ISSN 0029-3865

CBPF-NF-051/85

A BOSONIC POLYAKOV'S STRING WITH FERMIONIC BOUNDARIES FOR THE MAKEENKO-MIGDAL CONTOUR Q.C.D(SU(∞)) EQUATION

by

Luiz C.L. Botelho

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

In this note, a simple bosonic Polyakov's string with fermionic boundaries is proposed for the Makeenko-Migdal contour Q.C.D(SU(∞)) equation.

Key-words: Makeenko-Migdal countour Q.C.D equation; Polyakov's String with fermionic boundaries.

1 INTRODUCTION

In recent years, Makeenko and Migdal[1], have raised hopes that a string representation for Quantum Chromodynamics can be made possible by showing that the unrenormalized closed contour average (the Quantum Wilson Loop) in a four-dimensional Euclidean space-time.

$$W\left[C_{x(0)x(2\pi)}\right] = \frac{1}{N_c} \langle \mathrm{Tr}\,\mathbb{P}\left(\mathrm{Exp}\int_0^{2\pi} A_\mu\left(x_\mu(\sigma)\right)dx_\mu(\sigma)\right) \rangle \qquad (1)$$

satisfies the non linear string like equation in t'Hooft topological limit $N_c = +\infty$ [1].

$$\frac{\partial}{\partial x_\mu}\frac{\delta}{\delta\sigma_{\mu\nu}(x_\alpha)}\Bigg|_{x_\alpha = x_\alpha(\sigma)} W\left[C_{x(o)x(2\pi)}\right]$$

$$= (\lambda_0)^2\int_0^{2\pi} dx_\mu(\sigma')\,\delta^{(4)}(x_\mu(\sigma)-x_\mu(\sigma'))\,W\left[C_{x(o)x(\sigma)}\right]W\left[C_{x(\sigma)x(2\pi)}\right]; \qquad (2)$$

where $C_{x(o)x(2\pi)} = \{x_\mu(\sigma');\ 0 \leq \sigma' \leq 2\pi\}$ denotes an arbitrary closed contour in R^4.

The Eq. (2) is a closed equation for the called "Quantum Wilson Loop" (1) and an important problem consists to investigate its solutions at least at the formal level.

In this formal framework, A. Migdal[2] proposed a solution for (2) in terms of a non-linear Fermi String (the String's Elfin theory). However, the search for others solutions can be a useful step towards the understanding of the Makeenko-Migdal equation (2) as well as to all contour formalism.

Our aim in this note is to propose another formal string solution for (2) in the framework of Polyakov's Quantum Geometry and similar in its form to the strings model with fermionic boundaries as in ref. 3,4.

2 THE SOLUTION

Let us start our analysis by considering an arbitrary (topologicaly trivial) fixed surface Σ_C with the property of being bounded by the closed contour $C_{x(o)x(2\pi)}$. This surface may intersect itself as a consequence of the above $C_{x(o)x(2\pi)}$ contour have self-intersections as is implicit in the right hand side of Eq. (2). We consider such surface described by the parametric equation $\{x_\mu(\sigma,\zeta);\ 0 \le \sigma \le 2\pi;\ -\infty < \zeta < \infty\}$, and with boundary condition $x_\mu(\sigma,o) = C_{x(o)x(2\pi)}$. The self intersections of the surface arise at those points where $x_\mu(\sigma,\zeta) = x_\mu(\sigma',\zeta')$ with $(\sigma,\zeta) \neq (\sigma',\zeta')$. (see Fig. (1)).

Now let us introduce a unidimensional fermionic field $\{\psi(\sigma),\ 0 \le \sigma \le 2\pi\}$ which we will denominate as the "Reduced Elfin Field" and which interacts with the surface geometry through the action

$$S\left[\psi(\sigma);\ x_\mu(\sigma,\zeta)\right]$$

$$= \frac{1}{4}\lambda_o^2 \left\{ \int_0^{2\pi} d\sigma \int_{-\infty}^{+\infty} d\zeta \int_0^{2\pi} d\sigma' \int_{-\infty}^{+\infty} d\zeta' \psi(\sigma) d\sigma_{\mu\nu}(x_\alpha(\sigma,\zeta)) . \right.$$

$$\left. \delta^{(4)}(x_\mu(\sigma,\zeta) - x_\alpha(\sigma',\zeta').\psi(\sigma').d\sigma_{\mu\nu}(x_\alpha(\sigma,\zeta')) \right\} \qquad (3)$$

where $d\sigma_{\mu\nu}(x_\alpha(\sigma,\zeta))$ are the surface area tensors.

We consider, then, the contour functional

$$Z\left[C_{x(\tilde{\sigma})x(\tilde{\tilde{\sigma}})}\right] = \int \prod_{0\le\sigma\le 2\pi} (d\psi(\sigma))\exp\left(-\frac{1}{2}\int_0^{2\pi}\psi(\sigma)\ \frac{d}{d\sigma}\ \psi(\sigma)\right)$$

$$\psi(\tilde{\sigma}).\ \psi(\tilde{\tilde{\sigma}})\ \exp\left(-S\left[\bar{\psi}(\sigma)\ ;\ x_\mu(\sigma,\zeta)\right]\right) \tag{4}$$

A similar string ansatz without the fermionic uni-dimensional field $\psi(\sigma)$ was considered by Olesen and Petersen in Ref. 5. The formal proof that Eq. (4) for $\tilde{\sigma} = 0$, $\tilde{\tilde{\sigma}} = 2\pi$ satisfies the same contour equation (2) can be implemented as follows.

The Mandelstam Path Derivative is easily calculated following Ref. 5 - Eq. (3). We remark that this result holds true including the case of self-intersecting coutours. So:

$$\frac{\delta}{\delta\sigma_{\mu\nu}}(x_\alpha)\Bigg|_{x_\alpha = x_\alpha(\hat{\sigma})} Z\left[C_{x(0)x(2\pi)}\right]$$

$$= \int \prod_{0\le\sigma\le 2\pi} (d\psi(\sigma)).\exp\left(-\frac{1}{2}\int_0^{2\pi}\psi(\sigma)\ \frac{d}{d\sigma}\ \psi(\sigma)\right).\ \psi(0)\psi(2\pi)$$

$$\exp\left\{-S\left[\psi(\sigma),\ x_\mu(\sigma,\zeta)\right]\right\}.$$

$$\left(-\lambda_0^2\int_0^{2\pi}d\bar{\sigma}\int_{-\infty}^{+\infty}d\bar{\zeta}.\delta^{(4)}(x_\mu(\hat{\sigma}) - x_\mu(\bar{\sigma},\bar{\zeta}))\ \psi(\hat{\sigma})\psi(\bar{\sigma})\right) \tag{5}$$

Since the "Reduced Elfin Field" path measure has the propagation factorization property: we can re-write (5) in the "splitted" form:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x_\alpha)}\Bigg|_{x_\alpha=x_\alpha(\hat\sigma)} Z\left[C_{x(o)x(2\pi)}\right] = -\lambda_o^2\int_0^{2\pi} d\bar\sigma\int_{-\infty}^{+\infty} d\bar\zeta\, \delta^{(4)}(x_\mu(\hat\sigma)-x_\mu(\bar\sigma,\bar\zeta))$$

$$\left[\prod_{o\le\sigma\le\hat\sigma} d\psi(\sigma)\exp\left(-\frac{1}{2}\int_o^{\hat\sigma}\psi(\sigma)\,\frac{d}{d\sigma}\,\psi(\sigma)\right)\,.\,\psi(o).\psi(\hat\sigma).\right.$$

$$\exp\left\{-\frac{1}{4}\,\lambda_o^2\int_0^{\hat\sigma}d\sigma\int_{-\infty}^{+\infty}d\xi.\int_0^{\hat\sigma}d\sigma'\int_{-\infty}^{+\infty}d\zeta'\quad \psi(\sigma).d\sigma_{\mu\nu}(x_\alpha(\sigma,\zeta)).\right.$$

$$\left.\left.\delta^{(4)}(x_\alpha(\sigma,\zeta)-x_\alpha(\sigma',\zeta').\psi(\sigma').d\sigma_{\mu\nu}(x_\mu(\sigma,\zeta'))\right\}\right].$$

$$\left[\int_{\hat\sigma\le\sigma\le 2\pi}\prod d\psi(\sigma).\exp\left(-\frac{1}{2}\int_{\hat\sigma}^{2\pi}\psi(\sigma)\,\frac{d}{d\sigma}\,\psi(\sigma)\right).\psi(\bar\sigma).\psi(2\pi)\right.$$

$$\exp\left\{-\frac{1}{4}\,\lambda_o^2\int_{\hat\sigma}^{2\pi}d\sigma\int_{-\infty}^{+\infty}d\zeta\int_{\hat\sigma}^{2\pi}d\sigma'\int_{-\infty}^{+\infty}d\zeta'\psi(\sigma)d\sigma_{\mu\nu}(x_\alpha(\sigma,\zeta)\right.$$

$$\left.\left.\delta^{(4)}(x_\mu(\sigma,\zeta)-x_\mu(\sigma',\zeta'))\psi(\sigma')d\sigma_{\mu\nu}(x_\alpha(\sigma',\zeta'))\right\}\right] \qquad (6)\;.$$

Now we note that the surface $\sum_{C_{(x(o)x(2\pi))}}$ splits into two branches $\sum_{C_{x(o)x(\hat\sigma)}}$ and $\sum_{C_{(x(\bar\sigma),x(2\pi))}}$ as despicted in Fig. 1, so we get the result.

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x_\mu)}\Bigg|_{x_\mu=x_\mu(\sigma)} W\left[C_{x(o)x(2\pi)}\right]$$

$$= -\,\lambda_o^2\int_0^{2\pi}d\bar\sigma\int_{-\infty}^{+\infty}d\bar\zeta\;\delta^{(4)}(x_\mu(\sigma)-x_\mu(\bar\sigma,\bar\zeta))\,Z\left[C_{x(o)x(\hat\sigma)}\right].\,Z\left[C_{x(\bar\sigma)x(2\pi)}\right] \qquad (7)$$

Evaluating the ∂_μ^x derivative as in the Appendix of Ref. 5, Eq́. (A.1) we obtain that the proposed contour functional Eq.

(4), satisfies the Makeenko-Migdal contour equation

$$\partial^x_\mu \frac{\delta}{\delta\sigma_{\mu\nu}}(x_\alpha)\Big|_{x=x_\mu(\hat{\sigma})} Z\left[C_{x(o)x(2\pi)}\right]$$

$$= (\lambda_o^2)\cdot\oint_{C_{x(o)x(2\pi)}} \delta^{(4)}(x_\mu(\hat{\sigma}) - x_\mu(\bar{\sigma})).dx_\nu(\bar{\sigma}) Z\left[C_{x(o)x(\hat{\sigma})}\right]\cdot$$

$$\cdot Z\left[C_{x(\bar{\sigma})x(2\pi)}\right] \quad (8)$$

If we observe that the points $x_\mu(\hat{\sigma})$ and $x_\mu(\bar{\sigma})$ coincides in the space-time we see that the functional (4) satisfies the same Makeenko-Migdal $(Q.C.D)_{N_c=\infty}$ contour esquation.

Up to now we dealt with an arbitrary, but fixed surface $\sum_{C_{x(o)x(2\pi)}}$. And since the contour $C_{x(o)x(2\pi)}$ can be the boundary of a set of infinite surfaces, we have to sum over all such surfaces in order to obtain a quantum geometric solution. The natural formalism to implement this task is to use Polyakov's Quantum Geometry with boundaries[6,7]. So, we are lead to consider the Quantum Geometric Functional

$$\bar{W}\left[C_{x(o)x(2\pi)}\right] = \int d_\mu\left[\sum C_{x(o)x(2\pi)}\right]\cdot Z\left[C_{x(o)x(2\pi)}\right] \quad (9)$$

where the proper functional measure $d_\mu\left[\sum C_{x(o)x(2\pi)}\right]$ to implement the surface continuous sum is given in Ref. 6,7 and the covariant expression for the area tensor is given by $t_{\mu\nu}(x_\alpha(\sigma,\zeta)) = \frac{1}{\sqrt{g(\sigma,\zeta)}}\, g^{ab}(\sigma,\zeta)\,\partial_a x_\mu(\sigma,\zeta)\,.\,\partial_b x_\nu(\sigma,\zeta)$ and, consequently, the covariant version of Eq. (3) is now given by

$$\frac{1}{4}\lambda_o^2\left\{\int_0^{2\pi}d\sigma\int_{-\omega}^{+\infty}d\zeta\int_0^{2\pi}d\sigma'\int_{-\infty}^{+\infty}d\zeta'\psi(\sigma).\sqrt{g(\sigma,\zeta)}.\delta^{(4)}(x_\mu(\sigma,\zeta)-x_\mu(\sigma',\zeta'))\right.$$
$$\left.\psi(\sigma')\sqrt{g(\sigma',\zeta')}\right\} \qquad (10)$$

Here we have used the fact that only the self-intersecting surface points $x_\mu(\sigma,\zeta) = x_\mu(\sigma',\zeta')$ of the surface $\sum_{C_{x(o)x(2\pi)}}$ effectively contributes due to the delta function in Eq. (3) and by making use of the relation ship $t_{\mu\nu}(x_\mu(\sigma,\zeta)).t_{\mu\nu}(x_\mu(\sigma',\zeta')) = 1$ for $x_\mu(\sigma,\zeta) = x_\mu(\sigma',\zeta')$ we then get the covariant result Eq. (10).

It is instructive point out the self-supressing string surface interaction (see Ref. 8).

We emphasize that we do not know wheter this solution coincides with Migdal's Elfin Solution, since the fermionic degree in $\bar{W}\left[C_{x(o)x(2\pi)}\right]$ is defined only at the contour and not all over the surface as is in the above cited Migdal String.

This last Elfin property suggests another more interesting Ansatz for $(Q.C.D)_{N_c=+\infty}$ string. We at first, consider the Makeenko-Migdal equation continued for each closed contour $\tilde{C}_{x(o)x(2\pi)}$ lying at the surface $\sum_{C_{x(o)x(2\pi)}}$ (see Fig. 2) and then; consider the surface continued covariant solution (4).

$$\tilde{Z}\left[C_{x(o)x(2\pi)}\right] = \int \mathcal{D}\left[\psi(\sigma,\zeta)\right].\left\{\int_{-\infty}^{+\infty}d\zeta\ \psi(o,\zeta).\psi(2\pi,\zeta)\right\}$$
$$\exp\left\{-\int_0^{2\pi}d\sigma\int_{-\infty}^{+\infty}d\zeta\left[\frac{1}{2}\ i\psi(\gamma_\mu D_\mu)\psi\right](\sigma,\zeta)\right\}$$

$$\exp\left\{-\frac{1}{4}\lambda_0^2\int_0^{2\pi}d\sigma\int_{-\omega}^{+\infty}d\zeta.\int_0^{2\pi}d\sigma'\int_{-\omega}^{+\infty}d\zeta'\ \psi(\sigma,\zeta).\sqrt{g(\sigma,\zeta)}\ .\right.$$

$$\left.\delta^{(4)}(x_\mu(\sigma,\zeta)-x_\mu(\sigma',\zeta')).\psi(\sigma',\zeta').\sqrt{g(\sigma',\zeta')}\right\} \quad (11)$$

Here $\psi_\mu(\sigma,\zeta) = (\psi_1(\sigma,\zeta),\psi_2(\sigma,\zeta))$ being a two-dimensional Majorana Spinor describing the string fermionic degrees of freedom, $\gamma_\mu D_\mu$ the covariant Dirac operator and the functional measure $\mathcal{D}[\psi(\sigma,\zeta)]$ is obtained as the functional element of volume associated to the following functional Riemann metric $||\delta\psi||^2 = \int_0^{2\pi}d\sigma\int_{-\omega}^{+\infty}d\zeta\ \sqrt{g(\sigma,\zeta)}\ \delta\psi(\sigma,\zeta).\delta\psi(\sigma,\zeta)$ (2). Proceeding as above we consider the quantum geometric solution

$$\bar{W}\left[C_{x(o)x(2\pi)}\right] = \int d_\mu\left[\Sigma_{C_{x(o)x(2\pi)}}\right].\tilde{Z}\left[C_{x(o)x(2\pi)}\right] \quad (12)$$

To sumarize we have obtained a interacting bosonic string with fermionic boundary with a self supressing delta surface interaction at these points where the string surface crosses itself as another formal string Ansatz for Quantum Chromodynamics at $N_c = +\infty$. (see (9)).

In addition, we have considered a generalized contour meaning for the Makeenko-Migdal equation which can lead to a fermionic string (see Eq. 2) similar to the Migdal's Elfin Theory[2].

In would like to thank Edson P. Silva for enlightening discussions about the anstaz Eq. (5) and Professor J. Mignaco, Professor C. Bollini and Professor J. Giambiagi for some comments. This work was partially supported by CNPq-Brasil.

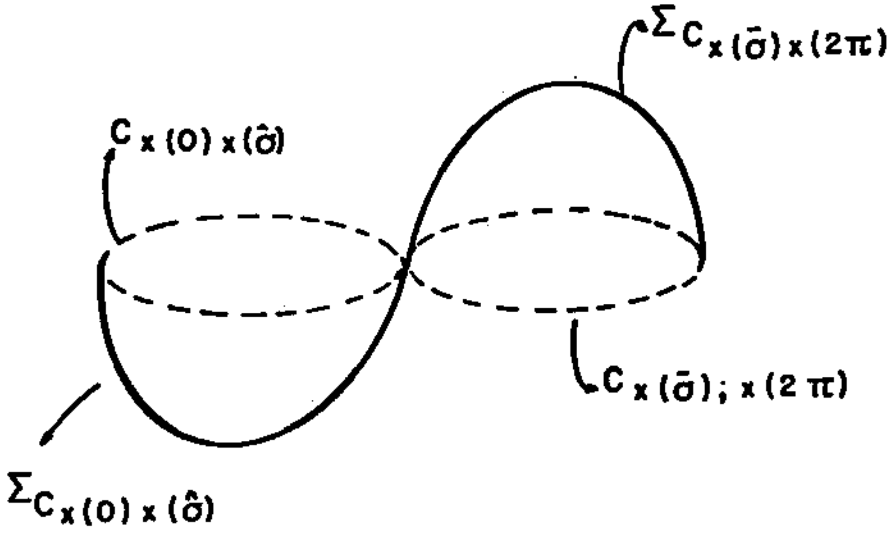

FIG.1

Fig. 1 - The surface $\sum_{C_{x(o)x(2\pi)}}$ and its branches $\sum_{C_{x(o)x(\hat{\sigma})}}$ and $\sum_{x(\bar{\sigma})x(2\pi)}$ associateds to the splitted contour $C_{x(o)x(\hat{\sigma})}$ and $C_{x(\bar{\sigma})x(2\pi)}$, respectively at the double contour point $x_\mu(\bar{\sigma}) = x_\mu(\hat{\sigma})$.

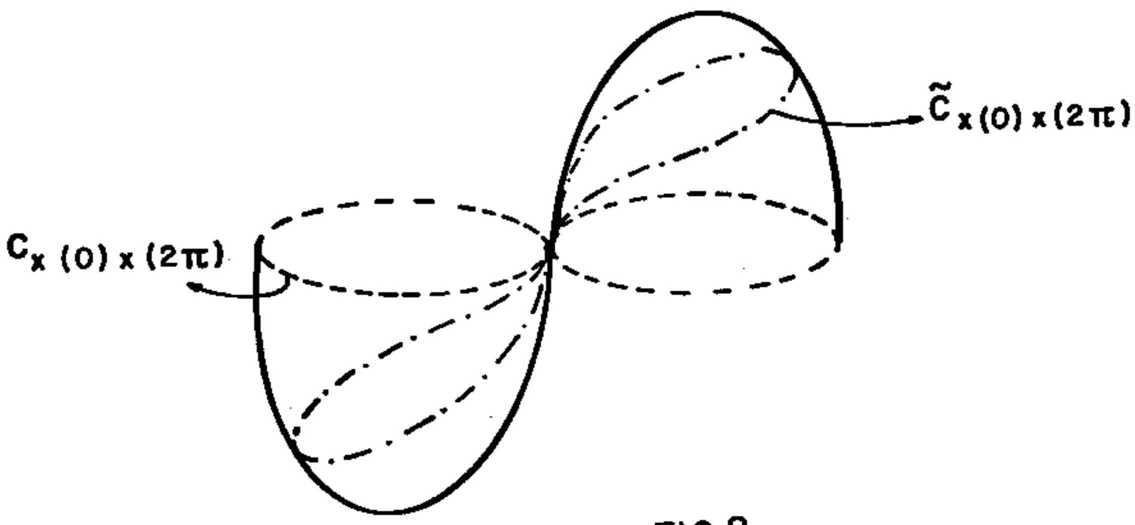

FIG.2

Fig. 2 - A closed contour $\tilde{C}_{x(o)x(2\pi)}$ lying at the surface $\sum_{C_{x(o)x(2\pi)}}$.

REFERENCES

1. A.A. Migdal, Phys. Reports 102, nº 4 (1983) - North Holland

2. A.A. Migdal, Nucl. Phys. B189 (1981), 253

3. J. Bars, A.J. Hanson, Phys. Rev. D13, 1744 (1976)

4. C.J. Burden, Il Nuovo Cimento, 80A, nº 4 (1984)

5. P. Olesen, J.L. Peterson, Nucl. Phys. B181, (5), (1981)

6. A.M. Polyakov, Phys. Lett. 103B (1981), 207

7. B. Durhuus at al., Nucl. Phys. B196 (1982) 498

8. Rafael I. Nepomechie at al., Phys. Lett. 105B (1981), 457