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ON A THEORY OF GRAVITATION IN MINKOWSKI SPACE

by

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ABSTRACT

It is shown that a theory of gravitation in Minkowski space, proposed by Logunov et al.[1], does not satisfy the well known results of the field in presence of a spherically symmetric distribution of matter (Schwarzschild field).

Key-words: Gravitation; Bimetric theory; Logunov theory.

Recently Logunov et al.[1] have proposed a theory of gravitation in Minkowski space, following an original idea due to Rosen[2]. Differently of general relativity where gravitation is essentially described by a Riemannian geometry, they propose to describe the gravitational field in Minkowski four-space by means of a physical tensor field $g_{ik}=g_{ik}(\gamma_{mn},\phi_{mn})$, where γ_{mn} is the flat spacetime metric tensor.

In this theory, the metric of four-space is γ_{ik}, but from a mathematical point of view we may say that we have a formulation with two "metric" tensors: γ_{ik} and g_{ik}, since two "connections" γ^i_{jk} and Γ^i_{jk} are used. They are defined by the expressions

$$\gamma^i_{jk} = \frac{1}{2}\gamma^{ia}\left(\frac{\partial\gamma_{aj}}{\partial x^k} + \frac{\partial\gamma_{ak}}{\partial x^j} - \frac{\partial\gamma_{jk}}{\partial x^a}\right) \qquad (1)$$

$$\Gamma^i_{jk} = \frac{1}{2} g^{ia}\left(D_k g_{aj} + D_j g_{ak} - D_a g_{jk}\right) \qquad (2)$$

In these formulae (x^k) denote curvilinear coordinates in Minkowski space. The symbol D_k indicates covariant derivatives for the metric γ_{ik}, with the usual condition $D_k\gamma_{mn}=0$. The objects γ^i_{jk} are third-rank tensors in Minkowski space, and

$$D_k g_{aj} = \partial_k g_{aj} - \gamma^b_{ak} g_{bj} - \gamma^b_{jk} g_{ab}$$

Obviously, in cartesian coordinates the γ^i_{jk} vanish and covariant derivatives become usual partial derivatives. Since the coordinate system is presently fixed by the condition that the metric is γ_{ik} (no coordinate condition is necessary), we may use any coordinates but the transition from one of these systems to another one is merely a mathematical convenience without further meaning. In Galilean coordinates γ_{ik}=diag.(-1,-1,-1,+1).

Since the choice of coordinates is not of importance, we may use Galilean coordinates and introduce the "Einstein-Hilbert" Lagrangian density for the gravitational field

$$L_g = \sqrt{-g}\, g^{ik} \left(\Gamma^m_{i\ell}\Gamma^{\ell}_{km} - \Gamma^{\ell}_{ik}\Gamma^m_{\ell m}\right)$$

Differently of what happens in general relativity, here L_g is a scalar density (in general relativity only δL_g is such an object). In this expression $\Gamma^m_{i\ell}$ have the same expression as in general relativity.

The connection between g_{ik} and the metric tensor γ_{ik} is chosen as

$$g_{ik} = \gamma_{ik} + \phi_{ik} \tag{3}$$

It should be mentioned that relation (3) holds in general, that means, even for the non-linear formulation. Another tensor f_{ik} is defined by

$$f_{ik} = \phi_{ik} - \frac{1}{2}\gamma_{ik}\phi \tag{4}$$

On the components of f_{ik} four conditions are imposed:

$$\frac{\partial f_{i\cdot}{}^{k}}{\partial x^k} = 0 \tag{5}$$

All indices are raised (lowered) with the use of $\gamma^{ik}(\gamma_{ik})$. The con ditions (5), which eliminate the spin 1 part of the components in f_{ik}, are presently taken as field equations.

The Lagrangian density for gravitation and for the matter may be written as

$$L = \sqrt{-g}\; g^{ik}(\Gamma^{m}_{i\ell}\Gamma^{\ell}_{km} - \Gamma^{\ell}_{ik}\Gamma^{m}_{\ell m}) + L_M(g_{ik},\Phi_A) + \lambda^a \partial^b f_{ab} \tag{6}$$

where λ^a are Lagrange multipliers introduced in order to obtain equations (5) as field equations.

Variation with respect to ϕ_{ij} and to the quantities λ^a generates the field equations[1]

$$R^{ij} - \frac{1}{2} g^{ij}R = 8\pi\, T^{ij} \tag{7}$$

$$\partial_k f_i^{\;k} = 0 \tag{8}$$

Since we have used Galilean coordinates in the Action Principle, the tensor R^{ij} and $R = g^{ij}R_{ij}$ have the same expressions as in general relativity.

The four conditions imposed on the g_{ik} (equations (8)) are not uniquely determined, since, as example we may also choose the non-linear conditions

$$\partial_k(\sqrt{-g}\; g^{ik}) = 0 \tag{9}$$

Presently, we want to verify if this formulation satisfies the known observational results for a relativistic theory of gravitation, namely, if it possesses the Schwarzschild solution in presence of spherically symmetric mass distribution.

Since equations (7) are similar to the Einstein equations we may directly obtain from them the exterior Schwarzschild solution in cartesian coordinates. Transforming to polar coordinates we have

$$ds^2 = (1-\chi)(dx^0)^2 - r^2(\mathrm{sen}^2\theta\, d\phi^2 + d\theta^2) - \frac{dr^2}{1-\chi}$$

$$\chi = \frac{2km}{c^2 r}$$

For the quantities ϕ_{ij} and f_{ij} we find the values

$$\begin{cases} \phi_{oo} = -\chi \\ \phi_{11} = \dfrac{-\chi}{1-\chi} \\ \phi_{22} = \phi_{33} = 0 \end{cases} \qquad \begin{cases} f_{oo} = \dfrac{\chi(\chi-2)}{2(1-\chi)} \\ f_{11} = \dfrac{\chi(\chi-2)}{2(1-\chi)} \\ f_{22} = \dfrac{r^2\chi^2}{2(1-\chi)}, \quad f_{33} = \dfrac{r^2 \mathrm{sen}^2\theta . \chi^2}{2(1-\chi)} \end{cases} \tag{10}$$

The subsidiary conditions in polar coordinates take the form

$$D^k f_{ik} = 0 \tag{11}$$

A straightforward calculation shows that the equations (11) for i=0,2,3 are identically verified, but for i=1 (the radial equation) we find

$$\frac{\chi^2}{2} + \chi - 1 = 0$$

since χ is a positive quantity, the solution has the form: χ=0.7. Since here all quantities are already determined (χ has no parameter to be fixed), we conclude that condition (11) is not satisfied in this case. Thus, the present formulation does not satisfy the observational results related to the field with spherical symmetry, as long as the subsidiary condition is taken under the form (11). For a further verification one takes the subsidiary condition as

$$D_k(\sqrt{-g}\; g^{ik}) = 0 \;, \tag{12}$$

or explicitly,

$$g^{ki}(\partial_k\sqrt{-g} - \sqrt{-g}\;\gamma^r_{kr}) + \sqrt{-g}(\partial_k g^{ki} + \gamma^k_{sk}g^{si} + \gamma^i_{sk}g^{ks}) = 0.$$

Again we verify that for i=0,2,3 these equations are identically satisfied, but for i=1 we get

$$\frac{\lambda}{r} = 0 \tag{13}$$

Clearly, such result cannot hold. Thus, the results presently reported seems to indicate that the present formulation of gravitation in Minkowski space is still not correct. In particular, condition (13) can be reproduced by taking $m=0$, that means: equations (7) and (12) are compatible in empty spaces, with solution $\phi_{ik}=0$, or $g_{ik}=\gamma_{ik}$, but this has no physical interest. Another possibility is the asymptotic region $r \to \infty$[3] where (7) and (12) are consistent. However, for finite values of r these equations are not consistent.

REFERENCES

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(3) This case is equivalent to set $m \to 0$, that means, asymptotically the metric g_{ik} is flat, so that it degenerates in the situation discussed previously.