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Effect of scalar nonlinearity on zonal flow generation by Rossby waves

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Abstract

Effects of scalar nonlinearity on the generation of zonal flow by Rossby waves in shallow rotating fluid are considered. Zonal flows are generated via the action of Reynolds stress due to vector nonlinearity together with the effects of scalar nonlinearity. It is shown that the scalar nonlinearity reduces the amplitude threshold of the zonal flow instability. In addition, it increases the range of wave vectors of unstable modes subjected to the instability. The growth rate of the instability as a function of the spectrum of primary waves is calculated. The spectrum is assumed to be arbitrary with emphasizing the case of two monochromatic waves.

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1. Introduction

Physics of Rossby waves in shallow rotating fluid in a gravitational field has been a subject of numerous theoretical and experimental studies [1–10]. Such studies have been stimulated, on one hand, by important applications of Rossby waves in astrophysics, space physics, physics of ocean and atmosphere [2]. On the other hand, general properties of Rossby waves are of interest because of their close analogy to the drift waves in magnetized plasmas [1].

Earlier nonlinear theory of Rossby waves was focused on studies of coherent solitary structures (solitons) [1,2]. Recently, nonlinear generation of zonal flows by the Rossby waves has been considered (see [5–10] and references therein) similar to the processes of zonal flow generation by drift waves in magnetized plasmas (see the review paper [11] with extensive bibliography). As a whole, this problem goes back to [12], see also [13].

In the theory of solitary coherent structures associated with the Rossby waves both effects of scalar and vector nonlinearities were considered [1]. Meanwhile, in the theory of zonal flows generation by the Rossby waves as well as drift waves in plasmas, as a rule, only the vector nonlinearity was mostly included. An important development therefore has been made in Ref. [8] where the scalar nonlinearity was taken into account for the problem of zonal flow generation by drift waves in magnetized plasmas. (The importance of scalar nonlinearity in the context of drift waves in plasmas containing density and electron temperature gradients was originally noticed in [14,15]. The excitation of zonal flows by drift waves in nonuniform magnetoplasma was properly considered in [16,17].) Similarly to Ref. [8] the present work aims to include both the scalar

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and vector nonlinearity into the problem of generation of zonal flows by the Rossby waves in the shallow fluid approximation. However, in contrast to Ref. [8], dealing with a monochromatic wave packet of the primary waves, we consider an arbitrary spectrum of these modes emphasizing the case of two monochromatic waves.

2. Basic equations and their analysis

The scalar nonlinearity can be properly considered within Rossby waves model described in review article [1]. Eq. (1.21) of Ref. [1] gives for the dimensionless depth of the fluid

$$\frac{\partial h}{\partial t} + V_R(1+h)\frac{\partial h}{\partial y} - r_R^2\left(\frac{\partial}{\partial t} + \mathbf{V}_D\cdot\nabla\right)\Delta_\perp h = 0. \tag{1}$$

Here $h = H(\mathbf{r},t)/H_0$, H_0 is the equilibrium depth of the fluid layer, $r_R = (gH_0)^{1/2}/f$ is the Rossby–Obukhov radius, $f(x)$ is the Coriolis parameter, $V_R = -(gH_0/f)\,d\ln f/dx$ is the Rossby velocity, g is gravity, and $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The velocity of the gravity drift $\mathbf{V}_D$ is defined by the expression

$$\mathbf{V}_D = \frac{g}{f}[\nabla H\times \mathbf{e}_z], \tag{2}$$

where $\mathbf{e}_z$ is the unit vector along the z direction.

Similar to [9] we represent

$$h = \tilde{h} + \hat{h} + \bar{h}, \tag{3}$$

where $\tilde{h}$, $\hat{h}$, and $\bar{h}$ describe primary modes, secondary (sideband) perturbations, and zonal flow, respectively. The zonal flow perturbation is taken in the form

$$\bar{h} = \bar{h}_0\exp(iq_x x - i\Omega t) + \text{c.c.}, \tag{4}$$

where Ω and q_x are frequency and radial wave number of the zonal flow perturbation, respectively, c.c. is the complex conjugative. The function $\tilde{h}$ is represented in the form

$$\tilde{h} = \sum_{\mathbf{k}}\left[\tilde{h}_+(\mathbf{k})\exp(i\mathbf{kr} - i\omega t) + \tilde{h}_-(\mathbf{k})\exp(i\omega t - i\mathbf{kr})\right], \tag{5}$$

where ω and $\mathbf{k}$ are the frequencies and wave vectors of the primary modes, $\tilde{h}_-(\mathbf{k})$ is complex conjugate of $\tilde{h}_+(\mathbf{k})$, $\tilde{h}_-(\mathbf{k}) = (\tilde{h}_+(\mathbf{k}))^*$, and the sum is taken over spectrum of the primary modes. At last, the function $\hat{h}$ is given by (cf. (5) of [9])

$$\hat{h} = \sum_{\mathbf{k}}\left[\hat{h}_+(\mathbf{k})\exp(i\mathbf{k}_+\mathbf{r} - i\omega_+ t) + \hat{h}_-(\mathbf{k})\exp(i\mathbf{k}_-\mathbf{r} - i\omega_- t) + \text{c.c.}\right]. \tag{6}$$

Here $\hat{h}_\pm(\mathbf{k})$ are the amplitudes of the secondary small-scale modes, $\omega_\pm = \Omega \pm \omega$, and $\mathbf{k}_\pm = (q_x \pm k_x, \pm k_y)$.

We assume that $q_x \ll k_x$, which is typical for the current theory of zonal flow generation by Rossby waves [9,18].

For primary modes Eq. (1) yields (cf. (6) of [9])

$$\omega = \frac{\omega_R}{1 + k_\perp^2 r_R^2}. \tag{7}$$

Here $k_\perp^2 = k_x^2 + k_y^2$, $\omega_R = k_y V_R$ is the Rossby wave frequency in the long wavelength limit, $k_\perp^2 r_R^2 \ll 1$.

Taking the zonal part of Eq. (1), we obtain in neglect of scalar nonlinearity

$$-i\Omega\bar{h}_0 - r_R^2\langle(\mathbf{V}_D\cdot\nabla)\Delta_\perp h\rangle = 0. \tag{8}$$

Taking into account Eq. (2) this equation is transformed to the form (cf. with (7) of [9])

$$-i\Omega\left(1 + r_R^2 q_x^2\right)\bar{h}_0 = f q_x^2 r_R^4\left\langle\frac{\partial h}{\partial x}\frac{\partial h}{\partial y}\right\rangle. \tag{9}$$

By using (5) and (6) this equation is transformed to

$$i\Omega\bar{h}_0 = -\frac{q_x^2 r_R^4}{1 + r_R^2 q_x^2}\sum_{\mathbf{k}} k_y\left[2k_x(\hat{h}_+\tilde{h}_- + \hat{h}_-\tilde{h}_+) + q_x(\hat{h}_+\tilde{h}_- - \hat{h}_-\tilde{h}_+)\right]. \tag{10}$$

Note that in the mentioned equation of [9] the sign minus has been missed under the summation over $\mathbf{k}$.

Now we turn to the part of Eq. (1) corresponding to the secondary small-scale modes. Then we obtain (cf. (9) of [9])

$$\hat{h}_\pm = \pm k_y V_0\hat{h}_\pm\frac{\mathbf{k}_\perp^2 r_R^2(1 - i\delta_{\mathbf{k}})}{(1 + \mathbf{k}_{\perp\pm}^2 r_R^2)D_\pm}. \tag{11}$$

Here

$$\delta_{\mathbf{k}} = \frac{1}{q_x k_\perp^2 r_R^2}\frac{d\ln f}{dx}. \tag{12}$$

The remaining definitions are the same as in [9], so that V_0 is the zonal part of the "cross-field" drift velocity,

$$V_0 = -i\frac{gH_0}{f}q_x\bar{h}_0, \tag{13}$$

$$\mathbf{k}_{\perp\pm}^2 = k_y^2 + (q_x \pm k_x)^2, \tag{14}$$

$$D_\pm = \omega_\pm \mp \omega_R/\left(1 + r_R^2\mathbf{k}_{\perp\pm}^2\right). \tag{15}$$

Following transformation of Eq. (10) is performed in complete analogy with Ref. [9]. Then we arrive at the following generalization of zonal flow dispersion relation of form (19) of Ref. [9]:

$$1 + \sum_{\mathbf{k}}\frac{(1 - i\delta_{\mathbf{k}})F(\mathbf{k})}{[\Omega - q_x V_g(\mathbf{k})]^2} = 0. \tag{16}$$

Here $F(\mathbf{k})$ is given by Eq. (20) of Ref. [9] (on the right-hand side of Eq. (20) of Ref. [9] the sign minus has been missed)

$$F(\mathbf{k}) = -gH_0\frac{q_x^2 r_R^4}{1 + q_x^2 r_R^2}\frac{V_g'}{\omega}k_y^2\mathbf{k}_\perp^2|\tilde{h}_+|^2. \tag{17}$$

Here V_g is radial group velocity for the primary modes given by (cf. with (14) of [9])

$$V_g(\mathbf{k}) = \frac{\partial\omega}{\partial k_x} = -\frac{2k_x\omega r_R^2}{1 + \mathbf{k}_\perp^2 r_R^2}, \tag{18}$$

$V_g' = \partial V_g/\partial k_x$ is its derivative, so that

$$V_g' = -\frac{2\omega r_R^2}{(1 + \mathbf{k}_\perp^2 r_R^2)^2}\left(1 + \mathbf{k}_\perp^2 r_R^2 - 4k_x^2 r_R^2\right). \tag{19}$$

Note that Eq. (16) is the generalization of Eq. (20) of [10].

According to (16), allowing for scalar nonlinearity in the problem considered is formally reduced to the renormalization

$$1 \to (1 - i\delta_{\mathbf{k}}). \tag{20}$$

Let us compare (16) with the result of Ref. [8]. In contrast to us, the authors of [8] have assumed that the primary modes are a single monochromatic wave. Then (15) reduces to

$$(\Omega - q_x V_g)^2 + (1 - i\delta_{\mathbf{k}})F(\mathbf{k}) = 0 \tag{21}$$

in this approximation.

Note that in the expression $\Omega - q_x V_g$ of Ref. [8] the term independent of the primary mode amplitude was also kept. If one omits this term the result of [8] (see Eq. (26) of [8]) transits qualitatively into (21). Here one should have in mind that the topic of [8] was not the Rossby waves but the electrostatic drift waves in magnetized plasma with cold ions. Then the Rossby radius is substituted by the ion Larmor radius calculated for the electron temperature, ρ_s, while, instead of our parameter $\delta_{\mathbf{k}}$, one should take the value $\delta_{\mathbf{k}}^{(d)}$, where

$$\delta_{\mathbf{k}}^{(d)} = -v_* \eta_T / \left(\omega_{ci} \rho_s^4 k_x^2\right). \tag{22}$$

Here v_* is the electron drift velocity, $\eta_T \equiv \partial \ln T / \partial \ln n_0$ is the relative temperature gradient, T is the electron temperature, n_0 is the equilibrium plasma number density, ω_{ci} is the ion cyclotron frequency, and the superscript (d) denotes "drift".

One can see from Eqs. (21) and (12) that in the problem of monochromatic primary modes the scalar nonlinearity is important only in the case when such modes are of sufficiently long wavelength (cf. [8])

$$\mathbf{k}_\perp^2 r_R^2 \lesssim \frac{1}{q_x} \frac{d \ln f}{dx}. \tag{23}$$

Then, according to Eqs. (17), (7), (18), and (19),

$$F(\mathbf{k}) = 2 q_x^2 k_y^2 \mathbf{k}_\perp^2 r_R^6 g H_0 |\tilde{h}_+|^2. \tag{24}$$

Therefore, our dispersion relation (18) can be considered as a generalization of similar dispersion relation of [8] for the case of non-monochromatic primary modes. Such a generalization allows one to analyze more extended class of problems compared with [8]. Let us consider an example of such a problem.

We assume that there are two monochromatic primary modes: the "main" and the "additional" denoted by the superscripts (1) and (2), respectively. We take that the main mode has not too small $\mathbf{k}_\perp^2$, so that for it one has the condition opposite to (23),

$$\left(\mathbf{k}_\perp^2 r_R^2\right)^{(1)} \gg \frac{1}{q_x} \frac{d \ln f}{dx}. \tag{25}$$

As for the additional mode, we assume it to be sufficiently long wavelength so that

$$\left(\mathbf{k}_\perp^2 r_R^2\right)^{(2)} \ll \frac{1}{q_x} \frac{d \ln f}{dx}. \tag{26}$$

Then dispersion relation (16) reduces to

$$1 + \frac{F^{(1)}}{(\Omega - q_x V_g^{(1)})^2} - \frac{i\delta_{\mathbf{k}}^{(2)} F^{(2)}}{\Omega^2} = 0. \tag{27}$$

Assuming $\Omega - q_x V_g^{(1)}$ to be a small parameter, we transform (27) to

$$\left(\Omega - q_x V_g^{(1)}\right)^2 = -\frac{F^{(1)}}{1 + i\delta_{\mathbf{k}}^{(2)} F^{(2)} / (q_x V_g^{(1)})^2}. \tag{28}$$

For sufficiently strong inequality (26) and for not too small amplitude of the additional mode, when

$$\delta_{\mathbf{k}}^{(2)} > \left(q_x V_g^{(1)}\right)^2 / F^{(2)}, \tag{29}$$

this equation takes the form

$$\left(\Omega - q_x V_g^{(1)}\right)^2 = -i \frac{(k_y^2 \mathbf{k}_\perp^2 |\tilde{h}_+|^2)^{(1)}}{(k_y^2 \mathbf{k}_\perp^2 |\tilde{h}_+|^2)^{(2)}}. \tag{30}$$

Dispersion relation (30) describes a new oscillating zonal flow instability of the Rossby waves due to the scalar nonlinearity. More detailed analysis of general dispersion relation (16) can be topic of following studies.

3. Discussions

We have analyzed the generation of zonal flows induced by a totality of non-monochromatic Rossby waves. A general dispersion relation taking into account both vector and scalar nonlinearities has been obtained. It is shown that scalar nonlinearity significantly modifies the process of zonal flow generation. In particular, the zonal flow instability extends into the short wavelength region. This is demonstrated by Eq. (30), when the scalar nonlinearity plays the leading role. Then the zonal flow generation may occur for smaller amplitude of primary waves and does not depend on the sign of the group velocity. It is shown that for the case of two primary waves the ratio of the wave amplitudes strongly affects the instability which is especially important at low amplitudes.

The generation mechanism described in this Letter can also be revealed for internal gravity waves in the atmospheres of planets as well as for Rossby waves turbulence in accretion disks [19–21].

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