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A QUANTUM FERMIONIC STRING REPRESENTATION FOR Q.C.D. (SU(Nc)) AT Nc=+∞

by

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ABSTRACT

We aim in this letter to suggest a covariant fermionic string theory representation for Q.C.D. (SU(∞)) in R^d. We point out that the Migdal'Elfin string theory can be obtained as a particular case from the proposed Q.C.D. (SU(∞)) string in the situation of "smooth" string world-sheet.

1. INTRODUCTION

It is expected that Q.C.D. (N_c) (at least at $N_c \to \infty$) should be described by some sort of string theory (see ref.[1] for an extensive review).

Our aim in this letter is to suggest that the bosonic loop average in Q.C.D. (SU(∞)) can be considered as a quantum string state of a self avoiding bosonic string theory with a intrinsic fermionic structure.

2. THE PROPOSED Q.C.D. (SU(∞)) STRING THEORY

One basic object in a covariant quantized string theory is the quantum string state which is represented by a functional $\Phi[\mathcal{C}_{X(-\pi),X(\pi)}] = \Phi[X_\mu(\sigma), e(\sigma)]$ of a parametrized contour $\{X_\mu(\sigma)\}$ $(-\pi \leq \sigma \leq \pi)$ and of a one dimensional intrinsic metric $\{e(\sigma)\}$ defined on $X_\mu(\sigma)$.

Dynamics being given by the evolution equation between a initial string state $\Phi[\mathcal{C}^{in}_{X(-\pi),X(\pi)}]$ for a final state $\Phi[\mathcal{C}^{out}_{X(-\pi),X(\pi)}]$

$$\Phi[\mathcal{C}^{out}] = \int d[\mathcal{C}^{in}]\, G[\mathcal{C}^{out};\mathcal{C}^{in}]\, \Phi[\mathcal{C}^{in}] \qquad (1)$$

The string propagation Kernel $G[C^{out}, C^{in}]$ is supposed to be expressed as the sum over all possible cylindrical string's surface S without holes and with its boundary fixed $\partial S = \{C^{in}; C^{out}\}$ ([2]).

Let us start our analyses describing the action of our proposed Q.C.D. (SU(∞)) covariant string theory (we work in euclidean space-time R^d) ([3]).

$$
\begin{aligned}
S[X_\mu, g_{ab}, \psi_{(k)}] &= \frac{1}{2} \int_{\mathcal{D}} (\sqrt{g}\, g^{ab} \partial_a X_\mu \partial_b X_\mu)(\sigma,\tau)\, d\sigma d\tau \\
&+ \frac{1}{2} \int_{\mathcal{D}} \left(\psi_{(k)}\, \gamma_a(\sigma,\tau) . \partial_a \overline{\psi}_{(k)} \right)(\sigma,\tau)\, d\sigma d\tau \\
&+ \frac{\lambda_0}{d} \int_{\mathcal{D}} (\sqrt{g}\, \psi_{(k)} \overline{\psi}_{(k)})(\sigma,\tau)\, T_{\mu\nu}(X(\sigma,\tau)) \\
&\left(\int_{\mathcal{D}} d\sigma' d\tau' \sqrt{g(\sigma',\tau')}\, T_{\mu\nu}(X(\sigma',\tau')) \times \frac{1}{\sqrt{g(\sigma',\tau')}} \delta^{(d)}(X_\mu(\sigma,\tau) - X_\mu(\sigma',\tau')) \right)
\end{aligned}
\qquad (2)
$$

The string's cylindrical surface parameter domain is the rectangle $\mathcal{D} = \{ (\sigma,\tau) ; -\pi \le \sigma \le \pi ; 0 \le \tau \le T \}$

The covariant degrees of freedom associated to the pure bosonic string sector are $\{ X_\mu(\sigma,\tau) , g_{ab}(\sigma,\tau) \}$ and they satisfy the conformal cylindrical gauge condition ([2])

$$g_{ab}(\sigma,\tau) = \rho(\sigma,\tau)\, \delta_{ab} \qquad (3)$$

so that

$$\mathcal{C}^{in} = \left\{ X_\mu^{in}(\sigma) = X_\mu(\sigma,0) \; ; \; e_{in}(\sigma) = \rho(\sigma,0) \right\}$$

$$\mathcal{C}^{out} = \left\{ X_\mu^{out}(\sigma) = X_\mu(\sigma,T) \; ; \; e_{out}(\sigma) = \rho(\sigma,T) \right\} \quad (4)$$

The pure string' fermionic structure is described by a set of O(m) majorana spinors defined on the string world sheet $\left\{ \psi_{(k)}(\sigma,\tau) \; ; \; (\sigma,\tau) \in \mathcal{D} \; ; \; (k) \in \{0,1,\dots,m\} \right\}$ These sectors interacts through a self suppresing interaction involving the string orientation tensor $T_{\mu\nu}(X_\mu(\sigma,\tau)) = \left(\epsilon^{ab}\partial_a X_\mu \partial_b X_\nu\right)(\sigma,\tau)$ and a (covariant) atractive delta function potential supported at the self intersect lines of the string surface $\{ X_\mu(\sigma,\tau) \}$ (see the λ_0 term in eq.(2)). In this proposed string theory we explicitly suppose that the string surface may intersect itself. These self intersections arise at those lines (or points) where $X_\mu(\sigma,\tau) = X_\mu(\sigma',\tau')$ with $(\sigma,\tau) \neq (\sigma',\tau')$

Let us now consider the following quantum string propagation Kernel

$$G[\mathcal{C}^{out};\mathcal{C}^{in}] = \int d\mu[X_\mu, g_{ab}]\, d[\psi_{(k)}]$$

$$\times \left(\psi_{(k)}(-\pi,0) \,.\, \bar{\psi}^{(k)}(\pi,0) \right)$$

$$\times \exp\left(- S[X_\mu, g_{ab}, \psi_{(k)}]\right) \quad (5)$$

Here, $d\mu[g_{ab}, X^{\mu}]$ denotes the bosonic surface quantum measure proposed by A.M.Polyakov ([2]; [4]). The fermionic measure $d[\Psi_{(k)}]$ is defined by the eigenvectors of the covariant Dirac operator $\gamma_a(\sigma,\xi)\partial_a$ in $\mathcal{D}$ which acts on the Majorana spinors satisfing the "parity-conservation" boundary condition

$$\gamma_5 \psi_{(k)}(\sigma,0) = \psi_{(k)}(\sigma,0)$$

$$\gamma_5 \psi_{(k)}(\sigma,T) = \psi_{(k)}(\sigma,T)$$

They are imposed in order to ensure in a explicitly fashion the physical invariance under the change of the orientation of the strings C^{in} or C^{out} in the transition amplitude eq.(5) besides we impose that the U(1) currents $J^{(k)}_a(\sigma,\xi) = (\bar{\Psi}_{(k)}\gamma_a\Psi_{(k)})(\sigma,\xi)$ associated to the U(1) symmetry of the action eq.(3) should vanish at the boundary, since there is no flow outside of the string's world sheet ([5]).

We shall now argue in the framework of Ref.[2] that the (closed) string quantum state $\Phi[C^{in}]$ associated to the string theory eq.(5) satisfies a covariant non-linear wave equation which reduces to a bosonic-like Q.C.D. (SU(∞)) loop wave equation by imposing the string proper-time gauge $e_{in}(\sigma) = |X'^{in}_{\mu}(\sigma)|^2 = \text{CONSTANT}$

So, in order to deduce the covariant wave equation for the string "Kernel" eq.(5), we have to consider variations on "time direction" of the C^{IN}-end (closed) string in eq.(3) (or in a equivalently fashion: boundary $g_{00}(\sigma,\zeta)$ variations at the C^{IN}-end) - ([2]). We, thus, obtain the following identity

$$0 = \int d\mu\,[g_{ab},X_\mu]\,d[\psi_{(k)}]\,\left(\psi_{(k)}(-\pi,0)\,\bar{\psi}^{(k)}(\pi,0)\right)$$
$$\times\ \exp\left(-S[g_{ab},X_\mu,\psi_{(k)}]\right)$$
$$\times \lim_{\zeta\to 0^+}\left(\mathcal{T}_{00}(X_\mu,\partial_\zeta X_\mu,\partial_\sigma X_\mu,g_{ab},\psi_{(k)})\right.$$
$$\left.+\frac{\delta}{\delta g_{00}(\sigma,\zeta)}\exp\left(-F[g_{ab}(\sigma,\zeta)]\right)\right) \qquad (6)$$

where $\mathcal{T}_{00}(X_\mu,\partial_\zeta X_\mu,\partial_\sigma X_\mu,g_{ab},\psi_{(k)})$ is the time component of the interacting string energy-momentum tensor. Its boundary-value at $\zeta\to 0^+$ and in the conformal gauge eq.(3) is given by

$$\lim_{\zeta\to 0^+}\mathcal{T}_{00}(X_\mu,\partial_\zeta X_\mu,\partial_\sigma X_\mu,g_{ab},\psi_{(k)}) =$$
$$\Pi_\mu^{IN}(\sigma)^2 - \frac{1}{2}\,|X_\mu'^{IN}(\sigma)|^2 - \left[\frac{\lambda_0}{d}\left(\psi_{(k)}(\sigma,0)\,\bar{\psi}_{(k)}(\sigma,0)\right)\right.$$
$$\left.\times\ X_\mu'^{IN}(\sigma)\int_{-\pi}^{\pi} dX_\mu(\sigma')\,\delta^{(d)}\left(X_\mu^{IN}(\sigma)-X_\mu^{IN}(\sigma')\right)\right] \qquad (7)$$

Here $\Pi_\mu^{IN}(\sigma) = \lim_{\zeta\to 0^+}\partial_\zeta X^\mu(\sigma,\zeta)$ denotes the canonical momentum associated to string vector position $X_\mu^{IN}(\sigma)$ and we have assumed that the formal limit below holds true yielding, thus, the λ_0 term in eq.(7). ([5]). Explicitly:

$$\lim_{\tau \to 0^+} \int d\sigma d\tau \, d\sigma' d\tau' \left(T_{\mu\nu}(X^\mu(\sigma',\tau')) \, . \, T^{\mu\nu}(X_\mu(\sigma,\tau)) \right)$$
$$\times \, \delta^{(d)} \left(X_\mu(\sigma,\tau) - X_\mu(\sigma',\tau') \right) W\left[(X_\mu(\sigma,\tau) \, ; X_\mu(\sigma',\tau')) \right]$$

$$= \int d\sigma \, d\sigma' \left(X'_\mu(\sigma,0) \, X'_\mu(\sigma,0) \right) . \delta^{(d)} \left(X_\mu(\sigma,0) - X_\mu(\sigma',0) \right)$$
$$. W \left[(X_\mu(\sigma,0) \, ; X_\mu(\sigma',0)) \right] \tag{8}$$

where $W\left[(X_\mu(\sigma,\tau) \, ; X_\mu(\sigma',\tau')) \right]$ denotes a arbitrary functional of the string vector position $X_\mu(\sigma,\tau)$.

The term $F[g_{ab}(\sigma,\tau)]$ takes into account the conformal anomaly factor of the string theory eq.(2). In the conformal gauge it can be written as

$$F[\rho(\sigma,\tau)] = \frac{26-(d-m)}{24\pi} \left\{ \int_D \frac{1}{2} \left(\partial_a \log \rho \right)^2 (\sigma,\tau) d\sigma d\tau \right\}$$
$$+ \lim_{\varepsilon \to 0^+} \frac{2+2m-d}{16\pi\varepsilon} \left\{ \int_D \rho(\sigma,\tau) \, d\sigma d\tau \right\} \tag{9}$$

where d is the number of space-time dimensions and m the number of components of the string Majorana Field which will be fixed by requiring that the infinite "cosmological term" $\lim_{\varepsilon \to 0^+} \frac{2+2m-d}{16\pi\varepsilon} \left\{ \int_D \rho(\sigma,\tau) d\sigma d\tau \right\}$ in eq.(5) vanishes ([5]), so we have the relationship $m = (d-2)/2$ for space-time with dimensionality even.

Inserting eq.(7) - eq.(9) in eq.(6), we obtain the following result for the physical space time R^4

$$\int d\mu[g_{ab},X_\mu]\Bigg[\int\Big(\prod_{\substack{-\pi\le\sigma\le\pi\\ 0\le\tau\le T}} d\psi(\sigma,\tau)\Big)\,\psi(-\pi,0)\,\overline{\psi}(\pi,0)$$

$$\times\ \exp - S[X_\mu,g_{ab},\psi]\Big(\Pi^\mu_{in}(\sigma)^2-\frac{1}{2}|X'^{\,in}_\mu(\sigma)|^2$$

$$+\ \frac{(d-m)-26}{24\pi}\Big(-\frac{1}{\rho(\sigma,\tau)}\partial^2\log\rho(\sigma,\tau)\Big)_{\tau\to 0^+}\Big)\Bigg]$$

$$=\frac{\lambda_0}{d}\int_{-\pi}^{\pi}dX_\mu(\sigma').\ X'^{\,in}_\mu(\sigma)\ \delta^{(d)}(X^\mu_{in}(\sigma)-X^\mu_{in}(\sigma'))$$

$$\Bigg\{\int d\mu[g_{ab},X_\mu]\Bigg[\int\prod_{\substack{-\pi\le\alpha\le\sigma\\ 0\le\tau\le T}} d\psi(\alpha,\tau)\ \exp - S^{(-\pi,\sigma)}[g_{ab},X_\mu,\psi]$$

$$\times\ \psi(-\pi,0)\,\overline{\psi}(\sigma,0)\Bigg]\times\Bigg[\prod_{\substack{\sigma\le\alpha\le\pi\\ 0\le\tau\le T}} d\psi(\alpha,\tau)\ \exp - S^{(\sigma,\pi)}[g_{ab},X_\mu,\psi] \tag{10}$$

$$\times\ \psi(\sigma,0)\,\overline{\psi}(\pi,0)\Bigg]\Bigg\}$$

where the "Splitted" string action $S^{(\omega,\omega')}[g_{ab},X_\mu,\psi]$ is defined on the restricted parameter domain $D^{(\omega,\omega')}=$

$$\{(\alpha,\tau)\ ;\ \omega\le\alpha\le\omega'\ ;\ 0\le\tau\le T\ \text{AND}\ -\pi\le\omega\ ;\ \omega'\le\pi\}$$

At this point of our study, <u>we introduce the pure transition amplitude of the string fermionic structure and depending on the string</u> $C_{X(\omega),X(\omega')}$ <u>and on the associated string world-sheet branche</u> $S_{C_{X(\omega),X(\omega')}}=\{X_\mu(\alpha,\tau)\ ;\ (\alpha,\tau)\in D^{(\omega,\omega')}\}$ <u>in a independent fashion</u>. Its expression reads

$$Z[\mathcal{C}_{X(\omega),X(\omega')};S_{\mathcal{C}_{X(\omega),X(\omega')}}] =$$

$$\int \Big(\prod_{\substack{\omega\leq\alpha\leq\omega' \\ 0\leq\zeta\leq T}} d\psi(\alpha,\zeta)\Big) \exp\Big(-S^{(\omega,\omega')}[g_{ab},X_\mu,\psi]\Big) \tag{11}$$

$$\times \;\big(\psi(\omega,0)\,.\,\overline{\psi}(\omega',0)\big)$$

On basis of the identity eq.(10) we, thus, obtain that $Z[\mathcal{C}_{X(-\pi),X(\pi)};S_{\mathcal{C}_{X(-\pi),X(\pi)}}]$ satisfies the loop wave equation written below

$$\Big(\Pi^{\mu}_{iN}(\sigma)^2 - \frac{1}{2}|X^{\prime;iN}_{\mu}(\sigma)|^2 + \frac{(d-m)-26}{24\pi}\Big(-\frac{1}{\rho(\sigma,\zeta)}\partial^2\log\rho(\sigma,\zeta)\Big)_{\zeta\to0^+}\Big)$$

$$Z[\mathcal{C}_{X(-\pi),X(\pi)};S_{\mathcal{C}_{X(-\pi),X(\pi)}}]$$

$$= \frac{\lambda_0}{d}\Big(\int_{-\pi}^{\pi} dX_\mu(\sigma').\;X^{\prime;iN}_{\mu}(\sigma)\,\delta^{(d)}(X^{\mu}_{iN}(\sigma)-X^{\mu}_{iN}(\sigma')) \tag{12}$$

$$\times\; Z[\mathcal{C}_{X(-\pi),X(\sigma)};S_{\mathcal{C}_{X(-\pi),X(\sigma)}}]$$

$$Z[\mathcal{C}_{X(\sigma),X(\pi)};S_{\mathcal{C}_{X(\sigma),X(\pi)}}]$$

Now let us identify in this loop equation the bare string coupling constant with the QCD (SU(∞)) bare gauge coupling constant (i.e.: $\lim_{N_c \to \infty} g^2 N_c = \lambda_0/\alpha$); impose the string proper-time gauge $e_{in}(\sigma) = \text{constant} = |X'^{in}_\mu(\sigma)|^2$ and use the Schroedinger <u>ordered</u> representation for the string "kinetic" operator

$$:\Pi^\mu_{in}(\sigma)^2: = \lim_{\delta \to 0^+} \int_{-\delta}^{\delta} d\varepsilon \, \frac{\delta^{(2)}}{\delta X^{in}_\mu(\sigma + \frac{1}{2}\varepsilon)\,\delta X^{in}_\mu(\sigma - \frac{1}{2}\varepsilon)} \tag{13}$$

As a result we find that the pure fermionic transition amplitude $Z[\mathcal{C}_{X(-\pi),X(\pi)}; S_{\mathcal{C}_{X(-\pi),X(\pi)}}]$ as functional of the contour $\mathcal{C}^{in} = \{X^{in}_\mu(\sigma); -\pi \le \sigma \le \pi\}$ satisfies the unrenormalized $Q.C.D\,(SU(\infty))$ loop wave equation (apart the string mass $-\frac{1}{2}|X'^{in}_\mu(\sigma)|^2$. See eq.(9) - [6] restricted to its bosonic sector). So, it appears natural to suggest that the QCD (SU(∞)) bosonic loop average $W[\mathcal{C}^{in}] = \frac{1}{N_c}\langle Tr\, P\{\exp \oint_{\mathcal{C}^{in}} A_\mu dX_\mu\}\rangle$ should be identified with the quantum string state $\Phi[\mathcal{C}^{in}]$ associated to the interacting string theory eq.(2).

At this point it is worth to note that we have to sum over the fluctuating strings world-sheet S_C in the functional $Z[C, S_C]$ before identify it with $W[C]$. Procedure needed to produce a pure loop functional. Simbolically $W[C] = \sum_{\{S_C\}} Z[C, S_C]$ .next, we remark that if we had imposed the Bianchi identities in our string theory, we would have had to consider the string surface S_C in eq.(11) as being the minimal area surface bounded by the loop C ([7]) and producing, then, a particular non-quantum string representation for QCD (SU(∞) ([4]) and this classical string representation for "static" fermions ($\psi(\sigma,\zeta)\bar{\psi}(\sigma,\zeta)$= constant) reduces to the particular area-behavior solution found in ref.[7].

Our next step is to obtain the migdal elfin string theory as a particular case of our proposed fermionic string theory for "smooth" string world-sheet S, i.e. the string surface $S = \{(X_\mu(\sigma,\zeta))\}$ does not pøsseses self-intersections and the intrinsic metric $\{g_{ab}(\sigma,\zeta)\}$ coincides with that one induced by the surface parametrization $X_\mu(\sigma,\zeta)$ (I.E: $g_{ab}(\sigma,\zeta) = (\partial_a X_\mu \partial_b X_\mu)(\sigma,\zeta) = \tilde{\rho}(\sigma,\zeta)\delta_{ab}$)

In the above cited "smooth" case we have that the self-supressing λ_0-potential in the action eq.(2) can be written in the form below, owing to the normalization condition of the orientation tangent tensor at non self-intersect points $X_\mu(\sigma,\zeta)$ ($T_{\mu\nu}(X(\sigma,\zeta)).T^{\mu\nu}(X(\sigma',\zeta')) = 1$ FOR $\sigma=\sigma'$ AND $\zeta=\zeta'$)

$$\frac{\lambda_0}{d} \int_{\mathcal{D}} d\sigma d\tau\, \tilde{\rho}(\sigma,\tau) \int_{\mathcal{D}} d\sigma' d\tau' \tilde{\rho}(\sigma',\tau') \; (\psi \bar{\psi})(\sigma,\tau)$$

$$\frac{\delta^{(2)}\big((\sigma,\tau)-(\sigma',\tau')\big).\ \delta^{(2)}(0)}{\tilde{\rho}(\sigma,\tau).\ \tilde{\rho}(\sigma',\tau')} \tag{14}$$

So, by evaluating the (σ', τ') integrations we get the (bare) elfin mass term

$$\left(M^{(0)}_{ELF}\right) \int_{\mathcal{D}} (\psi\ \bar{\psi})(\sigma,\tau)\, d\sigma d\tau \tag{15}$$

where the (bare) elfin mass is formally given by $M^{(0)}_{ELF} = \left(\frac{\lambda_0}{d}\right).\delta^{(2)}(0)$
It is remarkable that we have obtained the elfin mass in function of the QCD(SU(∞)) gauge coupling-constant without the use of

the delicate local limit implemented in the Ref.[5]. - 5 and appendix B. We notice that the appearence of the singular term $\delta^{(2)}(0)$ may be related to the existence of a natural cut-off in QCD and probabily being given by a gauge invariant quantized size domain characterising the QCD non perturbative vacum ([7]).

Finally, we comment that our choose for a vanished "cosmological term" eq.(9) is dictated by the physical fact that the only dimensional parameter present in Q.C.D. should be the dimensionally transmuted gauge coupling constant, i.e., the renormalized λ_0 coupling constant.

It will be a subject of a extended paper the investigation in full detail of the phase structure of our proposed QCD (SU(∞)) string SOLUTION. And, thus, we hope to show finally the equivalence between QCD (SU(∞)) and the string theory with fermionic structure eq.(5).

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Note added: we remark that recently we have proposed a fermionic loop space to accomplish in a more invariant way the quark's lorentz spin in the QCD loop formulation ([6], [8]). We, then, derived a closed fermionic looop wave equation which posseses a kind of loop supersymmetry ([6] so, on basis of this supersymmetry we expect that the direct supresymmetric extension of the action eq.(5) should leads to a more realistic string representation for QCD(SU(∞)).

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