ELECTRON CAPTURE AND STELLAR CORE IMPLOSION

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SUMMARY

In order to examine the role of electron capture as a trigger of implosion, the change of chemical composition in a highly evolved stellar core is followed. For simplicity, a one-shell model of presupernova is used to treat the hydrodynamics, and the problem of neutrino energy transfer is short-circuited by the introduction of a single parameter. The e-capture rates are obtained from the gross theory of nuclear beta decay.

Calculations are done for three different degrees of neutrino confinement, and the results show that the e-capture is very efficient as a trigger mechanism of the implosion.

Key words: electron capture - stellar collapse - supernova.

I. INTRODUCTION

At the presupernova stage of stellar evolution, nuclei and free protons in very dense stellar cores are to capture electrons (in the continuum) of the environment, causing a rapid neutronization of the core and the corresponding emission of neutrinos. These neutrinos are believed to be good carriers of energy from the inner shells to the outer shells of the star, so that it is legitimate to regard the e-capture as one of possible trigger mechanisms of the core implosion (gravitational collapse). See Chiu (1964) and Bahcall (1964).

In effect, e-capture was already considered in the earlier hydrodynamic calculations by Colgate and White (1966), and Arnett (1966), although none of them had coupled directly the e-capture to hydrodynamics. In other words, the effect of changes in the chemical composition was not treated consistently with the hydrodynamics of the stellar matter.

However, recently, Sato (1975) and Arnett (1977) have shown that the neutrinos would be trapped inside the stellar core, when the neutral currents in weak interactions are taken into account. If the neutrino confinement is the case, it may cause at least the following three basic effects: 1) heating up of the material due to the neutrino energy deposition; 2) pressure increase due to neutrinos; 3) suppression of capture rates due to the neutrino degeneracy.

In this way, the role played by the e-capture as the trigger mechanism of the core implosion is not so pacific, because the core would be opaque before the e-capture becomes efficient. Thus we think it is necessary to clarify the effect of e-capture at the onset of implosion by performing a detailed calculation which couples, in each step, the e-capture to the hydrodynamics, especially to investigate the consequences of the item 1 mentioned above. Items 2 and 3 will become important at the later stage of core collapse.

As we are interested in how the e-capture process induces the core collapse and heats the stellar matter up, we choose a rather low temperature presupernova model, where only few reactions other than e-capture take place. Therefore we do not assume nuclear statistical equilibrium for the nuclear abundances and for the equation of state. We start with a temperature equal to $T_9 = 1$ ($T_9 \equiv T/10^9\ {}^oK$), while the nuclear statistical equilibrium approximation is good only for $T_9 \gtrsim 3$ (Koebke et al.,1974; Yokoi et al., 1979).

In order to simplify the numerical calculations we have used a one-shell (homogeneous) model of presupernova and we have short-circuited the problem of energy transfer by neutrinos, introducing a parameter, which corresponds, in one limit, to the free escape of neutrinos and, in the other limit, to the total confinement. Beside the e-capture, we have also included several other processes, although not dominant in our calculations: the beta decay, the photo-emission of neutron and the delayed neutron emission in e-capture as well as in beta decay.

In this form, we constructed a reaction network of nuclei, going from $A = 56$ down to the $A \simeq 6$ family, involving about 280 nuclei.

II. THE MODEL

In this section, we present our model which substantially truncates the hydrodynamic calculation as well as the neutrino energy transport problem. These simplifications are due to the limitations, both in time and in memory size in numerical computation. However, we expect the main features of the problem to be described qualitatively by our model.

A. EQUATION OF MOTION

The homogeneous model of presupernova has the following Lagrangean

$$L = \frac{3}{10} M\dot{R}^2 - M\varepsilon/\rho + \frac{3}{5} GM^2/R \qquad (1)$$

where R, ε, ρ and M are radius, energy density, matter density and the total mass of the star, respectively. G is the gravitational constant and the dot denotes time derivative. In Eq. (1), ρ is related to R by $\rho = 3M/(4\pi R^3)$.

The equation of motion is

$$\ddot{R} = \frac{5P}{\rho R} - \frac{GM}{R^2} \qquad (2)$$

where $P = \rho \frac{\partial \varepsilon}{\partial \rho} - \varepsilon$ is the pressure.

To simulate a quasi-static stellar core at the presupernova stage, the initial condition should be consistent with Eq. (2), i.e,

$$\left[\frac{5P}{\rho R} - \frac{GM}{R^2}\right]_{t=0} = 0 \qquad (3)$$

which gives the core mass as a function of the initial values of density and temperatures ρ_0 and T_0, respectively. In our calculation, we take $\rho_0 = 2 \times 10^9$ g cm^{-3} and $T_0 = 10^9$ ^oK, so that, with the help of the equation of state described later, $M = 1.5\ M_\odot$.

B. REACTION NETWORK

In order to calculate the variation of nuclear abundances during the hydrodynamic motion we consider the following processes:

a) Electron Capture - Because we work in the low temperature region ($T_9 < 3$), we neglect the e-capture by free protons (Epstein and Arnett, 1975). So hereafter e-capture will mean e-capture by complex nuclei only, which cause a neutronization of the nuclei along A = constant, and proceeds untill they attain the neutron drip line, or are interrupted by (γ,n) reactions, thus emitting a free neutron.

b) Beta Decay - The emission of an electron occurs whenever the atomic mass-difference exceeds the electron Fermi energy ε_F, i.e.

$$Q = M(Z+1,A) - M(Z,A) > \varepsilon_F \tag{4}$$

so that a neutron in the nucleus decays into a proton, an electron and an antineutrino.

c) Delayed Neutron Emission - The emission of a neutron proceeds whenever the released energy in a β-decay or e-capture of a nucleus exceeds S_n, the neutron separation energy. Such neutron emissions generate abundances of lighter isobaric families.

d) (γ,n) Reactions - These reactions also provoke the gradual appearance of lighter isobaric families. However, along the whole nuclei network, they have very small contributions except near the neutron drip line and when $T_9 > 2.5$.

Thus, the change of the nuclear abundance $N_{Z,A}$ of the nucleus (Z,A) is expressed as

$$\frac{dN_{Z,A}}{dt} = - \sum_i \lambda_i N_{Z,A} + \lambda_{ec} N_{Z+1,A} + \lambda_\beta N_{Z-1,A} + \lambda_{\gamma n} N_{Z,A+1} + \lambda_{ed} N_{Z+1,A+1} + \lambda_{\beta d} N_{Z-1,A+1} \quad (5)$$

where the subscripts ec, β, γn, ed and βd denote, respectively, e-capture, beta decay, γ-n reaction, delayed neutron emission in e-capture and delayed neutron emission in beta decay. The subscript i in Eq. (5) runs through all the mentioned processes.

C. THERMODYNAMICS

In order to calculate the temperature change in time, we have to know how much nuclear binding energy is converted into heat. The binding energy released by the nuclear processes listed in the previous section is distributed in the form of kinetic energy of particles, radiation energy and neutrino energy. We assume that the kinetic and radiation energies attain immediately the thermodynamical equilibrium with matter. Thus, we are able to express these contributions to the temperature change in terms of the chemical potential of the constituents. As mentioned in the Introduction, we avoid the complicated calculation of the neutrino energy transfer problem, by introducing a parameter χ, which represents the degree of neutrino confinement in the matter.

Using the first law of thermodynamics (Arnett, 1966), we get

$$\frac{dT}{dt} = - \left\{ (1-\chi)\dot{Q} + \left[P + \left(\frac{\partial E}{\partial V}\right)_{T,\{N_{Z,A},n_e,n_n\}} \right] \frac{dV}{dt} + \right.$$
$$\left. + \sum_{Z,A} \mu_{Z,A} \frac{dN_{Z,A}}{dt} + \mu_e \frac{dn_e}{dt} + \mu_n \frac{dn_n}{dt} \right\} \Big/ \left(\frac{\partial E}{\partial T}\right)_{V,\{N_{Z,A},n_e,n_n\}} \quad (6)$$

where T is the temperature, E the specific energy, $V=1/\rho$ the specific volume, $\mu_{Z,A}$, μ_e and μ_n the chemical potentials of nucleus (Z,A), electron and neutron, respectively. $n_{e(n)}$ denotes the electron (neutron) specific number density. $\dot{Q}$ is the total energy released into neutrinos by e-capture or beta decay, and approximated as

$$\dot{Q} = \sum_{Z,A} \left[(\lambda_{ec} + \lambda_{ed}) \langle \varepsilon_\nu \rangle + (\lambda_\beta + \lambda_{\beta d}) \langle \varepsilon_{\bar{\nu}} \rangle \right] N_{Z,A} \quad (7)$$

where $\langle \varepsilon_\nu \rangle$ ($\langle \varepsilon_{\bar{\nu}} \rangle$) is the neutrino (antineutrino) mean energy associated with an e-capture (β-decay) of nucleus (Z,A). We approximate the chemical potentials for nucleus and neutron as

$$\mu_{Z,A} \simeq - \delta M(Z,A) \quad \text{MeV} \quad (8)$$
$$\mu_n \simeq 8.07 \quad \text{MeV} \quad ,$$

since $kT_9 \ll 1$ MeV. Electron chemical potential is numerically calculated from Fermi integrals (see Sec. IV). The parameter χ is introduced in the way that $\chi = 0$ corresponds to the free neutrino escape, and $\chi = 1$ corresponds to the complete neutrino confinement.

III. REACTION RATES

The transition rates of (γ,n) reactions were taken from Sato (1974) with minor modifications in order to incorporate recent results of giant dipole resonance theory (Myers et al., 1977).

The beta decay rates were determined, using the gross theory, such as prescribed in Kodama and Takahashi (1975), instead of the usual theory of beta decay (Hansen, 1968; Mazurek et al., 1974; Epstein and Arnett, 1975; etc.).As far as the astrophysical applications are concerned, the gross theory is advantageous because it takes into account the general trends of β-strength functions. However, see Pabst (1978).

The gross theory of beta decay is used also for computing the e-capture rates, first developped by Egawa et al. (1975).Some further calculations are done by Takahashi et al. (1978) and Hilf et al. (1979). Here, we essentially follow their calculations, neglecting the effect of finite temperature. This reduces enormously the computational time, without changing the global behavior of the collapse significantly, but only dislocating the threshold density to smaller values. We neglected also the forbidden transitions, as well as the effects of excited states of the nuclei.

Thus the e-capture rates are calculated by

$$\lambda_c \simeq \frac{m_e^5 c^4}{2\pi^3 \hbar^7} \int_{-Q}^{\varepsilon_F} \left[|g_F|^2 |M_F(E_T)|^2 + |g_{GT}|^2 |M_{GT}(E_T)|^2 \right] dE_T \int_{E_T+1}^{\varepsilon_F+1} w\sqrt{w^2-1}\; F(Z,A,w)(w-1-E_T)^2\, dw \qquad (9)$$

and the neutrino (antineutrino) mean energy is given by

$$\langle \varepsilon_{\nu(\bar\nu)} \rangle = \frac{\int_{-Q}^{\varepsilon_F} \left[|g_F|^2 |M_F(E_T)|^2 + |g_{GT}|^2 |M_{GT}(E_T)|^2 \right] dE_T \int_{E_T+1}^{\varepsilon_F+1} \varepsilon_{\nu(\bar\nu)}\, w\sqrt{w^2-1}\; F(Z,A,w)(w-1-E_T)^2\, dw}{\int_{-Q}^{\varepsilon_F} \left[|g_F|^2 |M_F(E_T)|^2 + |g_{GT}|^2 |M_{GT}(E_T)|^2 \right] dE_T \int_{E_T+1}^{\varepsilon_F+1} w\sqrt{w^2-1}\; F(Z,A,w)(w-1-E_T)^2\, dw} \qquad (10)$$

For notation and more details, we refer the reader to Egawa et al. (1975).

In order to save further computational time, we fitted the capture rates as a function of density ρ in the following form

$$\log \lambda_{ec} = \frac{1}{2\alpha} \left\{ - \beta + \left[\beta^2 + 4\alpha \log \frac{\rho}{\bar{\rho}_0}\right]^{1/2} \right\} + \log \gamma \qquad (11)$$

where α, β and γ are the fitting parameters, and $\bar{\rho}_0$ is related to the threshold density of the nucleus ρ_{th} by $\bar{\rho}_0 = 50\ \rho_{th}$.

The Eq. (11) reproduces the gross theory calculation within an error around 5% for most nuclei. However, for some nuclei errors become larger but not exceeding 40%, still being acceptable.

The delayed neutron emission rates are calculated in a similar manner described in Kodama and Takahashi (1975), both for β-decay and e-capture.

IV. EQUATION OF STATE

In the density range we are interested, the matter is composed of nuclei, electrons and neutrons. The nuclei are treated as nonrelativistic Boltzmann perfect gas and the electrons and neutrons as ideal Fermi gases. We neglect interactions among them (For interacting Fermi gas, see El Eid and Hilf (1977)). In this density region, electrons are relativistic and neutrons are nonrelativistic. Radiation energy and pressure are included, but they have appreciable contribution only in low density and low temperature.

We avoided to utilize the simplified formulae of completely degenerate gas for the electron and neutron gases, because in the density and temperature ranges in concern these gases are not completely degenerate. In order to include the semi-degeneracy effect of theses gases, we used the procedure given by Chiu (1968) for calculating the finite temperature Fermi integrals.

With respect to nuclear masses, we used experimental values (Wapstra and Bos, 1977) whenever possible. When not possible, judicious extrapolation, using Myers-Swiatecki mass formula (Myers and Swiatecki, 1969), was performed.

V. RESULTS AND DISCUSSION

We started the calculation with the density $\rho = 2 \times 10^9 g\ cm^{-3}$, $T_9 = 1$, for a pure ^{56}Fe core with radius $R = 7.4 \times 10^7$ cm and total mass $M = 1.5\ M_\odot$. This choice of initial condition has no particular significance but we took the density higher than the e-capture threshold density of ^{56}Fe ($\sim 10^9$ g cm^{-3}), in order to skip the prolonged quasi-static evolution stage.

In Fig. 1, we plotted the stellar radius as function of time for three values of the parameter X . It can be seen that in all cases the radius at beginning shrinks gradually in time, then leading to the collapsing stage. The greater the value of X is, the later the collapse takes place. In other words, the core collapse is favoured by the free neutrino escape, and slowed down by the neutrino confinement. In Fig. 2 , the radial velocities are plotted against time and in Fig. 3 , it is shown the variation of the density with respect to time. The Fig. 4 gives the abundance of the first three nuclei along the time for X = 1 (other values of X give essentially the same behavior). The abundance of ^{56}Mn is due to the e-capture processes of ^{56}Fe. From this figure we see that the chemical composition changes accompanying the density increase.

The e-capture of ^{56}Fe induces an effect of decreasing the electron pressure against the gravitational forces, then leads to the density increase of the core. In general, daughter nuclei have higher e-capture Q-values, but the increase of the density surpasses the threshold value, so that these daughter nuclei undergo e-capture. Thus, the compression continues to generate neutron-rich nuclei, leading to the collapsing stage. Furthermore, nuclei produced by the n-emission process have usually smaller e-capture Q-value than the precursors. This accelerate the mechanism of core collapse.

To see more clearly the effect of e-capture on the chemical composition, we plotted in Fig.5 the mean molecular weight per electron, $\mu_e = 1/\sum_i \frac{Z_i X_i}{A_i}$, where X_i is the mass fraction of the nucleus i. The abrupt increase at about t = 1.2 sec is perfectly in accordance with the density curves shown in Fig. 3.

In Fig. 6 , the evolution curves in ρ-T plane are shown for each X value. The dashed straight line is the 4/3 adiabatic boundary, which stands for the dynamic stability line. The three curves begin to deviate from the boundary due to e-capture processes, the decay heat of which contributes to increase the temperature more than the adiabatic compression. For $\rho > 3.2 \times 10^9$ g cm^{-3}, the effect of neutrino confinement splits appreciably in three curves according to the values of parameter X. When the temperature becomes about T_9=2.5, the three curves begin to degenerate. Then, the temperature tends to stabilize. This is due to endoenergetic (γ,n) reactions which become very active at this temperature.

The present work showed that, independent of the degree of neutrino confinement, the stellar core is led to collapse because of the e-capture of seed nuclei. It is also shown that the hydrodynamic aspects are essentially governed by their e-capture rates.

In our calculations, the neutrino degeneracy effect was neglected (see also Bethe et al., 1979). Assuming that all neutrinos are completely confined in the core from the beginning, the upper limit of neutrino Fermi energy is estimated as around 4.5 MeV at the onset of the implosion ($\rho \sim 10^{10}$ g cm^{-3}) where only 5% of all electrons are captured. At this density, the electron Fermi-energy is 11 MeV.

The e-capture rates with the neutrino degeneracy taken into account are given by

$$\lambda_{\nu\text{-deg}} \simeq \frac{m_e^5 c^4}{2\pi^3 \hbar^7} \int_{-Q}^{\varepsilon_F - \varepsilon_\nu} \left[|g_F|^2 |M_F(E_T)|^2 + |g_{GT}|^2 |M_{GT}(E_T)|^2 \right] dE_T$$

$$\int_{E_T+1+\varepsilon_\nu}^{\varepsilon_F+1} w\sqrt{w^2-1}\; F(Z,A,w)(w-1-E_T)^2 dw \qquad (12)$$

where ε_ν is the neutrino Fermi energy. For ε_ν = 4.5 MeV and ε_F = = 11 MeV, Eq. (12) yields

$$\lambda_{e\text{-deg}} \simeq 0.8\, \lambda_{ec} \quad ,$$

where λ_{ec} is given by Eq. (9).

Thus the effect of neutrino degeneracy on the e-capture rate is not important up to the onset of implosion. In other words, the stellar core is led to collapse before the neutrino degeneracy suppresses the e-capture process significantly.

We followed the core implosion caused by e-capture process up to $\rho = 3 \times 10^{10}$ g cm^{-3}, $T_9 = 3$. For higher density and temperature values, our calculation fails due to other nuclear reactions which become efficient. It is known that for $T_9 > 3$, the approximation of nuclear statistical equilibrium is fairly good. However, to apply the nuclear equilibrium for the equation of state, the correct value of N/P ratio is relevant (see e.g. Takahashi et al.,1978). This can be known only from the explicit calculation of all e-capture processes in the duration of the collapsing stage as we have demonstrated in this work.

Since the mechanism of supernova explosion is so sensible to the equation of state and to the hydrodynamics, the correct treatment of e-capture from the presupernova stage, coupled to the hydrodynamics, should be essential.

ACKNOWLEDGEMENT: *The authors would like to thank Prof. E.R. Hilf, for reading the manuscript and for his several critical comments and suggestions.*

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FIGURE CAPTIONS

Fig. 1 - The plot of the core radius R as a function of time. R_0 is the initial core radius, $R_0 = 7.4 \times 10^7$ cm.

Fig. 2 - The radial velocity $\dot{R}$ plotted against the time.

Fig. 3 - The plot of the density ρ against the time.

Fig. 4 - Abundance curves (normalized to the initial ^{56}Fe abundance) of the first three members of the A = 56 family, for the X = 1.

Fig. 5 - The relative variation of the chemical potential μ_e as a function of time. The superscripts (i) and (f) denote initial and final values.

Fig. 6 - The evolution curves in the ρ-T plane. The dashed line stands for the 4/3 adiabatic boundary.

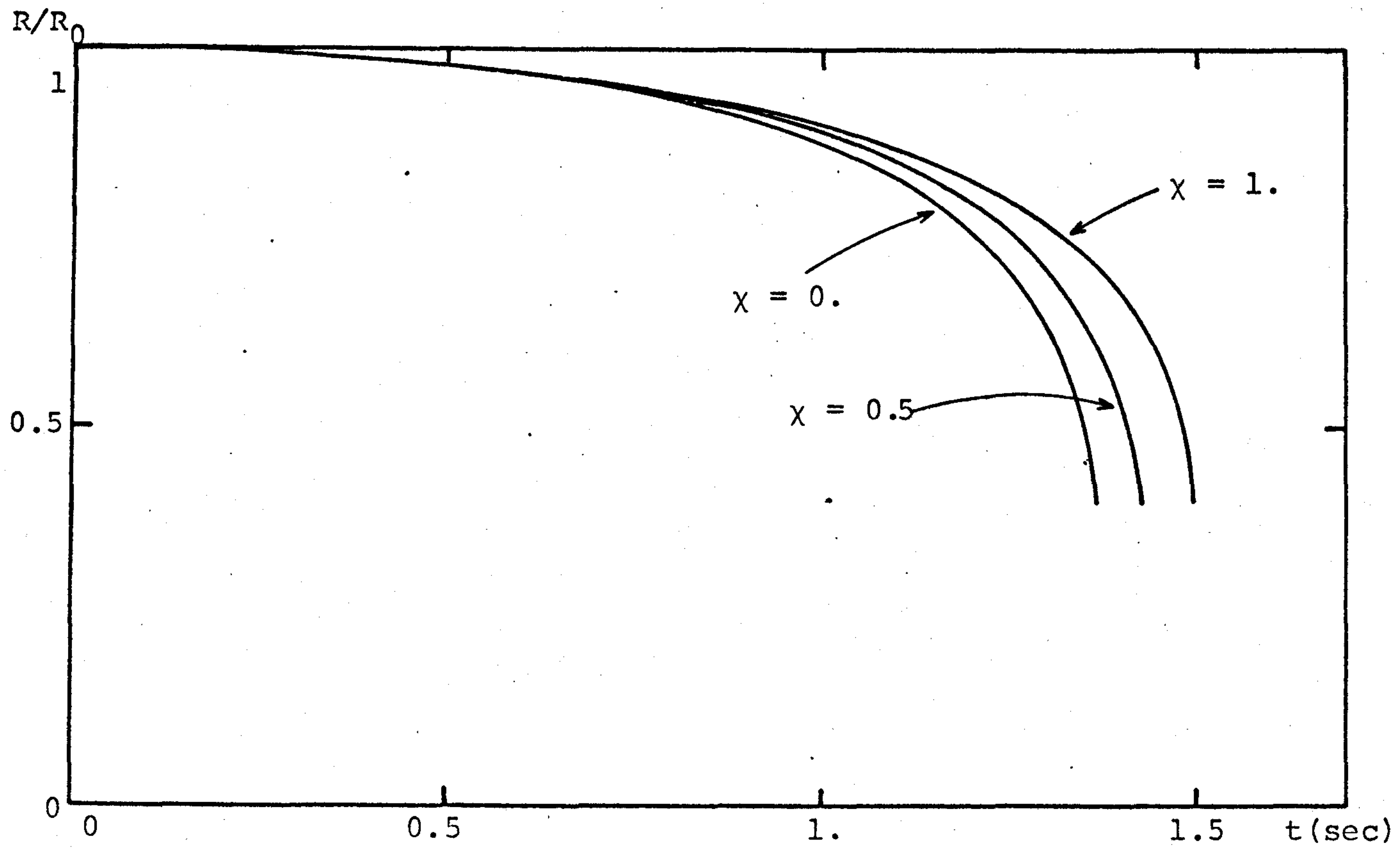

Fig. 1
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Fig. 2

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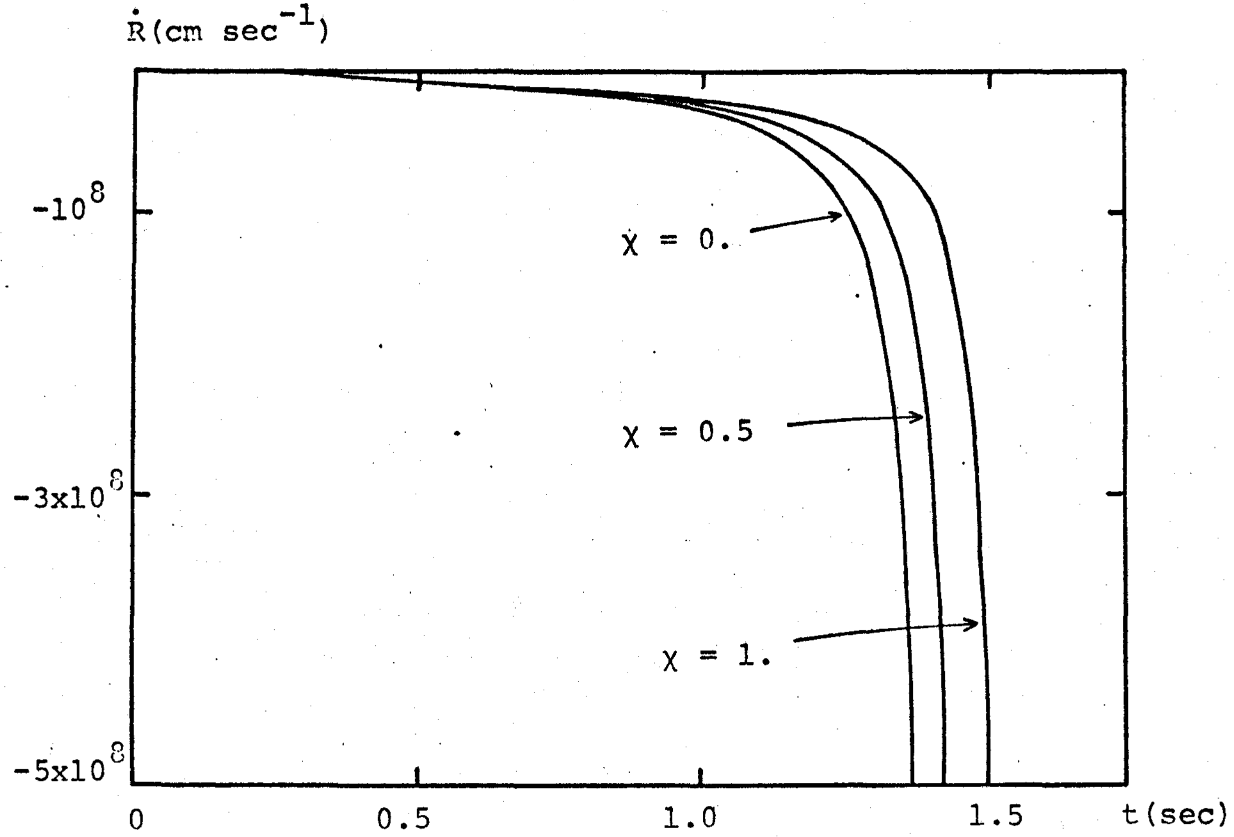

Fig. 3

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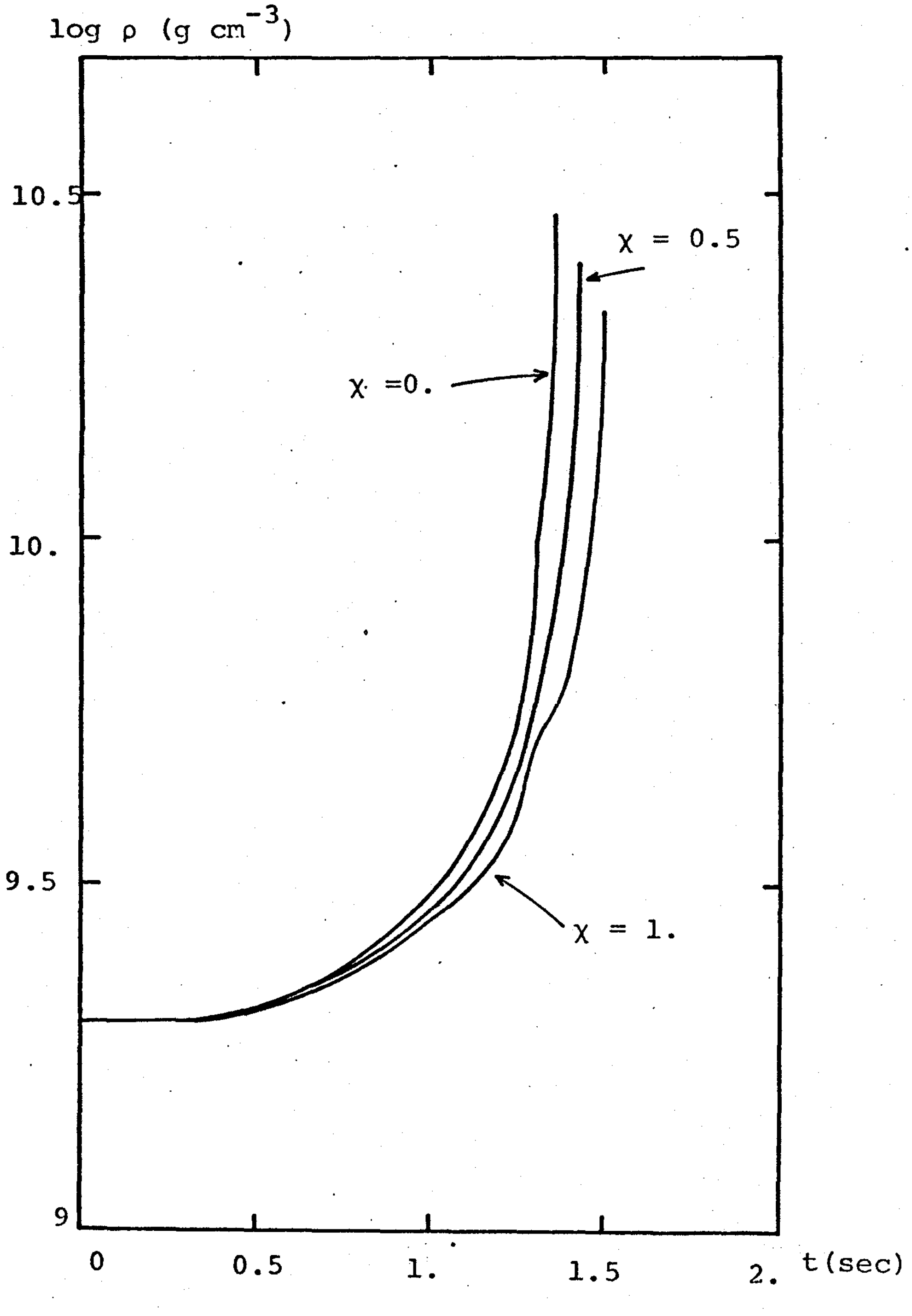

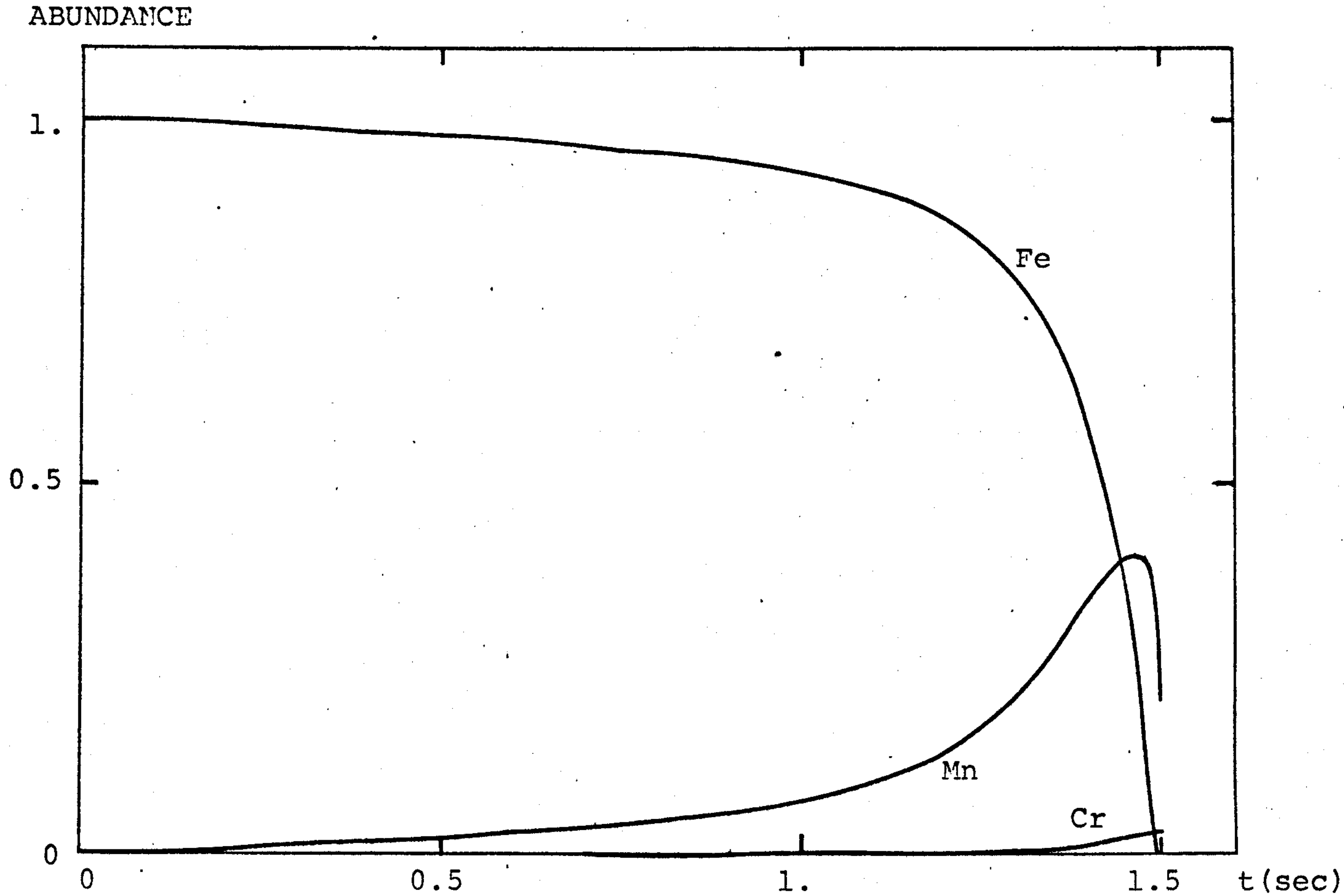

Fig. 4

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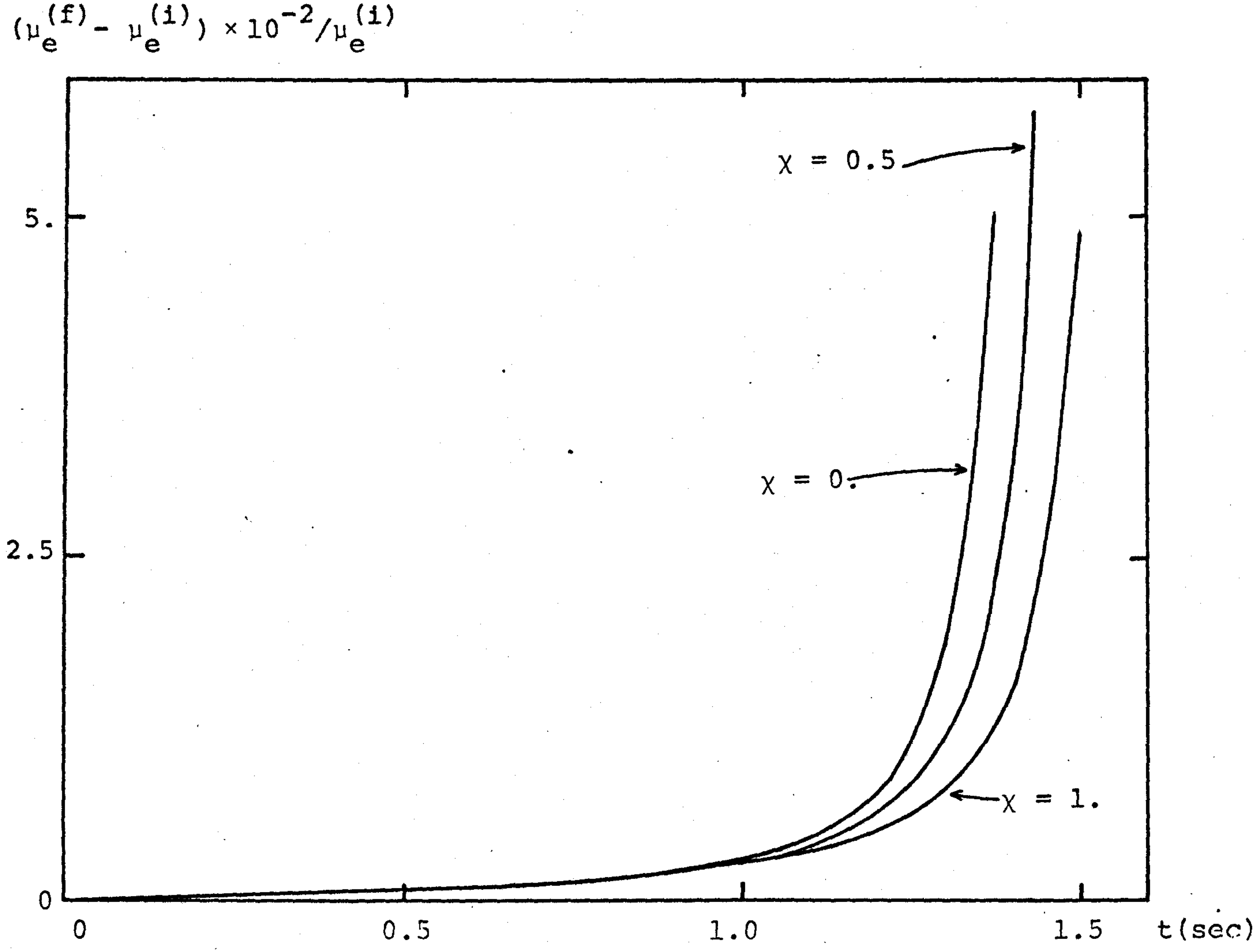

Fig. 5
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Fig. 6
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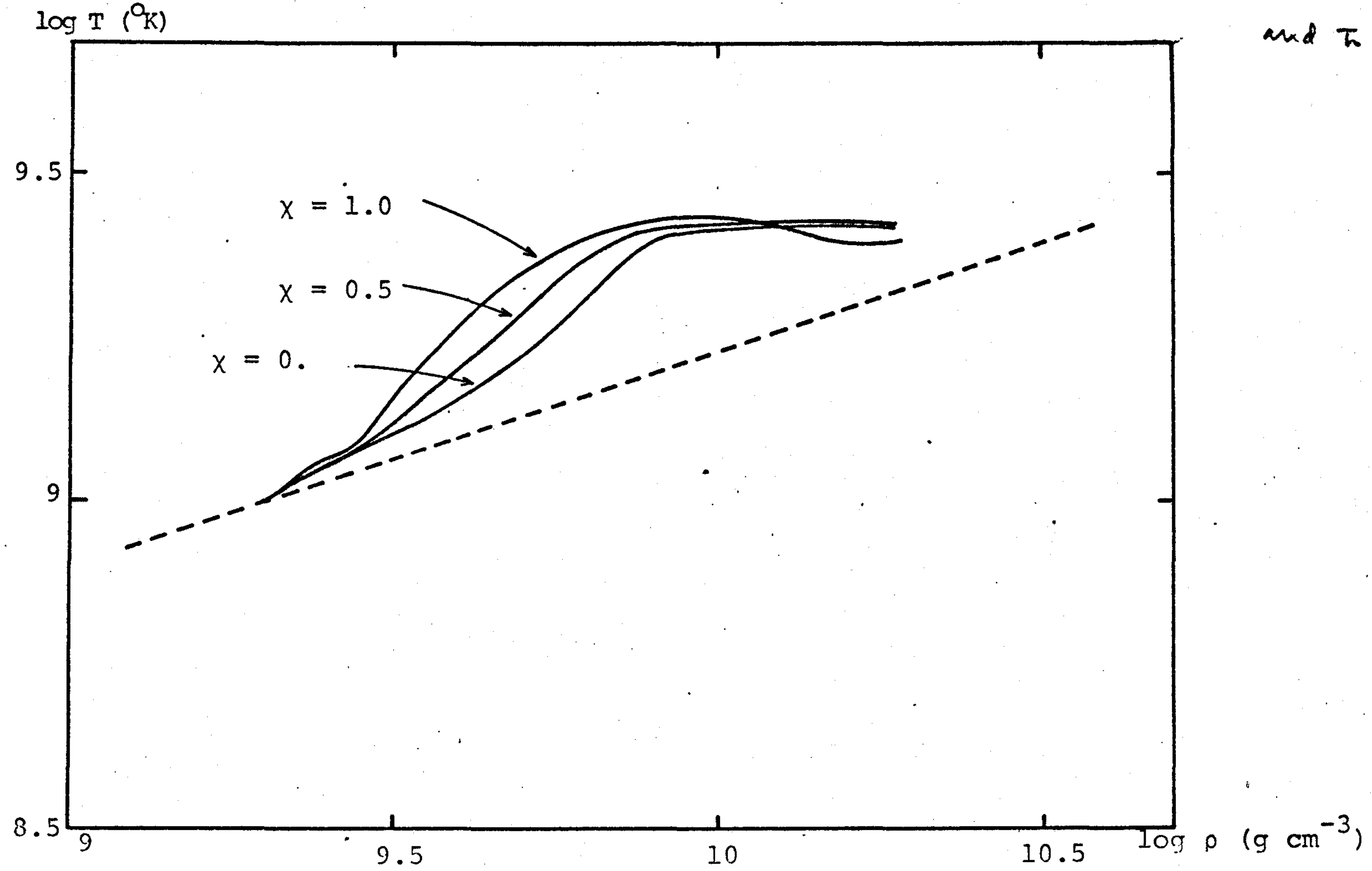

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