ISSN 0029 - 3865

CBPF-NF-068/85

SPIN-TWO EQUATIONS

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Abstract

Transformations that maintain invariant the spin-two equation presented by Galles are considered.

Key-words: Wave equations; Invariances.

In the proof of invariance of (1) under (3) the presence of the last term of (1) is essential, so that the equations considered by Drew-Greenberg[2] and Barut-Xu[3] are not invariant under (3). Some possible useful expressions of ϕ are $h_\rho^{\ \rho}$ and $\overline{h}_\rho^{\ \rho}$, where $\overline{h}_{\mu\nu}$ is a tensor different from $h_{\mu\nu}$.

Another kind of solution of (1) to be considered is $f_{\mu\nu} = \partial_\mu \partial_\nu \phi$ where, again, ϕ is an arbitrary scalar. As this function $f_{\mu\nu}$ is not, in general, a solution of (2), it is necessary to impose on ϕ the following condition

(4) $$\Box \phi = 0$$

Therefore, the transformation to be considered is

(5) $$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \partial_\nu \phi \quad ,$$

with ϕ obeying (4). In this case (1) and (2) are invariant under (5).

In the same way it can be shown that $x_\mu x_\nu$ is a solution of (1) and (2) so that the corresponding transformation is $h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + x_\mu x_\nu$.

Finally, the Pauli-Fierz transformation[4] is expressed as

(6) $$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\nu f_\mu + \partial_\mu f_\nu - 2g_{\mu\nu}\partial_\sigma f^\sigma \quad .$$

In the theory formulated in ref. (4) f_μ were considered to be arbitrary functions of x^μ. Presently it is assumed that f_μ must satisfy convenient equations that will be indicated. It can be shown that (1) is invariant under (6) if the following relation holds

(7) $$\partial_\mu F_\nu + \partial_\nu F_\mu = 0 \quad ,$$

where

(8) $$F_\mu = \Box f_\mu - \partial_\mu \partial_\sigma f^\sigma \quad .$$

As a consequence of (7) and (8) it can be seen that F_μ satisfies the following relations

(9) $$\Box F_\mu = 0 \quad , \quad \partial_\mu F^\mu = 0 \quad .$$

The knowledge of F_μ, however, is insuficient to determine f_μ for the relation (2) must be invariant under (6). In view of this restriction one must have

(10) $$\Box \Box \partial_\sigma f^\sigma = 0 \quad .$$

Instead of looking for the general solutions of the equations (7) and (10), a more simple solution, namely, the one that uses functions f_μ that verify

(11) $$\Box f_\mu = 0 \ ,$$

and

(12) $$\partial_\mu f^\mu = 0 \quad ,$$

will be considered. These functions f_μ satisfy (7) and (10) so that if they are inserted into (6) they give origin to a transformation that maintains (1) and (2) invariant. In view of the equation (12), of course, there is a simplification and (6) is reduced to

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\nu f_\mu + \partial_\mu f_\nu \quad .$$

References

1. C.D. Galles: Lettere Nuovo Cimento 42, 382 (1985).

2. M.S. Drew and J.D. Greenberg: Nuovo Cimento A,60,41(1980).

3. A.O. Barut and B.N. Xu: Phys. Lett. B,102,37(1981).

4. W. Pauli and M. Fierz: Proc. Roy.Soc. (London) A,173,211(1939).