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STATIC POTENTIALS FROM AN EXTENDED GAUGE SYMMETRY

by

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ABSTRACT

Static potentials derived from the inclusion of more than one vector field in a single simple group are calculated. A confinement mechanism including colourful unphysical particle is discussed.

Key-words: Two compensating fields in the same group.

1 INTRODUCTION

The past few years have witnessed the affirmation of gauge theories as a very systematic and mathematically elegant framework for representing the basic forces of nature. The extrapolation of the gauge principle from the class of the so-called internal symmetries to the category of space-time symmetries let to the formulation of simple and extended supergravity theories. Their properties are shedding light on our present understanding of how gravity can be accommodated together with the electroweak and strong interactions in a single unifying scheme. The momentum generated by the gauge formulations was also a decisive stimulation for the development of the Kaluza-Klein programme. It brought a way of systematically understanding the presence of certain local internal symmetries in the four-dimensional world as a manifestation of extra compact dimensions with a characteristic length of the order of the Planck scale.

Aside from these features which are of a more formal nature, gauge theories, both in their exact or spontaneously broken version, have also encountered pragmatic support from the domain of the experimental physics. The appearance of scaling-violation in deep inelastic processes as calculated in QCD is a good test in favour of the non-abelian structure, whereas the recent UA1 - and UA2-collaboration results on the masses of the intermediate gauge bosons $W^{\pm}$ and Z are a very sharp indication that gauge theories constitute indeed a very suitable way of formulating field-theoretical models for the fundamental interaction.

However, issues like the cancellation of infrared divergences and the confinement of the quantum numbers associated with the locally conserved charges in non-Abelian gauge theories are still lacking a definite proof and even a full conceptual understanding. Though of an essentially formal nature, these problems are of great interest for practical applications. The former, plays a crucial rôle for reintering these theories a mathematically consistent way of calculate processes [1]. The Bloch-Nordsieck mechanism with a coloured particle as the initial state does not cancel the infrared divergences. At high energies, where QCD effects can be considered measurable, the factorization theorems fail for quark-gluon scattering. The latter, has two aspects to be analysed. The algebraic framework and the dynamical aspect. The second approach allow us to have an insight about the confining potentials. However the tree-level shape $\frac{1}{k^2}$ for QCD propagator does not yield a rising potential. Therefore in order to find a satisfactory explanation for the quarkonium spectra it becomes necessary to introduce heuristic confining potentials. An alternative way is to consider non-perturbative numerical calculations. They are applied on the lattice or in the continuum [2].

In view of what was discussed above, our effort here is to enlarge the gauge principle [3] through the introduction of more than one family of gauge potentials in association with a single simple gauge troup. Consequently, it gives gauge invariant mass terms for the sector of vector bosons, the freedom to build up gauge invariant scalars increases reasonably and for the propagators there can appear a better behaviour in the ultra-violet limit without higher derivatives being introduced [4]. However, one has to control the introduction of negative-metric ghosts in the theory.

The central motivation of our work consists therefore in analysing the consequences of the introduction of more than a class of gauge potentials in a single simple gauge group. Thus mixed vector-field propagators are derived and discussed, and various types of potentials are worked out in the static limit. This work is one of the stages to guide us for Lagrangians which yield confining potentials.

Our paper is outlined as follows. In section 2, we discuss the introduction of families of gauge potentials, and explicitly write down for the case of two families the general gauge-invariant Lagrangian that contributes to the propagator. In section 3, the vector-propagators are derived. The various classes of potentials are obtained and discussed in section 4. Finally, in section 5, we conclude with a number of remarks, criticisms and comments on our results.

2 LAGRANGIAN

In this section, we wish to write down a general gauge-invariant Lagrangian describing the dynamics of two families of vector bosons, A_μ and B_μ, associated however with a single compact and simple group, G. Their transformation laws are proposed to be

$$A'_\mu = U A_\mu U^{-1} + \frac{1}{ig} U(\partial_\mu U^{-1}) \tag{1}$$

and

$$B'_\mu = U B_\mu U^{-1} + \frac{1}{ig} U(\partial_\mu U^{-1}) \tag{2}$$

They yield covariant tensors as

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu B_\mu + g[B_\mu . A_\nu] \tag{3}$$

At a first glance, it might appear that the fields A_μ and B_μ are physically undistinguishable. However the theory differentiates them through various aspects, as number of degrees of freedom and dynamics. A gauge field means a field with a transformation law that eliminates some of its unphysical degrees of freedom. The presence of an inhomogeneous term in (1) and (2) will reduce the number of degrees of freedom either from A_μ or from B_μ. Therefore there is just one gauge field. Although the calculation of the dynamical variables depends on the Lagrangians to be proposed from (1) and (2), it is easy to show cases where these fields will carry different numbers and types of canonical momenta. The explicit calculation of the equations of motion also shows the existence of a different dynamics for each field.

In spite of the presence of an inhomogeneous term in (1) and (2), one should notice that the gauge connection is unique. The field combination

$$D_\mu \equiv \frac{1}{2} (A_\mu + B_\mu) \quad , \quad D'_\mu = U D_\mu U^{-1} + \frac{1}{ig} U(\partial_\mu U^{-1}) \tag{4}$$

is the genuine gauge potential of the theory, whereas the combination

$$C_\mu \equiv \frac{1}{2} (A_\mu - B_\mu) \tag{5}$$

transforms homogeneously under the adjoint representation of G,

$$C'_\mu = U C_\mu U^{-1} \tag{6}$$

Phrasing in a different way, the fields A_μ and B_μ are vector fields parametrized by the gauge connection of the theory D_μ and a spin-1 matter field C_μ.

In the presence of these two families of vector fields, it is clear that the freedom one has in building up gauge invariant terms which finally contribute to the action increases in a reasonable way. We shall now present and discuss the most general class of terms that can be generated out of the fields A_μ and B_μ (or alternatively, C_μ and D_μ) and are compatible with gauge invariance and renormalizability. In the context of this work our objective will be just to analyse the propagator contribution from the total Lagrangian (5). The basic covariant terms are

$$\nabla_\mu = \partial_\mu + g D_\mu \tag{7}$$

$$\mathcal{D}_\mu = \nabla_\mu + c C_\mu \tag{8}$$

$$D_{\mu\nu} = [\nabla_\mu, \nabla_\nu] \tag{9}$$

$$C_{\mu\nu} = [\nabla_\mu, C_\nu] \tag{10}$$

$$Z_{\mu\nu} = c_1 C_{\mu\nu} + c_2 C_{\nu\mu} + c_3 [C_\mu, C_\nu] + d_1 D_{\mu\nu} \tag{11}$$

They yield an abundance of gauge scalars in the same group. The part contributing to the propagator is

$$\mathcal{L} = \mathrm{tr}[\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6] \tag{12}$$

where

$$\mathcal{L}_1 = x_1 Z_{\mu\nu} Z^{\mu\nu} + y_1 Z_{\mu\nu} Z^{\nu\mu} + z_1 Z^\mu_\mu Z^\nu_\nu \tag{13}$$

$$\mathcal{L}_2 = x_2 C_\mu \mathcal{D}_\nu Z^{\mu\nu} + y_2 C_\mu \mathcal{D}_\nu Z^{\nu\mu} + z_2 C_\mu \mathcal{D}^\mu Z^\nu_\nu \tag{14}$$

$$\mathcal{L}_3 = x_3 Z_{\mu\nu} \mathcal{D}^\mu C^\nu + y_3 Z_{\mu\nu} \mathcal{D}^\nu C^\mu + z_3 Z^\mu_\mu \mathcal{D}_\nu C^\nu \tag{15}$$

$$\mathcal{L}_4 = x_4 \mathcal{D}_\mu \mathcal{D}_\nu Z^{\mu\nu} + y_4 \mathcal{D}_\mu \mathcal{D}_\nu Z^{\nu\mu} + z_4 \mathcal{D}_\mu \mathcal{D}^\mu Z^\nu_\nu \tag{16}$$

$$\mathcal{L}_5 = x_5 C_\mu \mathcal{D}_\nu \mathcal{D}^\mu C^\nu + y_5 C_\mu \mathcal{D}_\nu \mathcal{D}^\nu C^\mu + z_5 C_\mu \mathcal{D}^\mu \mathcal{D}_\nu C^\nu \tag{17}$$

$$\mathcal{L}_6 = x_6 \mathcal{D}_\mu \mathcal{D}_\nu \mathcal{D}^\mu C^\nu + y_6 \mathcal{D}_\mu \mathcal{D}_\nu \mathcal{D}^\nu C^\mu + z_6 \mathcal{D}_\mu \mathcal{D}^\mu \mathcal{D}_\nu C^\nu \tag{18}$$

The coefficients c-d_1 and x_1-z_6 are called as the free parameters of theory. They are numbers. Thus (12) contains twenty three parameters that can take any values without breaking the gauge symmetry. Taking trace it appears three different structures in the adjoint representation of SU(N),

$$\mathrm{tr}\, t^a t^b = N \delta^{ab} \tag{19}$$

$$\mathrm{tr}\, t^a t^b t^c = \frac{i}{2} N C^{abc} \tag{20}$$

$$\mathrm{tr}\, t^a t^b t^c t^d = \delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc} + \frac{N}{4} (d^{abe} d^{cde} - d^{ace} d^{dbe} + d^{ade} d^{bce}) \tag{21}$$

where $[t^a, t^b] = i\, c^{ab} c^{tc}$.

Considering from (12) just the propagator part,

$$\mathcal{L}_p = a_1(\partial_\mu C^a_\nu)^2 + b_1\,\partial_\mu C_{\nu a}.\partial^\nu C^{\mu a} + c_1(\partial.C^a)^2 +$$

$$+ d_1(\partial_\mu D^a_\nu)^2 + e_1\,\partial_\mu D_{\nu a}.\partial^\nu D^{\mu a} + f_1(\partial.D^a)^2 +$$

$$+ g_1\,\partial_\mu C_{\nu a}.\partial^\mu D^{\nu a} + h_1\partial_\mu C_{\nu a}.\partial^\nu D^{\mu a} + i_1(\partial.C^a)(\partial.D^a) +$$

$$+ m_c C_{\mu a}C^{\mu a} \tag{22}$$

where

$$a_1 = x_1(c_1^2+c_2^2) + y_1\,2c_1c_2 - x_2c_2 - y_1c_1 + x_3c_1 + y_3c_2 +$$

$$- x_4\,cc_2 - y_4\,cc_1 - y_5 - y_6\,c \tag{23}$$

$$b_1 = x_1\,2c_1c_2 + y_1(c_1^2+c_2^2) - x_2c_1 - y_2c_2 + x_3c_2 + y_3c_1 +$$

$$- x_4cc_1 - y_4cc_2 - x_5c - x_6c \tag{24}$$

$$c_1 = z_1(c_1+c_2)^2 - z_2(c_1+c_2) + z_3(c_1+c_2) - z_4c(c_1+c_2) +$$

$$- z_5 - z_6c \tag{25}$$

$$d_1 = x_1\,2d_1^2 - y_1\,2d_1^2 \tag{26}$$

$$e_1 = - x_1\,2d_1^2 + y_1\,2d_1^2 \tag{27}$$

$$f_1 = 0 \tag{28}$$

$$g_1 = x_1\, 2d_1(c_1-c_2) - y_1\, 2d_1(-c_1+c_2) + x_2d_1 - y_2d_1 +$$

$$+ x_3d_1 - y_3d_1 + x_4(cd_1-gc_2) - y_4(cd_1+gc_1) - y_6g \qquad (29)$$

$$h_1 = x_1\, 2d(-c_1+c_2) + y_1\, 2d(c_1-c_2) - x_2d_i + y_2d_1 - x_3d_1 +$$

$$+ y_3d_1 - x_4(gc_1+d_1c) + y_4(-gc_2+dc) - x_6g \qquad (30)$$

$$i_1 = - z_4g(c_1+c_2) + z_7g(c_1+c_2) - z_6g \qquad (31)$$

The canonical momenta corresponding to (22) are

$$\pi^\mu(C) = 2a_1\, \partial^0C^\mu + 2b_1\, \partial^\mu C^0 + g_1\, \partial^0D^\mu + h_1\, \partial^\mu D^0 +$$

$$+ g^{0\mu}[2c_1\, \partial.C + i_1\partial.D] \qquad (32)$$

$$\pi^\mu(D) = 2d_1\, \partial^0D^\mu + 2e_1\, \partial^\mu D^0 + g_1\, \partial^0C^\mu + h_1\, \partial^\mu C^0 +$$

$$+ g^{0\mu}[2f_1\, \partial.D + i_1\partial.C] \qquad (33)$$

At this stage, one should notice that the distribution of on--shell degrees of freedom is clear. D_μ being a true gauge potential carries two physical degrees of freedom, whereas C_μ can be given a mass, m_c, consistently with gauge invariance and describes three physical degrees of freedom.

The introduction of a mass parameter for D_μ can be justified if, for example, by coupling D_μ to a multiplet of scalar fields and then invoking the Higgs mechanism to generate the above mass term. One could also avoid the gauge-symmetry breaking, and couple the field D_μ to a scalar field, ρ, taking values in the Lie group G (not in its Lie algebra). An effective theory can be obtained for a massive D_μ-field ([5]). Consider

$$\mathcal{L}(\Phi) = m^2 \, tr[\rho^+ (\nabla^\mu \nabla_\mu \rho) + h.c.]$$

where

$$\rho = e^{iw^a t a}$$

$$\rho \to U \rho U^{-1} \qquad (34)$$

Defining $\Phi = m\rho$

$$\mathcal{L}(\Phi) = tr[-2\Phi^+ \Box \Phi + 4ig\, D^\mu (\partial_\mu \Phi)\, \Phi^+ - 4ig\, \Phi^+ (\partial^\mu \Phi) D_\mu +$$

$$+ 4g^2 \Phi^+ A^\mu \Phi A_\mu - 4g^2 m_D^2 D_\mu D^\mu] \qquad (35)$$

The generating functional including (12) and (35) is

$$Z[J_\mu;J] = \int \mathcal{D} D_\mu \mathcal{D} C_\mu \mathcal{D}\Phi^+ \mathcal{D}\Phi \; e^{i\, S_{gauge}[D_\mu, C_\mu; J_\mu, J_\mu]} \,.$$

$$. \; e^{i\int d^4x \; \Phi^+ M \Phi + J^+ \Phi^+ + J\Phi} \qquad (36)$$

Performing the integration on $\mathcal{D}_\Phi \mathcal{D}_\Phi^+$

$$Z[J_\mu;J] = \int \mathcal{D} D_\mu \mathcal{D} C_\mu \; e^{i\, S_{eff}}$$

$$S_{eff} = S_{gauge} - 4g^2 \, m_D^2 \, D_\mu D^\mu - tr \, \ell n \, M$$

$$tr \, \ell n \, M = t_1 D^{\mu\nu} D_{\mu\nu} + t_2 D^\mu \Box D_\mu + t_3 D^{\mu\nu} [D_\mu, D_\nu] + t_4 [D_\mu, D_\nu][D^\mu, D^\nu] \qquad (37)$$

where t_1, t_2, t_3, t_4 are numerical coefficients. At this way both fields D_μ and C_μ contain a mass parameter. Propagators will depend on them.

3 PROPAGATORS

The part in (12) that contributes for the propagators is

$$[C_\mu, D_\mu] \; K^{\mu\nu} \begin{bmatrix} C_\nu \\ D_\nu \end{bmatrix}$$

where

$$K^{\mu\nu} = K_1 \Box \, \eta^{\mu\nu} + K_2 \, \partial^\mu \partial^\nu \tag{38}$$

In perturbation theory a physical particle is defined as the poles of the complete and renormalized two-point Green's function. However, this interpretation is not straightforward in the case of two fields. There appear mixing propagators that are originated from the non-diagonal elements in the kinetic term. The relevant poles are the ones originated from the matrix K_1; they will be called basic poles. The general expression for the propagators obtained from it is

$$\langle T(P^i_\mu P^j_\nu)\rangle \equiv -i \, \frac{\mathrm{cof}(\Box + K_1^{-1} m^2)_{ki}}{\det(\Box + K_1^{-1} m^2)} (K_1^{-1})_{kj} \tag{39}$$

where $P^1_\mu \equiv C_\mu$, $P^2_\mu \equiv D_\mu$.

Thus the physical masses are given by the eigenvalues of the matrix $K_1^{-1} m^2$. The physical fields will be determined by

$$\begin{pmatrix} x_\mu \\ y_\mu \end{pmatrix}_{\text{physical}} = R^T \begin{pmatrix} C_\mu \\ D_\mu \end{pmatrix} \tag{40}$$

where R^T is an orthogonal matrix which elements are given by the eigenvalues of the matrix $K_1^{-1}m^2$. It yields,

$$x_\mu = \frac{1}{\sqrt{1+\zeta_1^2}} (C_\mu+\zeta_1 D_\mu) \tag{41}$$

$$x_\mu = \frac{1}{\sqrt{1+\rho_1^2}} (C_\mu+\rho_1 D_\mu) \tag{42}$$

$$y_\mu = \frac{1}{\sqrt{1+\zeta_2^2}} (C_\mu+\zeta_2 D_\mu) \tag{43}$$

$$y_\mu = \frac{1}{\sqrt{1+\rho_2^2}} (C_\mu+\rho_2 D_\mu) \tag{44}$$

where

$$\zeta_{\left(\frac{1}{2}\right)} = - \frac{1}{g_1 m_D^2} \delta_{(\pm)}$$

$$\rho_{\left(\frac{1}{2}\right)} = \frac{g_1 m_C^2}{\delta_{(\pm)}}$$

with

$$\delta_{(\pm)} = a_1 m_D^2 - d_1 m_C^2 (\pm) \Delta$$

$$\Delta^2 = (d_1 m_C^2 - a_1 m_D^2)^2 + g_1^2 m_C^2 m_D^2 \tag{45}$$

Observe that (41) and (42) solutions are not independent. Similarly (43) and (44). Therefore any choice in each set is a candidate for physical fields. For instance, the situation $x_\mu = C_\mu, y_\mu = D_\mu$ (or vice-versa) is obtained through $\delta_+ = 0$ (or $\delta_- = 0$). It is also possible with $g_1 = 0$.

S-matrix is unaffected by a momentum-independent reparametrization. This is the case in (40). Therefore, for simplicity, we prefer to use the C_μ and D_μ basis. Propagators will be obtained through $K_{\mu\nu}$'s inverse. Defining,

$$\begin{bmatrix} P_{\mu\nu}^{C\to C} & P_{\mu\nu}^{C\to D} \\ P_{\mu\nu}^{D\to C} & P_{\mu\nu}^{D\to D} \end{bmatrix} = \begin{bmatrix} Q & T \\ R & S \end{bmatrix}_{\mu\nu}^{-1} \tag{46}$$

Shows that in order to invert (46) it is necessary to have at least two block matrices invertible.

The general form of the propagators for two fields in the same group representation is

$$P_{\mu\nu} = \frac{1}{D}\,[a(k^2,m^2)\eta_{\mu\nu} + b(k^2,m^2)k_\mu k_\nu]$$

where
$$D = \alpha_1 k^4 + \alpha_2 k^2 + \alpha_3 \tag{47}$$

The constants α_1, α_2 and α_3 depend on the free parameters of the_ory. Writing the gauge invariant form for vector fields

$$P_{\mu\nu} = \sum_{n=1}^{N} P_n(k^2,m^2)\eta_{\mu\nu} - \sum_{n=1}^{N} \frac{k_\mu k_\nu}{m_N^2} P_n(k^2,m^2) \quad ; \; N = 1,2 \tag{48}$$

Consider now for the function $P_n(k^2)$ the pole approximation (or Born approximation)

$$P_n(k^2) = \frac{\lambda_n}{k^2-m_N^2} \tag{49}$$

where $\lambda_n = (k^2-m_N^2)P_N(k^2)\Big|_{k^2=m_N^2}$ are residues. Substituting (49) in (47) yields

$$a(k^2,m^2) = D \sum_{n=1}^{N} \frac{\lambda_n}{k^2-m_n^2} \qquad (50)$$

and

$$b(k^2,m^2) = -D \sum_{n=1}^{N} \frac{\lambda_n}{m_n^2(k^2-m_n^2)} \qquad (51)$$

(47), (50) and (51) define the poles. Unphysical poles can be avoided through a suitable choice of the free parameters of the theory. The present model with two fields is expected to have just two poles in the propagator. Poles in (51) are necessary to avoid "time compensated" ghosts. Multiplying it by k^2 and decomposing through the partial fractions we have

$$k^2\, b(k^2) = -D.\frac{a(0)}{\alpha_3} + a(k^2) \qquad (52)$$

where a(0) is $a(k^2,m^2)\big|_{k^2=0}$. (52) shows the restriction conditions for (51).

In order to effectively investigate the pole structure for the propagators we are going to choose in (46) the case where T and R are invertible. It gives

$$P^{CC}_{\mu\nu} = \frac{1}{D}\,[a_{cc}(k^2,m^2)\eta_{\mu\nu} + b_{cc}(k^2,m^2)k_\mu k_\nu] \qquad (53)$$

with

$$a_{cc} = d_1 k^2 + m_D^2 \qquad (54)$$

$$b_{cc} = \frac{B_1 k^6 + B_2 k^4 + B_3 k^2 + B_4}{B_5 k^6 + B_6 k^4 + B_7 k^2 + B_8}$$

$$B_i = B_i(g_\alpha, m_\alpha^2) \qquad (55)$$

The coefficients in the k^2 polynomials at (54) and (55) have their values depending on the theory free parameters. For instance,

$$B_1 = -\frac{1}{2} a_1 g_1 (e_1+f_1)[a_1+b_1+c_1+\frac{1}{4}\frac{(h_1+i_i)^2}{e_1+f_1}] - \frac{g_1^3}{8}(b_1+c_1) \quad (56)$$

From (53), (54) we have

$$\frac{a_1 k^2+m_D^2}{\alpha_1 k^4+\alpha_2 k^2+\alpha_3} = \frac{\lambda_1}{k^2-m_1^2} + \frac{\lambda_2}{k^2-m_2^2}$$

where

$$\lambda_1 = \frac{a_1 m_1^2+m_D^2}{m_1^2-m_2^2}$$

$$\lambda_2 = \frac{-a_1 m_2^2-m_D^2}{m_1^2-m_2^2} \quad (57)$$

and

$$m^2_{(\frac{1}{2})} = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2-4\alpha_1\alpha_3}}{2\alpha_1}$$

$$\alpha_1 = a_1 d_1 - \frac{g_1^2}{4}$$

$$\alpha_2 = a_1 m_D^2 - \frac{g_1}{2} m_C^2$$

$$\alpha_3 = m_D^2 m_C^2 \quad (58)$$

Poles characterizing physical particles will be determined through a choice in the theory free parameters. It means that we can avoid the cases with resonances ($m_N = \mathrm{Re}\, m_n + i\, \mathrm{Im} m_n$), tackyons ($m_N^2 = -(m_N^2)$) and ghosts ($m_N = -m_n$). Negative probability states (ghosts) can also be avoided by having equations (57) and (58) giving the same sign for the residues. The consistency condition (52) in (55) yields

$$B_1 = \frac{\alpha_1}{\alpha_3} m_D^2 B_6$$

$$B_2 = \frac{\alpha_1}{\alpha_3} m_D^2 B_7 - (a_1 - \frac{\alpha_2}{\alpha_3} m_D^2) B_6$$

$$B_3 = - (a_1 - \frac{\alpha_2}{\alpha_3} m_D^2) B_7 + \frac{\alpha_1}{\alpha_3} m_D^2 B_8$$

$$B_4 = - (a_1 - \frac{\alpha_2}{\alpha_3} m_D^2) B_8$$

$$B_5 = 0 \tag{59}$$

The others propagators are

$$P_{\mu\nu}^{D\to D} = \frac{1}{P} [a_{DD}(k^2,m^2)\eta_{\mu\nu} + b_{DD}(k^2,m^2)k_\mu k_\nu]$$

with $$a_{DD} = a_1 k^2 + m_C^2 \tag{60}$$

$$P_{\mu\nu}^{C\to D} = \frac{1}{P} [a_{CD}(k^2,m^2)\eta_{\mu\nu} + b_{CD}(k^2,m^2)k_\mu k_\nu] \ .$$

with $$a_{CD} = - \frac{g_1}{2} k^2 \tag{61}$$

$$P_{\mu\nu}^{D\to C} = P_{\mu\nu}^{C\to D} \tag{62}$$

Observe that (53), (60) and (61) have the same basic poles structure. The calculations agree with (39). Similarly to (55) poles from b_{DD} and b_{DC} can be worked out to avoid the ghost originated from covariant quantitation.

The propagators structure is unchanged for the case where $(Q,S)_{\mu\nu}$ or $(Q,T,R,S)_{\mu\nu}$ matrices are invertible. The condition for diagonalize (38) is that the symmetric matrices K_1 and K_2 commute. It yields,

$$(h_1+i_1)(a_1-d_1) = g_1(e_1+f_1-b_1-c_1) \tag{63}$$

A second possibility for the pole structure is a double pole,

$$P^{(ij)}_{\mu\nu} = \frac{\beta k^2-m^2}{(k^2-m^2)^2}\,[\eta_{\mu\nu}+b^{(ij)}(k^2,m^2)k_\mu k_\nu] \tag{64}$$

where β is a number depending on the free parameters of the theory. In the tree level the imaginary part corresponding to the amplitude in (64) is

$$\text{Im } T = \beta\,\pi\,\delta(k^2-m^2) - (1-\beta)\pi\, m^2\,\delta'(k^2-m^2) \tag{65}$$

One can demonstrate that the second term on the r.h.s. corresponds to a ghost. Thus in physical amplitudes the ghost must be decoupled (or not correspond to a real asymptotic state) if this theory is to be consistent.

Briefly we will illustrate the case with three fields in a same group. In this case the propagators have the following form

$$P_{\mu\nu} = \frac{1}{D_3}\,[a(k^2,m^2)\eta_{\mu\nu} + b(k^2,m^2)k_\mu k_\nu]$$

where
$$D_3 = \beta_1 k^6 + \beta_2 k^4 + \beta_3 k^2 + \beta_4 \tag{66}$$

The consistence condition is

$$k^2 b(k^2,m^2) = -D_3 \frac{a(0)}{\alpha_4} + a(k^2) \tag{67}$$

The basic pole structure is

$$\frac{a(k^2,m^2)}{D_3} = \sum_{i=1}^{3} \frac{\lambda_i}{k^2-m_1^2} \tag{68}$$

where $a(k^2,m^2) = \gamma_1 k^4 + \gamma_2 k^2 + \gamma_3$; β_i, γ_i are constants depending on theory free parameters. The residues expressions are

$$\lambda_i = \frac{\gamma_1 m_i^4 + \gamma_2 m_i^2 + \gamma_3}{\beta_i m_i^4 + \beta_2 m_i^2 + \beta_3} \tag{69}$$

We can always choose residues with equal sign and one λ_i's equal zero. The equation $D_3=0$ gives the masses for the vector particles.

4 NON-RELATIVISTIC POTENTIAL

The discovery of the J/ψ and ϒ particles has stimulated much interest in potential models within the framework of non-relativistic quantum mechanics. For simplicity this work intends to study the case just with gluons. The non-abelian nature of gauge theories predicts the existence of hadrons with no quark content ([6]). From the phenomenology of these hadrons one expects to have a striking proof for non-abelian theories. Here we investigate the possibility of extending the idea that a colour index in the gluon field operator implies the existence of bound states without quarks.

Introducing two fundamental vector fields in the same group yields another method to generate the gluonic matter. From (12) the self-coupling of the gauge bosons is obtained by terms like

$$\partial_\mu C_{\nu a}[D^\mu, C^\nu]^a \quad , \quad [C_\mu, D_\nu]_a\, [D^\mu, D^\nu]^a$$

$$\{C_\mu, D_\nu\}_a\, \{D^\mu, D^\nu\}^a \quad , \quad C_{\mu a}.C^{\nu a} \;\; D^{\mu b}.C_{\nu b} \qquad (70)$$

These couplings are expected to contribute to the occurrence of hadronic states built of two different massive gluons. Observe that experimentally it is expected that about fifty percent of the nucleonic momentum is carried by gluons. This fact can support the argument that the gluons are massive and appear in different families. There are other arguments in favour of a low-mass gluon (7).

Traditionally, physical insights have been found through perturbation theory. A common hypothesis is that the simplest view should emerge from a linear potential (8). As a strong guide for such potential would be a tree level propagator yielding a linearly rising potential. After that one can get a justification for considering perturbativelly the self coupling (70) as a source to build up gluonic. The non-abelian character is expected to reproduce asymptotic freedom.

In the static charge approximation the fourier transform of the single poles in (57) yields the following Yukawa potential,

$$V(r) = \frac{G}{r}\,(e^{-m_1 r} + e^{-m_2 r}) \qquad (71)$$

where G is a constant. The coulomb potential appears as a particular case when m_1 and m_2 are simultaneously zero. Then the following relation must be satisfied

$$\frac{m_C^2}{m_D^2} = -\frac{a_1}{d_1} \tag{72}$$

(71) can be interpreted as having at the position of a singularity a particle, say a quark, acting as the source of the gauge boson fields. It produces a potential with finite range e,

$$e \simeq 2.10^{-11} \left[\frac{1}{m_1^2} + \frac{1}{m_2^2}\right]^{1/2} \tag{73}$$

where the distance is measured in fermion and the mass in GeV.

For the double pole case, the potential is a Bessel function of the third kind, $K_{-\frac{1}{2}}(mx)$ (9),

$$V(r) = G\sqrt{2}\ \pi^2 \frac{e^{-mr}}{m} \tag{74}$$

Observe that in the limit when $mr \to 0$ the potential is a constant. It tells that at small distances an "asymptotic freedom" property similar to that are obtained from renormalization group can be reproduced in a non-relativistic limit.

In order to make some simply application consider the Iota particle. Neglecting the bound state energy the so-called as effective mass of this particle is

$$m_1 + m_2 = \left(\frac{a_1 m_D^2 - \frac{g_1}{2}\ m_C^2}{a_1 d_1 - \frac{g_1^2}{4}}\right)^{1/2} \sim 1.4\ \text{GeV} \tag{75}$$

Note that in this model it will depend on the free parameters of theory and on the mass parameters m_C^2 and m_D^2.

Nevertheless, neither (71) nor (74) yield the desired linear potential. Therefore in order to build confined bound states for glueballs it is necessary to understand the consequences of the non-abelian properties in (12). However the intention on this work is not to study the Dyson-Schwinger equations properties. The motivation here is to explore straightforward arguments for confinement. Although it can contain colourful ghosts/tackyons. For instance, in order to get a linear potential from (71) and (74) it is necessary to expand them in series. Considering that in a hadron the concept of large distance is relative we can create a logic on it to in order to justify the serie expansion. Another method is to calculate the total energy to separate two gluons including the self energy. It is given by [8]

$$E(\vec{r}) = \frac{1}{(2\pi)^3} \int d\vec{k}\,(1-e^{i\vec{k}\cdot\vec{r}})\,P_{00}(k^2) \qquad (76)$$

Propagators including colourful ghosts and tackyons can also be derived from (47). For instance,

$$P_{00} \sim \frac{1}{(k^2-m_1^2)(k^2-m_2^2)} \qquad (77)$$

$$P_{00} \sim \frac{1}{k^2}\left[\frac{1}{(k^2-i\mu^2)} + \frac{1}{(k^2+i\mu^2)}\right] \qquad (78)$$

(77) and (78) in (76) respectively yields

$$E(r) \sim \frac{1}{m_1+m_2} + \frac{1}{r}\left[e^{-m_1 r} - e^{-m_2 r}\right]\frac{1}{m_1^2-m_2^2} \qquad (79)$$

$$E(r) \sim \frac{1}{r\mu^2} [1-(1+r\mu)e^{-\mu r}] \qquad (80)$$

A large amount of energy must be supplied to pull the involved particles to a certain radius of separation since small distances are considered for (79) and (80). Observe that the concept of a relative distance in a hadron must be imposed again. It must be interpreted at the level of suggestions. After that, for (79) the energy to pull them apart becomes constant. For (80), beyond $\frac{1}{\mu}$ the energy of separation decreases. The linear behaviour can be achieved by introducing three vector fields in the same group. Then, a triple pole appears that corresponds to a static potential with the form re^{-mr}. However it contains undesirable ghosts. Although it is a task for a future work, in this paper we have not studied how to control the presence of ghosts. Moreover the confined aspect leaves space for more than one speculation.

Colourful matter is being a block-box for experimental physics. Therefore the present status is to construct a mechanism to justify the confinement of colour. Actually it is characterized by three aspects. There is no particle state with colour quantum number [10], the dynamics of quarks inside the hadrons is non-relativistic [11], and third, there is experimental evidence for heuristic potentials involving linearly rising potentials [12]. Take for example the P-wave charmonium states of the quark-antiquark interaction. These three facts are the sources for speculative mechanisms. Our point of view is that unphysical particles should be allowed to participate in one of such mechanisms since the potential that they intermediate does not have asymptotic limit. They would not be detected but would influence the boundary limits for the colourful matter.

Concluding this section, we would note the relevance of the free parameters of the theory. Depending on their values the propagator in (47) can generate Coulomb, Yukawa and exponential potentials. It shows that a symmetry does not determine the potential structure univocally.

5 CONCLUSION

Quarks can be detected indirectly through jets. The evidence for the existence of gluons is slim compared with that for quarks. However the gluonic force is the responsible for the binding of quarks into colour singlet. Therefore one way of looking for gluons is through the study of the forces between quarks. Under this point of view the presence of a linear potential should be the guide about the gluon structure. The discussion in section 4, shows that the introduction of a second kind of gluon belonging to the same octet improves, relatively to QCD, the potential shape. For the time being several glueball candidates have already being reported (6), but the situation concerning their nature seems to be controversial and confused (13). An effective mass is included. Therefore we would not be moving so far by postulating the presence of massive gluons. Building blocks made by gluons with mass have as consequence the inclusion of candidates for gg bound states with quantum numbers J=1. These states are not allowed when two identical massless bosons are assumed. Nevertheless to have these phenomenological arguments, a model must first to carry substance for interpreting confinement.

Confinement is a subtle subject. It is a new and unexpected matter behaviour. Therefore we are just studying the scenario (s) for confinement. As far as we know there is no basic principle underlying it. The basic question is if coloured objects can exist or not as free particles. Right now the only principle underlying colour physics is $SU(3)_C$ symmetry. Therefore we have to try a variety of heuristic notions as bags, strings, constituent gluons, solitions, etc. Thus using these trial and error prescriptions we would like to look at confinement with two different approaches. They are:

i) Strong force method. There is experimental evidence to believe that colour forces between coloured objects decrease at small distance but increases at large distances. A way to describe this situation would be to associate with the force a non-heuristic quantum mechanical potential originated from a gauge-invariant Lagrangian.

ii) Algebraic approach - The non-commutative relations for the non-abelian charges are a basic problem for colour measurement [14].

In this work we have attempted to investigate the possibility the confining potentials exist because of two families of vector fields in a single gauge group. Thus we would like to present some considerations regarding the relationship between rising potentials and actual confinement of particles of a theory.

If in a given Lagrangian model one obtains that the fourier transform of some propagator gives a rising potential, the first conclusion one might draw is that the particles associated to the fields which interact by interchanging that propagator would never appear as asymptotic free states of the theory. This would be a first motivation, or hint, for justifying the confinement of particles of a theory. However this is not a suficient argument. If one is able to project out of the $\mathcal{H}$-space of states of the theory a subspace of physical states which in the asymptotic limit ($t \to \pm\infty$) appear as free states, then the fact that the potential is growing with the particle separation is of no relevance. Incidentally this is what happens in Q.E.D. whenever one works in the Landau gauge. There, the presence of a term like $\frac{1}{k^4}$ in the photon propagator leads to a linearly rising interparticle potential, though the electron and the positron are not confined. Indeed, one can project out of the $\mathcal{H}$-space electron and positron states which have a free asymptotic limit (up to infrared problems that however one perfectly knows how to deal with). The growing character of the potential is due to the $\frac{1}{k^4}$ term. It corresponds to a ghost which completely decouples from the theory. It means that the very asymptotic states can be well-defined.

In our case, the situation is not as simple as in Q.E.D., as we have a non-Abelian gauge theory and moreover two families of intermediate vector bosons. Therefore, the growing behaviour of our potential may have more severe consequences than the QED case. Though we may really have confinement (that is, one cannot single out colored asymptotic states), one has less control of the unitarity of the theory. If we manage to show that we cannot really define particle states which are free in some asymptotic region (confined states) and that the S-matrix is unitary in the physical subspace of the full $\mathcal{H}$-space, then we can say that we really have a confining theory.

If this is actually the case, the colourful unphysical particles though of not directly observable would lead to remarkable physical consequences. The confining character of the interparticle potential would be attributed to their presence. In some sense, this turns similar to what happens in the case spontaneously breaking of a continuous local symmetry. There the Goldstone particles do not appear in the physical spectrum but are the responsible for the presence of massive vector bosons.

This work should be understood as one of the stages for building up an alternative model in gauge theories. The introduction of more than one potential under the same group requires anothers step to be controlled. They are under study. Globally speaking, a present achievement it that from a same group we can get different physical situations. Depending on the free parameters of theory physical and unphysical particles be played. Similarly different kinds of potentials as the Coulomb, Yukawa and exponential can appear. We could compare this situation with QED. There the electric and magnetic fields are parts of the same tensor $F_{\mu\nu}$.

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