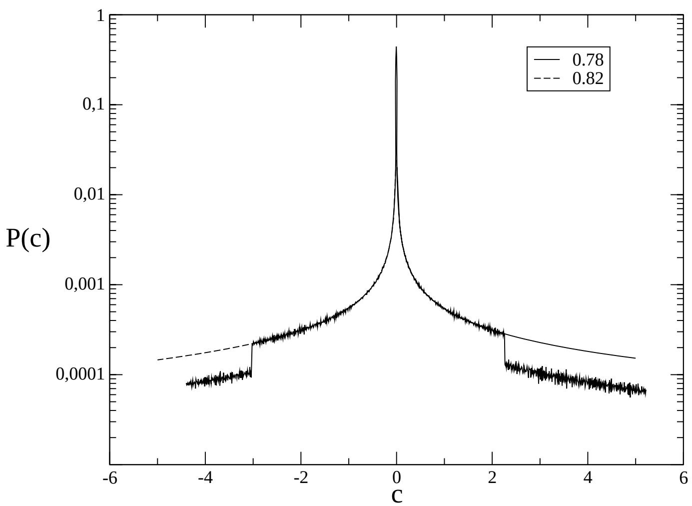
P. Grosfils and J.P. Boon, 2005



P. Grosfils and J.P. Boon, 2005

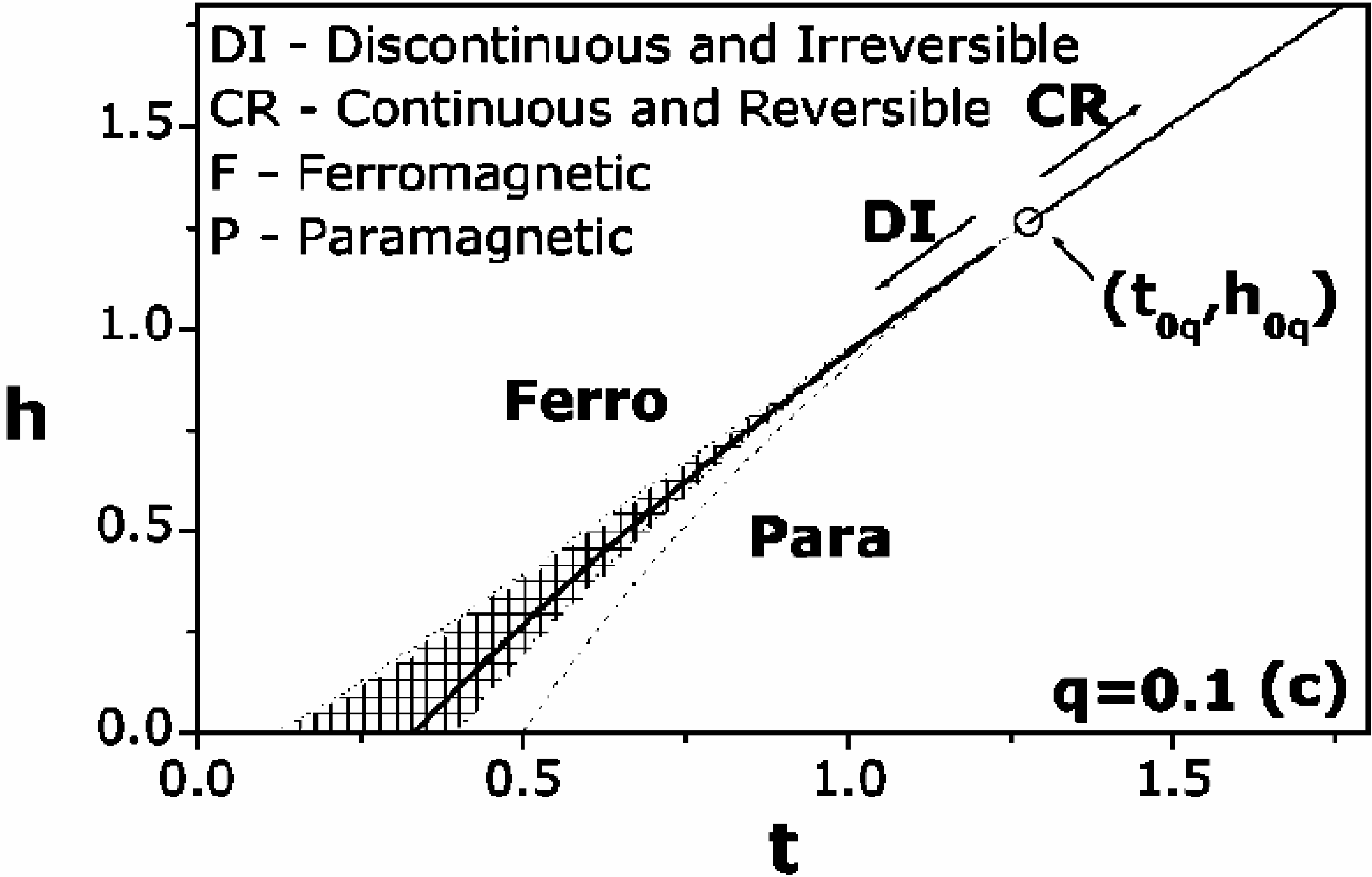




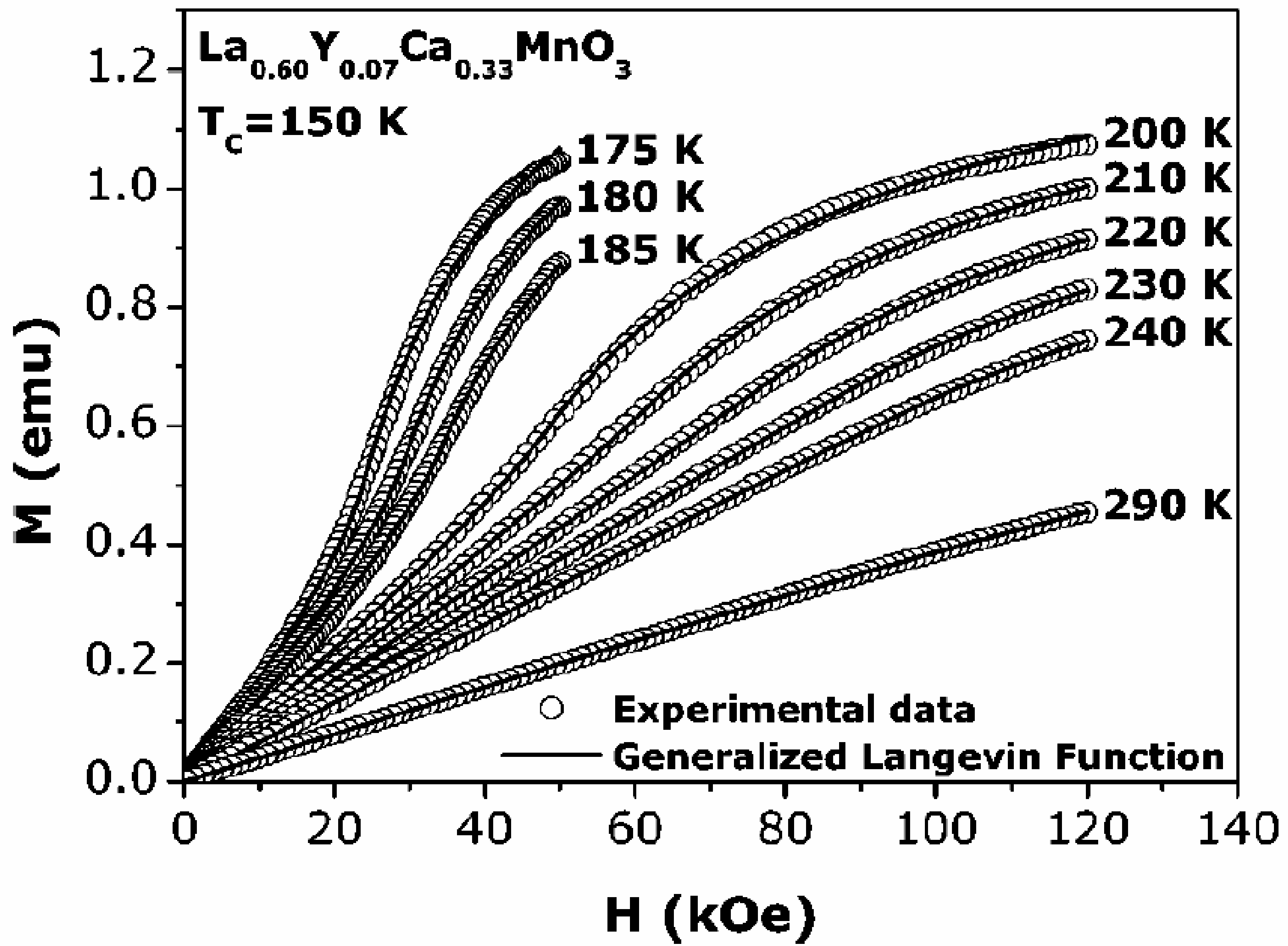


P. Grosfils and J.P. Boon, 2005

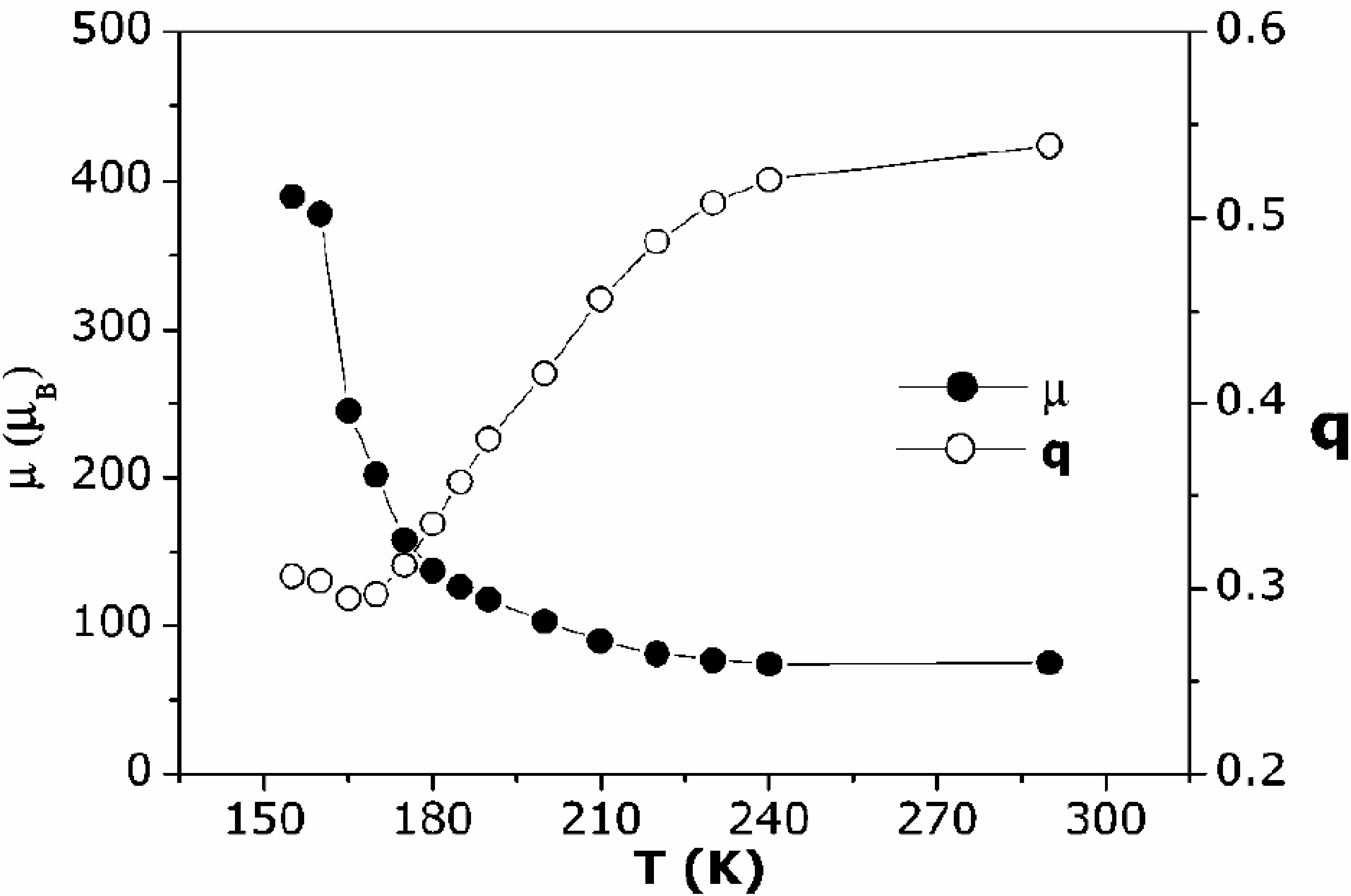
MANGANITES



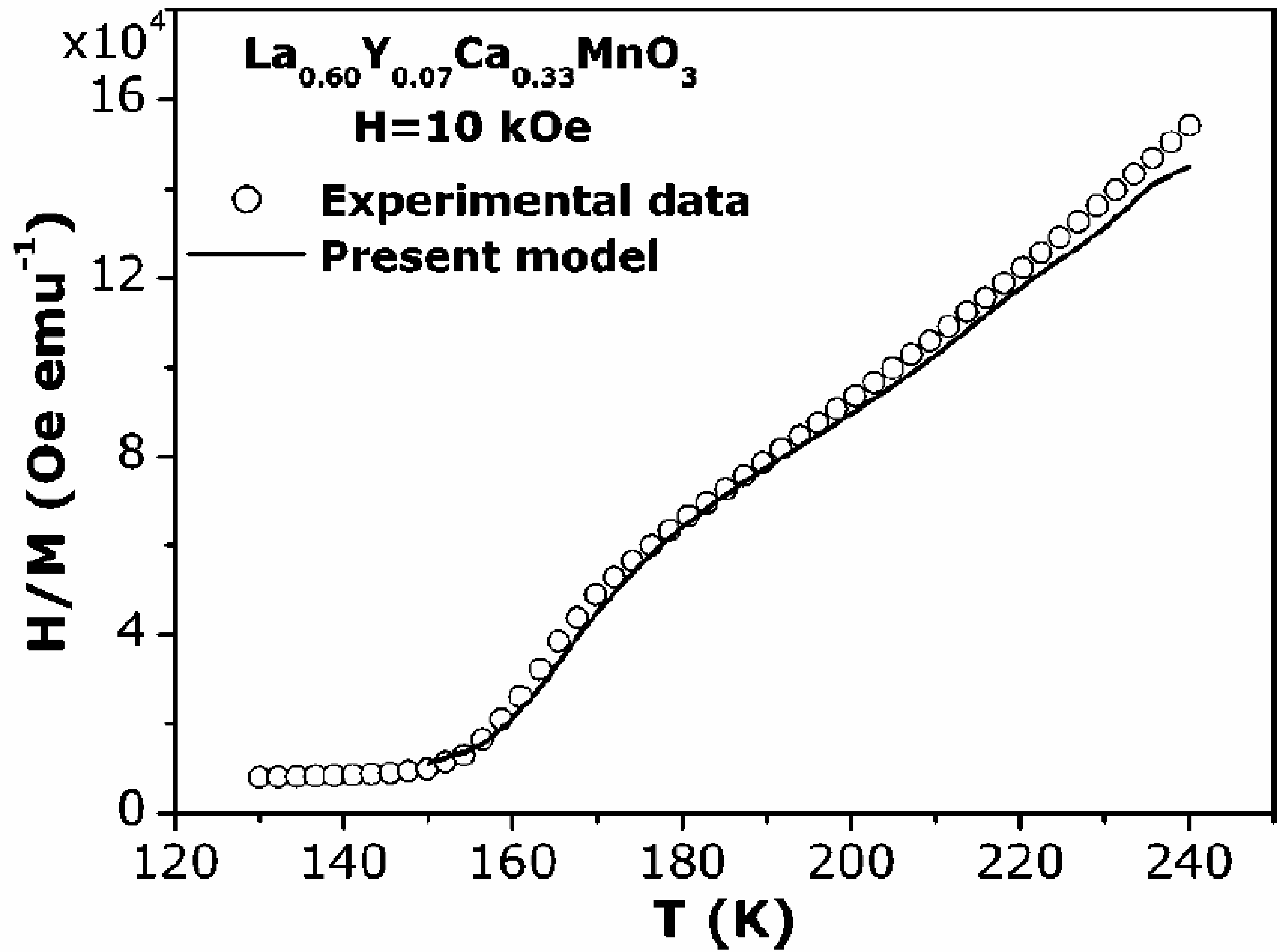
Reis et al, Phys Rev B (2003)

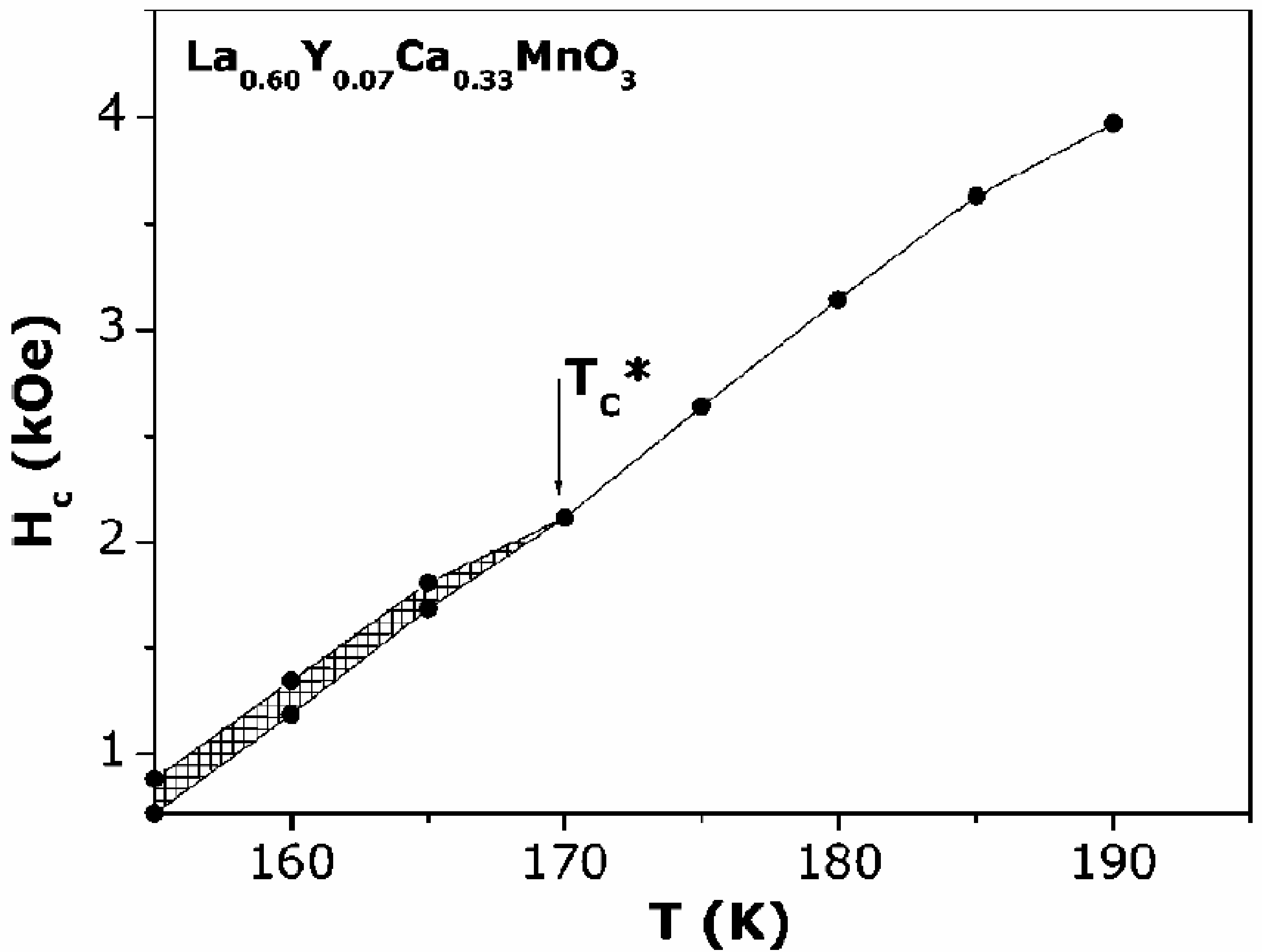


La0.60Y0.07Ca0.33MnO3
TC=150 K
175 K
180 K
185 K
200 K
210 K
220 K
230 K
240 K
290 K
Experimental data
Generalized Langevin Function
M (emu)
H (kOe)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
20
40
60
80
100
120
140



500
400
300
200
100
0
μ (μ$_B$)
0.6
0.5
0.4
0.3
0.2
q
μ
q
150
180
210
240
270
300
T (K)


x10^4
La0.60Y0.07Ca0.33MnO3
H=10 kOe
Experimental data
Present model
16
12
8
4
0
H/M (Oe emu-1)
120
140
160
180
200
220
240
T (K)



$La_{0.60}Y_{0.07}Ca_{0.33}MnO_3$
T_C*
H_c (kOe)
T (K)
1
2
3
4
160
170
180
190

ECONOMICS

$$dv = -\gamma \left(v - \frac{\alpha + 1}{\beta} \right) dt + \sqrt{2\, v\, \frac{\gamma}{\beta}}\, dW_t,$$

$$P(v) = \frac{1}{Z} \left(\frac{v}{\theta} \right)^{\alpha} \exp_q \left(-\frac{v}{\theta} \right) \qquad e_q^x \equiv [1 + (1-q)\, x]^{\frac{1}{1-q}} \qquad (e_1^x \equiv e^x)$$

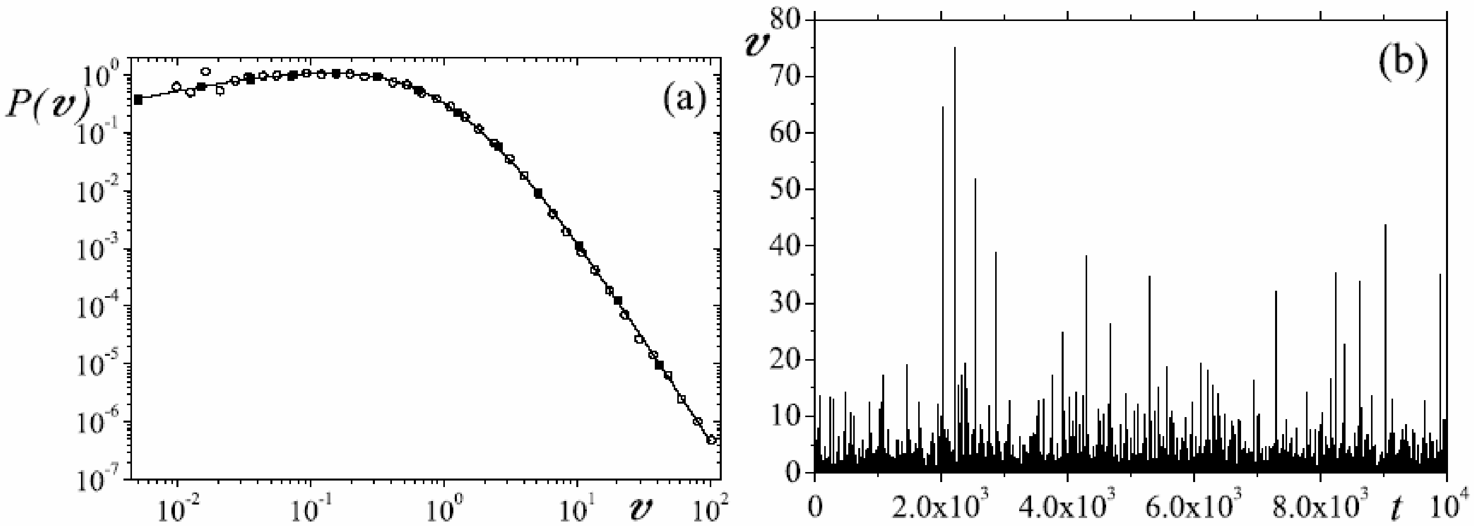
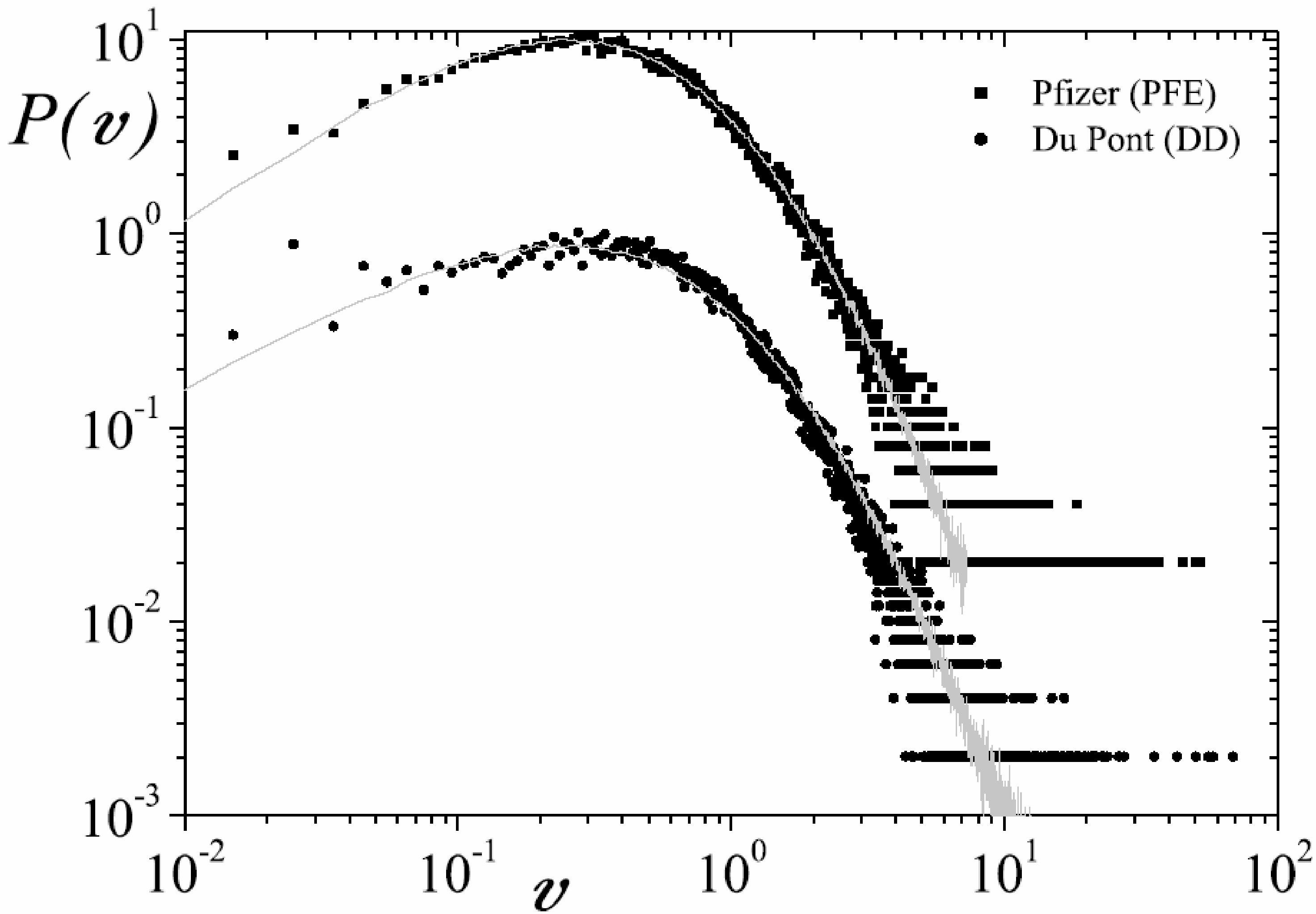


Figure 6. In panel (a) open symbols represents the PDF for the ten-high 1 minute traded volume stocks in NYSE exchange; solid symbols represent the PDF obtained for the numerical realization depicted in panel (b) and line the theoretical PDF Eq. (28). Parameters are $q = 1.17$, $\alpha = 1.79$, $\lambda = 1.42$ and $\delta = 3.09$.

[STOCK VOLUMES:]

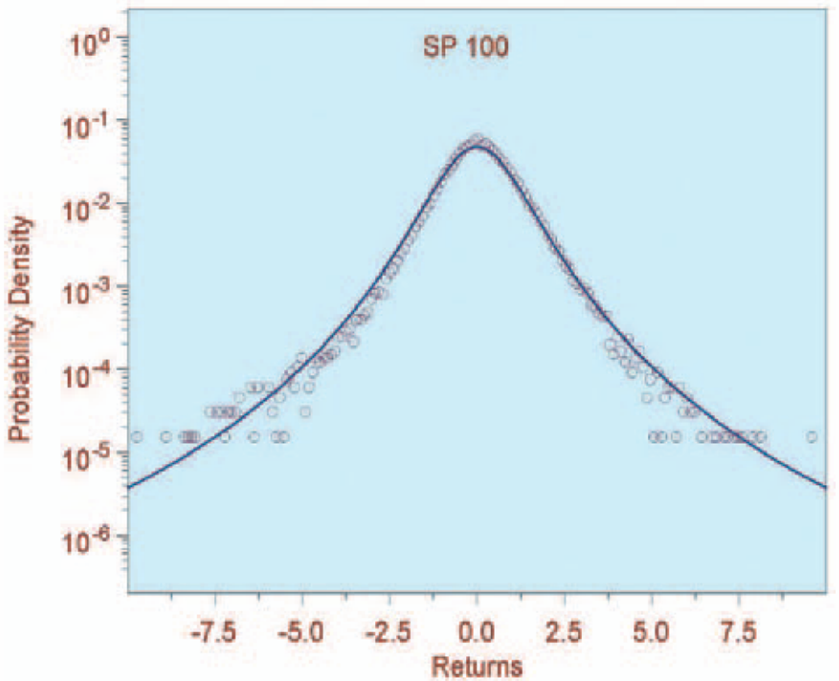


$$p\left(u\right)=\frac{1}{Z}\left(\frac{v}{\theta}\right)^{-\alpha-2}\exp_{q}\left[-\frac{\theta}{v}\right]$$

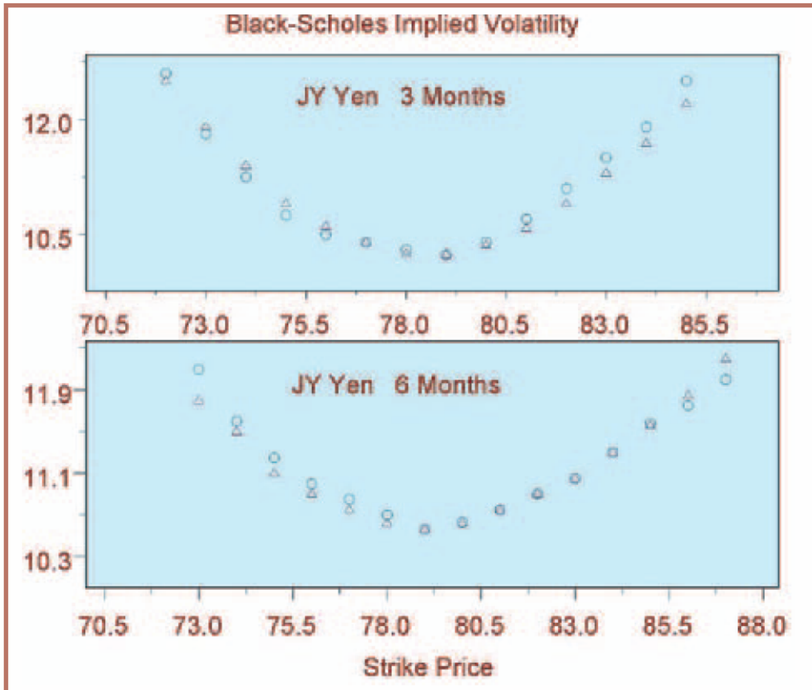
J de Souza, LG Moyano and SMD Queiros, Eur Phys J B **50**, 165 (2006)

q-GENERALIZED BLACK-SCHOLES EQUATION:

L Borland, Phys Rev Lett **89**, 098701 (2002), and Quantitative Finance **2**, 415 (2002)
L Borland and J-P Bouchaud, Quantitative Finance **4**, 499 (2004)
L Borland, Europhys News **36**, 228 (2005)
See also H Sakaguchi, J Phys Soc Jpn **70**, 3247 (2001)
C Anteneodo and C T, J Math Phys **44**, 5194 (2003)



▲ Fig.2: The empirical distribution of daily returns from the stocks comprising the SP 100 (red) is fit very well by a q-Gaussian with $q = 1.4$ (blue).

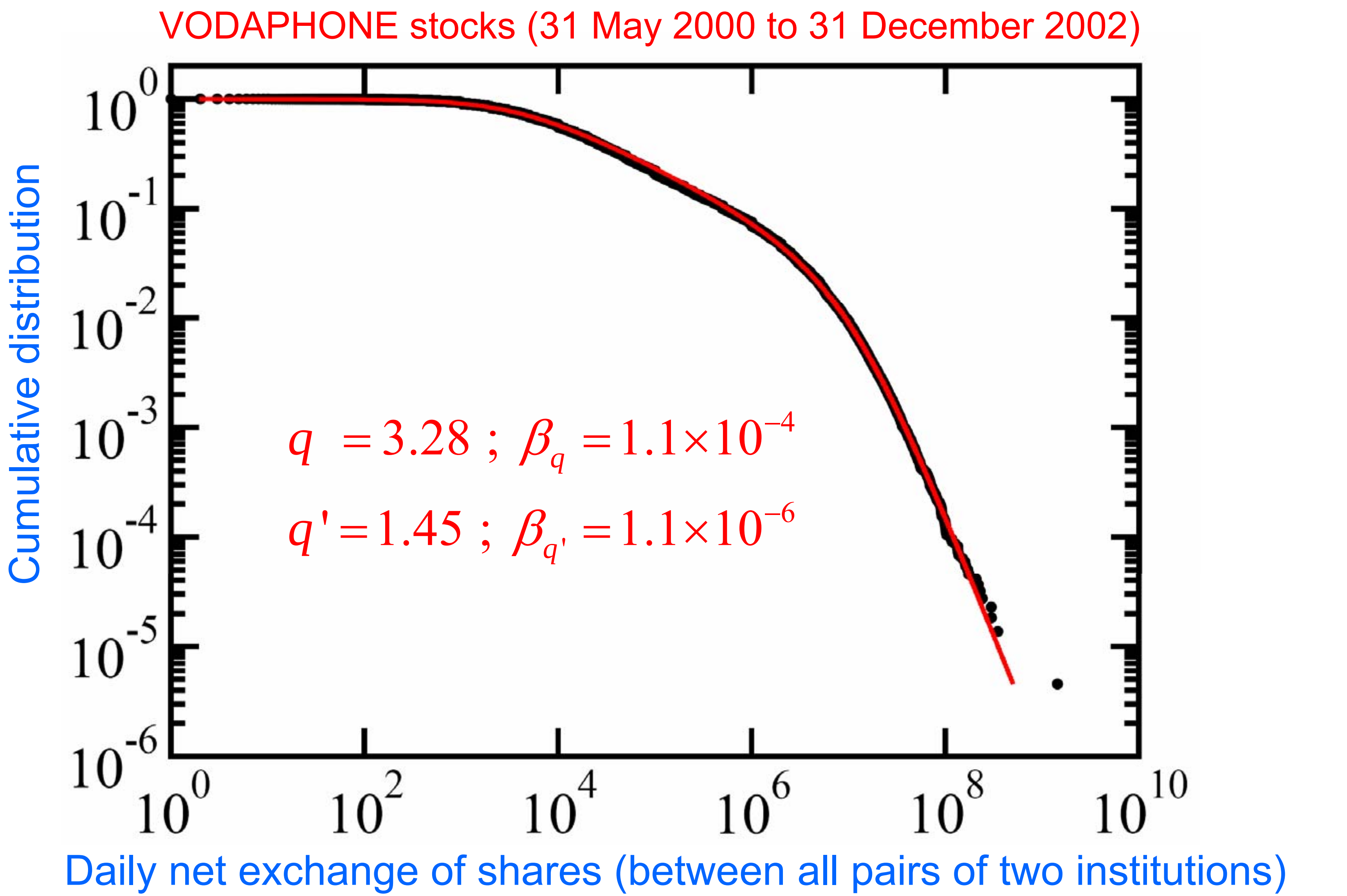


▲ Fig.3: Theoretical implied Black-Scholes volatilities from the $q = 1.4$ model (triangles) match empirical ones (circles) very well, across all strikes and for different times to expiration.

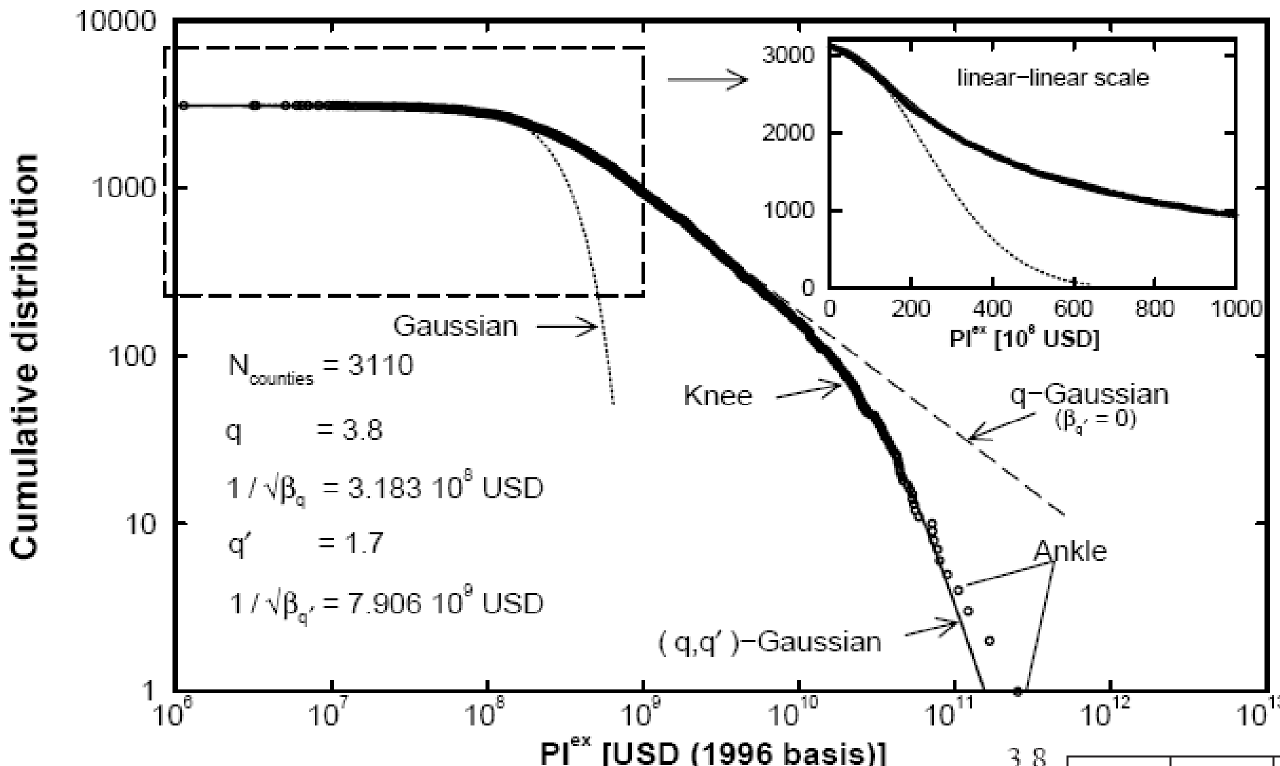
[*REMARK: Student t−distributions are the particular case of q−Gaussians when* $q = \frac{n+3}{n+1}$ *with n integer*]

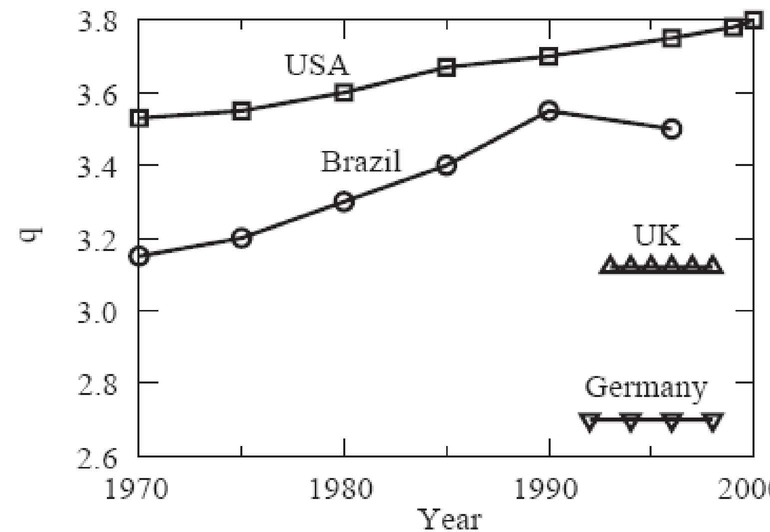
LONDON STOCK EXCHANGE (Block market):

Data: I.I. Zovko; **Fitting**: E.P. Borges (2005)



WEALTH DISTRIBUTION: E.P. Borges, Physica A **334**, 255 (2004)

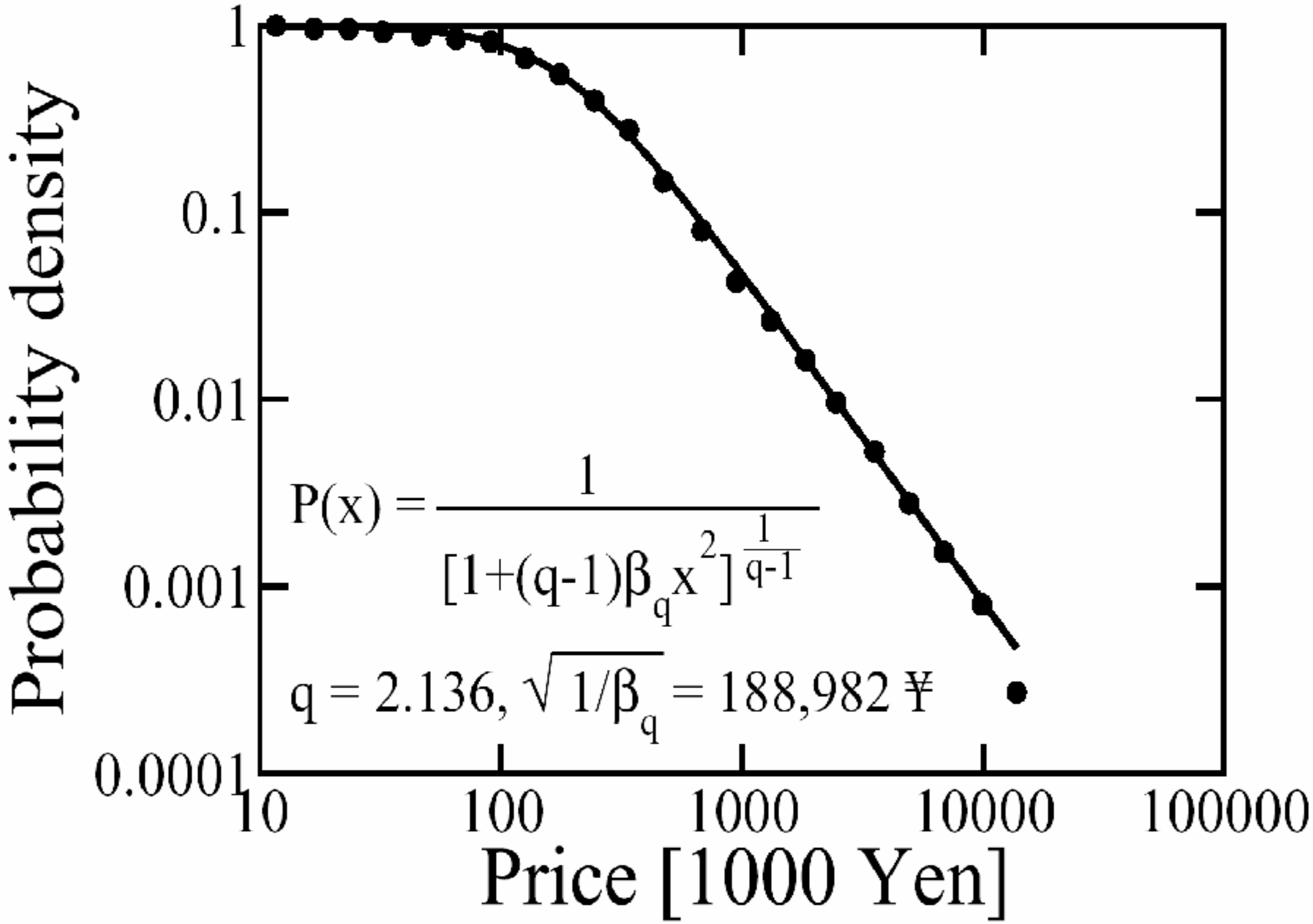




LAND PRICES IN JAPAN

(cumulative distribution)

Data: T. Kaizoji, Physica A **326**, 256 (2003)
Curve: E.P. Borges (2003)



COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

Theoretical predictions by E. Lutz, Phys Rev A **67**, 051402(R) (2003):

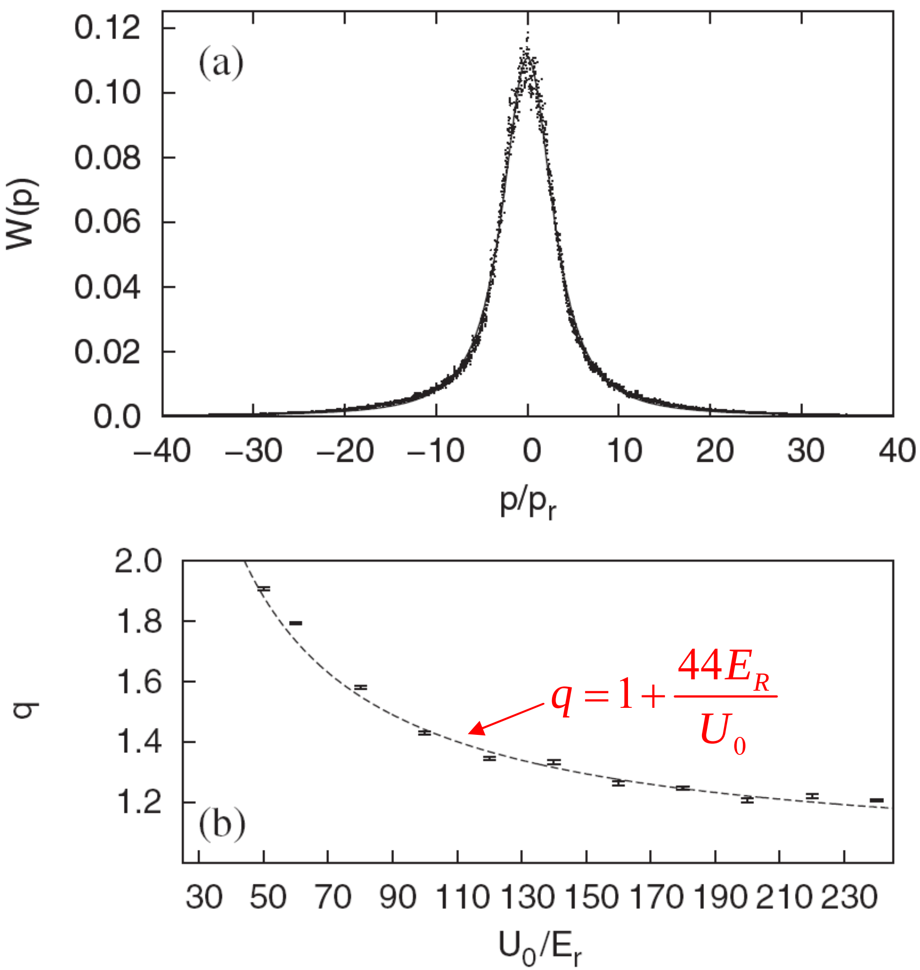
(i) The distribution of atomic velocities is a *q*-Gaussian;

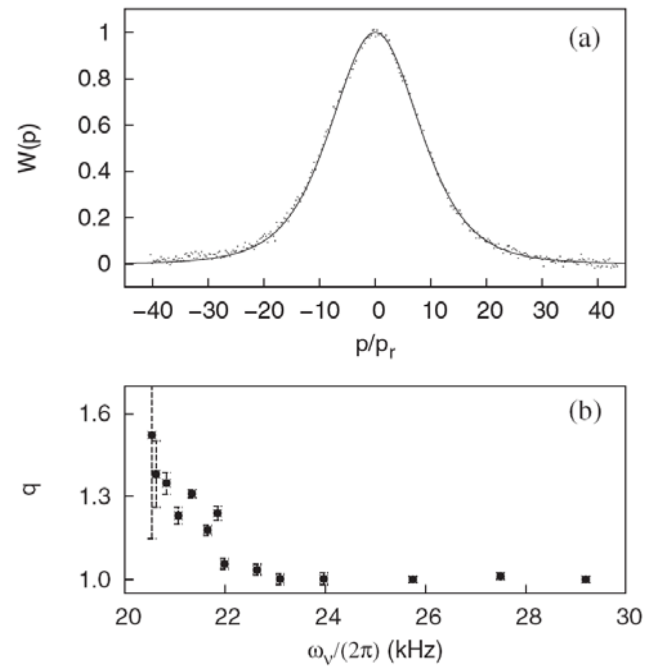
(ii) $q = 1 + \frac{44 E_R}{U_0}$ where $E_R \equiv$ recoil energy

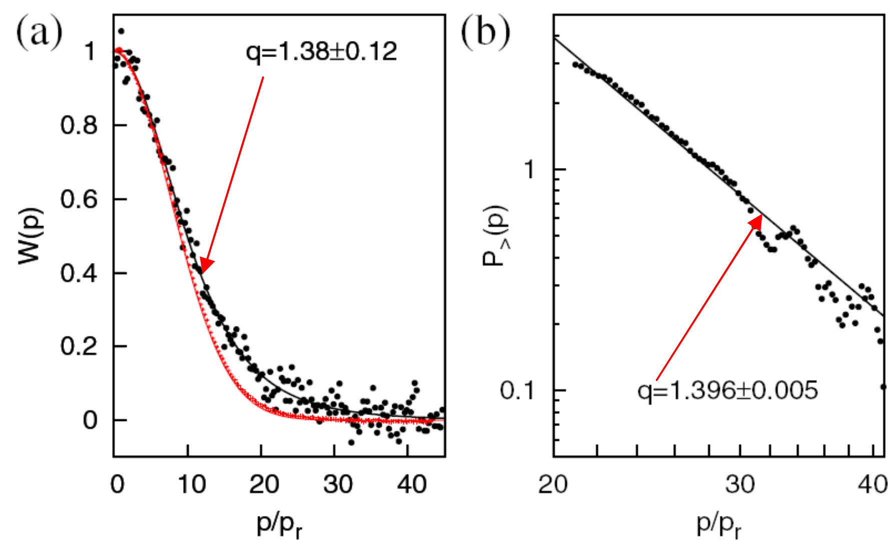
$U_0 \equiv$ potential depth

Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)

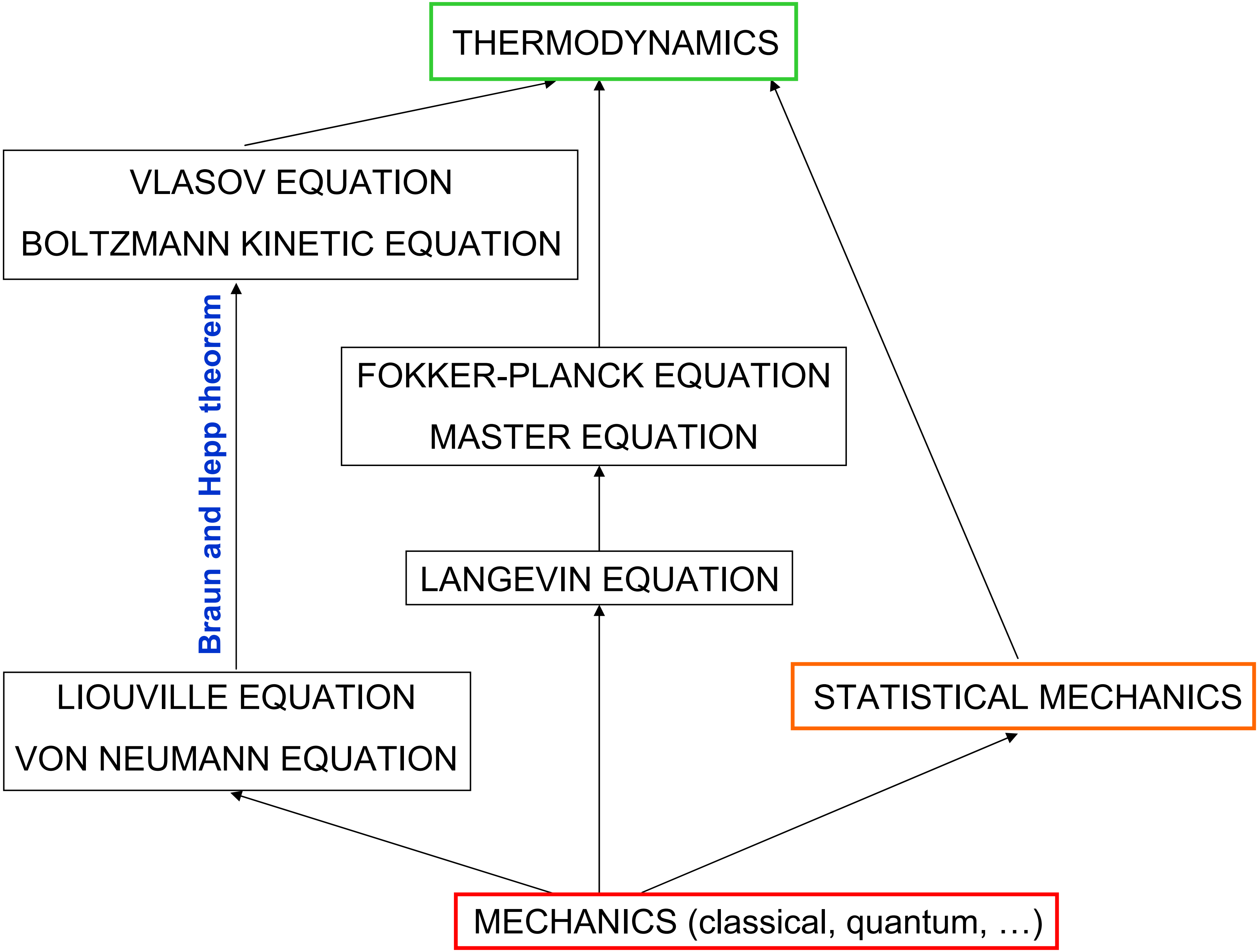






(Computational verification: quantum Monte Carlo simulations)

(Experimental verification)


THERMODYNAMICS
VLASOV EQUATION
BOLTZMANN KINETIC EQUATION
Braun and Hepp theorem
FOKKER-PLANCK EQUATION
MASTER EQUATION
LANGEVIN EQUATION
LIOUVILLE EQUATION
VON NEUMANN EQUATION
STATISTICAL MECHANICS
MECHANICS (classical, quantum, …)

Nonextensive statistical mechanics appears to be consistent with the

0^{th}, 1^{st}, 2^{nd}, 3^{rd} and 4^{th} principles of thermodynamics,

hence

the thermodynamical principles appear to be **stronger** than the role attributed to them by Boltzmann - Gibbs statistical mechanics.

Satellite conference of STATPHYS 23

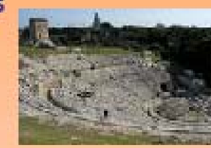
International Conference on
Complexity, Metastability and Nonextensivity

Dipartimento di Fisica e Astronomia - Universita' di Catania

1-5 July 2007

Main topics of the conference:

- Models and dynamics of complex systems
- Nonextensive statistical mechanics
- Glassy dynamics and metastability
- Networks and synchronization
- Interdisciplinary applications



Main speakers

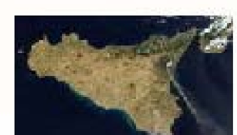
S. Abe (Japan)	*C. Beck* (UK)	*J.-P. Boon* (Belgium)	*T. Bountis* (Greece)
L. Burlaga (NASA, USA)	*F. Caruso* (Italy)	*G. Casati* (Italy)	*A. Carati* (Italy)
P.H. Chavanis (France)	*E.G.D. Cohen* (USA)	*K. Dawson* (Ireland)	*J. Soares Andrade* (Brazil)
L. de Arcangelis (Italy)	*J.D. Farmer* (USA)	*A. Giansanti* (Italy)	*I. Giardina* (Italy)
G. Kaniadakis (Italy)	*H.J. Herrmann* (Switzerland)	*V. Latora* (Italy)	*R.N. Mantegna* (Italy)
M. Marsili (Italy)	*M. Paczuski* (Canada)	*A.R. Plastino* (South Africa)	*A. Politi* (Italy)
A. Pluchino (Italy)	*P. Quarati* (Italy)	*A. Robledo* (Mexico)	*S. Ruffo* (Italy)
B. Spagnolo (Italy)	*H.E. Stanley* (USA)	*F. Tamarit* (Argentina)	*U. Tirnakli* (Turkey)
S. Thurner (Austria)	*C. Tsallis* (Brazil)	*S. Umarov* (USA)	*C. Vignat* (France)

International Organizing Committee

S. Abe, H.J. Herrmann, P. Quarati, C. Tsallis, A. Rapisarda (Chairman)

www.ct.infn.it/ctnext07

For more information write to: *catania-next07@ct.infn.it*



Sofia e la scoperta delle fragole (Marco Bersanelli)

A Gutenberg, tra le verdissime colline austriache, una mattina saliamo per il sentiero che attraversa il bosco scuro e profumato alle spalle del paese. Dopo mezz'ora di cammino troviamo sulla destra una sorgente presso una radura e ci fermiamo a bere. Con una grande espressione di felicità ad un tratto Sofia, la piccola di tre anni, esclama: «Mamma, mamma!! una fragola!!». Gli altri due accorrono e, constatato che la sorellina ha prontamente raccolto e inghiottito il frutto della sua scoperta, si mettono a cercare, presto seguiti dai genitori. «Un'altra!» e dopo un po': «Guarda qui, ce ne sono altre tre, quattro...». La caccia è aperta. Cercando in quel prato abbiamo presto riempito un bicchiere di fragole di bosco. Poi al ritorno, con mia sincera sorpresa, ripercorrendo lo stesso sentiero dalla sorgente in giù ne abbiamo trovate altrettante! Zero fragole all'andata, forse un centinaio al ritorno: un effetto statisticamente schiacciante. Cos'era cambiato?

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A Gutenberg, tra le verdissime colline austriache, una mattina saliamo per il sentiero che attraversa il bosco scuro e profumato alle spalle del paese. Dopo mezz'ora di cammino troviamo sulla destra una sorgente presso una radura e ci fermiamo a bere. Con una grande espressione di felicità ad un tratto Sofia, la piccola di tre anni, esclama: «Mamma, mamma!! una fragola!!». Gli altri due accorrono e, constatato che la sorellina ha prontamente raccolto e inghiottito il frutto della sua scoperta, si mettono a cercare, presto seguiti dai genitori. «Un'altra!» e dopo un po': «Guarda qui, ce ne sono altre tre, quattro...». La caccia è aperta. Cercando in quel prato abbiamo presto riempito un bicchiere di fragole di bosco. Poi al ritorno, con mia sincera sorpresa, ripercorrendo lo stesso sentiero dalla sorgente in giù ne abbiamo trovate altrettante! Zero fragole all'andata, forse un centinaio al ritorno: un effetto statisticamente schiacciante. Cos'era cambiato?

Eravamo cambiati noi.

HOKKAIDO UNIVERSITY – SAPPORO

WILLIAM S. CLARK (1826-1886)

1877: BOYS, BE AMBITIOUS!

$than_q$

Aulas do Prof. Andrea Rapisarda:

www.ct.infn.it/rapis/rio-lectures

Aulas do Prof. Constantino Tsallis:

http://www.cbpf.br/NextCurso2007/AulasTsallis.pdf

Aulas do Prof. Alberto Robledo:

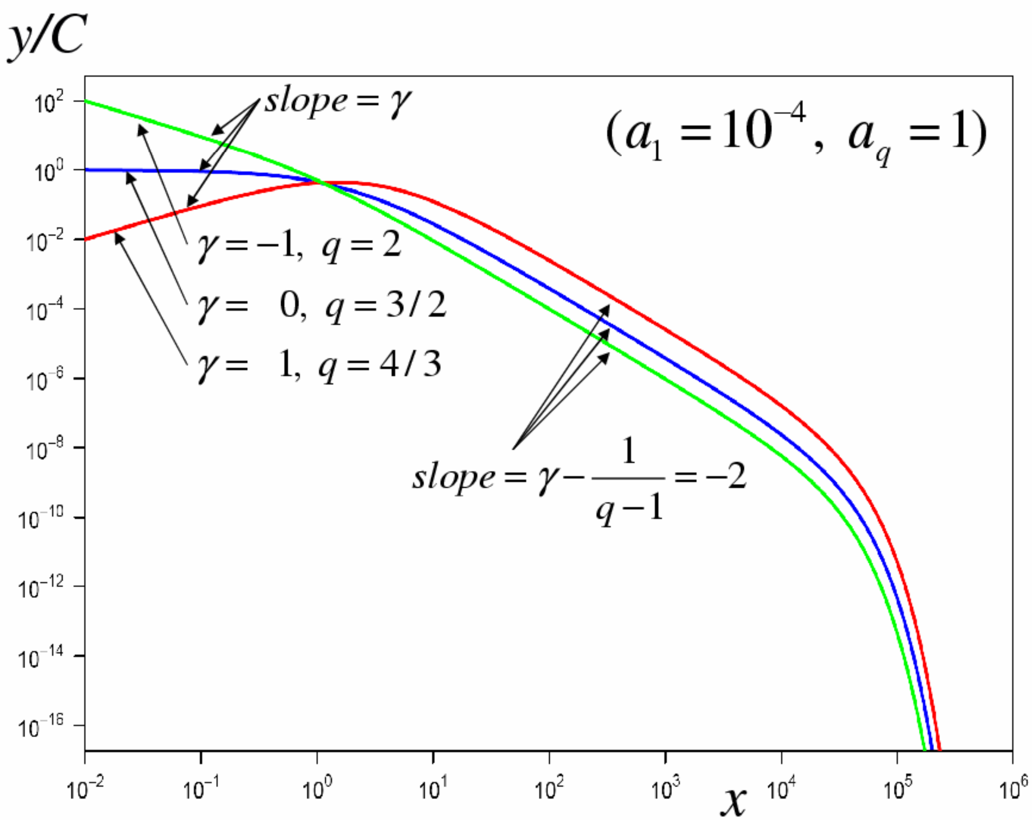
Foi distribuido em cada pasta de participante

O conjunto permanecera disponivel (a partir de uma semana) em

http://tsallis.cat.cbpf.br/NextCurso2007

We may introduce a "density of states" Cx^{γ} $(\gamma \in R)$

$$\Rightarrow y = \frac{C\,x^{\gamma}}{\left[1 - \frac{a_q}{a_1} + \frac{a_q}{a_1} e^{(q-1)a_1 x}\right]^{\frac{1}{q-1}}}$$



Logos Quotes 26-JAN-2007,
Every day a new quotation translated into many languages.

Quotation of the day:
Author - Mark Twain

The man with a new idea is a crank until the idea succeeds.

English - the man with a new idea is a crank until the idea succeeds
Albanian - njeriu që ka një ide të re është një i çmendur deri sa ajo ide të ketë sukses
Basque - ideia berri bat duen gizakia ero bat da, ideiak arrakasta lortzen duen arte
Bolognese - un òmen con un'idê nôva l é un mât, infénna che cl'idê la n à suzès
Brazilian Portuguese - um homem com uma ideia nova é um louco até que a ideia tenha sucesso
Breton - ken na zeu e vennozh da vat, un den ideet eo an hini en deus ur mennozh nevez
Calabrese - 'n uomu cu 'n'idea nuova è nu pacciu finu a quannu l'dea nun teni successu
Catalan - un home amb una idea nova és un boig fins que la idea no triomfa
Croatian - čovjek s novom idejom je čudak sve dok ideja ne uspije
Danish - en mand med en ny ide er skør, indtil ideen lykkes
Dutch - iemand met een nieuw idee is een dwaas, totdat het slaagt
English - the man with a new idea is a crank until the idea succeeds
Esperanto - homo kun nova ideo estas frenezulo ĝis kiam la ideo sukcesas
Estonian - inimest, kellel on uued ideed, peetaske hulluks senikaua, kuni tema ideid kroonib edu
Finnish - ihminen jolla on idea on hullu, kunnes ideasta tulee menestys
French - un homme avec une nouvelle idée est un fou tant que l'idée n'a pas de succès
Furlan - un omp con une gnove idee al è un mat fintremai che chê idee no à sucess
Galician - un home cunha idea nova é un tolo ata que a idea teña éxito
German - ein Mensch mit einer neuen Idee ist so lange ein Spinner, bis die Idee zum Erfolg wird
Griko Salentino - nan àntrepo me mian idea nea (cinùria) ene na ppàccio sara ka cin idea en echi successo
Hungarian - az új ötlettel rendelkező embert mindaddig hülyének nézik, amíg az ötlet nem lesz sikeres
Italian - un uomo con un'idea nuova è un matto finché quell'idea non ha successo
Judeo Spanish - tun ombre kon una idea mueva es un loko asta ke la idea triunfe
Latin - homo quidam cum nova cogitatione stultus est antequam cogitatio firmetur
Latvian - cilvēks ar jaunu ideju ir jucis, kamēr ideja nav realizēta
Leonese - un home cun una idega nueva ye un home alloriáu fasta que la idega triunfa
Mudnés - un àmm ch'àl ghà n'idèa nôva l'è un mât fintânt che c'l'idèa l'àn ghà sucês
Neapolitan - n'ommo cu na penzata nova è nu schirchio nfin'a quanno chella penzata nun trionfa
Papiamentu - un persona ku un idea nobo ta un loko te ora e idea logra
Portuguese - um homem com uma ideia nova é um louco até que a ideia tenha sucesso
Roman - ´n´ omo co´ ´n´ idea nova è solo un matto, fino a che l´ idea nun ciabbia successo
Spanish - un hombre con una idea nueva es un loco hasta que la idea triunfa
Umbro-Sabino - n'omo co' n'idea nòa è sciurnu finaquanno ell'idea nun c'à succiessu
Venetian - l'omo co na idèa el xe un móna fin che l'idèa no ła ga suceso
Wallon - l' sakî k'a ène noûve idêye dimère on sot djusk' à c'k' èle fuchisse riconèxheuwe come boune
Welsh - hyd nes bod ei syniad yn llwyddo, dyn â chwilen yn ei ben yw'r un â syniad newydd
Zeneize - un òmmo con unn'idea neuva o l'é un sciòllo scin che quell'idea a no l'à successo

EUROPHYSICS LETTERS 15 May 2005

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Dynamical correlations as origin of nonextensive entropy

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(Phenomenological model for collisions in a diluted gas with probability *r* of forming clusters of *q* correlated particles)