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A QUANTUM FERMIONIC STRING FOR THE THREE DIMENSIONAL ISING MODEL

by

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ABSTRACT

We aim in this letter to point out a fermionic string theory which is formally equivalent (on the lattice) to the three dimensional ising gauge model (3D - I.M).

1. INTRODUCTION

It is known that the partition functional of the three dimensional ising gauge model (3D - I.M.) can be expressed in the form ([1])

$$Z = (ch\beta)^N \sum_{\{S\}} \exp\left(-A(S)\log\frac{1}{th\beta}\right)\Phi[\tilde{C}(S)]$$

$$; \quad \Phi[\tilde{C}(S)] = \sum_{\tilde{C}(S)} (-1)^{\ell(\tilde{C}(S))} \tag{1}$$

where the summation $\sum_{\{S\}}$ is carried out over the closed two dimensional surfaces S on a three dimensional regular lattice, $\beta = J/\kappa T$ denotes the ratio of the 3D - I.M. coupling constant and the temperature, N is the number of the plaquettes of the lattice, A(S) is the area of the lattice surface S. The presence of the functional $\Phi[\tilde{C}(S)]$: the sum over the lines of self-intersection $\tilde{C}(S)$ of the surface S weighted with sign factor $(-1)^{\ell(\tilde{C}(S))}$ is introduced to ensure the necessary cancellation of self-intersect surface in eq.(1). Here, $\ell(\tilde{C}(S))$ denotes the total lenght of these selfintersection lines.

The explicity dependence of eq.(1) on the area of the surface S leads to the hope that near its critical point, a (fermionic) string representation should be possible ([1], [2]).

Our propose in this letter is to present a bosonic string theory, which upon being fermionised posseses formally on the lattice the 3D - I.M. partition functional eq.(1).

2. THE STRING THEORY

The simplest model of fluctuating closed surfaces in the continuum R^d is given by the Polyakov's Closed Bosonic String Theory with partition functional $(i=1,\ldots,d)$

$$Z = \int d\mu[g_{ab}, X^i] \; \exp\left(-\frac{1}{2\pi\alpha'} S_0[g_{ab}, X^i]\right) \tag{2}$$

where

$$S_0[g_{ab}, X^i] = \int_D (\sqrt{g}\, g^{ab} \partial_a X^i \partial_b X^i)(\xi)\, d^2\xi \tag{3}$$

denotes the covariant Brink - Di Vecchia - Howe string action and $d\mu[g_{ab}, X^i]$ the covariant functional measure proposed by A.M. Polyakov ([3]).

It is expected that by using a lattice regularization of R^d, eq.(2) should leads to a theory of closed lattice surfaces with statistical weight depending only on the area of these surfaces.

$$Z^{(\mathrm{LATTICE})} = \sum_{\{S\}} e^{-\frac{1}{2\pi\alpha'} A(S)} \tag{4}$$

Proceeding by analogy, it is a natural idea to consider the 3D - I.M. partition functional as defining either a new sort of quantum string theory. However, this quantum string should possesses a more rich structure than the Polyakov's string due to the additional presence of the functional $\bar{\Phi}[\hat{C}(S)]$ in its formal lattice definition eq.(1).

Our basic point to define this 3D - I.M. string is to consider as another string's degrees of freedom (besides the usual ones $\{X^i(\xi), g_{ab}(\xi)\}$; the orthonormal triple of vector $\{e^k(\xi)\}, k=1,2,3\}$ determining the tangent plane and the normal line at a point $\{X^i(\xi)\}$ of the string world sheet.

Since these vectors are orthonormal $\left(g_{ab}(\xi)\cdot e^p_a(\xi).e^q_b(\xi) = \delta^{pq}\right)$ and possessing a SO(3) invariance, they can be considered as the components of a SO(3) σ -field $\Omega(\xi) = e_k(\xi).\lambda_k + 0\cdot\mathbb{1}$ with $\{\lambda_k\}$ denoting the normalized generators of the SO(3) lie algebra. A dynamics can be given naturally by the associated covariant SO(3) σ-action with a Wess - Zumino functional $\Gamma_{W2}(\Omega(\xi))$ ([4]).

$$S_1[\Omega; g_{ab}] = \frac{1}{2g^2}\int_D \left(\sqrt{g}\, \mathrm{Tr}\,(\Omega^{-1}\partial_a\Omega)^2\right)(\xi)\,d^2\xi + 4\pi i\,\Gamma_{W2}[\Omega(\xi)] \tag{5}$$

where g^2 is the σ -coupling constant.

Let us now consider the partition functional of the proposed string theory with complete action $S[X^i, g_{ab}, \Omega] = \frac{1}{2\pi\alpha'} S_0[X^i, g_{ab}] + S_1[\Omega; g_{ab}]$ (see eq.(3) and eq.(5)).

$$Z = \int d\mu[g_{ab}, X^i]\, d^{(H)}[\Omega]\, \exp\left(-S[X^i, g_{ab}, \Omega]\right) \tag{6}$$

where $d^{(H)}(\Omega)$ being the covariant SO(3) - functional Haar measure needed to define in the path integral framework the quantum SO(3) - σ -model eq.(5).

At this point, it is basic for our propose that the bosonic string theory eq.(6) can be fermionized due to the presence of the multivalued Wess-Zumino functional $\Gamma_{W2}[\Omega(t)]$ ([4], [5]) and, thus, leading to a string possessing a (real) fermionic structure in its world-sheet. The fermionized version is given by the following partition functional

$$Z = \int d\mu[g_{ab}, X^i]\, d^{(H)}[\Omega]\, d[\psi]$$
$$\exp\left(-\frac{1}{2\pi\alpha'} S_0[X^i, g_{ab}] - S_2[\Omega, g_{ab}, \psi]\right) \tag{7}$$

with the (covariant) fermionic action $S_2[\Omega, g_{ab}, \psi]$ written down below ($g_{ab}(z) = (e^{\mu}_{\nu} e^{\nu}_{b})(z)\delta_{\mu\nu}$)

$$S_2[\Omega, g_{ab}, \psi] = \frac{1}{2}\int_{D}\left(\bar{\psi}\, e^{a}_{\mu}\gamma_a\left(i\partial_a + g\,\Omega^{-1}\partial_a\Omega\right)\psi\right)(z)\,d^2z \tag{8}$$

Here, the Majorana fermion two dimensional field $\psi(z)$ belongs to the fundamental representation of the SO(3) group.

We shall argument that the above quantum fermionic string theory in R^3 is <u>formally equivalent to the 3D - I.M. at the strong coupling limit</u> $g^2 \to 0$ <u>and with the identification</u> $\alpha' = 2\pi \cdot \lg \frac{1}{th\beta}$

In order to suggest this equivalence, we use the (random) Wilson lattice approximation ([6]) for the action $S_2[\Omega, g_{ab}, \psi]$ (see eq.(8)) and evaluate the (covariant) fermionic functional integration $d[\psi]$ in eq.(7). We, thus, obtain that in the strong coupling limit $g^2 \to 0$, the fermionic functional determinal: det ($e^{a}_{\mu}\gamma_a(i\partial_a + g\Omega^{-1}\partial_a\Omega)$) can be expressed by a sum over closed contours C(S) defined on the string world-sheet S weighted by the SO(3) Wilson Loop factor $W[C(S)] = \mathrm{Tr}\, P\{\exp \oint_{C(S)} \Omega^{-1}\partial_a\Omega\}$ ([6]). Now due to the topological meaning of W[C(S)] as being a topological invariant associated to the immersion of the string surface S into the space-time R^3 ($W[C(S)] \in \pi_1(SO(3)) = Z_2$), we can follow the homogical-homotopical argument of Sedrakyan-

Kavalov ([7], [8]) to see that the lattice version of the above quoted sum over C(S) coincides with the 3D - I.M. functional $\Phi[\tilde{\zeta}(S)]$ (see eq.(1)), and, then, concluding our argument.

It will be a subject of another paper to investigate in full detail the phase structure of the above proposed quantum string theory.

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