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INVARIANT PATH INTEGRATION AND THE FUNCTIONAL MEASURE FOR EINSTEIN GRAVITATION THEORY*

by

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ABSTRACT

We propose an invariant path integral approach for the Einstein gravitation theory suitable to the analysis of the associated functional measure problem. We use the proposed formulation to analyse the phenomenon of quantum gravity in two dimensional space times.

Key-words: Invariant path integration; Functional path measure; Einstein theory; Induced two dimensional gravity.

1 INTRODUCTION

The path integral for gravitational interactions has been discussed several times in the past and most recently the important problem of the gravitational path integral measure has been reexamined (1,2,3).

In this paper, we intend to propose an approach for the quantization of Einstein's gravitational theory in the framework of path integrals suitable to the analysis of the above mentioned problem of the paths' local measure.

The basic idea in our discussion (4) is the introduction of a Riemann structure into the functional manifold of the metrical field variables compatible with the invariance group of the theory and, then, consider the associated partition functional as an infinite dimensional version of a invariant integral in a Riemman manifold (5). As a result we will not need to introduce the "ad-hoc" insertion of the Faddev-Popov unity resolution into the path integral measure in order to extract the gauge orbit volume (6), since we will be able to implement this calculation in a pure geometric way. So, in the proposed framework, it is not necessary the use a posteriori of a constraint hamiltonian path integral (7) to justify the Faddev-Popov procedure, besides our approach leads to a natural and adequate local path measure. Finally, we analyse in the proposed framework the phenomenon of the gravity in two dimensional space times produced by quantum effects.

I INVARIANT INTEGRATION

We start our analyses by briefly reviewing the basic results of the theory of invariant integrals in Riemman manifolds ([5]).

Let T be a homomorphism of a compact Lie group G in the isometry group of a given Riemman manifold M. Let us consider the integral.

$$\int_M f(x)\,[d\mu](x) \tag{1}$$

where f(x) is invariant under the action of G $(f(T(g)x) = f(x), \forall g \in G)$ and $[d\mu]$ is the measure in M induced by its Riemman metric. The orbit of a point $x \in M$ (the sub manifold of M formed by all the points $\{T(g)x\}, g \in G)$ will be denoted by O(x). The called orbit quotient space M/G can be realized as a sub-manifold of M formed by all those points of M which are not related by a group element. The measure induced by the M-Riemman metric in M/G is denoted by $[d\bar{\mu}]$ and that induced in O(x) by $[d\nu]$. Now we can state the basic result of the theory ([5]). We have the following relationship between the integral (1) and an integral defined only over the orbit quotient space M/G

$$\int_M f(x)\,[d\mu](x) = \int_{M/G} f(x)\,[d\bar{\mu}](x).v(x) \tag{2}$$

with

$$v(x) = \int_{O(x)} [d\nu](x) \tag{3}$$

We remark that $[d\nu](x)$ is a G-invariant measure over the group G, since O(x) can be realized as a manifold "copy" of G. This result is fundamental for our analysis.

Another result of differential geometry which we will use is the coordinate expression for the induced metric in a given sub manifold of M. Let $\{g_{hj}(x)\}$ denote the matrix of the metric tensor in M with $1 \leq h,j \leq N$ (N being the dimension of M). Here, x belongs to a M' coordinate domain. Let H be a sub manifold of M described by the parametric equations

$$X_j = R_j(z_\ell) \tag{4}$$

with $\{z_\ell\}$ $(1 \leq \ell \leq k\ ;\ k \leq N)$ belonging to a domain $\mathcal{D}$ (coordinate domain for H). Assuming that the matrix $[A]_{jk}(z_\ell) = \left[\frac{\partial R_j}{\partial z_k}\right](z_\ell)$ has maximal characteristic k in $\mathcal{D}$, the metric $\{g_{hj}(x)\}$ induces the following metric in H

$$\left\{g_{pq}^{(ind)}(z_k)\right\} = \left\{g_{hj}A^{hp}A^{jq}\right\}(z_k) \tag{5}$$

with the volument element given by

$$[d\rho](z_k) = \sqrt{\mathrm{Det}\left\{g_{pq}^{(ind)}(z_R)\right\}}.dz^1 \ldots dz^k \tag{6}$$

After having displayed the basic results of invariant integration we pass to the problem of the path integral quantization for the Einstein theory.

II A QUANTUM PATH MEASURE FOR EINSTEIN THEORY

Let us start our analyses writing the Hilbert-Einstein action for the theory of gravitation defined in a d-dimensional Minkowski space-time manifold E with fixed topology and without boundary (see Ref. (8); for the case of an open space-time).

$$S[\{g_{\alpha\beta}(x)\}] = \frac{1}{16\pi g} \int_E (\sqrt{-g}\ R)(x) d^D x \tag{7}$$

where the field variables are given by those metrical tensors $\{g_{\alpha\beta}(x)\}$ that can be defined in E, i.e.: compatible with its topological structure, $-g(x) = \det\{g_{\mu\nu}(x)\}$, $R(x)$ being the scalar of curvature induced by $\{g_{\mu\nu}\}$ in M and g the Newton gravitational constant.

The starting point of the Feynman's path integral quantization for the Einstein theory is the continuous sum over histories under the influence of a field external source $J^{\mu\nu}(x)$.

$$Z[j_{\mu\nu}(x)] = \sum_{\{g_{\mu\nu}(x)\}} e^{\frac{i}{\hbar} S[\{g_{\mu\nu}\}(x)] + \frac{i}{\hbar}\int_E g_{\mu\nu}(x) . j^{\mu\nu}(x)\sqrt{-g}(x) d^D x} \tag{8}$$

The precise meaning for the continuous sum (8) is achieved by introducing a path measure in the functional space of all possible field configurations, (denoted by M); $[d\mu](g_{\mu\nu}(x))$, such that (8) can be written as

$$Z[j_{\mu\nu}(x)] = \int_M [d\mu](g_{\mu\nu}(x))\, e^{\frac{i}{\hbar} S[g_{\mu\nu}(x)\}] + \frac{i}{\hbar}\int_E g_{\mu\nu}(x) . j^{\mu\nu}(x)\sqrt{-g(x)} d^D x} \tag{9}$$

The fundamental problem in Eq. (9) is to define appropriately the path measure since the Einstein action posseses the physical invariance under the action of the group of the coordinate transformations in M (the Einstein general relativity principle) denoted by $G^{DIFF}(E)$:

$$x^{\mu} \to \ell^{\mu}(x^{\mu}) \tag{10}$$

$$g_{\mu\nu}(x) \to \frac{\partial\ell^{\mu}(x^{\mu})}{dx_{\alpha}} g_{\alpha\beta}(\ell^{\mu}(x^{\mu})) . \frac{\partial\ell^{\nu}(x^{\nu})}{\partial x_{\beta}} = (L\, g_{\alpha\beta}(x))\mu\nu \tag{11}$$

and which in its infinitesimal version ($G^{DIFF}_{INF}(E)$) is given by

$$\delta x^{\mu} = \varepsilon^{\mu}(x^{\mu}) \tag{12}$$

$$\delta g_{\mu\nu}(x^{\mu}) = (\nabla_{\mu}\varepsilon_{\nu} + \nabla_{\nu}\varepsilon_{\mu})(x^{\mu}) \tag{13}$$

where ∇_{α} the usual covariant derivatives.

This invariance property lead us to treat the above path integral as an infinite dimensional version $G^{DIFF}(E)$ - invariant integral in M(see Eq. (1)).

So, we intend to use the fundamental relation Eq. (2); Eq. (3) in its functional version in order to get its expression in the physical path manifold $M/G^{DIFF}(E)$, where we can implement for instance: a Feynman diagramatic analysis. As a first step to implement the invariant integration theory we have to introduce a metrical structure in M compatible with the group $G^{DIFF}(E)$. By following B. de Witt' analysis we introduce a metric (functional) tensor $\gamma^{(\mu\nu;\alpha\beta)}[g_{\Sigma P}(x)]$ on the functional path space M for which the actions of $G^{DIFF}(E)$ are isometries. The unique functional metric which the above condition is given by the following expression (4), (9): (the so Called De Witt metric).

$$ds_F^2 = \int_E d^D x \sqrt{-g}(x) \cdot \int_E d^D x' \sqrt{-g}(x') \cdot (\delta g_{\mu\nu})(x) \cdot$$

$$\cdot \gamma^{(\mu\nu,\alpha\beta)}(x, x') \cdot (\delta g_{\alpha\beta})(x') \tag{14}$$

where the tensor density $\gamma^{(\mu\nu;\alpha\beta)}(x,x')$ is explicitly given by $(c \neq -\frac{2}{D})$

$$\gamma^{(\mu\nu,\alpha\beta)}(x,x') = \frac{1}{\sqrt{2}} \frac{\delta^{(D)}(x-x')}{\sqrt{-g}(x')} (g^{\mu\alpha} g^{\nu\beta} + c g^{\mu\nu} g^{\alpha\beta})(x) \tag{15}$$

and $(\delta g_{\Sigma p})(x)$ denotes the functional infinitesimal displacements on M. It is instructive to point out the condition $c \neq -\frac{2}{D}$ in Eq. (15) insures the positivity of the De Witt metric Eq. (14).

After introducing a Riemann structure on the path functional manifold M we can use the basic relationship Eq. (21-Eq.(3) to give a precise meaning for the path integral

$$Z = \int_M [d\mu](g_{\mu\nu}(x)) \cdot e^{\frac{i}{h} S[g_{\mu\nu}(x)]} \tag{16}$$

As a first step, we have to realize the abstract orbit quotient space $M/G^{DIFF}(E)$ in M. For this task we consider a set of D functionals $f^{\mu}(g_{\delta\Sigma}(x))$ defined in M and in a such way that the equations (see Eq. (11)) in $G^{DIFF}(E)$

$$f^{\mu}((L.g_{\delta\Sigma})_{\alpha\beta}(x)) \equiv 0 \qquad (17)$$

have only the identity solution for a given $\{g_{\gamma\Sigma}(x)\}$ i.e.: we have fixed our gauge. In order to simplify the discussion below we restrict our analysis to the class of the linear functionals $f^{\mu}(g_{\delta\Sigma}(x))$ satisfying the following condition:

$$\frac{\delta f^{\mu}(g_{\tau\Sigma}(x))}{\delta g_{\delta\rho}(x')} \text{ is a functional independent of the fiel variables } \{g_{\alpha\beta}(x)\} \qquad (18)$$

For instance, the well known harmonic gauge $\partial^{\alpha}g_{\mu\alpha}(x) \equiv f^{\mu}(g_{\alpha\beta}(x))$ belongs to the above cited class $\left(\frac{\delta f^{\mu}}{\delta g_{\sigma\rho}(x')} \equiv \partial_{\alpha}\delta^{\rho\alpha}\delta^{\delta\mu}\delta_{\sigma\rho}\right)$

Thus, we can realize the orbit quotient space $M/G^{DIFF}(E)$ in M as the path inequivalent manifold solution of the equations in M.

$$\bar{g}_{\alpha\beta}(x) \in M/G^{DIFF}(E) \rightleftarrows f^{\mu}(\bar{g}_{\alpha\beta}(x)) \equiv 0 \qquad (19)$$

With this implicit $M/G^{DIFF}(E)$ parametrization; (we assume formally its global validity in what follows - see $\oint$ 14.5.3 - Ref. (9)); the induced path measure is, then, given by the well known De Witt result ((9) - $\oint$ 14.52)

$$[d\bar{\mu}](\bar{g}_{\alpha\beta}(x)) = \prod_{(x\in E)} (dg_{\alpha\beta}(x)).(\text{DET}\,\delta^{\mu\nu,\alpha\beta}(x,x')\,\delta_F(f^{\mu}(g_{\delta\Sigma}(x)) \qquad (20)$$

where

$$\mathrm{DET}\,\gamma^{(\mu\nu,\alpha\beta)}(x,x') = (-1)^{D-1}\left(1+\frac{CD}{2}\right)\left(\sqrt{-g}(x)\right)^{\frac{1}{4}(D-4)(D+1)} \tag{21}$$

and the functional delta $\delta_F(f^\mu(g_{\delta\Sigma}(x))$ in the functional measure (20) restrict its support to the manifold of inequivalents metrics.

Now we have to evaluate the orbit (functional) volume defined by a given inequivalent configuration $\{\bar{g}_{\alpha\beta}(x)\}\in M/G^{DIFF}(E)$.

For this purpose we need an explicit parametrization of the orbit sub manifold $O(\bar{g}_{\alpha\beta}(x)\}$. Such expression is given by the path integral

$$Y_{\mu\nu}[L,\bar{g}_{\gamma\Sigma}(x)] = \int_M \Big(\prod_{x\in E}(dg_{\rho\delta}(x))\Big)\cdot g_{\mu\nu}(x)\cdot\delta_F(f^\mu(g_{\rho\sigma})(x)-f^\mu((L.\bar{g})(x)_{\rho\sigma}) \tag{22}$$

We remark that the $\{g_{\rho\sigma}(x)\}$ functional integration in Eq. (22) is carried out over the whole path manifold M and the group $G^{DIFF}(E)$ is the parameter domain for the orbit manifold $O(\{\bar{g}_{\alpha\beta}(x)\})$.

The functional integration over M gives the result

$$Y_{\mu\nu}[L,\bar{g}_{\gamma\Sigma}(x)] = (L.\bar{g})_{\mu\nu}(x)\cdot\left\{\prod_{\mu=1}^{D}\mathrm{DET}_F\left\{\frac{\delta f^\mu(g_{\gamma\Sigma})}{\delta_{\sigma\tau}}(x)\right\}\right\}^{-1} \tag{23}$$

and since the functional determinants involved in Eq. (23) are $g_{\sigma\zeta}(x)$-independent by the condition (18) we obtain that $Y_{\mu\nu}[L,\bar{g}_{\delta\Sigma}(x)]$ is an explicit parametrization of the orbit $O\{(\bar{g}_{\mu\nu}(x))\}$.

In order to evaluate the induced metric in $O\{(\bar{g}_{\mu\nu}(x)\}$ by the De Witt metric Eq. (14) we use the functional version of Eq. (5) with Eq. (22) playing the role of Eq. (4). So, the dif ferential line element in $O\{(\bar{g}_{\mu\nu}(x)\}$ is given by

$$\tilde{d}s^2 = \int d^\nu x\, d^\nu x' \left\{ \frac{\delta}{\delta\varepsilon_\rho} Y_{\mu\nu}\left[\{\varepsilon^\gamma(x), \bar{g}_{\delta\Sigma}(x)\right]\right\} . (\delta\varepsilon^\rho)(x)$$

$$\sqrt{-\bar{g}(x)} . \gamma^{(\mu\nu,\alpha\beta)}(x,x')\sqrt{-\bar{\delta}(x')}$$

$$. \left\{ \frac{\delta}{\delta\varepsilon_{\rho'}} Y_{\alpha\beta}\left[\{\varepsilon^\gamma(x'), \bar{g}_{\gamma\Sigma}(x')\right]\right\} . (\delta\varepsilon^{\rho'})(x') \qquad (24)$$

where we have considered the group transformation $L \in G^{DIFF}(E)$ being infinitesimal and characterized by the infinitesimal generators $\{\varepsilon^\gamma(x)\}$ (see Eq. (2)-Eq. (3)).

The functional derivative in Eq. (24) yields the result

$$\tilde{d}s^2 = \int d^D x\, d^D x' \sqrt{-\bar{g}(x)} . DET_F \left(\frac{\delta f^\mu (L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_\rho}(x) \right) . (\delta\varepsilon^\rho(x))$$

$$. TR\left[\hat{\gamma}^{(\mu\nu,\alpha\beta)}(x)\right] . \sqrt{-\bar{g}(x')} . \delta^{(D)}(x-x')$$

$$DET_F \left\{ \left(\frac{\delta f^\mu ((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho'}} \right) \right\}(x') \quad . (\delta\varepsilon^{\rho'}(x)) \qquad (25)$$

with

$$Tr\left[\hat{\gamma}^{(\mu\nu,\alpha\beta)}(x)\right] = \sum_{(\sigma_1,\sigma_2,\sigma_3,\sigma_4)} (\delta_\mu^{\delta_1} \delta_\nu^{\delta_2} (\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} + c\bar{g}^{\mu\nu}\bar{g}^{\alpha\beta}))\delta_\alpha^{\delta_3}\delta_\beta^{\delta_4})(x) \qquad (26)$$

The functional measure induced by Eq. (25) in $O\{(\bar{g}_{\mu\nu}(x)\}$ is then given by (see Eq. (3) - Eq. (6))

$$[d\nu][\bar{g}_{\mu\nu}(x)] = \int \prod_{(x\in E)} \sqrt{-\bar{g}(x)}.\left(\mathrm{Tr}[\hat{\gamma}^{(\mu\nu,\alpha\beta)}(x)]\right)^{\frac{1}{2}}(d\varepsilon^{\rho}(x))$$

$$\left\{\mathrm{DET}_F\left(\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho}}(x)\right)\right\} \quad (27)$$

Since we are considering the infinitesimal group transformations only in Eq. (27) we can use the Taylor expansion for the functional $\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho}}(x)$

$$\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho}}(x) = \left.\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho}}\right|_{\varepsilon_{\rho}\equiv 0}(x) + O(\varepsilon^2) \quad (28)$$

and as a consequence of Eq. (28), we get the result where the invariant group volume is factorized (the well known result due to Faddev-Popov [6])

$$[d\nu](\bar{g}_{\mu\nu}(x)) = \mathrm{Tr}[\hat{\gamma}^{(\mu\nu,\alpha\beta)}(x)].\mathrm{DET}_F\left(\left.\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta\varepsilon_{\rho}}\right|_{\varepsilon_{\rho}\equiv 0}(x)\right)$$

$$\left(\int_{G^{DIFF}} \prod_{x\in E} \sqrt{-\bar{g}(x)}.(d\varepsilon^{\rho})(x)\right) \quad (29)$$

where the last right hand functional integral is the orbit volume.

Finally by grouping together the obtained results Eq.(20) and Eq. (29) (see Eq. (2) and Eq. (16)) we obtain the proposed path measure for Einstein gravitation theory

$$[d\mu](g_{\mu\nu}(x)) = \left(\prod_{x\in E}(dg_{\alpha\beta}(x)).\right)(\mathrm{DET}\,\gamma^{(\mu\nu,\alpha\beta)}(x,x'))$$

$$\cdot \delta_F (f^{\mu}(g_{\gamma\Sigma}(x)) . (TR[\hat{\gamma}^{(\mu\nu,\alpha\beta)}(x)])^{\frac{1}{2}}$$

$$\cdot \left\{ DET_F \left(\frac{\delta f^{\mu}((L.\bar{g})_{\alpha\beta})}{\delta \varepsilon_\rho} \Bigg|_{\varepsilon_\rho \equiv 0} \right) (x) \right\} . \tag{30}$$

which differ from the De Witt proposed quantum measure by the term $(TR[\hat{\gamma}^{(\mu\nu,\alpha R)}(x)])^{1/2}$ (see Eq. 14.107 - Ref. (9)).

III THE INDUCED QUANTUM GRAVITY IN TWO DIMENSIONAL SPACE-TIMES

Now, we analyse the phenomenon of induced gravity with non zero cosmological constant in two dimensions closed space time manifolds diffeomorphic to a unitary disc S^2 (see Ref. (10) for a study with zero cosmological constant. It is well known that in two dimensions gravity is apparently without physical meaning because the Einstein density $(\sqrt{g}\, R)(\xi)(\xi \in S^2)$ is a total divergence; so the associated Einstein equation is an identity

$$\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right)(\xi) = 0 \qquad (\xi \in S^2) \tag{31}$$

However, at the quantum level the physical object is the quantum path measure (see Eq. (16) and Eq. (30).) $[d\mu](g_{\mu\nu}(\xi))$. Thus, let us consider the partition functional

$$Z = \int [d\mu](g_{\mu\nu}(\xi)) . e^{\frac{i}{h} \frac{1}{16\pi g} \int_{S^2} (\sqrt{-g}\, R)(\xi) d^2\xi} e^{-i\mu_0^2 \int_{S^2} \sqrt{-g}\, d\xi^2} \tag{32}$$

$$= e^{\frac{i}{\hbar 4 g}} . \int [d\mu](g_{\mu\nu}(\xi) . e^{-i\mu_0^2 \int_{S^2} \sqrt{-g}\, d^2\xi} \tag{33}$$

where $(\mu_0)^2$ is the (bare) cosmological constant.

In order to evaluate the functional path measure $[d\mu](g_{\mu\gamma}(\xi))$ we follow A.M. Polyakov ([11]) by chosing the global conformal gauge in the Minkowski 1 +1 space time

$$g_{ab}(\xi) = \rho(\xi)\delta_{ab} \tag{34}$$

From Eq. (20), Eq. (21), Eq. (30) we obtain explicitly the expression for the above cited functional path measure

$$[d\nu](g_{ab}(\xi)) = \prod_{\xi\in S^2}\left(d\rho(\xi)(-1(1+c))(\rho(\xi))^{-3/2}\right)\mathrm{DET}_F(L) \tag{35}$$

where L, denote the Faddev-Popov operator associated to the conformal gauge obtained by the first time in (Ref. (11) - Eq. (21)- Eq. (24) in a Euclidean S^2):

$$\ell og\ \mathrm{DET}_F(L) = -\frac{26i}{24\pi\hbar}\int_{S^2}\left[\frac{1}{2}\left(\frac{\partial_\mu\rho}{\rho}\right)^2(\xi)d^2\xi + \lim_{\varepsilon\to 0+}\frac{2i}{4\pi\varepsilon\hbar}\int_{S^2}\rho(\xi).d^2\xi\right.$$

$$(i = \sqrt{-1}) \tag{36}$$

Grouping together the Eq. (36) - Eq. (35) and absorbing the infinite part of the Polyakov determinant L in a renormalization of the bare cosmological constant, we get the induced two dimensional gravity:

$$Z = \int\left(\prod_{\xi\in s^2}d\rho(\xi)-1(1+c)(\rho(\xi))\right)^{-\frac{3}{2}}.\exp\left\{\frac{i}{\hbar}\int_{S^2}\frac{26}{24\pi}\left(\frac{1}{2}\left(\frac{\partial_\mu\rho}{\rho}\right)^2\right)(\xi)d^2\xi\right.$$

$$\left. + \frac{i}{\hbar}\mu_R^2.\int_{S^2}\rho(\xi).d^2\xi\right\} \tag{37}$$

We remark the non-flat path measure in Eq. (37). Compare with the path measure for D = 4, Eq. (21)).

Now the classical gravity ($\hbar \to +\infty$ in Eq. (37)) is governed by the Liouville equation instead of the Einstein equation, i.e:

$$\Box_{\xi}(\ell og\ \rho(\xi) = \mu_R^2 . \frac{24\pi}{26} . \rho(\xi) \qquad (38)$$

which is exactly solvable ([11])

$$\rho(\xi) = \frac{|f'(z)|^2}{\left(\frac{13\pi}{26}\mu_R^2 - |f(z)|\right)^2} \qquad (39)$$

with $f(z) = f(\xi_1 + i\xi_2)$ a holomorphic function in S^2.

So, we conclude that the gravity is non trivial in 1 + 1 space times and is a purely quantum phenomenon ([12]).

REFERENCES

[1] K. Fujikawa - Nucl. Physics B226 (1983), 437-443

[2] Ron K. Unz - SLAC-PUB - 3656 (Preprint) (1985)

[3] Ron K. Unz - SLAC-PUB - 3663 (Preprint)(1985)

[4] B. de Witt - Jour. of Math. Phys. vol. 3 nº 6 (1962)

[5] Hsiang, W.J. In: Proceedings of Conference on Compact Transformation groups, springer (1967).

[6] L.D. Faddev and V.N. Popov - Phys,Lett. 25B (1967), 29

[7] L.D. Faddev - Teor. Mat. Fiz. 1,3 (1969).

[8] S.W. Hawking - in General Relativity: an Einstein Centenary Survey, edited by S.W. Hawking and W. Israel, Cambridge (1969).

[9] B.S. De Witt - in General Relativity: an Einstein Centenary Survey, edited by S.W. Hawking and W. Israel, Cambridge (1979).

[10] S.G. Rajeev - Phys. Lett. 113B (1982), 146

[11] A.M. Polyakov - Phys.Lett. 103B (1981), 207

[12] S. Adler - Phys. Rev. Lett. 44 (1980), 1567