ISSN 0029-3865

CBPF-NF-034/85

# A FERMIONIC LOOP WAVE FUNCTIONAL FOR QUANTUM CHROMODYNAMICS AT $N_c=+\infty$

by

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ABSTRACT

A fermionic loop wave functional for euclidean QCD in the t'Hooft topological limit is considered. Arguments are given that this equation leads to a fermionic (supersymmetric) string representation for the above theory.

Key-words: Loop wave functional; Fermionic loop space; Supersymmetric string Ansatz.

In the last years a new quantization of Yang-Mills gauge fields has been pursued by several authors which seems appropriate for handling its confining phase. It makes use of the so called "Quantum Wilson Loop" as dynamical variable (see ref. [1] for an extensive review) which have the meaning of being the probability amplitude of a bosonic (Klein-Gordon) coulored particle propagating along a closed world line $X_\mu(s)$ and in the presence of the vacuum of a pure gauge theory.

A closed wave functional for this dynamical variable at the t'Hooft topological limit was derived: the Migdal-Makeenko equation ([1],[2]) which supports a string solution ([3]).

In this letter, we consider the case of the above particle possesses Dirac Spin degrees by making use of the Pseudo Classical Mechanics Formalism as exposed in ref. ([4]).

The basic dynamical variable in the loop space formulation for Euclidean $(QCD)_{N_c}$ at $N_c = +\infty$ is the amplitude for a quark loop propagate in the vacuum of a pure Yang-Mills. At this point our idea is implemented. Since the quark possesses Dirac spin degrees of freedom, its (Euclidean) world line should reveals the existence of these fermionic degrees. A natural framework to implement these idea is the pseudo-classical mechanics ([4], [5], [6]) where the world line of a spinning particle is described by a fermionic vector position $X_\mu^{(F)}(s,\theta) = X_\mu^{(B)}(s) + i\theta\psi_\mu(s)$ with s being the evolution parameter, $X_\mu^{(B)}(s)$ the ordinary (bosonic) position coordinate and $\psi_\mu(s)$ Grassman variables associated to the spin coordinates.

In this framework, the quark loop amplitude associated to a given spinning closed world line $\{X_\mu^{(F)}(s,\theta); 0 \leq s \leq 1; X_\mu^{(B)}(0) = X_\mu^{(B)}(1) = X \in \mathbb{R}^D\}$ in the presence of the vacuum of a pure U(N) Yang-Mills gauge theory is proportional to the following dynamical factor (the fermionic version of the usual (bosonic) Wilson loop) (see eq.(25) - ref. [4]):

$$W^{(F)}[X_\mu^{(F)}(s,\theta)] = \langle \mathrm{Tr}\,\mathbb{P}\,\{\exp\{\int_0^1 ds \int d\theta\, A_\mu(X_\mu^{(F)}(s,\theta)) DX_\mu^{(F)}(s,\theta)\}\}\rangle \tag{1}$$

where $A_\mu(x)$ denotes the usual U(N) Yang-Mills potential, $\mathbb{P}$ the path ordenation of the U(N) matrix indices of the exponent in (1) along the bosonic path $X_\mu^{(B)}(s)$ and $D = \frac{\partial}{\partial\theta} + i\theta\frac{\partial}{\partial s}$ the covariant derivative. The quantum average < > is defined by the partition functional of the pure Yang-Mills theory.

A important remark to be used below, is that (1) possesses the fermionic mixing symmetry ([4])

$$\begin{aligned} \delta X_\mu^{(B)}(s) &= i\,\epsilon\,\psi_\mu(s) \\ \delta\psi_\mu(s) &= \epsilon\, X_\mu^{(B)}(s) \end{aligned} \tag{2}$$

with $\epsilon$ a Grassmanian spinor parameter.

We note that by realizing the $\theta$-integration in the phase in (1) we get in addition to the usual term $\int_0^1 ds\, A_\mu(X_\mu^{(B)}(s))\, dX_\mu^{(B)}(s)$, the term responsible for the interaction between the spin degrees and the field strenght, namely: $\frac{1}{2} i\, [\psi_\mu,\psi_\nu]_+(s)\, F_{\mu\nu}(X_\mu^{(B)}(s))$.

In order to deduce a closed functional for the fermionic Wilson loop (1) we shift the $A_\mu(x)$ - variable and get the result [2].

$$\frac{1}{2g^2N} < \mathrm{Tr}\{ \mathbb{P} \{D_\mu F_{\mu\nu}\}(x) \exp \int_0^1 ds \int d\theta \, A_\mu (X_\mu^{(F)}(s,\theta)) \; DX_\mu^{(F)}(s,\theta)\}\}>$$

$$= \int_0^1 d\sigma \int d\theta \;\; \delta^{(D)} (X_\mu^{(F)}(\sigma,\theta)-x) \;\; DX_\mu(\sigma,\theta)$$

$$<\mathrm{Tr}\; \mathbb{P}\{\exp \int_0^\sigma ds \int d\theta \; A_\mu(X_\mu^{(F)}(s,\theta))DX_\mu^{(F)}(s,\theta)\}>$$

$$<\mathrm{Tr}\; \mathbb{P}\{\exp \int_\sigma^1 ds \int d\theta \; A_\mu(X_\mu^{(F)}(s,\theta))DX_\mu^{(F)}(s,\theta)\}> \tag{3}$$

Now we note the crucial fact that we are in presence of a very irregular path $X_\mu^{(B)}(s)$ which intercept itself at every point [7] and, further, insures the gauge invariance of each fermionic Wu-Yang factor in the right-hand side of eq.(3). As a consequence of this remark, the relation (3) takes the closed form at t'Hooft limit $N_c \to \infty$ $(\lim_{N_c\to\infty} g^2N_c = \lambda^2)$ ([1])

$$<\mathrm{Tr}\{ \mathbb{P}\{(D_\mu F_{\mu\nu})(x)\psi[X_\mu^{(F)}(s,\theta); 0 \le s \le 1]\}\}>$$

$$= 2\lambda^2 \int_0^1 d\sigma \int d\theta \;\; \delta^{(D)} (X_\mu^{(F)}(\sigma,\theta) - X) \; DX_\mu^{(F)}(\sigma,\theta)$$

$$<\psi[X_\mu^{(F)}(s,\theta); 0 \le s \le \sigma]><\psi[X_\mu^{(F)}(s,\theta); \sigma \le s \le 1]> \tag{4}$$

where we have introduced the more compact notation for the fermionic Wu-Yang factors in eq.(3)

$$\psi[X_\mu^{(F)}(s,\theta);\; \sigma_1 \le s \le \sigma_2] = \mathbb{P}\{\exp \int_{\sigma_1}^{\sigma_2} ds \int d\theta \; A_\mu(X_\mu^{(F)}(s,\theta))\, DX_\mu^{(F)}(s,\theta)\} \tag{5}$$

At this point of the analysis it is convenient to multiply both sides of eq. (4) by the fermionic current density $j_\nu(x)$ =

$= \delta^{(D)}(X - X_\mu^{(F)}(\bar\sigma,\theta) DX_\nu(\bar\sigma,\theta)$ and integrate out the result in relation to space time variable X. So, we get

$$<\mathrm{Tr}\{\mathbb{P}\{(D_\mu F_{\mu\nu})(X_\mu^{(F)}(\bar\sigma,\theta))DX_\nu^{(F)}(\bar\sigma,\theta)\psi[X_\mu^{(F)}(s,\theta);0\le s\le 1]\}\}>$$
$$= 2\lambda\int_0^1 d\sigma\int d\theta\ \delta^{(D)}(X_\mu^{(F)}(\sigma,\theta) - X_\mu^{(F)}(\bar\sigma,\theta))DX_\mu^{(F)}(\sigma,\theta)DX_\nu^{(F)}(\bar\sigma,\theta)$$
$$<\psi[X_\mu^{(F)}(s,\theta)\ ;0\le s\le\sigma]><\psi[X_\mu^{(F)}(s,\theta)\ ;\sigma\le s\le 1]> \qquad (6)$$

In order to write the left-hand side of relation (6) in a form similar to the usual strings equations, we note the relations:

$$\frac{\delta}{\delta X_\mu^{(F)}(\bar\sigma,\theta)}\mathrm{Tr}(\psi[X_\mu^{(F)}(s,\theta);0\le s\le\sigma])$$
$$= \mathrm{Tr}\,\mathbb{P}\{F_{\mu\nu}(X_\mu^{(F)}(\bar\sigma,\theta))DX_\nu^{(F)}(\bar\sigma,\theta)\psi[X_\mu^{(F)}(s,\theta);0\le s\le\sigma]\} \qquad (7)$$

and consequently (compare with the bosonic similar relation eq. (5-11) - eq. (5-12) ([2]):

$$\frac{\partial^2}{\partial^2 X_\mu^{(F)}(\bar\sigma,\theta)}\mathrm{Tr}(\psi[X_\mu^{(F)}(s,\theta);0\le s\le 1]])=$$
$$\lim_{\zeta\to 0^+}\int_{-\zeta}^{+\zeta} d\zeta\ \frac{\delta^2}{\delta X_\mu^{(F)}(\bar\sigma+\frac{1}{2}\zeta,\theta)\,\delta X_\mu^{(F)}(\bar\sigma-\frac{1}{2}\zeta,\theta)}\mathrm{Tr}(\psi[X_\mu^{(F)}(s,\theta);0\le s\le 1])$$
$$= \mathrm{Tr}\{\mathbb{P}\{(D_\mu F_{\mu\nu})(X_\mu^{(F)}(\bar\sigma,\theta))DX_\nu^{(F)}(\bar\sigma,\theta)\psi[X_\mu^{(F)}(s,\theta);0\le s\le 1]\}\} \qquad (8)$$

So, we can rewrite eq. (6) in the form

$$\frac{\partial^2}{\partial X_\mu^2(\bar{\sigma},\theta)} W^{(F)}[X_\mu^{(F)}(s,\theta), 0 \le s \le 1]$$

$$= 2\lambda \int_0^1 d\sigma \int d\theta \; \delta^{(D)}(X_\mu^{(F)}(\sigma,\theta) - X_\mu^{(F)}(\bar{\sigma},\theta)) DX_\mu^{(F)}(\sigma,\theta) DX_\mu^{(F)}(\bar{\sigma},\theta)$$

$$W^{(F)}[X_\mu^{(F)}(s,\theta), 0 \le s \le \sigma] . W^{(F)}[X_\mu^{(F)}(s,\theta); \sigma \le s \le 1] \tag{9}$$

This is the proposed fermionic loop wave functional for QCD at $N_c = +\infty$.

Note the initial condition imposed to the solutions of eq.(9) and related to the asymptotic freedom of QCD

$$W^{(F)}[X_\mu^{(F)}(s,\theta) \equiv 0] = 1 \tag{10}$$

Since our equation is deduced formally, the important problem of its regularization and renormalization shows up. At first, we note that in loop dynamics the paths $X_\mu^{(B)}(s)$ are very irregular geometric objects in euclidean space, so all Feynman diagramatic perturbative analyses break down ([8]). A probable useful scheme should be the introduction of its discrete version as in ref. [9] and the continuous limit is taken together with other kinematical factors ([4]).

Another more interesting point of view is to solve formally eq.(9) - eq.(10) in terms of the functional integral of a String Theory [3]. Due to the fermionic mixing symmetry (2) of the fermionic Wilson loop (1), it appears naturally to consider as a string Ansatz a supersymmetric string ([10], [11]) with all of its spectrum good features ([12]).

To summarize, we propose a fermionic loop wave functional for

QCD at $N_c = +\infty$ which support the hope about the existence of a QCD supersymmetric string Ansatz.

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq-Brasil).

REFERENCES

[1] A.A. Migdal, Phys. Reports 102, nº 4 (1983) - North Holland

[2] A.M. Polyakov, Nucl. Phys. B164 (1979), 171.

[3] A.A. Migdal, Nucl. Phys. B189 (1981), 253.

[4] P.D. Vecchia, F. Ravndal, Phys. Lett. 73A (1979), 371.

[5] F.A. Berezin and M.S. Marinov, JETP Lett. 21 (1975), 320;
A. Barducci, R. Casalbuoni and L. Lusanna, Nuovo Cimento 35A (1976) 377.

[6] L. Brink, P. Di Vechhia and P. Howe, Nucl. Phys. B118 (1977) 76;
P.A. Collins and R.W. Tucker, Nucl. Phys. B121 (1977),307.

[7] A. Dvoretzky, P. Erdös and S. Kakutani, Acta Sci. Math.(Szeged) 12B (1950) 75.

[8] R.A. Brandt, F. Neri and M. Sato, Phys. Rev. D24 (1981),879.

[9] A.A. Migdal, Phys. Lett., 147B (1984), 347.

[10] A.M. Polyakov, Phys. Lett. 103B (1981), 211.

[11] J.L. Gervais, A. Neveu, Nucl. Phys. B192 (1981), 463.

[12] Luiz C.L. Botelho, Phys. Lett., 152B (1985),358.