ISSN 0029-3865

CBPF-NF-037/85

GLUON AND $q\bar{q}$ MIXING IN THE η-η'G ($\equiv \iota$(1440) SYSTEM

by

A.C.B. Antunes[1*], F. Caruso[2**], E. Predazzi[2]
and J. Tiomno[1]

[1]Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[2]Instituto di Fisica Teorica dell'Università-Torino, Italy
Instituto Nazionale di Fisica Nucleare - Sezione di Torino

*Also at Instituto de Física da Universidade Federal do Rio de Janeiro, Brasil

**On leave of absence from the Instituto de Física da Universidade Estadual do Rio de Janeiro, Brasil. Fellow of CNPq.

Abstract

A mixing scheme among three pseudoscalar mesons η, η' and $\iota(1440)$ is proposed in order to analyze their $q\bar{q}$ and gluonic components. Inequalities follow from requiring the consistency of the scheme and the range permitted for the mass of the pure gluonic state (glueball) turns out to be rather small. The resulting picture is in good agreement with the overall phenomenology and with the (rather scarce) data on the subject.

Key-words: Pseudoscalar mesons; Glueball.

1 INTRODUCTION

Glueballs, i.e. bound states made solely of gluons, are expected within the framework of QCD[1]. They should have properties quite similar to those of "ordinary" (quark-make) hadrons[2], which makes it difficult to find a well-defined signature to characterize them. As a consequence, in spite of a rather impressive literature on the subject, the experimental status of glueballs is rather confused and their theoretical properties and predictions are strongly model dependent. Some reviews on the subject are given in ref. (3).

Strictly speaking, to search and identify these states we have only a few phenomenological guides which are usually based on the so-called Okubo-Zweig-Iizuka (OZI) rule[4] i.e. one should search for reactions forbidden by this rule but experimentally not suppressed such as, for example $\pi^- p \to \phi\phi n$ [5],[2], $\pi^- Be \to \phi\phi +$ anything[6] etc.

This idea can be traced back to ref. (7) where small OZI rule violations in the $\phi \to \rho\pi$ reaction are interpreted introducing a vector meson O which does not contain quarks and mixes with ω, ϕ and ψ mesons.

A promising channel for positive C-parity glueball is represented by the J/ψ (or ψ) radiative decays with a large branching ratio compared to that of $J/\psi \to$ hadrons[8]. Here again, the OZI rule plays an important role.

Another appealing possibility to study the properties of glueballs comes from looking for mixing effects between $q\bar{q}$ and gluonic components in ordinary hadrons[9]. Considering the aforementioned difficulties in the direct detection of glueballs, this approach could be very fruitful. This procedure has been used, for instance, to derive observable consequences on the f(1270) and to predict the existence of an orthogonal partner of it, named f^{i}, using the MIT bag model prediction of a $J^{PC} = 2^{++}$ glueball around 1.3 GeV[10]. It has also been used[11] to interpret the ØØ enhancement reported around 2.2 GeV[5] as due to a tensor meson state described as an admixture of $q\bar{q}$ and gg components. Also the state named $g_s(1240)$ can be included in this program[12].

The basic point, however, is that, as already stated, different predictions are obtained from different models and the choice of one of them is largely matter of taste. To make the point, we shall briefly review the situation on the predicted glueball mass spectrum which is quite emblematic in this respect. The MIT bag model[13] predicts a glueball ground state around 0.8 GeV whereas potential models[14] tend to predict it in the mass range of 1.0-1.3 GeV. There are also lattice gauge calculations claiming a pseudoscalar glueball mass between 1.25 to 1.66 GeV centered at 1.42 GeV[14.a]. QCD sum rule arguments[15], finally, do not lend any support to the above estimates but put the mass of a 0^{-+} glueball in the somewhat higher region of 2-2.5 GeV[16].

As there are no *a priori* compelling reasons for any of the above predictions, it is our contention that one should resort to phenomenology to shed light on the subject and we shall show how this can be done within an extended mixing scheme. If, indeed, there is a pseudoscalar glueball around 1 GeV, this could drastically effect the standard η-η' mixing and perhaps solve some of its standard problems. The success or failure of this extended scheme could thus be an indirect indication for or against the existence of this pseudoscalar glueball in the 1 GeV region.

A scheme for η-η' - glueball mixing has in fact already been suggested[17] showing that a consistent numerical analysis can be carried out when a state G(1440) is assumed to modify the traditional η'-η mixing. In the new scheme it is found that the usual singlet-octet mixing angle is changed from about 10^0 to about 30^0 [17c]. The main assumption in this new scheme is that the G(1440) is a "glueball" and that the $\eta_c(2980)$ and $\eta_c'(3592)$ contributions can be ignored being their masses too high up from the ones under consideration.

Within this same assumption, in this paper we shall take a different approach to the $q\bar{q}$-gluon mixing in the pseudoscalar meson sector around 1 GeV which, in our opinion, gives a better insight to the problem.

First, we shall briefly review the experimental and phenomenological arguments that suggest a gluonic component in both η and η' therefore supporting the new mixing scheme (Sec. 2).

Based on these arguments, we will thus assume the existence of a third pseudoscalar meson G in the 1 GeV region which leads then naturally to a new admixture between the "traditional" quark components and a new gluonic component.

The key point to get quantitative estimates and predictions is, therefore, that a third pseudoscalar meson should exist not far from the η and η' states. We shall thus consider here this point.

A state $\eta(1275)$ has recently been reported but has been included only in the Data Card listing of the last edition of the

Review of Particle Properties[18] and not in the Meson Tables. We shall, for the time being, ignore the possibility that this should be the proper candidate for our scheme. The only serious candidate for being the third pseudoscalar meson needed in our scheme appears to be the celebrated $\iota(1440)$ which we shall tentatively identify with G. As we shall argue, this identification could indeed solve the longstanding ambiguities connected with the physical nature and the properties of the state $\iota(1440)$[19] which we shall briefly review here.

The possibility of interpreting the $\iota(1440)$ as a $q\bar{q}$ states seems a remote one[20] since, in this case, it should be the isoscalar member of the radially excited pseudoscalar nonet (ζ'). This hypothesis seems to be incompatible with the complete experimental picture[21]. On the other hand, the identification of the $\iota(1440)$ as a glueball[22,23] depends on actually finding the ζ'[21]. Several arguments, and especially the fact that the B.R. ($J/\psi \to \gamma\iota$) is the largest of all decays of the type $J/\psi \to \gamma P_s$ (P_s = pseudoscalar), lead to the conclusion that the ζ' has not yet been detected and, at the same time, that the $\iota(1440)$ is largely of gluonic content (we shall come back to this point in Sec. 2). Furthermore, several authors[24] argue against the interpretation of the $\iota(1440)$ as a "pure" glueball. The fact that three collaborations[24.a] give some evidence of a decay $\iota \to \gamma\rho$, in particular, suggests rather strongly that the $\iota(1440)$ meson could indeed be a mixing of $q\bar{q}$ and gg components. This is exactly what we need for our scheme and this is what we are going to assume in the rest of this paper.

In Sec. 4 we will discuss the phenomenological consequences and the numerical predictions that follow from our assumption that there exists a third pseudoscalar state that mixes with η and η'.

First, we shall find the range of values of the "pure" glueball mass which makes the overall mixing scheme consistent with the data. This will lead us to upper limits for the decay constants F_η and $F_{\eta'}$, and to predict the decay widths for $G\rightarrow 2\gamma$ and $G\rightarrow \rho^0\gamma$. Assuming $F_\eta = F_{\eta'} = F_\pi$ as usually done, we shall restrict to a very small interval the allowed mass of the pure glueball state which turns out very close to that predicted by potential models[14]. In particular, choosing this mass 1.2 GeV (as suggested in Ref.(14c)) as the one that gives the best overall agreement with the data, the new mixing angles are calculated. This allow us to calculate the mass of the octet component η_8 which is found in good agreement with that predicted from the Gell-Mann Okubo mass formula. Also the calculated mass difference $\eta'-\eta$ turns out to be in excellent agreement with the data. The gluonic components of η, η' and G is given together with the fraction of (u,d) quarks in these mesons. Various phenomenological points are discussed. The overall picture turns out to be quite encouraging. Some conclusions are drawn in Sec. 5.

2 EVIDENCES FOR A GLUONIC COMPONENT IN THE $\eta-\eta'$ SYSTEM

Traditionally, the η and η' mesons are known as a singlet--octet mixing of the type[25]:

$$|\eta\rangle = \cos\theta\,|\eta_8\rangle + \sin\theta\,|\eta_1\rangle$$
$$|\eta'\rangle = -\sin\theta\,|\eta_8\rangle + \cos\theta\,|\eta_1\rangle \qquad (2.1)$$

with $|\theta| \approx 10^{\circ}$ [18] and where

$$|\eta_8\rangle = \frac{1}{\sqrt{6}} |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle \qquad (2.2)$$

and

$$|\eta_1\rangle = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} - s\bar{s}\rangle$$

In this framework we obtain for the pseudoscalar nonet, from the invariance of the trace of the mass matrix,

$$\eta_1 + \eta_8 \equiv \eta + \eta' = 2K \qquad (2.3)$$

The last equality (which follows from general arguments[25]) is not satisfied by the data. Indeed the left hand side is about 40% greater than the right hand side. (We use the letters of each meson to denote its mass). This is to be contrasted with the analogous equation for the vector nonet

$$\omega_1 + \omega_8 \equiv \omega + \phi = 2K^* \qquad (2.4)$$

which is well satisfied.

It has been emphasized by several authors[26] that the reason for the abnormal pseudoscalar mixing pattern may be found in QCD, where a new term, corresponding to the annihilation of the $q\bar{q}$ component into gluons, has to be added to the mass matrix of isoscalar mesons. This procedure gives rise to a positive contribution to the right hand side of eq. (2.3). Indeed a consistent picture is found only when this contribution is assumed to be mass dependent[26.b]. This kind of mechanism must be negligible in the vector nonet when eq. (2.4) is well satisfied, and this conjecture is supported by the OZI-suppression in the ϕ ($s\bar{s}$) decay where the $s\bar{s}$ component mixes very little with $u\bar{u}$ and $d\bar{d}$. From this point of view it could be said[21] that the large deviation from ideal mixing of the light pseudoscalar shows that the OZI rule is not satisfied in the J=0 two gluons channel in the mass range of $\sim$1 GeV; differently stated, we could expect a gluonic admixture in the η-η' system.

The possibility seems to overcome the difficulties introduced by considering a $c\bar{c}$ contribution to the η-η' system, which would lead to observable[25-27] decays ($J/\psi \rightarrow \eta$-η'+X) and ($\chi \rightarrow \eta$-η'+X) contrary to the experimental observation.

In a qualitative way we can argue whether the hierarchy of branching ratios for radiative decay modes of the J/ψ e.g.[18], B.R.$(J/\psi \rightarrow \gamma\pi^0) = 0.007 \pm 0.005\%$; B.R.$(J/\psi \rightarrow \gamma\eta) = 0.086 + 0.009\%$ B.R.$(J/\psi \rightarrow \gamma f) = 0.15 \pm 0.04\%$; B.R.$(J/\psi \rightarrow \gamma\eta') = 0.36 \pm 0.05\%$; B.R.$(J/\psi \rightarrow \gamma\iota)$ B.R.$(L \rightarrow K\bar{K}\pi) = 0.42 \pm 0.12\%$, is not in itself indicative that, at least η' and ι have some gluonic contribution. As shown above, in fact, the branching ratios $J/\psi \rightarrow \gamma\eta'$ and $J/\psi \rightarrow \gamma\iota$ are roughly the same (recall that there are some indications that the decays $\iota \rightarrow \eta\pi\pi$ and $\iota \rightarrow 4\pi$ have substantially smaller branching ratios than $\iota \rightarrow K\bar{K}\pi$[28], although there are some difficulties also in this respect[21]). A decay of the type $J/\psi \rightarrow \gamma$ gluonium was estimated by using the relations B.R.$(J/\psi \rightarrow \gamma\iota)$/B.R.$(J/\psi \rightarrow \gamma\eta_c) \gtrsim 1.9 - 3.7$ and B.R.$(J/\psi \rightarrow \gamma\eta_c) \approx$ $\approx 0.7 - 1.5\%$[31]. It was obtained B.R.$(J/\psi \rightarrow \gamma\iota) \approx 1.3 - 5.6\%$. This value is too large compared to the data[18] but is quite in agreement with B.R.$(J/\psi \rightarrow \gamma X) = \Gamma(J/\psi \rightarrow \gamma gg)/\Gamma(J/\psi \rightarrow ggg) \approx 0(5$-$10\%)$ expected from perturbative QCD[32], if the ι is a pure glueball. This indicates that ι has also a $q\bar{q}$ component.

Another qualitative argument is based on the decays $P_s \to 3\pi^0 (\pi^+\pi^-\pi^0)$; $\eta\pi\pi$; $K\bar{K}\pi$, where $P_s = (\eta), \eta', \iota$. It is well known that B.R.$(\eta \to 3\pi^0) \approx 32\%$ and B.R.$(\eta \to \pi^+\pi^-\pi^0) \approx 24\%$, while η' and ι are not seen to decay in these channels. The dominant decay mode of η' is $\eta' \to \eta\pi\pi$ with a B.R. $\approx 65\%$, while the dominant decay mode for the ι is in $K\bar{K}\pi$ channel not $\eta\pi\pi$ (28). This picture could support a mixing scheme for these pseudoscalar mesons since, in such a scheme, we could expect that when one of the three states has a *dominant* decay channel, the decay of the other two, in this same channel, must be *suppressed*. This is well known to be the case in the vector nonet, where w and Ø are orthogonal states, and B.R.$(\omega \to \pi^+\pi^-\pi^0) \approx 90\%$ and B.R.$(\phi \to \pi^+\pi^-\pi^0) \approx 15\%$. It is exactly because the OZI rule works well in this case that we have no reason to alterate the "ideal mixing" for ω-ϕ, while this is not the case in the η-η' system, as we have pointed out above.

A more quantitative evidence of the content of both $q\bar{q}$ and gluonic components in η and η' is contained in the analysis by the Mark III collaboration(33). They use the data on

$$J/\psi \to V + P_s$$

where $V = \omega, \phi$ and $P_s = \eta, \eta'$. They set

$$|\eta\rangle = X_\eta |N\rangle + Y_\eta |S\rangle + Z_\eta |g\rangle$$
$$|\eta'\rangle = X_{\eta'} |N\rangle + Y_{\eta'} |S\rangle + Z_{\eta'} |g\rangle \tag{2.5}$$

with $|N\rangle = (1/\sqrt{2})|u\bar{u} + d\bar{d}\rangle$ and $|s\rangle = |s\bar{s}\rangle$. Using the ratio of the widths $\Gamma(J/\psi \to \omega\eta)$ and $\Gamma(J/\psi \to \phi\eta)$ together with the information coming also from $V \to \gamma + P_s$, one gets $|X_\eta| \approx 0.6 \pm 0.1$ and

$$\begin{aligned} X_\eta^2 + Y_\eta^2 &\approx 1.1 \pm 0.1 \\ X_{\eta'}^2 + Y_{\eta'}^2 &\approx 0.65 \pm 0.1 \end{aligned} \tag{2.6}$$

The above analysis strongly suggests a sizeable gluonic component of η'. We shall see (Sec. 4) that our conclusions are in excellent agreement with these findings.

3 THE MIXING SCHEME. GENERAL RESULTS

Following the motivations discussed in the previous Sections we can, instead of (2.1), consider and admixture of the following type for the physical states $|\eta\rangle$, $|\eta'\rangle$ and $|G\rangle$:

$$|\eta\rangle = (\cos\gamma\cos\alpha - \cos\beta\sin\alpha\sin\gamma)|\eta_8\rangle + (\cos\gamma\sin\alpha + \cos\beta \times$$

$$\times \cos\alpha\sin\gamma)|\eta_1\rangle + \sin\gamma\sin\beta|g_0\rangle \equiv a_{11}|\eta_8\rangle + a_{12}|\eta_1\rangle + a_{13}|g_0\rangle$$

$$|\eta'\rangle = -(\sin\gamma\cos\alpha + \cos\beta\sin\alpha\cos\gamma)|\eta_8\rangle + (-\sin\gamma\sin\alpha + \cos\beta \times$$

$$\times \cos\alpha\cos\gamma)|\eta_1\rangle + \cos\gamma\sin\beta|g_0\rangle \equiv a_{21}|\eta_8\rangle + a_{22}|\eta_1\rangle + a_{23}|g_0\rangle$$

$$|G\rangle = \sin\beta\sin\alpha|\eta_8\rangle - \sin\beta\cos\alpha|\eta_1\rangle + \cos\beta|g_0\rangle \equiv$$

$$\equiv a_{31}|\eta_8\rangle + a_{32}|\eta_1\rangle + a_{33}|g_0\rangle \quad , \tag{3.1}$$

where, together with the singlet and octet $|\eta_1>$ and $|\eta_8>$, given by (2.2), we postulate the existence of a pure gluonic state $|g_o>$. From the orthogonality of the transformation we have the inverse transformation

$$|\eta_8> = b_{11}|\eta> + b_{12}|\eta'> + b_{13}|G>$$

$$|\eta_1> = b_{21}|\eta> + b_{22}|\eta'> + b_{23}|G> \quad (3.2)$$

$$|g_o> = b_{31}|\eta> + b_{32}|\eta'> + b_{33}|G>$$

where $b_{ij} = a_{ji}$. This scheme reduces to (2.1) in the limit $\beta=\gamma=0$ when $|G> \equiv |g_o>$. In this limit α reduces to θ. It follows that the masses of the pure states in terms of the masses of the physical states are

$$\eta_8 \equiv <\eta_8|H|\eta_8> = b_{11}^2\eta + b_{12}^2\eta' + b_{13}^2 G$$

$$\eta_1 \equiv <\eta_1|H|\eta_1> = b_{21}^2\eta + b_{22}^2\eta' + b_{23}^2 G \quad (3.3)$$

$$g_o \equiv <g_o|H|g_o> = b_{31}^2\eta + b_{32}^2\eta' + b_{33}^2 G$$

which immediately give the relation between the masses:

$$\eta_8 + \eta_1 + g_o = \eta + \eta' + G \quad (3.4)$$

In the present paper, we will determine the three angles of the transformation (3.1) by considering the decay modes $\eta \to 2\gamma$ and $\eta' \to 2\gamma$ because the electromagnetic decay is sensitive to non $q\bar{q}$

admixtures. Indeed, if we assume that the electromagnetic decay of η and η' is due to the $q\bar{q}$-components, (recall that the quarks only are charged, not the gluons), we can easily generalize the expressions obtained for the ratios

$$R_\eta \equiv \Gamma(\eta \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma) \quad \text{and} \quad R_{\eta'} \equiv \Gamma(\eta' \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma)$$

considering the standard mixing scheme[26-d]. The standard amplitudes for those decays are given in ref. (25) and the generalized ones are

$$A_{[\eta \to 2\gamma]_{q\bar{q}}} = \frac{1}{\sqrt{3}}\,(a_{11} + 2\sqrt{2}\; a_{12})$$

$$A_{[\eta' \to 2\gamma]_{q\bar{q}}} = \frac{1}{\sqrt{3}}\,(a_{21} + 2\sqrt{2}\; a_{22})$$

$$A_{[G \to 2\gamma]_{q\bar{q}}} = \frac{1}{\sqrt{3}}\,(a_{31} + 2\sqrt{2}\; a_{32})$$

In this case we have the following branching ratios

$$R_\eta = \frac{1}{3}\left(\frac{\eta}{\pi^0}\right)^3 \left(\frac{F_\pi}{F_\eta}\right)^2 (a_{11} + 2\sqrt{2}\; a_{12})^2$$

$$R_{\eta'} = \frac{1}{3}\left(\frac{\eta'}{\pi^0}\right)^3 \left(\frac{F_\pi}{F_{\eta'}}\right)^2 (a_{21} + 2\sqrt{2}\; a_{22})^2 \qquad (3.5)$$

$$R_G = \frac{1}{3}\left(\frac{G}{\pi^0}\right)^3 \left(\frac{F_\pi}{F_G}\right)^2 (a_{31} + 2\sqrt{2}\; a_{32})^2$$

where here again π^0, η, η' and G denote the masses and F_π, F_η, $F_{\eta'}$ and F_G are the decay constants of these four mesons.

Using eqs. (3.5) and (3.1) we obtain in a straightforward way the equations relating α and β to γ. From the first of the two eqs. (3.5) we can obtain an equation for $\cos\beta$ as function of α and γ. Substituting this value into the second eq. (3.5) it follows immediately that

$$\sin(\alpha+x) = \pm \frac{1}{3}\left(\sin\gamma\sqrt{\tilde{R}_{\eta'}} - \cos\gamma\sqrt{\tilde{R}_{\eta}}\right), \qquad (3.6)$$

where $x = \arcsin 1/3$.

With (3.6) we eliminate the dependence on α in the equation for $\cos\beta$ and then obtain β only as a function of γ, R_η and $R_{\eta'}$. It is more convenient to write the equation for $\sin^2\beta$ and we get

$$\sin^2\beta = \frac{9-\tilde{R}_{\eta'}-\tilde{R}_{\eta}}{9-\left(\sin\gamma\sqrt{\tilde{R}_{\eta'}}-\cos\gamma\sqrt{\tilde{R}_{\eta}}\right)^2} \qquad (3.7)$$

where we have defined

$$\tilde{R}_M = 3R_M\left(\frac{\pi^0}{M}\right)^3\left(\frac{F_M}{F_{\pi^0}}\right)^2 \quad , \quad M=\eta,\eta' . \qquad (3.8)$$

As the left side of eqs. (3.6) and (3.7) are trigonometric functions, they can be used to establish constraints concerning the unknown decay constants F_η and $F_{\eta'}$. The point is that the mixing scheme is a consistent one only if $|\sin(\alpha+x)| \leq 1$ and $0 \leq \sin^2\beta \leq 1$. These conditions lead to the statement that the values of the ratios F_η/F_π and $F_{\eta'}/F_\pi$ must be bounded by an eliptic equation, i.e.,

$$\frac{1}{3}R_{\eta'}\left(\frac{\pi^0}{\eta'}\right)^3\left(\frac{F_{\eta'}}{F_\pi}\right)^2 + \frac{1}{3}R_\eta\left(\frac{\pi^0}{\eta}\right)^3\left(\frac{F_\eta}{F_\pi}\right)^2 \leq 1 \qquad (3.9)$$

Using (3.5) we can express the ratio

$$R_G \equiv \frac{\Gamma(G\to 2\gamma)}{\Gamma(\pi^0\to 2\gamma)} = 3\left(\frac{G}{\pi^0}\right)^3 \left(\frac{F_\pi}{F_G}\right)^2 \quad \sin^2\beta\cos^2(\alpha+x) \qquad (3.10)$$

For evaluating the three transformation angles, it is still necessary an equation for γ. For this we use the third eq. (3.3) to obtain

$$\sin^2\beta = \frac{g_o - G}{\eta\sin^2\gamma + \eta'\cos^2\gamma - G}$$

which can be used together with (3.7) to get the equation for γ:

$$[(g_o-G)(9-\mathcal{R}_{\eta'}) + (G-\eta)(9-\mathcal{R}_{\eta'}-\mathcal{R}_\eta)]\,tg^2\gamma +$$

$$+ 2(g_o-G)\sqrt{\mathcal{R}_{\eta'}}\sqrt{\mathcal{R}_\eta}\,tg\gamma + [(g_o-G)(9-\mathcal{R}_\eta) +$$

$$+ (G-\eta')(9-\mathcal{R}_{\eta'}-\mathcal{R}_\eta)] = 0 \qquad (3.11)$$

with $\mathcal{R}_M$ given by (3.8).

At this stage γ is given as function of four "unknown" constants. If we assume g_o to be given by some model (of the types discussed in Sec. 1) and if F_η and $F_{\eta'}$ are either measured or derived in some theoretical way, then eq. (3.9) becomes a one-to-one relation between γ and the G mass. However, we have already discussed the opportunity of taking a different view-point and identifying G as the ι(1440) meson. In this case we can fix the value of G and take, for simplicity, $F_{\eta'} \approx F_\eta \approx F_\pi$, as it is usually done.

By demanding that the mixing scheme be compatible with all the phenomenological constraints, we will be able to derive the mass range for the pure glueball mass g_o and to make comparisons with the models discussed in Sec. 1 (13,14.15). This will enable us to calculate the three mixing angles and make numerical predictions for the widths $\Gamma(G\to 2\gamma)$ (given by (3.10) and $\Gamma(G\to\rho^0\gamma)$. For the latter, we assume again that this decay proceeds only via the $q\bar{q}$ components. It is convenient to calculate the ratio $\Gamma(\eta'\to\rho^0\gamma)/$ $/\Gamma(G\to\rho^0\gamma)$. The amplitudes for these processes are:

$$A_{[\eta'\to\rho^0\gamma]_{q\bar{q}}} = \frac{1}{\sqrt{3}}\,(\sqrt{2}\,a_{22}+a_{21})$$

$$A_{[G\to\rho^0\gamma]_{q\bar{q}}} = \frac{1}{\sqrt{3}}\,(\sqrt{2}\,a_{32}+a_{31})$$

and the phase space is proportional to $(m_M-m_\rho^2/m_M)$, $M=\eta',G$. Then we have

$$\frac{\Gamma(\eta'\to\rho^0\gamma)}{\Gamma(G\to\rho^0\gamma)} \doteq [-\mathrm{cotg}\beta\cos\gamma+\mathrm{cossec}\beta\sin\gamma \quad x$$

$$x\ \mathrm{tg}(\alpha+y)]^2(m_{\eta'}-m_\rho^2/m_{\eta'})^3/\,(1-m_\rho^2/{}_G^2)^3\ ,$$

where $y=\mathrm{arc}\sin(1/\sqrt{3})$.

The numerical results are presented in Sec. 4.

4 NUMERICAL RESULTS

We will start this section making some comments about the decay constants. Using the experimental values(18) π^0=0.135, η=0.549 and η'=0.958 GeV and $R_{\eta'}$=700, R_η=40.4, we obtain from (3.9)

$$0.65 \left(\frac{F_{\eta'}}{F\pi}\right)^2 + 0.20 \left(\frac{F_\eta}{F\pi}\right)^2 \leq 1$$

giving the upper values $F_{\eta'} \lesssim 1.24\ F_\pi$ and $F_\eta \lesssim 2.24\ F_\pi$. Notice that the equality sign corresponds to $\beta=0$, case when there is no mixing for the gluonic component (see (3.1)). As it is not our aim in this paper to calculate the exact values of these constants, we will set $F_{\eta'} \simeq F_\eta \simeq F_\pi$, as it is generally done. With this choice and taking G=1.44 GeV we obtain a constraint on the g_0 value for which eq. (3.11) has a real solution We find

$$0.64 \leq g_0 \leq 1.36\ \text{GeV} \quad .$$

Thus, within the present philosophy, we see that the mass scale of a pseudoscalar glueball can not be in the range 2-2.5 GeV.

We can further restrict the range of g_0 if we notice that should we choose $g_0 \lesssim$ .8 GeV (MIT bag model prediction) we would get either $tg\gamma = -1.28$ or $= -3.83$.

However, if we look back to eqs. (3.1), we see that $|tg\gamma|$ is the ratio of the gluonic components in η and η' respectively. We have already discussed the reasons that indicate that the gluonic component in η' is expected to be greater than in η. This reduces the possible values of g_0 to the rather limited interval $1 < g_0 <$ < 1.36 GeV. In order to test some value in this interval we take $g_0 = 1.2$ GeV which gives the best results when comparing with the overall phenomenological picture. This value is, incidentally, the value suggested in a potential model[14c]. In this case the three angles turn out to be

$$\gamma = -30.6^0 \quad ; \quad \beta = \pm 39^0 \quad ; \quad \alpha = 33^0 \quad . \qquad \textbf{(4.1)}$$

Substituting these values into eqs. (3.1) we obtain:

$$|\eta\rangle \approx 0.93|\eta_8\rangle + 0.14|\eta_1\rangle \mp 0.32|g_o\rangle$$

$$|\eta'\rangle \approx 0.06|\eta_8\rangle + 0.84|\eta_1\rangle \pm 0.54|g_o\rangle \qquad (4.2)$$

$$|G\rangle \approx \pm 0.34|\eta_8\rangle \mp 0.53|\eta_1\rangle + 0.77|g_o\rangle$$

As a consequence, the gluonic content of each meson is the following: for η we have $\approx$ 10%, $\approx$ 30% for η' and $\approx$ 60% for G. This is quite in agreement with the findings of ref. (33), (see eq. (2.6)).

For the mass of the octet component η_8 we find (using (3.3)) η_8=0.645 GeV which compares quite well with the value η_8=0.613 GeV, obtained by using the Gell-Mann-Okubo mass formula(25). For the singlet component we have η_1=1.09 GeV. The resulting mass difference $m=\eta'-\eta$=0.420 GeV, also obtained from (3.3), compares also very well with the experimental value 0.410 GeV.

The next point concerns the predictions for the decay rates of G, quoted in Sec. 3. We consider first the decay $G \to 2\gamma$. From eq. (3.8) and using the experimental value for $\Gamma(\pi^0 \to 2\gamma)$ (18) we have

$$\Gamma(G \to 2\gamma) = 4.27 \ (F_\pi/F_G)^2 \text{ KeV}. \qquad (4.3)$$

In order to discuss the above result, we have, first of all, to understand the mechanism which determines the relative size of the various electromagnetic decays within the present scheme. In particular, we note that the contribution that comes from the $|\eta_8\rangle$ and $|\eta_1\rangle$ couplings to $|2\gamma\rangle$ for the ratio $R_{\eta'}/R_\eta$ is $(a_{21}+2\sqrt{2}\,a_{22})^2/(a_{11}+2\sqrt{2}\,a_{12})^2 \approx 3.3$ while the contribution from phase space is of the order of $(\eta'/\eta)^3 \approx 5.3$, which reproduces the experimental ratio $R_{\eta'}/R_\eta \approx 17$, value that was used as an input in determining α and β. Although η' has a gluonic component three times greater than that of η and as we have assumed that these electromagnetic decays are due only to the $q\bar{q}$-component of the mesons, η' has also a large singlet component which, together with the large phase space contribution, is the origin of $R_{\eta'}/R_\eta > 1$. Going back to the discussion of eq. (4.3), if we use eq. (3.5) we get ≈ 0.75 for the ratio $(a_{31} + 2\sqrt{2}\,a_{32})^2/(a_{11} + 2\sqrt{2}\,a_{12})^2$, but the phase space contribution is $(G/\eta) \approx 17$. Thus, if $F_G \approx F_\eta$ we should have $\Gamma(G \to 2\gamma) > \Gamma(\eta \to 2\gamma)$. Recalling that $\Gamma(\eta' \to 2\gamma) > \Gamma(\eta \to 2\gamma)$ the expected hierarchy for the various radiative decays is

$$\Gamma(\eta \to 2\gamma) < \Gamma(G \to 2\gamma) \lesssim \Gamma(\eta' \to 2\gamma) \ .$$

Assuming $F_G \approx F_\pi$ eq. (4.3) gives us $\Gamma(G \to 2\gamma) \approx 4.3$ KeV, which is indeed roughly comparable to $\Gamma(\eta' \to 2\gamma)$ as expected ($\Gamma(\eta' \to 2\gamma) \approx$ ≈ 6 KeV [(18)]). This result is about one order of magnitude smaller than the prediction of ref. (17-c), which gives the unreasonably large value $\Gamma(G \to 2\gamma) \approx 70 \pm 30$ KeV.

The previous result is relevant when applied to predictions of the reaction $e^+e^- \to e^+e^- G$.

Recently the cross section for η' production in the reaction $e^+e^- \to e^+e^-\eta'$ (and $\eta' \to \pi^+\pi^-\gamma$) has been measured[(34)] in the range of beam energies $E \approx 2.0$–3.7 GeV. If we use the same equivalent-photon-approximation for the cross section for production

$$\sigma(e^+e^- \to e^+e^- X) = 16\alpha^2 \frac{(2J+1)}{M_x^3} \Gamma(X \to \gamma\gamma) [\ell n \frac{E}{m_e} - \frac{1}{2}]^2 \, f\left(\frac{M_x}{2E}\right)$$

with $f(y)=(2+y^2)^2 \ln(1/y)-(1-y^2)(3+y^2)$ (and m_e is the electron mass), and our prediction for $\Gamma(G\to 2\gamma)$ given by (4.3), we can estimate the cross section $\sigma(e^+e^-\to e^+e^-G)$. Taking the values J=0 and M_x=G we obtain

$$\sigma(e^+e^-\to e^+e^-G) \simeq 55 \left(\frac{F_\pi}{F_G}\right)^2 \text{pb} \quad , \quad \text{for } E=2.0\text{ GeV} \quad \text{and}$$

$$\sigma(e^+e^-\to e^+e^-G) \simeq 136 \left(\frac{F_\pi}{F_G}\right)^2 \text{pb} \quad , \quad \text{for } E=3.7\text{ GeV}.$$

Thus, when the cross section $\sigma(e^+e^-\to e^+e^-G)$ will be measured at different energies, the above estimates will give a specific determination of F_G and represent an explicit check to the scheme proposed. A search was made for production of E(1420) in two-photon interactions[34] looking for a final state $K^0K^\pm\pi^\mp$; no signal was observed. However we would like to point out that the measured cross sections for the reaction $e^+e^-\to e^+e^-\eta'$ are one order of magnitude greater than our prediction if $F_G \approx F_\pi$.

Going now to the prediction for the decay rate $G\to\rho^0\gamma$, we have

$$\Gamma(G\to\rho^0\gamma) \approx 0.21\ \Gamma(\eta'\to\rho^0\gamma)$$

where eq. (3.10) has been used. Experimentally we have[18] B.R.$(\eta'\to\rho^0\gamma)$= =30 ± 1.6% and then $\Gamma(G\to\rho^0\gamma) \approx 261$ KeV, leading to B.R.$(G\to\rho^0\gamma)\approx 10^{-3}$ which does not seem to be incompatible with the present limits[24.a].

We end this phenomenological overview with an evaluation of the $q\bar{q}$ content in the various pseudoscalar mesons, in order to compare with the findings of ref. (33). We set

$$P_s = \chi_{P_s}(u\bar{u} + d\bar{d})/\sqrt{2} + Y_{P_s}(s\bar{s}) + Z_{P_s} \quad (gluonic)$$

with $P_s = \eta, \eta'$ and G and we find

$$|\chi_\eta| \gtrsim 0.65 \quad ; \quad |\chi_{\eta'}| \gtrsim 0.72 \quad ; \quad |\chi_G| \gtrsim 0.24 \; .$$

This result together with the predictions for gluonic components in each P_s are in good agreement with the values quoted previously (eq. (2.6)).

5 CONCLUSIONS

In this paper we have discussed the gluon and $q\bar{q}$ mixing in the three pseudoscalar meson system η-η'-G by neglecting the charm sector. The general formalism has been reduced to numerical comparison arguing that the third pseudoscalar meson G has to be identified with the celebrated ι(1440).

First of all, inequalities for the ratios of decay constants F_η and $F_{\eta'}$ to F_π have been derived and the range of masses allowed for the "pure" glueball has been obtained by the requirement that the mixing scheme be compatible with the data. The result puts our estimate in the range predicted by potential models[14] rather than by the MIT bag model[13] or by the QCD sum rule calculations[15]. In particular, we point out the result from ref. (14-c), where the lowest energy glueball state, obtained from a relativistic wave equation with a one-parameter potential setting the gluon mass equal zero, has a mass of 1209 MeV.

The overall numerical picture that emerges from our scheme taking the glueball mass of 1200 MeV is quite nicely consistent with the (scarce) phenomenological information we have at present on this subject and with analogous analysis of the data[33].

Better data are certainly needed to give a sounder ground to this investigation but the approach seems promising and deserves further attention.

ACKNOWLEDGEMENTS

One of us (F.C.) would like to thank the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brasil for financial support.

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