ISSN 0029 - 3865

CBPF-NF-048/86

A CONSISTENCY QUESTION IN THE RENORMALIZATION GROUP ANALYSIS OF DEEP INELASTIC SCATTERING

by

Sebastião Alves Dias and Juan Alberto Mignaco

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

The application of renormalization group methods in perturbation theory results in a definite behavior for terms of higher orders under changes of renormalization prescription. Apparently, this fails to be true for the one-loop correction to the moments of the non-singlet structure functions, which is believed to be a renormalization group invariant. We point a possible explanation for this contradiction in the treatment of the terms taking into account non-perturbative effects.

Key-words: Deep inelastic scattering; Moments of structure function in QCD; Renormalization group.

PACS numbers: 11.10 Gh, 12.38 Aw, 13.60 Hb

In a series of nice articles Stevenson[1] gave a complete characterization of the renormalization group in perturbative theory. Other authors[2,3,4] studied later the extension of the theoretical framework.

One of the most successful applications of Quantum Chromodynamics (QCD) is the determination of the ratios of the moments of the structure functions in deep inelastic experiments, the so-called Perkins plot[5a]. These results are obtained already at the leading order in the perturbative regime of high Q^2.

The calculation was soon extended to the next, one loop term. Though the experimental results don't allow yet to compare them with the predictions of the theory, it is a current problem that one finds scattered in the literature how this term behaves with respect to transformation of renormalization prescriptions.

In this short note, we want to raise the issue of an apparent contradiction between what renormalization group analysis determines to be behavior of the perturbation theory contribution and what emerges out of the procedures used to calculate the first (one loop) corrections to the moments of the structure functions.

Let's give a sketchy description of how the coefficients of the perturbative expansion of a physical observable behave under changes of renormalization scheme (RS). According to Stevenson[1] a full description of RS can be given in terms of a infinite set of parameters $\{\tau, c_2, c_3, \ldots\}$ where

$$\tau = b \ln(\mu/\Lambda) \tag{1}$$

$$\frac{\partial a}{\partial \tau} = -a^2\left(1 + ca + c_2 a^2 + c_3 a^3 + \ldots\right) \tag{2}$$

(here μ is the massive parameter introduced in the definition, at one loop level, of the renormalization constants; b= $\beta_0/2$, C= $\beta_1/4\beta_0$, etc are the β - function coefficients related with those of the traditional function; a= $g^2/4\pi^2$, where g is the coupling constant of QCD; Λ is the integration constant for the equation that defines a).

In order to investigate how a physical observable R transforms under changes of RS, let us calculate the following derivatives , sticking to Stevenson [1a].

$$D_i R = \frac{dR}{dc_i} = \left(\frac{\partial}{\partial c_i} + \beta_i(a) \frac{\partial}{\partial a} \right) R = 0 \tag{3}$$

with

$$\beta_i(a) = \frac{\partial a}{\partial c_i} = N_i a^{i+1} \left(W_0^i + W_1^i a + W_2^i a^2 + \ldots \right) \qquad i = 2,3,\ldots$$

$$\frac{\partial a}{\partial c_1} = \frac{\partial a}{\partial \tau} \ , \ N_1 = -1 \ , \ W_0^i = 1 \ , \ W_1^2 = c \ , W_2^1 = c_2 \ , \ldots \tag{4}$$

Using

$$R = a^N \left[1 + r_1 a + r_2 a^2 + \ldots \right] \tag{5}$$

we have

$$D_1(r_1) = -N_1 N W_0^1 \ , \quad D_k(r_1) = 0 , \ k = 2,3,\ldots \tag{6}$$

$$\begin{aligned} D_1(r_2) &= -N_1 N W_1^1 - N_1 (N+1) W_0^1 r_1 \ , \\ D_2(r_2) &= -N_2 N W_0^2 \ , \quad D_k(r_2) = 0 , \ k = 3,4,\ldots \end{aligned} \tag{7}$$

etc.

In words, up to order n in perturbation theory, only n-1 parameters ($\tau, c_2, \ldots, c_{n-1}$) are relevant for the characterization of the renormalization prescription.

Let us, equally sketchy, condense the current description of the moments of the non-singlet structure functions

$$M_N^{i,rs}(Q^2) \equiv \int_0^1 dx \;\; x^N \; F_i^{rs}(x,Q^2) \tag{8a}$$

$$= \sum_{k=1} \tilde{C}_{i,k}^{N,rs}\left(Q^2/\mu^2, a(\tau)\right)\bar{O}_k^N \tag{8b}$$

$$= \sum_{k=1} a^{d_N}\left(1 + r_{1,k}^{N,rs} a + \ldots\right)\bar{O}_k^N \tag{8c}$$

Here i is the label of the structure function (1,2 for charged leptons ,1,2,3 for neutrinos); r,s are the flavour indices of the hadrons intervening; $x \equiv Q^2/2\nu$ is the dimensionless Bjorken variable; C are the dimensionally reduced Wilson coefficients for the light cone operator product expansion (OPE) for the product of two currents; $\bar{O}_k^N$ are defined from a basis set $\hat{O}^k_{\mu_1 \ldots \mu_N}$ in the OPE through

$$\langle h | \hat{O}^k_{\mu_1 \cdots \mu_N} | h \rangle = i^N h_{\mu_1} \ldots h_{\mu_N} \bar{O}_k^N \left(h^2/\mu^2, a(\tau)\right) \tag{9}$$

where h indicates the momentum of a hadron state and $d_N \equiv \gamma_0^N/2\beta_0$ with γ_0^N the lowest order coefficient in the perturbative expansion of the anomalous dimension for the (composite) operator 0.

For simplicity, let us consider a single term in (8b). Forgetting about indices, consider the relevant perturbative expansion:

$$M_N(Q^2) = M_0^N \left[a(\tau_0)\right]^{d_N}\left(1 + r_1^N a(\tau_0) + \ldots\right) \tag{10a}$$

with

$$M_0^N = \left[a(\tau)\right]^{-d_N}\left(1 + \left(\frac{\gamma_0^N}{4b} - \frac{\gamma_1^N}{16b}\right)a(\tau)\right)\bar{O}^N\left((h^2/\mu^2), a(\tau)\right)$$

$$= f(\tau)\,\bar{O}_1^N(\tau) \tag{10b}$$

$$r_1^N = \frac{\gamma_1^N}{16b} - \frac{\gamma_0^N c}{4b} + \frac{\varepsilon_1^N}{4} \tag{10c}$$

$$\tilde{C}^N\left(1, a(\tau_0)\right) = 1 + \frac{1}{4}\varepsilon_1^N a(\tau_0) + \ldots \tag{10d}$$

(where $\tau = b\ell n(Q/\Lambda)$). Let us assume that we perform a change in the renormalization point through a dilation:

$$\mu \longrightarrow \mu e^{\kappa} \tag{11a}$$

This is a genuine transformation under the renormalization group, which brings a change in the value of the effective coupling constant in the theory.

Correspondingly, we should expect from (6) the change

$$r_1^N \longrightarrow r_1^N + d_N b \kappa \tag{11b}$$

We shall consider the problem in the same way as other authors do, with a substancial difference: their transformation do not change the value of the coupling constant, and this amounts to work in practically the same renormalization scheme.

As defined in (10c), the coefficient r_1^N has two terms which are dependent on the renormalization scheme: ε_1^N and γ_1^N

The first is obtained from the calculation of the structure functions for quark-lepton scattering, by comparing the left and right hand sides in Eqs. (8c) and (10b). They read:

$$1 - \frac{1}{4} a \left(\frac{1}{2} \gamma_0^N \ln \frac{q^2}{p^2} - \rho^N \right) + O(a^2) =$$

$$= \left(1 + \varepsilon_1^N \frac{a}{4} \right) \cdot \left(1 - \frac{1}{8} a \gamma_0^N \ln \left(\frac{q^2}{\mu^2} \right) \right) \cdot$$

$$\cdot \left(1 + \frac{1}{8} a \gamma_0^N \ln \left(\frac{p^2}{\mu^2} \right) + \frac{a}{4} \delta^N \right) + O(a^2) \tag{12}$$

Notice that in order to have the correct renormalization group transformation for the third bracket in the right hand side, δ^N should not depend on the renormalization parameter, μ. (See below Eq. (21c)), We have, then:

$$\varepsilon_1^N = \rho^N - \delta^N \tag{13}$$

Calling A the actual renormalization scheme with parameter μ, and B the scheme obtained from (11c), we have:

$$O_N^{(A)} = 1 + \left(\frac{1}{2}\gamma_0^N \ln\left(\frac{p^2}{\mu^2}\right) + \delta^N\right)\frac{a(\mu^2)}{4} + \dots \tag{14a}$$

$$O_N^{(B)} = 1 + \left(\frac{1}{2}\gamma_0^N \ln\left(\frac{p^2}{\mu'^2}\right) + \delta^N\right)\frac{a(\mu'^2)}{4} + \dots$$

$$= 1 + \left(\frac{1}{2}\gamma_0^N \ln\left(\frac{p^2}{\mu^2}\right) + \delta^N - \gamma_0^N \kappa\right)\frac{a(\mu^2)}{4} + O(a^2) \tag{14b}$$

The last step used the relation

$$a(\mu'^2) = a(\mu^2) - a^2(\mu^2)\, b \ln\left(\frac{\mu'^2}{\mu^2}\right) + O(a^2) \tag{15}$$

From Eqs. (12)-(15) results:

$$\varepsilon_1^{N(B)} = \varepsilon_1^{N(A)} + \gamma_0^N \kappa \tag{16}$$

For what regards γ_1^N, from the scratch, the definition

$$\gamma_0^N(a) = \gamma_0^N \frac{a}{4} + \gamma_1^N \left(\frac{a}{4}\right)^2 + \dots \tag{17}$$

gives, from (15),

$$\gamma_1^{N(B)} = \gamma_1^{N(A)} - \gamma_0^N x \tag{18}$$

This means, then:

$$r_1^{N(B)} = r_1^{N(A)} \tag{19}$$

and it turns out that r_1^N is invariant under the transformation (11a). This contradicts (11b) clearly.

The point is that a term is lacking in (10a) which should come from the $\overline{O}^N$ term in (10b), expressing the nontrivial quark dependence of the hadron.

Remark that this dependence lies essentially out of any perturbation analysis, which is the only framework where an accurate characterization of the renormalization group is available. Were it possible to perform a complete field theoretical calculation of bound states in QCD, where the renormalization group dependence is correctly accounted for, a comprehensive relation with the perturbative formulation might be established.

This feature is not unique to the deep inelastic case. The similar problem in the photon structure function has arisen recently renewed interest[6].

Assuming that a plausible perturbative description is possible, we may try to reconcile our findings as follows. Since a physical observable should be invariant under renormalization group, and we know that (5b,5c).

$$\frac{d}{d\tau}\tilde{C}_N = -\frac{\gamma(O_N)}{b}\tilde{C}_N \tag{20}$$

we find

$$\frac{d}{d\tau}R = 0 = -\frac{\gamma(O_N)}{b}\tilde{C}_N\overline{O}^N + \tilde{C}_N\frac{d}{d\tau}\overline{O}^N$$

$$\Longrightarrow \quad \frac{d}{d\tau}\overline{O}^N = \frac{\gamma(O_N)}{b}\overline{O}^N \tag{21a}$$

and for its (formal) perturbative expansion*

$$\bar{O}^N = 1 + O_1^N\, a(\tau) + \dots \tag{21b}$$

we easily find

$$\frac{dO_1^N}{d\tau} = \frac{\gamma_0^N}{4b} = d_N \implies O_1^N \to O_1^N + d_N b x \tag{21c}$$

(under $\mu \to \mu e^x$)

This way, coming back to (8c) we recover for a redefined r_1^N the correct transformation (11b).

Let us add a few comments in conclusion.

The analysis based on a renormalization group improvement for the evaluation of moments at leading order provides a meaningful result, as the Perkin's plots wit. However, the whole quantitative analysis for the moments in the next perturbative order is inaccurate if no quantitative estimate is possible for the magnitude of the matrix element of the composite operator. A gross conjecture is that all calculations are safe provided $O_1^N \lesssim \frac{r_1}{10}$. This number comes from looking at the figures obtained in the application of $\overline{\text{PMS}}$[7] to the current known values in MS, demanding the stability of the perturbative expansion.

We have not regarded the problem of factorization, since we focused our aim in the problem of how well the perturbative results are determined from the theory. Factorization, however, is important for practical calculations, and in this respect we believe it should be accounted for properly when calculating the quantity

$$\mathbb{R} = -\frac{4}{\gamma_0^N} \frac{d \ln M_N}{d \ln (Q^2/\Lambda^2)}$$

---

* A phenomenological ingredient is hidden here; the zeroth order term in O_1^N depends quite middly on a (τ). With this assumption, the Perkins plot is preserved.

since, as we have known, the matrix element of the operator (truly factorization scheme dependent)enters.

We wish to thank Profs. José de Sá Borges Filho and Paul M. Stevenson for helpful comments on the subject.

REFERENCES

1) P.M. Stevenson a) Phys. Review D23, 2916 (1981)
b) Phys. Lett. 100B, 61 (1981)

2) M.R. Pennington, Rep. Prog. Phys. 46, 393 (1983)

3) D.N. Duke, R.G. Roberts, Phys. Reports, 120, 275 (1985)

4) H.D. Politzer, Nucl. Phys. B194, 493 (1982).

5a) A. Peterman, Phys. Rep. 53, 157 (1979)

5b) E.G. Floratos, P.A. Ross, C.T. Sachrajda, Nucl.Phys. B129, 66 (1977)

5c) W.A. Bardeen, A.J. Buras, D.W. Duke, T. Muta, Phys. Rev. D18, 3998 (1978).

6) G. Grunberg, preprint, Centre de Phys. Théorique, École Polytechnique A717.0586.
J.H. Field, F. Kapusta e L. Poggioli, preprint DESY 86-073.

7) J.A.Mignaco and I. Roditi, Phys. Lett. 126B (1983) 481.
J.C. Wrigley, Phys. Rev. D27 (1983) 1965.