On the nonextensive character of some magnetic systems

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Abstract. During the past few years, nonextensive statistics has been successfully applied to explain many different kinds of systems. Through these studies some interpretations of the entropic parameter q, which has major role in this statistics, in terms of physical quantities have been obtained. The aim of the present work is to yield an overview of the applications of nonextensive statistics to complex problems such as inhomogeneous magnetic systems.

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INTRODUCTION

The Nonextensive statistics attempts to handle some anomalies that appear in physical problems which are not completely described by the usual Boltzmann-Gibbs statistics. Its applications run through different systems; magnetic systems are just one of them. In this work we show that Nonextensive statistics, due to the agreement between theoretical models and experiments, is a interesting alternative to first principle models to the study of complex magnetic systems.

NONEXTENSIVE STATISTICS OF MAGNETIC SYSTEMS

The Tsallis generalized statistics, or Nonextensive statistics, is based on the definition of the generalized entropy $S_q = k(1 - Tr\{\hat{\rho}^q\})/(q-1)$, where k is a positive constant, $q \in \mathscr{R}e$ is the entropic parameter and $\hat{\rho}$ is the density operator. This entropy functional recovers the Boltzmann-Gibbs one on the limit $q = 1$ $(q \to 1)$. From the maximization of this entropy functional, under a correct q-normalized constraint [1], is straightforward to express the density operator as $\hat{\rho} = \left[1 - (1-q)\beta\hat{\mathscr{H}}\right]^{1/(1-q)}/Z_q$, in which Z_q is the nonextensive partition function, $\beta = 1/kT$ and $\hat{\mathscr{H}}$ is the systems Hamiltonian. Through the same reasoning, the magnetization of a system in the nonextensive approach is given by [2]

$$\mathscr{M}_q = \frac{Tr\{\hat{\mu}\,\hat{\rho}^q\}}{Tr\{\hat{\rho}^q\}} \tag{1}$$

where $\hat{\mu}$ is the magnetic moment operator. In Fig.1, it is illustrated the fit of Eq.(1), within the mean field approximation, to experimental data. On the main graph it is

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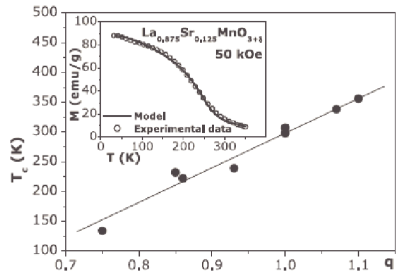

FIGURE 1. T_c versus q shows a linear dependence.

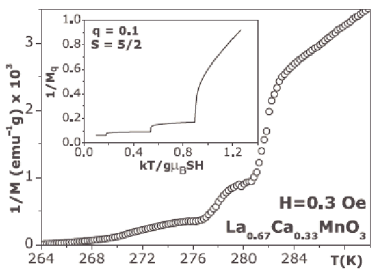

FIGURE 2. Inverse of the susceptibility versus temperature for a paramagnetic system. The inset represents the same quantity calculated for a nonextensive magnetic model where $q = 0.1$ and spin value $S = 5/2$

plotted different values of T_c, also taken from experimental data, as a function of q, which shows a simple straight line dependence, $T_c^{(q)} = q\,T_c^{(1)}$ (For details see Ref. [2]).

Furthermore, one can define the q-generalized magnetic moment thermal average as $\langle\hat{\mu}_z\rangle_q = g\,\mu_B\,\langle\hat{S}_z\rangle_q = q\,\mu_B\,S\,\mathscr{B}_S^{(q)}$, in which $\langle\hat{S}_z\rangle_q$ is the q-generalized thermal average spin operator and $\mathscr{B}_S^{(q)}$ is the *generalized Brillouin function* given by

$$\mathscr{B}_S^{(q)} = \frac{1}{S}\langle\hat{S}_z\rangle_q = \frac{1}{S}\frac{\sum_{m_S=-S}^{+S} m_S\left[1+(1-q)\frac{m_S}{S}x\right]^{q/(1-q)}}{\sum_{m_S=-S}^{+S}\left[1+(1-q)\frac{m_S}{S}x\right]^{q/(1-q)}} \tag{2}$$

where $x = g\,\mu_B\,\beta\,S B$. Using a general expression to define the magnetic susceptibility such as $\chi_q = \lim_{B\to 0}\partial_B\langle\hat{\mu}_z\rangle_q$, one can see in Fig.2 that the inverse of the susceptibility predicts correctly the behavior of the paramagnetic systems (For details see Ref. [3]).

On the other hand, considering a classical spin $\vec{\mu}$ under a homogeneous magnetic field $\vec{H}$, whose Hamiltonian is given by $\mathscr{H} = -\mu\,H\cos\theta$, it is then possible to determine the *generalized Langevin function* from Eq.(1), which yields:

$$\mathscr{L}_q = \frac{\mu}{(2-q)}\left[\coth_q\left(\frac{\mu H}{kT}\right) - \frac{kT}{\mu H}\right] \tag{3}$$

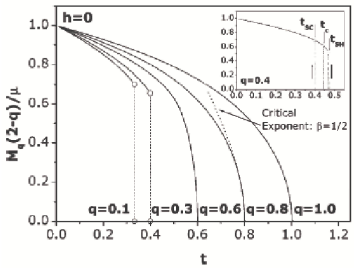

FIGURE 3. Magnetization versus temperature for several values of q and $h = 0$. For $q < 0.5$ the transition is of first-order type, whereas for $q > 0.5$ the transition is of second-order type.

in which $\coth_q$ is the q-hyperbolic cotangent. This expression can be obtained also taking the limit of large spin values ($S \to \infty$) on the generalized Brillouin function. As can be seen on Fig.3, this magnetization, also within the mean field approximation, predicts a first-order type phase transition for $q < 0.5$ and a second-order type for $q > 0.5$ which which occurs in certain kinds of magnetic systems (For details see Ref. [4]).

INTERPRETATION OF THE ENTROPIC PARAMETER

Now, with these magnetic quantities defined in a nonextensive approach, one may ask the question: If is possible to connect a homogenous nonextensive system to a inhomogeneous one described by the Boltzmann-Gibbs statistics could the entropic parameter be interpreted through some magnetic quantities?

So, to answer this question, lets us consider an inhomogeneous system (described by the Boltzmann-Gibbs statistics) composed by magnetic clusters distributed in size, therefore in their net magnetic moment, and the $f(\mu)$ representing this distribution. The average magnetization will be given by

$$\langle \mathscr{M} \rangle = \int_0^\infty \mathscr{M} f(\mu)\, d\mu \tag{4}$$

From Eqs.(3) and (4), the average and the nonextensive susceptibilities can be derived as $\langle \chi \rangle = \langle \mu^2 \rangle / 3kT$ and $\chi_q = q\mu^2/3kT$ respectively. Calculating saturation values of the magnetization one finds $\langle \mathscr{M} \rangle_{sat} = \langle \mu \rangle$ for the average and $\mathscr{M}_{q,sat} = \mu/(2-q)$ for the nonextensive. Thus, equating those limits ($\langle \chi \rangle = \chi_q$ and $\langle \mathscr{M} \rangle_{sat} = \mathscr{M}_{q,sat}$), a microscopic analytical expression to the q parameter is found

$$q(2-q)^2 = \frac{\langle \mu^2 \rangle}{\langle \mu \rangle^2} \tag{5}$$

where $\langle \mu \rangle$ and $\langle \mu^2 \rangle$ are the first and second moments of the distribution $f(\mu)$. This result is valid for any distribution $f(\mu)$ and gives to the entropic parameter an interpretation, which involves magnetic quantities, that can experimentally determined (For details see Refs.[5, 6]).

However, it is possible to try a more fundamental magnetic quantity considering a ferromagnetic N spins system. In this case, let consider that each spin interacts with z neighbors in an inhomogeneous medium, in the Boltzmann-Gibbs framework [7]. As the interactions are inhomogeneous, one must consider a distribution $f(J)$ of the system exchange integrals on the system. Thus, the average magnetization variation per unit of volume is given by

$$\langle \Delta m \rangle = \frac{\zeta(3/2) g \mu_B}{8\pi^{3/2}} \left(\frac{k_B T}{a^2 z S} \right)^{3/2} \int_0^\infty dJ \frac{f(J)}{J^{3/2}} = \frac{\zeta(3/2) g \mu_B}{8\pi^{3/2}} \left(\frac{k_B T}{a^2 z S} \right)^{3/2} \langle J^{-3/2} \rangle \quad (6)$$

Now, calculating the same quantity but on a homogeneous nonextensive system one can find [7]

$$\langle \Delta m \rangle_q = \frac{g \mu_B}{2\pi} \int_0^\infty dk k^2 \frac{Tr\{n_k \rho^q\}}{Tr\{\rho^q\}} = \frac{g \mu_B}{4\pi} \left(\frac{k_B T}{a^2 S \mathcal{J}} \right)^{3/2} F(q) \quad (7)$$

where $F(q)$ is a dimensionless integral that depends only on the value of the entropic parameter q. From the mean field approximation it is then possible to find the critical temperature $T_c^{(1)} = zS(S+1)\langle J \rangle / 3k_B$ for the inhomogeneous Boltzmann-Gibbs and $T_c^{(q)} = zS(S+1) q \mathcal{J} / 3k_B$ for the homogeneous nonextensive. Using the relation showed above between these two critical temperatures, one can find that $\mathcal{J} = \langle J \rangle$. Thus, comparing Eqs.(6) and (7), another expression is obtained

$$F(q) = \frac{\sqrt{\pi}}{2} \zeta\left(\frac{3}{2}\right) \frac{\langle J^{-3/2} \rangle}{\langle J \rangle^{-3/2}} \quad (8)$$

revealing a relation between the q parameter and moments of the exchange interaction distribution $f(J)$ (For details see Ref.[7]).

CONCLUSIONS

Summarizing, we showed that the nonextensive statistics is an useful alternative for the description of complex behavior of some kinds of magnetic systems. It was also showed that the entropic parameter q can be interpreted through magnetic quantities, and can be seen as a measurement of the inhomogeneity of a magnetic system.

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