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Image thresholding using Tsallis entropy

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Abstract

Image analysis usually refers to processing of images with the goal of finding objects presented in the image. Image segmentation is one of the most critical tasks in automatic image analysis. The nonextensive entropy is a recent development in statistical mechanics and it is a new formalism in which a real quantity q was introduced as parameter for physical systems that present long range interactions, long time memories and fractal-type structures. In image processing, one of the most efficient techniques for image segmentation is entropy-based thresholding. This approach uses the Shannon entropy originated from the information theory considering the gray level image histogram as a probability distribution. In this paper, Tsallis entropy is applied as a general entropy formalism for information theory. For the first time image thresholding by nonextensive entropy is proposed regarding the presence of nonadditive information content in some image classes. Some typical results are presented to illustrate the influence of the parameter q in the thresholding.

Keywords: Tsallis entropy; Entropic segmentation; Image thresholding

1. Introduction

Segmentation consist in subdividing an image into its constituent part and extracting those of interest. Many techniques for global thresholding have been developed over the years to segment images and recognize patterns (e.g. Kapur et al., 1985; Sahoo et al., 1988; Pal, 1996; Li, 1993; Rosin, 2001). The principal assumption of the use of global thresholding as a segmentation technique is that "objects" and "backgrounds" can be distinguished by inspecting only image gray level values.

Recent developments of statistical mechanics based on a concept of nonextensive entropy, also called Tsallis entropy, have intensified the interest of investigating a possible extension of Shannon's entropy to Information Theory (Tsallis, 2001). This interest appears mainly due to similarities between Shannon and Boltzmann/Gibbs entropy functions. The Tsallis entropy is a new proposal in order to generalize the Boltzmann/Gibbs's

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traditional entropy to nonextensive physical systems. In this theory a new parameter q is introduced as a real number associated with the nonextensivity of the system, and it is system dependent.

This paper shows the application of Tsallis entropy as a new method of image segmentation. Our work is based on the entropic thresholding technique of Kapur et al. (1985). We use Tsallis entropy form due to the presence of nonadditive information in some classes of images. The method substitutes a purely additive expression found in Kapur's original method for a pseudo-additive expression defined by Tsallis theory for statistical independent systems. The paper is organized as follows: Section 2 presents some fundamental concepts of nonextensive systems and Tsallis entropy; Section 3 describes the mathematical settings of the threshold selection for the proposed method; Section 4 gives some examples of thresholding using the proposed method and discusses the influence of the nonextensive parameter q in the image segmentation and Section 5 is dedicated to presenting some concluding remarks.

2. Nonextensive entropy

From a conventional point of view, the entropy is a basic thermodynamic concept that is associated with the order of irreversible processes in the universe. Physically it can be associated with the amount of disorder in a physical system. Shannon redefined the entropy concept of Boltzmann/Gibbs as a measure of uncertainty regarding the information content of a system. He defined an expression for measuring quantitatively the amount of information produced by a process.

The entropy of a discrete source is often obtained from the probability distribution, where $p = \{p_i\}$ is the probability of finding the system in each possible state i. Therefore, $0 \leqslant p_i \leqslant 1$ and $\sum_{i=1}^{k} p_i = 1$, where k is the total number of states. The Shannon entropy may be described as

$$S = -\sum_{i=1}^{k} p_i \ln(p_i) \tag{1}$$

This formalism has been shown to be restricted to the domain of validity of the Boltzmann–Gibbs–Shannon (BGS) statistics. These statistics seem to describe nature when the effective microscopic interactions and the microscopic memory are short ranged. Generally, systems that obey BGS statistics are called extensive systems. If we consider that a physical system can be decomposed into two statistical independent subsystems A and B, the probability of the composite system is $p^{A+B} = p^{\mathrm{A}} \cdot p^{\mathrm{B}}$, it has been verified that the Shannon entropy has the extensive property (additivity):

$$S(A+B) = S(A) + S(B) \tag{2}$$

However, for a certain class of physical systems, which entail long-range interactions, long time memory and fractal-type structures, some kind of extension appears to become necessary. Inspired by multifractals concepts, Tsallis has proposed a generalization of the BGS statistics. The Tsallis statistics is currently considered useful in describing the thermostatistical properties of nonextensive systems, and it is based on a generalized entropic form,

$$S_q = \frac{1 - \sum_{i=1}^{k} (p_i)^q}{q-1} \tag{3}$$

where k is the total number of possibilities of the system and the real number q is an entropic index that characterizes the degree of nonextensivity. This expression meets the BGS entropy in the limit $q \to 1$. The Tsallis entropy is nonextensive in such a way that for a statistical independent system, the entropy of the system is defined by the following pseudo additivity entropic rule

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q) \cdot S_q(A) \cdot S_q(B) \tag{4}$$

Considering $S_q \geqslant 0$ in the pseudo-additive formalism of Eq. (4), three entropic classifications are defined as follows

- Subextensive entropy ($q < 1$)
  $S_q(A+B) < S_q(A) + S_q(B)$
- Extensive entropy ($q = 1$)
  $S_q(A+B) = S_q(A) + S_q(B)$
- Superextensive entropy ($q > 1$)
  $S_q(A+B) > S_q(A) + S_q(B)$

Taking into account similarities between Boltzmann/Gibbs and Shannon entropy forms, it is interesting to investigate the possibility of generalization of the Shannon's entropy to the Information Theory, as recently shown by Yamano (2001). This generalization can be extended to image processing areas, specifically for the image segmentation, applying Tsallis entropy to threshold images, which have nonadditive information content.

3. Entropic segmentation

In image processing, the most commonly used method to distinguish objects from background is "thresholding". Over the years, many methods of automatic threshold selection based on optimization of some criterion function have been proposed. Pun (1981) assumed that an image is the outcome of a symbol source. Thus, in order to select the threshold he maximized an upper bound of the total a posteriori entropy of the binary image. Kapur et al. (1985) assumed two probability distributions, one for the object and the other for the background. Next they maximized the total entropy of the partitioned image in order to obtain the threshold level. Abutaleb et al. (1989) extended the method using two-dimensional entropies. Li (1993) and Pal (1996) used the directed divergence of Kullback for the selection of the threshold, and Sahoo et al. (1988) used the Reiny entropy model for image thresholding.

In this section, a new thresholding method will be proposed based also on the entropy concept. This method is similar to the maximum entropy sum method of Kapur et al.; however, we used the nonextensive Tsallis entropy concepts customized for information theory.

For an image with k gray-levels, let $p_i = p_1, p_2, \dots, p_k$ be the probability distribution of the levels. From this distribution we derive two probability distributions, one for the object (class A) and another for the background (class B). The probability distributions of the object and background classes, A and B, are given by

$$p_A: \quad \frac{p_1}{P^A}, \frac{p_2}{P^A}, \dots, \frac{p_t}{P^A} \tag{5}$$

$$p_B: \quad \frac{p_{t+1}}{P^B}, \frac{p_{t+2}}{P^B}, \dots, \frac{p_k}{P^B} \tag{6}$$

where $P^A = \sum_{i=1}^{t} p_i$ and $P^B = \sum_{i=t+1}^{k} p_i$.

The a priori Tsallis entropy for each distribution is defined as

$$S_q^A(t) = \frac{1 - \sum_{i=1}^{t} \left(\frac{p_i}{p^A}\right)^q}{q-1} \tag{7}$$

$$S_q^B(t) = \frac{1 - \sum\limits_{i=t+1}^{k} \left(\frac{p_i}{p^B}\right)^q}{q-1} \tag{8}$$

The Tsallis entropy $S_q(t)$ is parametrically dependent upon the threshold value t for the foreground and background. It is formulated as the sum of each entropy, allowing the pseudo-additive property for statistically independent systems, defined in Eq. (4).

$$S_q(t) = \frac{1 - \sum_{i=1}^{t} (p_A)^q}{q-1} + \frac{1 - \sum_{i=t+1}^{k} (p_B)^q}{q-1} + (1-q) \cdot \frac{1 - \sum_{i=1}^{t} (p_A)^q}{q-1} \cdot \frac{1 - \sum_{i=t+1}^{k} (p_B)^q}{q-1} \tag{9}$$

We maximize the information measure between the two classes (object and background). When $S_q(t)$ is maximized, the luminance level t is considered to be the optimum threshold value. This can be achieved with a cheap computational effort.

$$t_{opt} = \operatorname{argmax}\left[S_q^A(t) + S_q^B(t) + (1-q) \cdot S_q^A(t) \cdot S_q^B(t)\right] \tag{10}$$

4. Experimental results

In order to analyze this segmentation technique we simulate different histograms describing the "object" and "background" by two gaussian peaks. As presented previously, the segmentation process looks for a luminance value t that separates these two regions. This procedure allows an evaluation of the segmentation result as function of amplitude, position and width of the peaks in the histograms. All these parameters have a significant role in the characterization of the image, as for example: homogeneity of the scene illumination (gray level contrast), image and object size, "object" and "background" texture, noisy images, etc.

Starting from two well defined gaussian peaks we classify four simulation cases. In each one we analyze the threshold level for 256 different histograms. An iterative procedure changes the parameter of the right side peak until it overlaps the left one. In each step we calculate the entropic segmentation threshold t. These four testing processes are presented in Fig. 1: (a) peak position; (b) peak width; (c) peak height and (d) noisy image. The purpose of these tests is to check how good the nonextensive entropic segmentation is for different classes of images, and also to analyze the influence of Tsallis parameter q in the segmentation result.

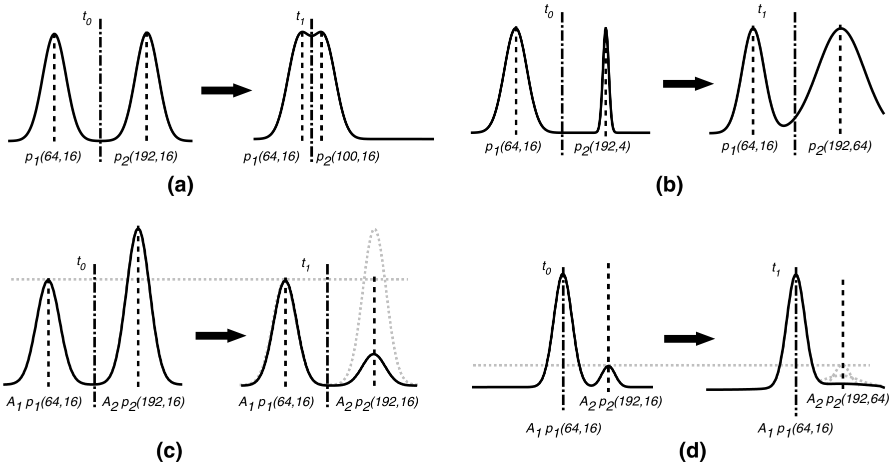

Fig. 1. Four gray level segmentation testing schemes: (a) peak position; (b) peak width; (c) peak height and (d) noisy image. We define a position and a width for each peak.

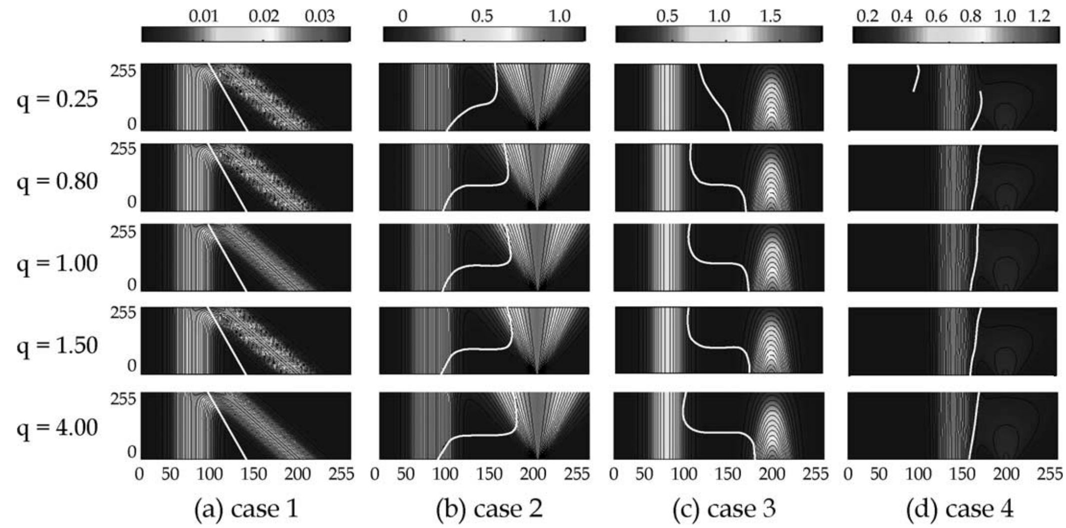

Fig. 2. Results of segmentation for simulations cases: $q = 0.25$; 0.80; 1.00; 1.50 and 4.00.

4.1. Simulation results

The result of all simulation cases is shown on Fig. 2. For all proposed cases we present four contour plots for five q values. In each contour plot we show 256 simulated histograms used to generate the entropic curve, Eq. (4). The threshold level is obtained at the maximum argument of each entropic curve, see Eq. (10). Thus one can follow the "trajectory" of the thresholding point at these

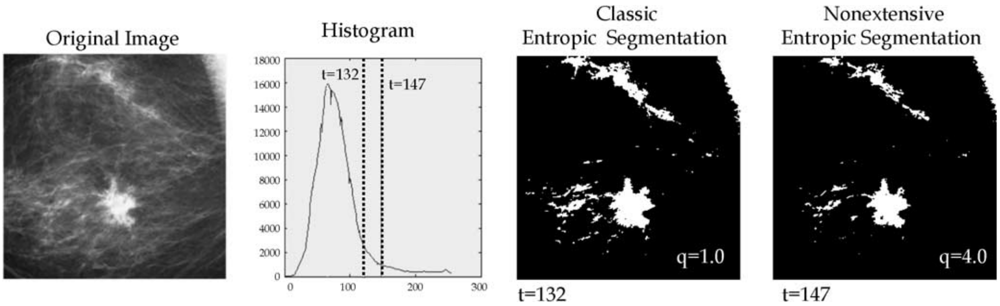

Fig. 3. Example of entropic segmentation for mammography image with an inhomogeneous spatial noise. Two image segmentation results are presented for $q = 1.0$ (classic entropic segmentation) and $q = 4.0$.

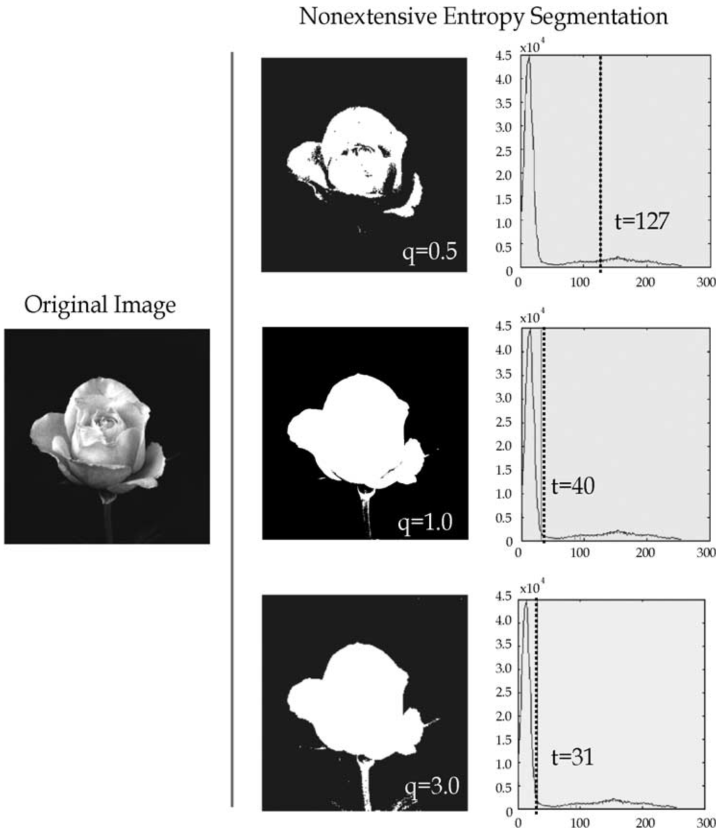

Fig. 4. Influence of parameter q in natural images: $q = 0.5$, $q = 1.0$ (classical entropic segmentation) and $q = 3.0$.

maximum values in each case. This figure shows the behavior of the nonextensive entropic segmentation in all cases. Specially, in cases (b—peak width), (c—peak height) and (d—noisy image) we observe a dependence of the segmentation result on the entropic parameter q. In the case "a—peak position" the nonextensive entropic segmentation method can be used as a classic entropic segmentation.

4.2. Real image results

We have evaluated the proposed method with several real images, following the schemes presented in the simulation section. Three of them were chosen to demonstrate the method. In Fig. 3 we show a mammography image with a bright region (tumor) surrounded by a noisy region.

The thresholding of this type of image is usually a very difficult task. The histogram is almost an unimodal distribution of the gray level values. The entropic method will search for regions with uniform distribution in order to find the maximum entropy. This will often happen at the peak limit. The entropic approach can give good results in these types of histograms. In Fig. 4 we show an image of a flower with an inhomogeneous distribution of light around it, leading to an irregular histogram of two peaks. The nonextensive entropic method can be very useful in such applications where we define a value for the parameter q to adjust the thresholding level to the correct point. In Fig. 4 we segment the image with q equal to 0.5, 1.0 and 3.0 respectively. With the latter we can cut the histogram in a more appropriate level.

Fig. 5 shows a third example of nonextensive image segmentation. We present an image of a magnetic domain structure observed by optical microscopy with a spatial background scattering noise and its gray level histogram. Four results are presented for four q values. For $q = 4.0$ we obtain a more "clean" segmentation.

4.3. The influence of parameter q in image segmentation

The parameter q in Tsallis entropy is usually interpreted as a quantity characterizing the degree of nonextensivity of a system. An appropriate choice of the entropic index q to nonextensive physical systems still remains an open field of study. In some cases the parameter q has no

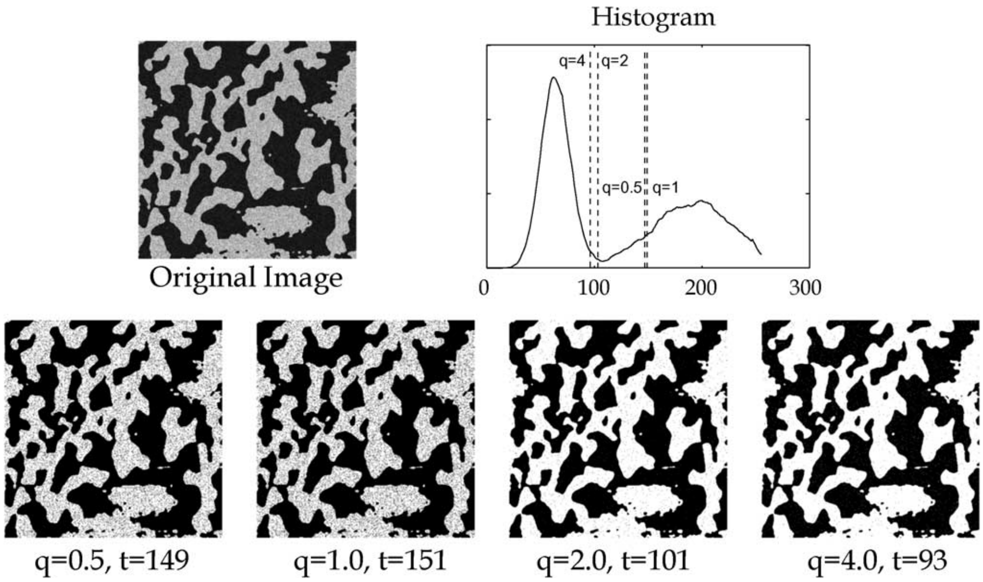

Fig. 5. Example of nonextensive entropic segmentation. Image of a magnetic domain structure with a regular spatial noise. Result of segmentation for $q = 0.5$, 1.0 (classical entropic segmentation), 2.0 and 4.0.

physical meaning, but it gives new possibilities in the agreement of theoretical models and experimental data, Tsallis and Albuquerque (2000). In other cases, q is solely determined by constraints of the problem and thereby q may have a physical meaning, Tatsuaki (2001).

In image segmentation, the nonextensivity of a system can be justified by the presence of correlations between pixels of the same object in the image. These correlations can be classified as long-range correlations in the case of images that present pixels strongly correlated in luminance levels and space fulfilling.

In the presented method we use positive values for q parameter and investigate the threshold level by visual inspection. As we are interested in automatic quantitative image analysis, thresholding "objects" with the same entropy can be an important characteristic in the image processing chain when treating the same type of images. Nevertheless, there is no reliable evidence between the parameter q and image categories.

5. Conclusions

Nonextensive entropy image thresholding is a powerful technique for image segmentation. The presented method has been derived from the generalized entropy concepts proposed by Tsallis. Entropic segmentation can give good results in many cases and works better when applied to noisy images, those in which gray level distribution are typically composed by an unimodal histogram. The advantage of the method is the use of a global and objective property of the histogram and this method is easily implemented. The Tsallis q coefficient can be used as an adjustable value and can play an important role as a tuning parameter in the image processing chain for the same class of images. This can be an advantage when the image processing tasks depend on an automatic thresholding.

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