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TWO POTENTIALS, ONE GAUGE GROUP:
A POSSIBLE GEOMETRICAL MOTIVATION

by

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ABSTRACT

By studying the purely gravitational sector of a higher dimensional matter-gravity coupled theory, one can see that in the case of non-vanishing torsion the effective 4-dimensional theory exhibits two gauge potentials that transform under the action of a single gauge group.

Key-words: Kaluza Klein; Two potential fields.

Group-theoretical concepts have been widely applied in the formulation of physical theories. Very remarkably, the requirement that a theory is invariant under the transformations of a certain local gauge group leads to a dynamical theory for spin-1 massless particles. The standard procedure consists in associating a single potential A^i_μ to each simple group or subgroup of interest. In other words, this means that the number of vector fields should be equal to the number of group generators. Successful applications of such a framework are the standard electroweak theory of Glashow-Salam-Weinberg and QCD.

A possible way to enlarge the benefits from the current gauge methodology is the introduction of more than a single gauge potential in association with a common *simple* Lie group. The case where two fields are present postulates the following transformation laws:

$$\vec{A}_\mu \longrightarrow U \vec{A}_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \tag{1}$$

and

$$\vec{B}_\mu \longrightarrow U \vec{B}_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} . \tag{2}$$

Notice that (1) and (2) have a common inhomogeneous part: a consequence of the fact that they both transform under the same group - the actual motivation of our proposal. This consequently means that just *one genuine* gauge field is present (in the sense that a gauge field is such a field whose transformation law allows the elimination of some of its unphysical degrees of freedom). The presence of the inhomogeneous piece in (1) and (2) will reduce the number of degrees of freedom *either* from A_μ *or* from B_μ, not from both simultaneously. This analysis becomes evident if one performs a simple field redefinition:

$$C_\mu = A_\mu - B_\mu \, , \tag{3}$$

$$D_\mu = A_\mu + B_\mu \, , \tag{4}$$

where D_μ is the true gauge potential of the theory and C_μ can be given a gauge-invariant mass term.

This remark raises a question of primary importance, namely the discussion about the physical distinction between the fields A_μ and B_μ. This characterization can be achieved through dynamical or geometrical methods. The facts that the fields C_μ and D_μ obey different equations of motion, carry different numbers of degrees of freedom and differ by quantum numbers such as mass, should suffice to accomplish the distinction between A_μ and B_μ both at the kinematical and dynamical levels. These aspects have already been discussed in ref. [1].

The main effort in this note is an attempt to justify the presence of two families of gauge potentials and a *single simple* gauge group on more geometrical grounds.

To pursue our investigation, we shall concentrate at the purely gravitational sector of a higher-dimensional matter-gravity interacting theory and see how the fields A_μ and B_μ, with the transformation laws (1) and (2), show up in the 4-dimensional world after a spontaneous compactification (triggered by the matter fields [2,3]) of the full theory takes place.

The extended D-dimensional gravity theory (D=4+K) can be entirely described by the vielbein and spin-connection fields, E_M^A and B_{MN}^A respectively. (The middle alphabet M, N,... are world indices, whereas the early alphabet A, B,... are local frame labels). Its Lagrangean density is

$$\mathcal{L}_{gr} = E\, E_A^M\, E_B^N\, R_{MNAB}\ , \tag{5}$$

where $E = \det E_M^A$.

In the general case of a non-vanishing torsion tensor, the fields E_M^A and B_{MN}^A are treated as being independent of each other, in which case the torsion and curvature tensors are respectively given by

$$T_{MN}^A = \partial_M E_N^A - \partial_N E_M^A + E_N^C\, B_{MC}^A - E_M^C\, B_{NC}^A \tag{6}$$

and

$$R_{MN[AB]} = \partial_M B_{N[AB]} - \partial_N B_{M[AB]} - B_{M[AC]}\, B_{N[CB]} + B_{N[AC]}\, B_{M[CB]}. \tag{7}$$

At this point, we should stress the importance of having a higher dimensional theory with a *non-zero* torsion: it will be crucial for our characterization of the fields A_μ^i and B_μ^i that E_M^A and B_{MAB} be actually independent fields. Whenever matter fields are present, the spin connection appears not only in $\mathcal{L}_{gr}$, but also in the kinetic terms of those fields (through covariant derivatives), and so a non-vanishing torsion is induced, as seen from the spin-connection equation of motion. Typical examples of matter fields would be fermions transforming in the spinor representation of SO(1, 3+K) or gauge fields A_M which are (4+K)-vectors.

Now, suppose that the matter fields coupled to the metric tensor g_{MN} induce the mechanism of spontaneous compactification[2], that is, the equations of motion of the coupled system factorise the ground state geometry according to $M^4 \times B^K$, where M^4 is the 4-dimensional Minkowski space (with coordinates x^μ) and B^K is a K-dimensional compact manifold (with coordinates y^m).

The compactification ansatz actually consists in taking a background vielbein configuration of the form

$$<E_M^A(x;y)> = \begin{pmatrix} \delta_\mu^a & 0 \\ 0 & \bar{e}_m^\alpha(x;y) \end{pmatrix}, \tag{8}$$

where $\bar{e}_m^\alpha$ denotes the vielbein of the internal manifold B^K. Notice that μ and m stand for world indices whereas a and α label the frame indices of the 4-dimensional and compact spaces respectively.

Denoting the fluctuations of the vielbein and spin connection with respect to the ground state by δE_M^A and δB_{MN}^A, one can generally write

$$E_M^A = <E_M^A> + \delta E_M^A \tag{9}$$

and

$$B_{MN}^A = <B_{MN}^A> + \delta B_{MN}^A \ . \tag{10}$$

Assume now, as it is currently done, that B^K is a compact *homogeneous* space, $B^K = G/H$ (invariant under the left-action of some compact Lie group G and such that any point can be transformed into any other by the action of a group element of G).

The background geometry specified by $\langle E_M^A(x;y)\rangle$ has the usual Poincaré symmetry; its local G-invariance consists of x-dependent left-translations, $(x;y) \longrightarrow (x;y' = F(g(x);y))$ and local frame rotations in B^K. This combination of coordinate transformations and frame rotations leaving the vielbein form-invariant is called an isometry. Under the isometries of the internal manifold, the following transformations hold[3]:

$$\bar{e}'^{\alpha}_{m}(x;y) = \frac{\partial y^n}{\partial y'^m} \; \bar{e}^{\beta}_{n}(x;y) \; \Lambda^{\alpha}_{\beta}(g(x);y) \tag{11}$$

and

$$\bar{B}'_{m\alpha\beta}(x;y') = \frac{\partial y^n}{\partial y'^m} \; [\bar{B}_{n\gamma\delta}(x;y) \; \Lambda^{\gamma}_{\alpha}(g(x);y) \; \Lambda^{\delta}_{\beta}(g(x);y) + $$

$$+ (\Lambda^{-1} \partial_n \Lambda)_{\alpha\beta}] \; . \tag{12}$$

g(x) being a transformation of the group G at the point x of space-time. Infinitesimally, the isometries of the internal space B^K are specified by the Killing vectors K^i_m (i is an index of the adjoint representation of G) and a certain number of parameters known as the Killing angles[4].

Under the isometry transformations of the internal manifold, the components e^{α}_{μ} and $B_{\mu\alpha\beta}$ of the vielbein and spin-connection transform according to the following expressions[3]:

$$e'^{\alpha}_{\beta}(x;y') = e^{\beta}_{\mu} \, \Lambda^{\alpha}_{\beta}(g(x);y) + \frac{\partial y^n}{\partial x^{\mu}} \, e^{\beta}_{n} \, \Lambda^{\alpha}_{\beta} \tag{13}$$

and

$$B'_{\mu\alpha\beta}(x;y') = B_{\mu\gamma\delta}\,\Lambda^{\gamma}_{\alpha}\,\Lambda^{\delta}_{\beta} + (\Lambda^{-1}\,\partial_{\mu}\Lambda)_{\alpha\beta} +$$

$$+ \frac{\partial y^{n}}{\partial x^{\mu}}\,[B_{n\gamma\delta}\,\Lambda^{\gamma}_{\alpha}\,\Lambda^{\delta}_{\beta} + (\Lambda^{-1}\,\partial_{n}\,\Lambda)_{\alpha\beta}]\ . \qquad (14)$$

These are the components of the higher dimensional vielbein and spin-connection that are relevant for what we wish to do: they carry spin-1 and transform inhomogeneously, therefore they are the higher dimensional sources of massless spin-1 modes associated to the gauge group of the effective 4-dimensional theory.

Using now the Killing vectors that are associated to the left-action of G on B^K, one can define as in ref. (3):

$$e^{\alpha}_{\mu} = A^{i}_{\mu}(x)\,K^{m}_{i}\,\bar{e}^{\alpha}_{m} + \ldots.. \qquad (15)$$

and

$$B_{\mu\alpha\beta} = B^{i}_{\mu}(x)\,K^{m}_{i}\,\bar{B}_{m\alpha\beta} + \ldots..\ . \qquad (16)$$

The coefficients $A^{i}_{\mu}(x)$ and $B^{i}_{\mu}(x)$ are to be interpreted as vector fields propagating in the 4-dimensional space-time. Notice they naturally carry the index i of the adjoint representation of G.

Finally, following what is done in the Appendix III of ref. (3), one can show that the coefficients $A^{i}_{\mu}(x)$ and $B^{i}_{\mu}(x)$ transform according to

$$\vec{A}'_{\mu} = g\,\vec{A}_{\mu}g^{-1} + g(\partial_{\mu}g^{-1})\quad , \qquad (17)$$

$$\vec{B}'_{\mu} = g\,\vec{B}_{\mu}g^{-1} + g(\partial_{\mu}g^{-1})\quad , \qquad (18)$$

At this point, our remark of non-zero torsion becomes clear: the components e_μ^α and $B_{\mu\alpha\beta}$ are independent fields, therefore so are the potentials A_μ^i and B_μ^i. In the 4-dimensional world, one therefore has that $D_\mu^i = A_\mu^i + B_\mu^i$ are the massless gauge bosons and $C_\mu^i = A_\mu^i - B_\mu^i$ are the massive vector bosons transforming homogeneously under G.

So, the two gauge potentials proposed to transform in an ad hoc way as in eqs. (1) and (2) can be thought of as arising from the gravity sector of a Kaluza-Klein theory with non-vanishing torsion and their origin can be traced back to the vielbein and spin-connection fields of the higher-dimensional world. The requirement of non-zero torsion is necessary to guarantee that the fields A_μ^i and B_μ^i of the low-energy effective theory actually describe independent spin-1 degrees of freedom. Therefore, since A_μ^i and B_μ^i were seen to be located in two different geometrical structures of a higher dimensional space-time, we consider that this could be sufficient to distinguish them in a more geometrical way as proposed at the beginning of this note.

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