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A SCHEME TO UNIFY GRAVITY AND ELECTRO-WEAK INTERACTIONS

by

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ABSTRACT

The theory of electro-weak interactions and Jordan's description of Einstein's Theory of gravity are unified in a unique scheme. A short-range counterpart of gravity, envolving massive tensor bosons of masses 365 GeV and 520,3 GeV, is obtained within the new scheme.

Key-words: Gravity-electro-weak; Unified theory.

I INTRODUCTION

The strong equivalence principle demands that gravitation couples to all matter in the Universe in the same way. In Einstein's theory of gravitation this is achieved by requiring that the source of gravity be just the energy-momentum tensor (of all the matter in the Universe)(*):

$$(1) \qquad G_\mu{}^\nu = R_\mu{}^\nu - \frac{1}{2} R\, \delta_\mu{}^\nu = -\kappa\, T_\mu{}^\nu$$

As is well-known equation (1) can be derived from a variational principle

$$(2) \qquad \delta \int \sqrt{-g}\, L\, d_4x = 0$$

where the total Lagrangian splits into a sum

$$(3) \qquad L = \frac{1}{\kappa} L_E + L_M$$

$L_E \equiv R$ and L_M is the matter Lagrangian from which $T_{\mu\nu}$ is obtained by variation

$$\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta g_{\mu\nu}} = T^{\mu\nu}$$

The specific splitting of the Lagrangian of the form (3) is called minimal coupling and it guarantees the strong equivalence principle to be obeyed. The Ricci scalar R plays the role of the free field part.

The key objects in Einstein's theory are all of geo-

(*) We use convention in which the metric has signature (+---).

metrical significance: $g_{\mu\nu}$ are the metric coefficients in the 4-dimensional space-times, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$; $\Gamma^\mu_{\alpha\beta}$ are the connection coefficients and $R_{\alpha\beta\mu\nu}$ are the components of the Riemann curvature tensor.

In not so close analogy with other forces, the $g_{\mu\nu}$ are the potentials; $\Gamma^\mu_{\alpha\beta}$ are the forces and $R_{\alpha\beta\mu\nu}$ are the tidal forces. The requirement that the Lagrangian be invariant under general coordinate transformation leads to the Bianchi identities

(4) $$G^{\mu\nu}{}_{;\nu} = 0$$

and to the conservation of energy and momentum in the form of an equation of motion for the bulk matter

(5) $$T^{\mu\nu}{}_{;\nu} = 0 \quad .$$

A series of beautiful experiments with ever increasing accuracy has shown Einstein's theory to be the only viable gravitational theory (disregarding such esotheric theories where torsion is introduced microscopically, for instance) and there is therefore not the slightest need, for the time being, for a modification of Einstein's theory. So leave Einstein's theory intact.

On the other hand the success of the so called gauge theories has led many workers to try to incorporate gravity into a unified scheme. As a matter of fact we know that Einstein himself made every attempt to unify gravity with electromagnetic forces. As is well know he and all others have not succeeded in such a program. Parallel to this evolution and independent of it, weak and electromagnetic forces have been shown to admit a sort of unification by means of an appeal to a gauge structure. One is therefore tempted in order to unify all these forces, to try to turn Einstein's theory into a gauge theory. Unfortunately until nowadays such enterprise has been unsuccessful. The main reason for this rests on the fact that the choice of primary objects in electro-weak and gravity theory is profoundly different, due to the special role of gravity in the description of the structure of metric properties of space-time.

In a recent paper(1) it has been suggested that the use of Jordan's formulation of gravity theory could be of great help in providing an alternative description of Einstein's theory which could well be adapted to a scheme to accomodate gravity and electro-weak forces. In the present paper we describe the model in more detail.

II JORDAN'S FORMULATION OF EINSTEIN'S THEORY OF GRAVITY

The Weyl conformal tensor $W_{\alpha\beta\mu\nu}$ can be written as

$$(6)\qquad W_{\alpha\beta\mu\nu} = R_{\alpha\beta\mu\nu} - \frac{1}{2}\,[g_{\alpha\mu}R_{\beta\nu} + g_{\beta\nu}R_{\alpha\mu} - g_{\alpha\nu}R_{\beta\mu} - g_{\beta\mu}R_{\alpha\nu}] + \frac{1}{6}\,R\,g_{\alpha\beta\mu\nu}$$

in which

$$g_{\alpha\beta\mu\nu} \equiv g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\ ,$$

and possess a number of algebraic and physical properties(2) which led Jordan and co-workers[3] to the idea to rewrite Einstein's equations by means of the Weyl tensor.

Using Bianchi identities in the form

$$W^{\alpha\beta\mu\nu}{}_{;\nu} = \frac{1}{2} R^{\mu[\alpha;\beta]} - \frac{1}{12} g^{\mu[\alpha}R^{,\beta]} \tag{7}$$

where [ ] i.e. antisymmetrization means $A_{[ik]} = (A_{ik}-A_{ki})$ and differentiating Einstein's equation (1) in the form

$$R_{\mu\nu} = \kappa\,(-T_{\mu\nu} + \frac{1}{2}\,T\,g_{\mu\nu})$$

one arrives at the equation

$$W^{\alpha\beta\mu\nu}{}_{;\nu} = \kappa\left(\frac{1}{2}\,T^{\mu[\alpha;\beta]} - \frac{1}{12}\,g^{\mu[\alpha}T^{,\beta]}\right) = J^{\alpha\beta\mu} \tag{8}$$

At the price of one more derivative we have thereby arrived at an equation which contains Weyl's conformal tensor and the energy momentum distribution. Remark that in general the set (7,8) of equations is not equivalent to Einstein's theory, since a third order differential equation which is derived from a second order differential equation has of course in general more solutions than the second order equation.

At this point one should then ask the following question: under what conditions will equations (7-8) be equivalent to Einstein's theory ? The answer to this question was given by Lichnerowicz[4] in the early sixties. After the examination of the Cauchy problem he showed that, if we admit Einstein's equations to be valid on an space-like hypersurface Σ, that is, $(R_{\mu\nu} - \frac{1}{2}\,Rg_{\mu\nu} + \kappa T_{\mu\nu})_{\Sigma} = 0$ as initial conditions for Jordan's equations, then this system (7-8) becomes completely equivalent to General Relativity. Thus using Einstein's equation in its standard form (eq. (1)) or in Jordan's formulation (eq. (7-8)) is just a matter of taste and/or simplicity, which should be dictated by the examination of the problem under investigation.

III THE LANCZOS FIELD $H_{\mu\nu\lambda}$

The difficulty in finding a true tensor as a potential of the Weyl object $W_{\alpha\beta\mu\nu}$ has been one of the main obstacles of Jordan's formulation: Does there exist a third order tensor from which, by first order derivatives, the Weyl conformal tensor can be constructed ?

Although this problem had been solved in principle some 20 years ago by Cornelius Lanczos[5] his result found little practical application by the scientific community. The mathematical situation was recently remedied by the work of Bampi and Caviglia[6], who proved a theorem which guarantees the existence of this potential $H_{\mu\nu\lambda}$ in any manifold endowed with a Riemannian structure.

Following Lanczos we set

$$(9) \qquad W_{\alpha\beta\mu\nu} = H_{\alpha\beta\mu;\nu} - H_{\alpha\beta\nu;\mu} + H_{\mu\nu\alpha;\beta} - H_{\mu\nu\beta;\alpha} +$$

$$+ \frac{1}{2}\Big[(H_{\nu\alpha} + H_{\alpha\nu})g_{\beta\mu} + (H_{\beta\mu} + H_{\mu\beta})g_{\alpha\nu} -$$

$$- (H_{\alpha\mu} + H_{\mu\alpha})g_{\beta\nu} - (H_{\beta\nu} + H_{\nu\beta})g_{\alpha\mu} \Big] +$$

$$+ \frac{2}{3} H^{\sigma\lambda}{}_{\sigma;\lambda}\, g_{\alpha\beta\mu\nu}$$

in which

(10) $$H_{\alpha\mu} \equiv H_{\alpha}{}^{\sigma}{}_{\mu;\sigma} - H_{\alpha}{}^{\lambda}{}_{\lambda;\mu}$$

This tensor $H_{\alpha\beta\mu}$ has the properties

(11a) $$H_{\alpha\beta\mu} = -H_{\beta\alpha\mu}$$

(11b) $$H^{*}_{\alpha\beta\mu} g^{\alpha\mu} = 0$$

Such tensor has then only 20 independent components. This implies that expression (9) admits a gauge symmetry. Indeed, (9) does not change under the map

(12) $$H_{\mu\nu\lambda} \to H'_{\mu\nu\lambda} = H_{\mu\nu\lambda} + M_{\mu} g_{\nu\lambda} - M_{\nu} g_{\mu\lambda}$$

for arbitrary vector M_{μ}. This means that the trace $H_{\mu\lambda\nu} g^{\nu\lambda} \equiv H_{\mu}$ is completely free for given $W_{\alpha\beta\mu\nu}$. Furthermore, (9) is invariant under the map

(13) $$H_{\mu\nu\lambda} \to \tilde{H}_{\mu\nu\lambda} = H_{\mu\nu\lambda} + S_{\mu\nu\lambda} \quad , \text{ for } S_{\mu\nu\lambda} \equiv Q_{\mu\lambda;\nu} - Q_{\nu\lambda;\mu}$$

in which $S_{\mu\nu\lambda}$ has the same symmetries as $H_{\mu\nu\lambda}$, is traceless and is a ghost Weyl field — that is, the Weyl tensor constructed with such $S_{\mu\nu\lambda}$ vanishes identically: it has only six degrees of freedom.

Thus the specification of the four M_μ and the six independent components of $S_{\mu\nu\lambda}$ yields a definite characterization of the additional 10 degrees of freedom of $H_{\mu\nu\lambda}$. There remain thus 10 degrees of freedom of our disposal not specified by the Weyl tensor.

It should be stressed (as was already pointed out by Lanczos) that although $H_{\alpha\beta\mu}$ and the metric tensor $g_{\mu\nu}$ are related, a functional dependence of $H_{\alpha\beta\mu}$ in terms of $g_{\mu\nu}$ is not defined in a local basis. This as will also become clear below somehow hampers the use of the Lanczos field.

However, there is a special case of interest in which the explicit dependence of $H_{\alpha\beta\mu}$ on $g_{\mu\nu}$ can be achieved locally: the case of a weak gravitational field. Indeed, as shown by Lanczos[5], if $g_{\mu\nu} \cong \eta_{\mu\nu} + \varepsilon\psi_{\mu\nu}$ (for $\varepsilon^2 << \varepsilon$) then in first order approximation we can write

$$H_{\mu\nu\lambda} \cong \frac{\varepsilon}{4} \{\psi_{\mu\lambda,\nu} - \psi_{\nu\lambda,\mu} + \frac{1}{6} \psi|_\mu \eta_{\nu\lambda} - \frac{1}{6} \psi|_\nu \eta_{\mu\lambda}\} \tag{14}$$

in which $\psi \equiv \psi_{\mu\nu}\eta^{\mu\nu}$.

In this case it is easy to show that Jordan's equation reduces to the Pauli-Fierz equation for a spin-2 particle represented by $\psi_{\mu\nu}$.

IV THE LAGRANGIAN

By now we have arrived at the following: we have derived field equations which are equivalent to the original Einstein's equations, provided the initial geometry satisfies

Einstein's equation on a Cauchy surface. The field equations can be formulated by means of a three index Lanczos field and this field contains enough freedom such that a vector and an anti-symmetric second order tensor can be incorporated at will. Can equation (8) be derived from a Lagrangian ? The answers is yes. We treat the free field first.

Consider

$$(15) \qquad L = - \frac{\ell_0^2}{8k} \sqrt{-g} \quad W^{\alpha\beta\mu\nu} W_{\alpha\beta\mu\nu}$$

where $W_{\alpha\beta\mu\nu}$ is given by (9) and ℓ_0^2 is a constant with dimension length. Three different kinds of terms appear in the Lagrangian. The Lanczos field itself $H_{\alpha\beta\mu}$, the metric $g_{\mu\nu}$ and $\Gamma^{\mu}_{\alpha\beta}$ in the covariant derivative of the Lanczos field. Note that the primary variables are $H_{\alpha\beta\gamma}$ and $H_{\alpha\beta\gamma|\delta}$.

We can show the algebraic variation

$$(16) \qquad \frac{\delta}{\delta g_{\mu\nu}} (\sqrt{-g} \quad W^{\alpha\beta\lambda\sigma} W_{\alpha\beta\lambda\sigma}) = 0 \qquad ,$$

since this is a direct consequence of the identity[7]

$$(17) \qquad W_{\alpha}{}^{\mu\nu\lambda} W_{\beta\mu\nu\lambda} = \frac{1}{4} W_{\rho\sigma\mu\nu} W^{\rho\sigma\mu\nu} g_{\alpha\beta}$$

Variation with respect to $H_{\mu\nu\lambda}$ gives the left hand side of equation (8) $\delta \int \sqrt{-g}\ L d^4x = \int \sqrt{-g} \cdot W^{\alpha\beta\mu\nu}{}_{;\nu}\ \delta H_{\alpha\beta\mu}$ and

and $\delta L // \delta\Gamma^{\alpha}_{\mu\nu}$ vanishes identically due to the gauge symmetry (13)[(*)].

V THE MATTER LAGRANGIAN AND A CURIOUS SCALLING LENGTH

All we know about the matter Lagrangean is that

$$\frac{\delta L_m}{\delta g_{\mu\nu}} = T_{\mu\nu}$$

But we do not know how L_M depends on $H_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda,\rho}$. Nor do we know how variations of the metric tensor $\delta g_{\mu\nu}$ are related to variations $\delta H_{\alpha\beta\mu}$. So let us see what is needed to obtain equation (8) by varying $H_{\mu\nu\lambda}$. Starting from

$$\delta L_M = \ell_0^2 J^{\alpha\beta\mu}\delta A_{\alpha\beta\mu} + \mathrm{div} = T^{\mu\nu}\delta g_{\mu\nu}$$

and integrating by parts it is easy to show that

$$\delta g_{\mu\nu} = \frac{\ell_0^2}{2}\,[\delta h_{(\mu}{}^{\alpha}{}_{\nu);\alpha} - \frac{2}{3}\, g_{\mu\nu}\;\delta[h_{\alpha\beta\mu}]_{;\sigma}\; g^{\beta\mu} g^{\alpha\sigma}\;] \tag{18}$$

is a sufficient condition and ℓ_0 is introduced by dimensional arguments. However $\delta g_{\mu\nu}$ could in general be different from the right-hand side of equation (18).

(*) Indeed let $A^{\mu\nu}_{\alpha} := \frac{\partial L}{\partial\Gamma^{\alpha}_{\mu\nu}}$

Partial integration leads then from $\delta S_\Gamma = \int A^{\mu\nu}_{\alpha}\,\delta\Gamma^{\alpha}_{\mu\nu}$ to $\delta S_\Gamma = \int M^{\mu\nu}\delta g_{\mu\nu}$ where $M^{\mu\nu}$ contains products of $W_{\alpha\beta\gamma\delta}$ and $H_{\alpha\beta\gamma}$. By means of Noether's theorem this term reduces to a surface integral which can then be neglected.

Taking into account the freedom of four coordinate conditions it is clear that $\delta g_{\mu\nu}$ contains six arbitrary functions and we have as a matter of fact exactly six arbitrary functions at hand with our gauge (13). Therefore, equation (18) fixes this gauge.

The choice of constant ℓ_0 in expression (18) is obviously related to the fact that although it must be present in Lagrangean (15), for dimensional reasons, this constant must dissappear in the final Jordan's equation (8). We use natural units $\hbar = c = 1$ and choose (arbitrarily) $\ell_0 = \ell_{Planck}$ - once this is the unique scale which can be naturally constructed with Newton's constant.

VI MAXWELL LAGRANGIAN

We have arrived thus at our first important result: by means of the Lanczos field $H_{\mu\nu\lambda}$ a Lagrangian can be constructed wich gives rise to the Jordan form of Einstein's equations. The only freedom left in the Lanczos field will be associated to the electromagnetic potential $A_\mu \equiv eH_\mu{}^\lambda{}_\lambda$. That is

(19a) $$A_\mu \to A_\mu + \phi_{,\mu}$$

(19b) $$H_{\mu\nu\lambda} \to H_{\mu\nu\lambda} + \phi_{,[\mu}g_{\nu]\lambda}$$ [ ]: antisymmetrization

all other gauges are fixed. We have introduced the electron charge e for dimensional reasons.

At this point it should be mentioned remarkable property of formula (9). Instead of dealing with $H_{\mu\nu\lambda}$, one can use $h_{\mu\nu\lambda}$, the traceless part of $H_{\mu\nu\lambda}$ and one obtains the same expression (9).

That is, defining the irreducible decomposition

(20) $$H_{\mu\nu\lambda} = h_{\mu\nu\lambda} + \frac{1}{3} H^{\sigma}_{[\mu\sigma} g_{\nu]\lambda}$$

it is a trivial exercise to show that expression (9) can be equivalently written with $h_{\mu\nu\lambda}$ instead of $H_{\mu\nu\lambda}$. That is

$$W_{\alpha\beta\mu\nu}[H] = W_{\alpha\beta\mu\nu}[h] \quad .$$

This means that Lagrangian (15) gives no information for the dynamics of the trace part of Lanczos field. We then add Maxwell's Lagrangian L_M to (15)

(21) $$L_M = - \frac{1}{4} f^{\mu\nu} f_{\mu\nu} \quad ,$$

with $f_{\mu\nu} \equiv A_{\mu,\nu} - A_{\nu,\mu}$.
We have thereby united gravity with electromagnetism and we note that exactly these two fields could be united within the Lanczos potential.

VII SALAM-WEINBERG GENERALIZED

We have pointed out in the preceeding section that Lanczos field has sufficiently degrees of freedom to contain both Jordan's potential ($h_{\mu\nu\lambda}$) and Maxwell's potential ($A_\mu \equiv eH_\mu{}^\lambda{}_\lambda$).

However, such (classical) result should not be restricted to those infinite range interactions. We know today that for instance, in general electromagnetic forces do not appear in Nature independently: weak forces co-participate in their processes – and thus the traditional will of unifying gravity and electromagnetism cannot be really accomplished if weak forces are not taken into account(8,9). So, we must now show that indeed it is possible to enlarge our scheme in this direction.

Guided by the success of SU(2)×U(1) Salam-Weinberg(10) unification, we start by considering a set of SU(2)×U(1) Lanczos tensors $A^{(i)}_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ in which (i) is an SU(2) index. Following decomposition (20) we set

$$A^{(i)}_{\mu\nu\lambda} = a^{(i)}_{\mu\nu\lambda} + \frac{g}{3} A^{(i)}_{[\mu} g_{\nu]\lambda} \tag{22}$$

in which $gA^{(i)}_{\mu} \equiv A^{(i)}_{\mu\nu\lambda} g^{\nu\lambda}$ and

$$B_{\mu\nu\lambda} = b_{\mu\nu\lambda} + \frac{g'}{3} B_{[\mu}g_{\nu]\lambda} \tag{23}$$

in which [dim g] = [dim g'] = [dim e].

Under a SU(2) transformation $a^{(i)}_{\mu\nu\lambda}$ transforms as a vector and $A^{(i)}_{\mu\nu\lambda}$ as a connection, that is

$$a^{(i)}_{\mu\nu\lambda} \rightarrow \tilde{a}^{(i)}_{\mu\nu\lambda} = S\ a_{\mu\nu\lambda}S^{-1} \tag{24a}$$

$$A_{\mu\nu\lambda} \rightarrow \tilde{A}_{\mu\nu\lambda} = S\ A_{\mu\nu\lambda}\ S^{-1} - \frac{1}{3}\ \partial_{[\mu}S\ g_{\nu]\lambda}\ S^{-1} \tag{24b}$$

$B_{\mu\nu\lambda}$ has a U(1) freedom:

$$B_{\mu\nu\lambda} \rightarrow \tilde{B}_{\mu\nu\lambda} = B_{\mu\nu\lambda} - \frac{g'}{3}\ \partial_{[\mu}\Lambda g_{\nu]\lambda} \quad . \tag{25}$$

We define then the corresponding generalized Weyl tensor

$$(26) \qquad W^{(i)}_{\alpha\beta\mu\nu} = a^{(i)}_{\alpha\beta[\mu\|\nu]} + a^{(i)}_{\mu\nu[\alpha\|\beta]} + \frac{1}{2}\Big[a^{(i)}_{(\mu\beta)}\, g_{\nu\alpha} + $$

$$+ a^{(i)}_{(\nu\alpha)} g - a^{(i)}_{(\mu\alpha)} g_{\nu\beta} - a^{(i)}_{(\nu\beta)} g_{\mu\alpha}\Big]$$

in which

$$a^{(i)}_{\mu\nu\lambda\|\sigma} = a^{(i)}_{\mu\nu\lambda;\sigma} + g\varepsilon^{ijk} A^j_\sigma\, a^k_{\mu\nu\lambda}$$

$$a^{(i)}_{\alpha\beta} = a^{(i)\sigma}{}_{\alpha\ \beta\|\sigma} \qquad ,$$

and also

$$(27) \qquad C_{\alpha\beta\mu\nu} = b_{\alpha\beta[\mu;\nu]} + b_{\mu\nu[\alpha;\beta]} + \frac{1}{2}\Big[b_{(\mu\beta)} g_{\nu\alpha} + $$

$$+ b_{(\nu\alpha)} g_{\mu\beta} - b_{(\mu\alpha)} g_{\nu\beta} - b_{(\nu\beta)} g_{\mu\alpha}\Big]$$

in which

$$b_{\mu\alpha} = b_\mu{}^\sigma{}_{\alpha;\sigma} \qquad .$$

We remark that in a complete analogy with the U(1) case we have also:

$$F^{(i)}_{\mu\nu} = A^{(i)}_{\mu,\nu} - A^{(i)}_{\nu,\mu} + g\varepsilon^{ijk} A^j_\mu A^k_\nu$$

$$B_{\mu\nu} = B_{\mu,\nu} - B_{\nu,\mu} \qquad .$$

The generalized Lagrangian which should include the vector fields of the Salam-Weinberg theory, the gravitational field, the interaction with matter and the coupling with a Higgs bosonic field to become responsible for providing conveniently masses for the fields is given by the sum of three terms:

$$\text{(28a)} \qquad L_1 = \sqrt{-\det g_{\mu\nu}} \left[-\frac{1}{8} \left(\vec{W}^{\alpha\beta\mu\nu} . \vec{W}_{\alpha\beta\mu\nu} + C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu} \right) + - \frac{1}{4} \vec{F}_{\mu\nu} \vec{F}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \right]$$

$$\text{(28b)} \qquad L_2 = \sqrt{-\det g_{\mu\nu}} \left[\frac{i}{6} \bar{R}\, \Sigma^{\mu\nu} \gamma^\lambda \left(g_{\nu\lambda} \nabla_\mu - g_{\mu\lambda} \nabla_\nu + 3B_{\mu\nu\lambda} \right) R + \frac{i}{6} \bar{L}\, \Sigma^{\mu\nu} \gamma^\lambda \left(g_{\nu\lambda} \nabla_\mu - g_{\mu\lambda} \nabla_\nu + 3\vec{\tau}.\vec{A}_{\mu\nu\lambda} + \frac{3}{2} B_{\mu\nu\lambda} \right) L \right]$$

$$\text{(28c)} \qquad L_3 = \sqrt{-\det g_{\mu\nu}} \left[\frac{1}{6} \left(\nabla_\mu \phi g_{\nu\lambda} - \nabla_\nu \phi g_{\mu\lambda} - 3i\vec{\tau}.\vec{A}_{\mu\nu\lambda} \phi + + 3iB_{\mu\nu\lambda} \phi \right)^2 + \sigma (\phi^+ \phi)^2 - m^2 \phi^+ \phi - g_\ell (\bar{L}\phi R + \bar{R}\phi^+ L) \right]$$

in which we are using the standard notation as in SU(2)×U(1), that is

$$L = \frac{1+\gamma_5}{2} \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$R = \frac{1-\gamma_5}{2}\, e$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} .$$

A simple inspection of these Lagrangians shows that in the vector part (A^i_μ, B_μ) we obtain the standard theory of electroweak unification.

Let us then turn our attention to the tensor part.

We remark first of all that due to property (11b) L_{II} reduces to

$$L_{II} = \sqrt{-\det g_{\mu\nu}}\left[i\bar{R}\gamma^\mu\nabla_\mu R + i\bar{L}\gamma^\mu\nabla_\mu L + g\bar{L}\gamma^\mu\vec{\tau}L.\vec{A}_\mu + - g'\left(\frac{1}{2}\bar{L}\gamma^\mu L + \bar{R}\gamma^\mu R\right)B_\mu\right] \quad . \tag{29}$$

That is the pure traceless tensor parts disapear from $L_{(2)}$. The Higgs mechanism gives then masses for the $(W^\pm_\mu, Z_\mu)$ bosons and the tensors. Setting $\phi = \begin{pmatrix} 0 \\ \lambda+\chi/\sqrt{2}\end{pmatrix}$ in which $\lambda = \langle 0|\phi|0\rangle$ = constant and redefining the tensor fields by the expressions

$$W^\pm_{\alpha\beta\mu} = \frac{a^{(1)}_{\alpha\beta\mu} \pm i\, a^{(2)}_{\alpha\beta\mu}}{\sqrt{2}} \tag{30a}$$

$$Z_{\alpha\beta\mu} = \frac{a^{(3)}_{\alpha\beta\mu} + b_{\alpha\beta\mu}}{\sqrt{2}} \tag{30b}$$

$$h_{\alpha\beta\mu} = \frac{-a^{(3)}_{\alpha\beta\mu} + b_{\alpha\beta\mu}}{\sqrt{2}} \tag{30c}$$

we obtain from L the values for the masses

$$m\,(W^\pm_{\alpha\beta\mu}) = \frac{3\lambda}{2} \quad ,$$

$$m\,(Z_{\alpha\beta\mu}) = \frac{3\sqrt{2}}{2}\lambda \quad , \qquad m\,(h_{\alpha\beta\mu}) = 0 \quad .$$

Knowing the value of λ from the masses of the vector bosons, we can evaluate that m $(W^{\pm}_{\alpha\beta\mu})$ = 369 GeV and m $(Z_{\alpha\beta\mu})$ = 520,3 GeV.

Remark that the fact that we are conducted to rotate by 45^{o} the tensors $(a^{(3)}_{\alpha\beta\mu}, b_{\alpha\beta\mu})$ in order to get a massless tensor $(h_{\alpha\beta\mu})$ is a consequence of the absence of a previleged direction in the space of tensors due to the absence of any new constant in the present theory. We should stress that this situation could be changed without affecting the formal body of the present model — although having distinct observational consequences. This is a matter which should be regulated by future experiments.

The above remarks induce us to naturally associate the massless tensor $h_{\mu\nu\lambda}$ to the long range gravitational field and to the inclusion of new short-range effects of gravitational interaction: the massive counter-part of it.

Let us make a final comment. After inserting the definition (30) into the total Lagrangian and looking for the equation of motion for $h_{\alpha\beta\mu}$ we note that besides the standard Jordan's expression, there appear extra terms envolving products of tensors $(W^{\pm}_{\alpha\beta\mu}, Z_{\alpha\beta\mu})$ with vectors $(A^{\pm}_{\alpha}, Z_{\alpha})$. Although these terms should be taken into account in the quantum version of the theory, its classical effects can be completely neglected since it envolves point products of the short range fields.

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10 - We can modify the structure of the fundamental group at our will without changing the unified scheme, by just considering an enlarged set of generalized Lanczos tensors.