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STATISTICAL

Influence of the Investor's Behavior on the Complexity of the Stock Market

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Abstract One of the pillars of the finance theory is the efficient-market hypothesis, which is used to analyze the stock market. However, in recent years, this hypothesis has been questioned by a number of studies showing evidence of unusual behaviors in the returns of financial assets ("anomalies") caused by behavioral aspects of the economic agents. Therefore, it is time to initiate a debate about the efficient-market hypothesis and the "behavioral finances." We here introduce a cellular automaton model to study the stock market complexity, considering different behaviors of the economical agents. From the analysis of the stationary standard of investment observed in the simulations and the Hurst exponents obtained for the term series of stock index, we draw conclusions concerning the complexity of the model compared to real markets. We also investigate which conditions of the investors are able to influence the efficient market hypothesis statements.

Keywords Behavioral finance · Efficient-market hypothesis · Cellular automaton · Hurst exponent

1 Introduction

Motivated by the seminal work of H. E. Stanley and collaborators [1], a new domain of research has developed—econophysics. Using mostly statistical physics tools and the increasing calculational power of computers, physicists have introduced several methods to interpret noisy and stochastic data. One of the most promising areas to apply this interdisciplinary knowledge to are stochastic time series such as the stock market index. Apart from finding a way to predict the index, which was the first "gold rush" of econophysics, we focus more on trying to understand the dynamic process of the market and how the local behavior of the investors can influence the worldwide tendency of the index.

The finance theory uses the efficient-market hypothesis as the ideal model system; it states that the investors are rational and immediately incorporate the new information of the market to find fair prices. In this way, the market becomes efficient and, because of this rational behavior, there would be no psychological features such as excess of confidence, indecision, and exaggeration and, therefore, a crisis would never occur in this market. A basis for a new theory, named behavioral finance, customizes concepts of psychology, sociology, and other sciences to model the investors, aiming to approximate the finance theory to the stock market reality. Empirical studies show that the behavior of the investors frequently deviates from the rational standards.

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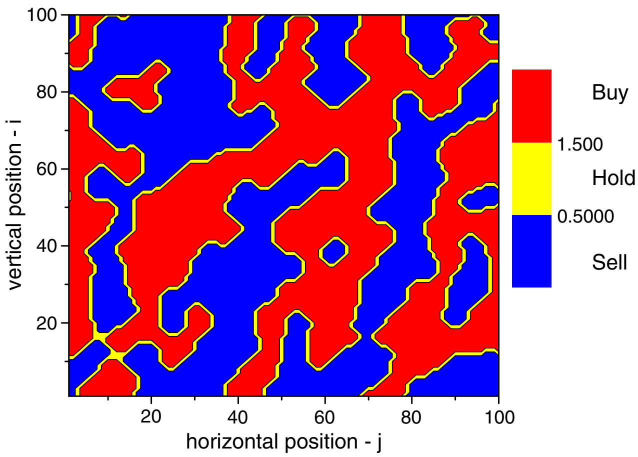

Fig. 1 Stationary configuration for the option of the investors; the initial condition was a random distribution of 33% for each option of investment and all investors imitate the neighborhood with $P_1 = 1$

Compared to a complex ecosystem, the stock market is a dynamic system in a complex network in which each individual (investor) searches to protect his (we will write "he" for "the investor" throughout for convenience, although we are well aware that there are many female investors as well) own safety (investment). In current economic models, the agent is generally perfectly rational and his target is to maximize his profit. Models based on agents, as those treated by statistical physics, can go beyond the theories of the "agent" prototype of the traditional economy because these statistical physics models can be used to incorporate collective behavior and develop a better understanding of the dynamic aspects of the financial market.

Recently, a cellular automaton model, a common technique in statistical physics, was proposed to simulate an artificial stock market, allowing one to study how the market depends on the investors' psychology [2, 3]. Yi-ming and co-authors considered two types of

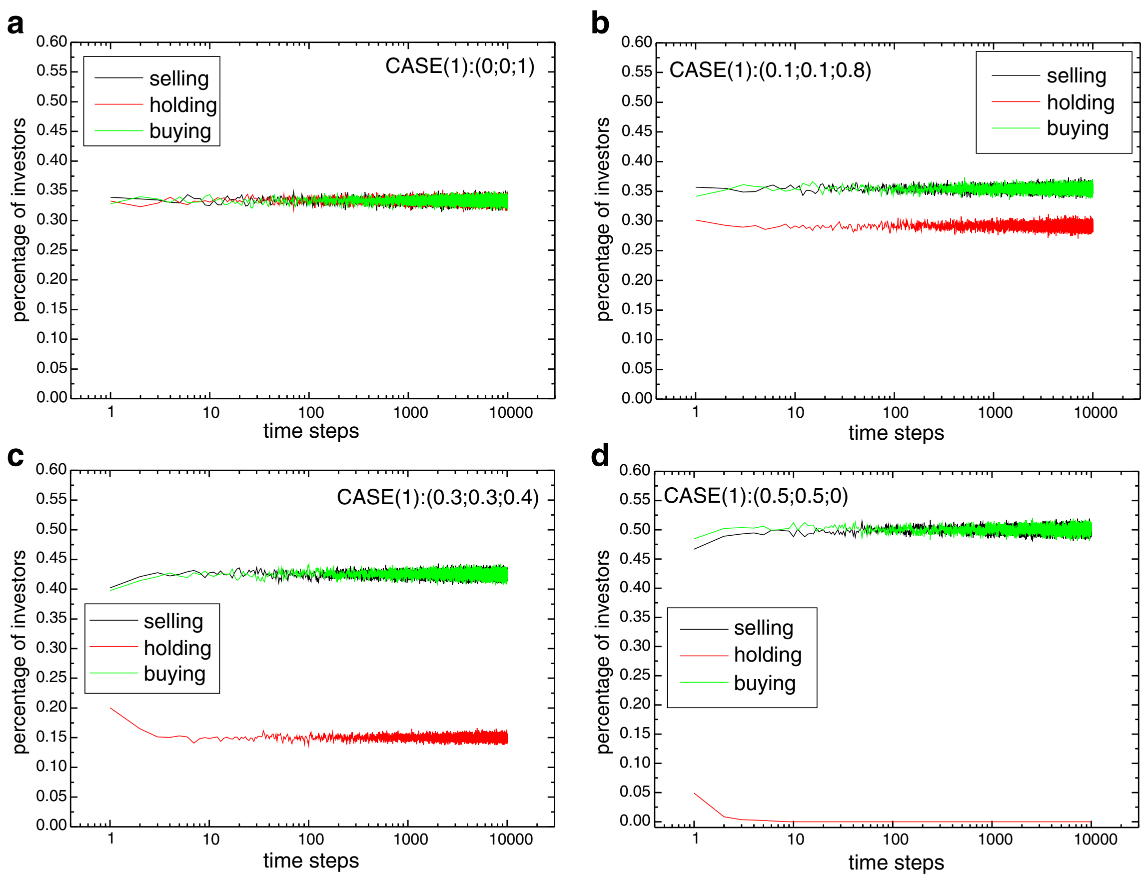

Fig. 2 Evolution of the investor's behavior during the time for case (1). In all cases, the initial state is random with 33% for each option of investment

behavior: the *imitation*, in which the investor follows the preference of the majority of his neighbors, and the *anti-imitation*, in which he adopts the opposite position of the majority of his neighbors.

We used a cellular automaton model to study the market behavior based on the investors' preferences. In addition to the types of investors mentioned above, we considered a third type, the *indifferent*, who makes his decisions regardless of his neighbors. To characterize the proposed model, we used a statistical physics techniques [1], such as the Hurst exponent, to analyze the temporal series of the stock index produced by the model and to verify whether the complexity of the results is statistically coherent with real markets. We also compared the results with the prediction from the efficient-market hypothesis.

2 Numerical Procedure

The cellular automata were originally proposed by J. Von Neumann and were thoroughly studied by Wolfram [4]. They are discrete dynamic systems of a

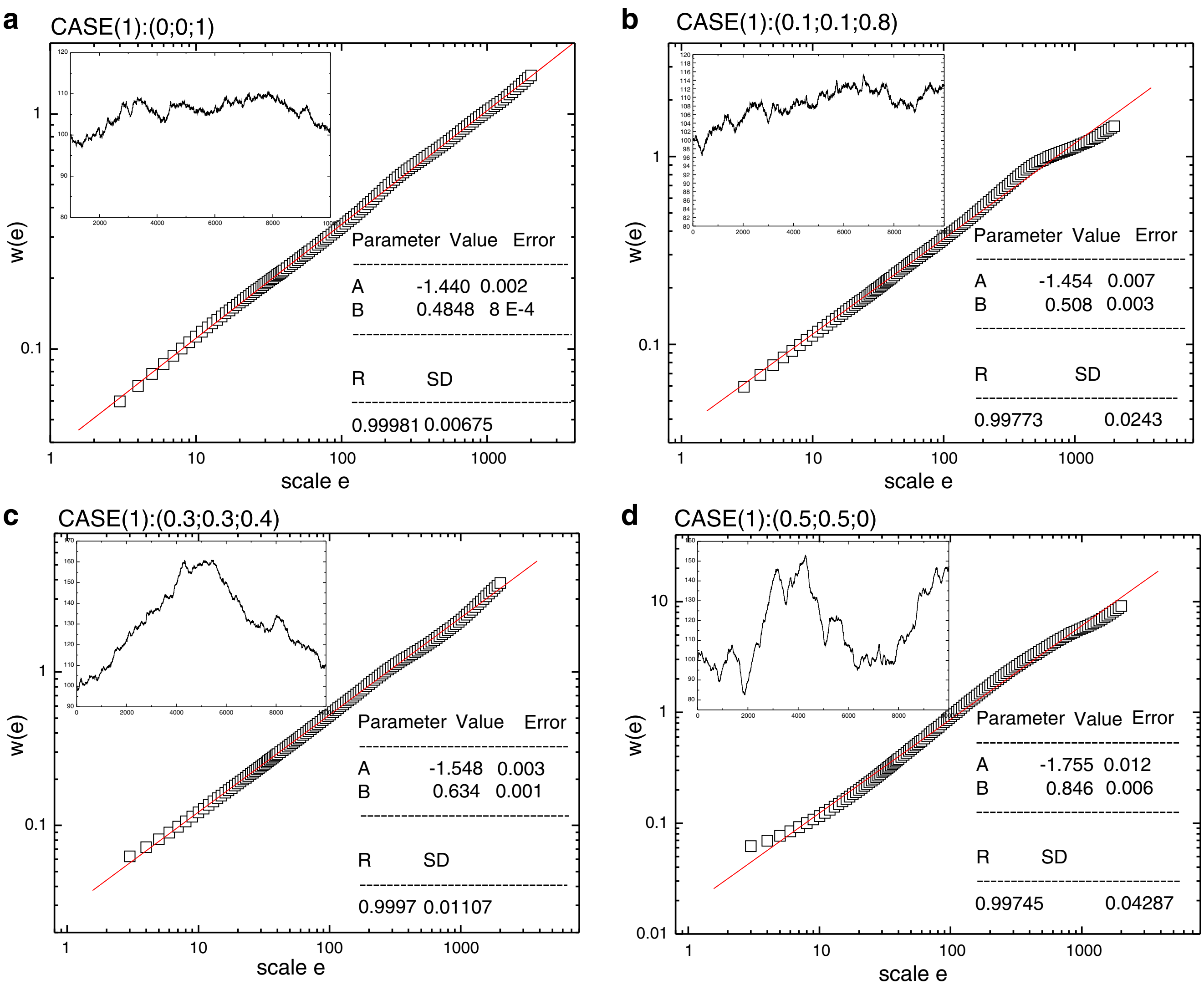

Fig. 3 Evolution of the investor behavior during the time for case (2). In all the cases, the initial state is random with 1/3 for each option of investment

simple construction, used as mathematics models to investigate problems in different areas because they are the simplest models that are able to describe complex systems [5]. In a general way, they consist of a uniform lattice of discrete elements (sites) that can be in k states. The condition of each site in a time step is determined by its neighborhood at the previous step, and the transition rules can be deterministic or probabilistic [6].

We represent the market as a cellular automaton in two dimensions where each site represents an investor. The linear extension of the system is $L = 100$, with $N = 10{,}000$ sites (investors). We use deterministic or probabilistic transition rules depending on the considered implementation and periodic boundary conditions in both directions. The variable $S_t(i, j)$ denotes the option of the investor at the site (i, j), at time t, from the following choices: S_b (to buy), S_h (to hold), and S_s (to sell). A Moore neighborhood with eight neighbors is used, and different evolution rules are associated to each behavior type of investors. In a general way, the state is determined by the prevailing state of the neighborhood in a specific step of time. In this way, the option of the investor in the moment $t + 1$ is a function in the majority state among its neighbors at the moment t, represented by the expression

$$S_{t+1}(i, j) = f\Big(S_t(i-1, j-1), S_t(i-1, j), S_t(i-1, j+1), S_t(i, j-1), S_t(i, j+1), S_t(i-1, j+1), S_t(i, j+1), S_t(i+1, j+1)\Big). \quad (1)$$

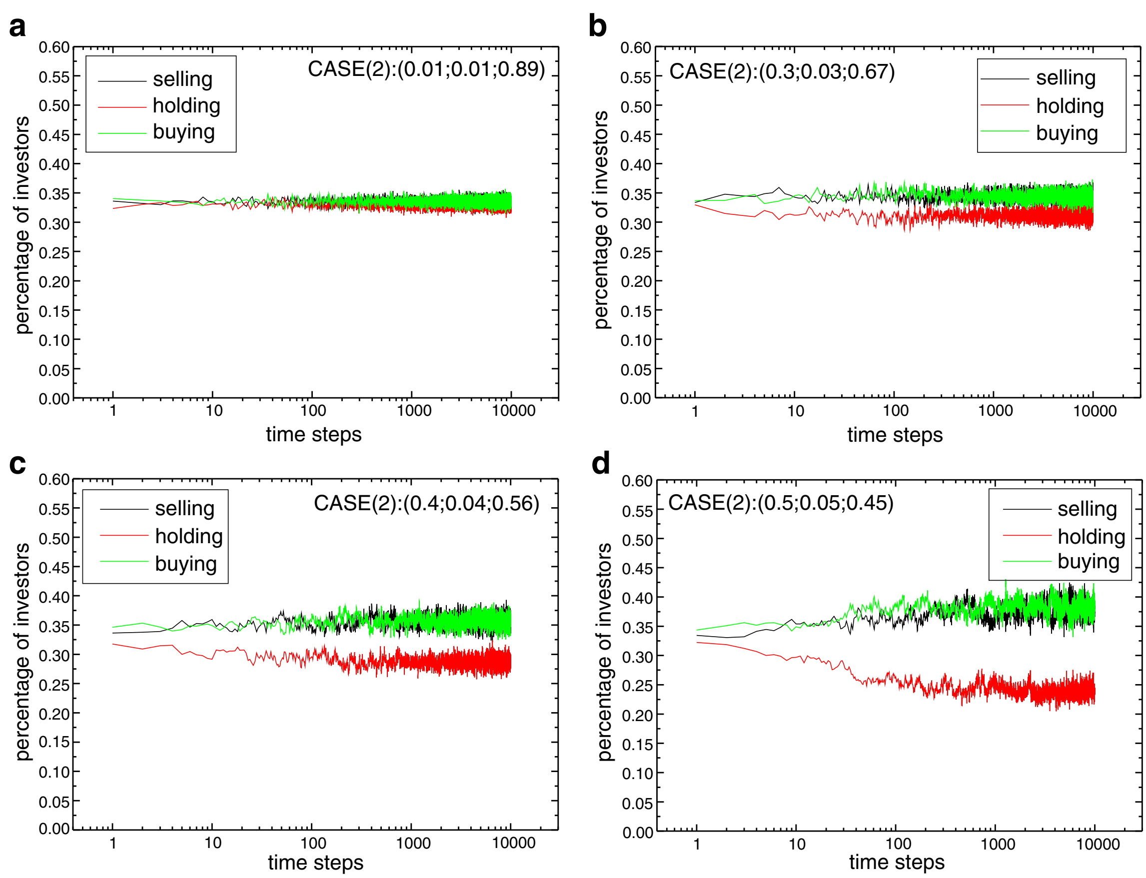

Fig. 4 Evolution of the investors behavior as a function of time for case (3). In all the cases, the initial state is random with 1/3 for each option of investment

The initial condition for each investor was determined considering a random distribution with probability of 1/3 for each option (to buy, to sell, to hold).

Three types of investor behavior were considered: "imitation," when the choice of the investor will be identical to the prevailing choice of its neighborhood; "anti-imitation," when the investor will choose the opposite of the prevailing tendency of the neighborhood (if the prevailing choice is to keep, the anti-imitation will choose, the less prevailing of the options to buy and to sell). In the third type, named "indifferent," the choice is independent of the tendency of the neighbors and is determined randomly among the three possible choices. This third type increases the randomness of the system.

In simulations, the quantity of money and stocks are limited. The stock market index is altered in each step depending on the options of the investors, in other words, depending on the supply and demand of stocks. The variable $P_1(i, j)$ represents the probability that the agent (i, j) imitates the neighborhood, $P_2(i, j)$ represents the probability (i, j) of anti-imitation, and $P_3(i, j) = 1 - P_1 - P_2$ is the probability of the agent (i, j) to be indifferent.

We considered several different combinations of these probabilities. First, we investigated the behavior of the model when all investors behave identically, considering each one of the three types [7]. Interestingly, when all the investors chose to imitate or anti-imitate their neighbors, the system froze into random

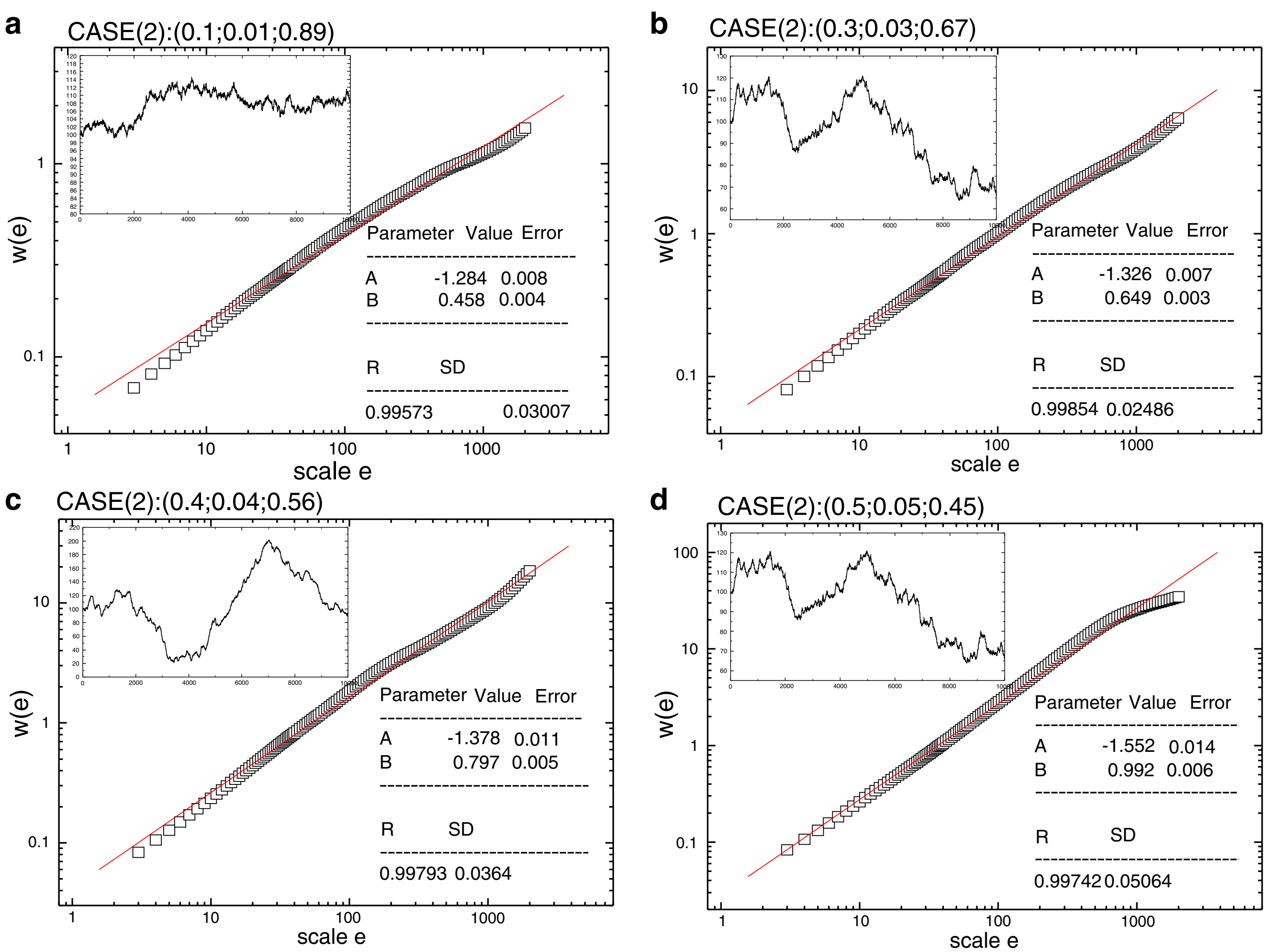

Fig. 5 Calculation of the Hurst exponent for the indices of the stocks in the case (1). The parameter B represents the H value. In the *inset in each panel*, we show the temporal evolution of the stock index. In all cases, the initial state is random with 1/3 for each option of investment

configurations that looked like patterns of Ising models close to criticality. Figure 1 shows an example of the pattern obtained for the case in which all investor follows the majority. The more interesting cases were obtained when the indifferent type was included in the system. The results of these simulations are summarized in the following figures and were obtained considering the following choices for the probabilities of each type of behavior:

$$(1)\, P_2 = P_1;\ (2)\, P_2 = 0.1 P_1;\ (3)\, P_2 = 10 P_1;$$

3 Results and Discussion

The Hurst exponent has been introduced through the work of the biologist Harold E. Hurst in 1951, where he proposed a new statistic to distinguish a random series from a non-random one, while examining the various regimes of water flow in a dyke. In 1972, B. Mandelbrot [8] introduced the Hurst exponent as a tool to analyze temporal series and calculate their fractal dimension. Edgar E. Peters used the Hurst exponent in explaining economic phenomena and in finance temporal series [9].

The method used most often for the calculation of the Hurst exponent is the detrended fluctuation analysis [10], which calculates the roughness around the line that better fits a set of points. We calculated the Hurst exponent for the temporal series of the stock index produced by the artificial market. In the model, the index value is denoted by $X(t)$, and each time step is considered as a market-closing day.

To calculate the Hurst exponent, we divided the temporal series into a set of points that were not superimposed, I_n, and calculated for each gap ϵ the best linear fit to the points inside the gap. Then, we calculated the

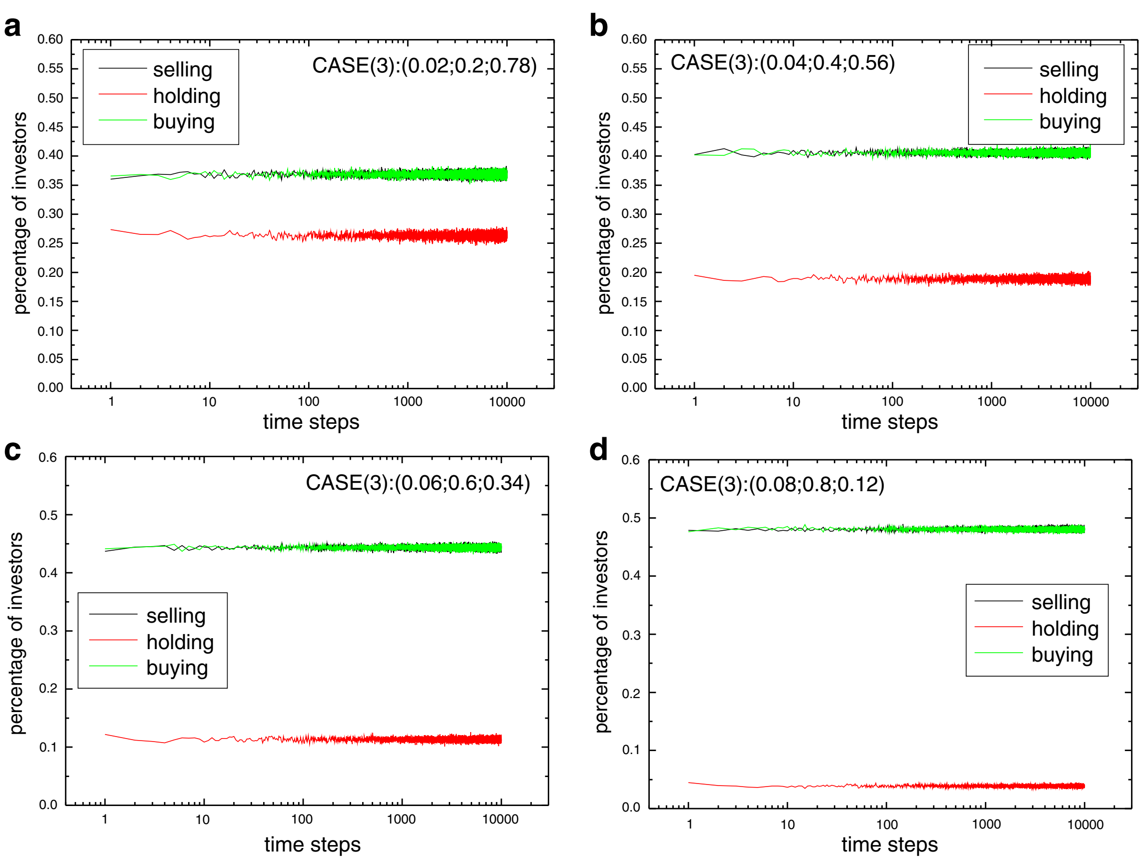

Fig. 6 Calculation of the Hurst exponent for the indices of the stocks produced in case (2). The parameter *B* represents the value of *H*. In the *inset of each panel*, we show the graphics of the stock index. In all cases, the initial state is random with 1/3 for each option of investment

local roughness at the scale ϵ as the standard deviation around the fitting line

$$w(\epsilon) = \frac{1}{N_\epsilon} \sum_{I_n} \sqrt{\frac{1}{T} \sum_t \left[X(t) - (a_n(\epsilon) - b_n(\epsilon)t)\right]^2} . \quad (2)$$

a_n and b_n are the linear fit coefficients in the gap ϵ and N_ϵ is the number of gaps. The value of the Hurst exponent H is obtained from the relation

$$w(\epsilon) \sim \epsilon^H , \quad (3)$$

where the value of the Hurst exponent can be estimated as the angular coefficient of the linear regression of the data from a log–log plot. It is known that for $H = 1/2$, the series is decorrelated, and different values indicate the existence of long-range correlations. When $1/2 < H < 1$, the series is positively correlated and shows persistence. For $0 < H < 1/2$, the series is negatively correlated and shows anti-persistence. Hurst exponents were calculated for all series of stock index for each of the samples.

We now consider the results from ten samples during 10,000 steps of time in each case. In Figs. 2, 3, and 4, we show the temporal distribution of the options of investment for the cases 1, 2, and 3, respectively. In Figs. 5, 6, and 7, the Hurst exponent is calculated from the market index for the cases 1, 2, and 3, respectively, and the market index behavior is shown in detail in each of the panels. As expected, the number of investors in each choice fluctuates randomly, and the stock index reflects this behavior, showing self-similar characteristics.

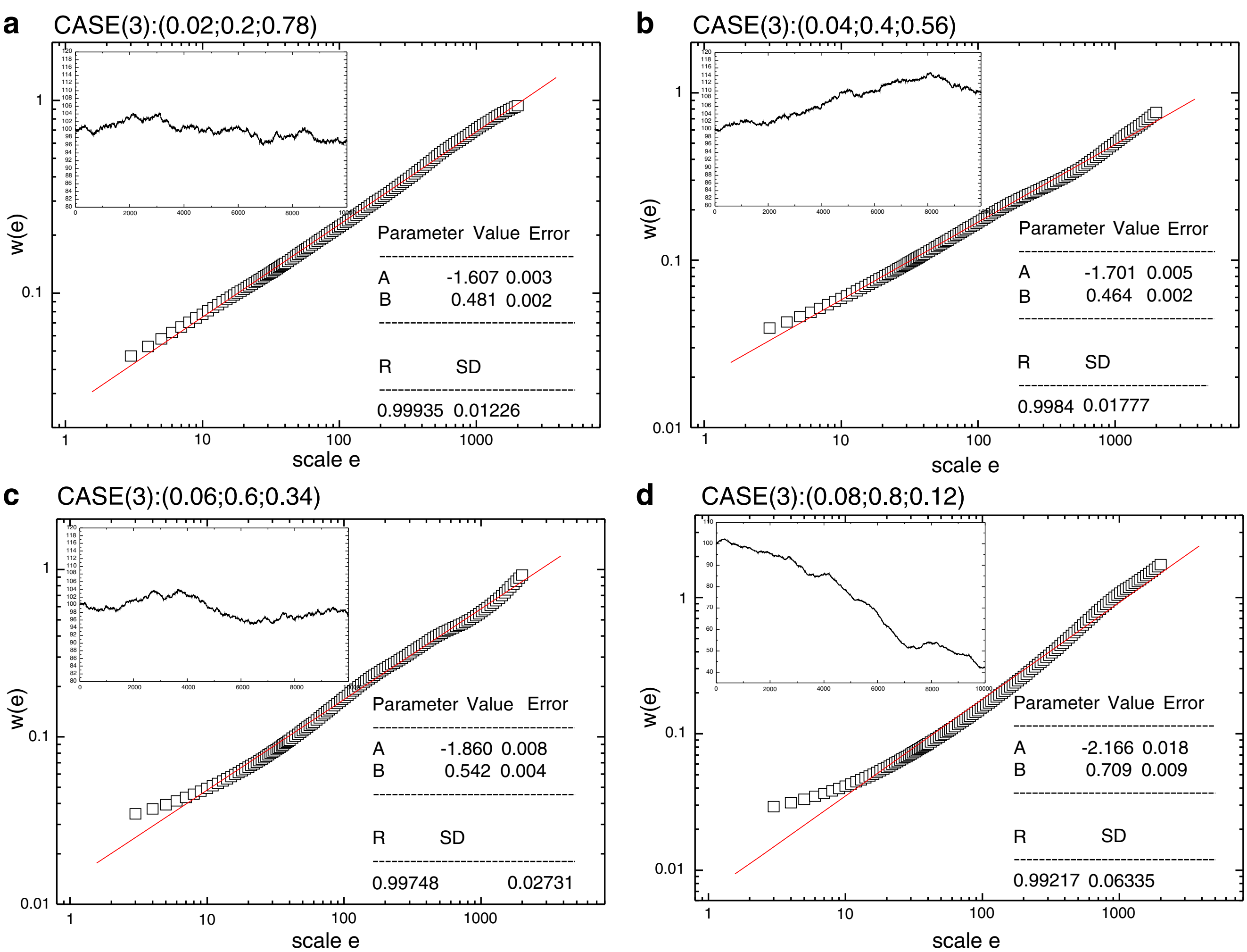

Fig. 7 Calculation of the Hurst exponent of the indices from the stocks produced in case (3). The parameter B corresponds to the H estimation. In the *inset of each panel*, we show the graphics of the stock index. In all cases, the initial state is random with 1/3 for each option of investment

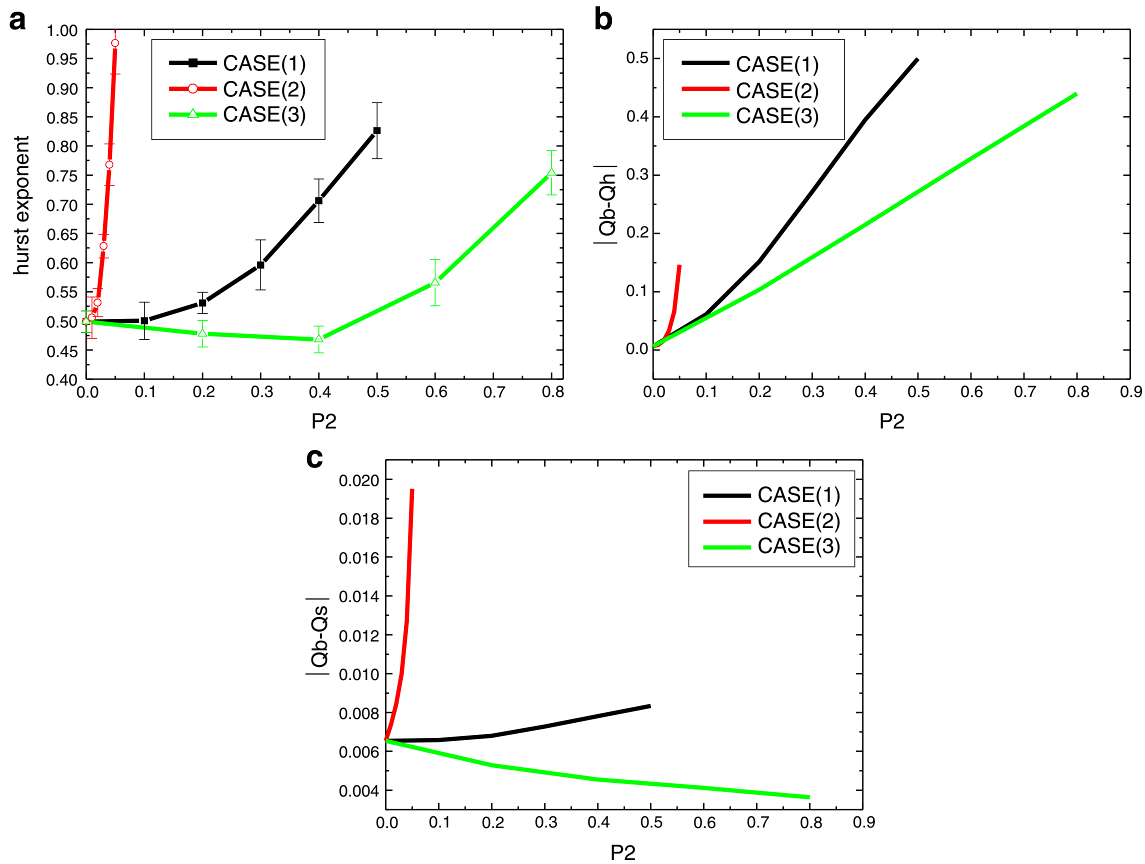

Fig. 8 **a** The average value of the Hurst exponent as a function of P_2 for each case considered. **b** The average difference between the number of buying, Q_b, and holding, Q_h, positions as a function of P_2. **c** The average difference of the buying Q_b and the selling Q_s positions

In most cases, we verified that the number of agents who are selling and buying are generally the same, and as P_2 increases, the number of agents in holding position decreases. This decreasing in the number holding positions directly influences the market prices, causing huge flows. When very many people hold, the market index varies less.

In case 2, the effect of crowd behavior increases for the choices to buy or to sell as P_1 increases. In case 3, we observe that if the number of independent investors is increased and the number of imitations decreases, the market fluctuates less.

In Fig. 8, we observe the tendencies of the Hurst exponent and the option of the investors as a function of the anti-imitation probability, P_2. We observe some correlation between the value of the Hurst exponent and the collective crowd behavior. In case 1, clearly the increasing of P_1 (imitation investors) increases the tendency for $H > 0.5$, which is associated to emerging markets. In case 2, in which the percentage of the imitation investors is ten times higher than that of anti-imitation investors, this crowd effect rises faster as a function of P_1 than in the other cases, and H converges quickly to 1.

In case 3, in which the number of anti-imitations is ten times higher than the number of imitations (and the lower the number of anti-imitations, the higher the number of indifferent people), we observe the tendency of $H \sim 0.5$, as expected for a completely decorrelated market. As P_2 increases, the number of investors who hold decreases, and this alters the stability of the market, implying $H > 0.5$.

4 Conclusions and Perspectives

The Hurst exponent is proposed in the literature to classify the market indices of the countries as a measure of the efficiency of the market. If long-range persistence is present in the return of the financial assets, the hypothesis of the efficient market is invalid. Following Cajueiro [11, 12], we can assume that the value of the Hurst exponent in developed markets—where the efficient-market hypothesis should be valid—must be close to $H = 0.5$. In emerging markets, its value is less than this. In our artificial market, the value of the Hurst exponent stays close to these expected values. In this context, the model was able to reproduce the complexity of the market to a degree comparable to reality.

The imitation behavior in the model can be associated to the rational investor of the efficient-market hypothesis. The anti-mimic investors and the indifferent ones are the "irrational" investors of behavioral finance. Our results showed that the market stability is affected by the investment preference. When the investor behavior is affected only by other investors' behaviors, huge market fluctuactions are expected—a crowd effect. When there are many imitation investors, the Hurst exponent exceeds 0.5; the efficient-market hypothesis is then invalid. The Hurst number is closest to 0.5 the higher the number of indifferent investors and the lower the number of imitation investors, as expected for the developed markets (efficient markets). Thus, the stock market stability is the highest when the investor's behavior is most independent.

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