ISSN 0029 - 3865

CBPF-NF-031/85

ATOMIC IONIZATION BY STRONG ELECTRIC FIELDS: ONE DIMENSIONAL MODEL

by

H.S. Antunes Neto[1,2] and J.C. de Mello[1]

[1]Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[2]Departamento de Física
Universidade Federal do Pará
60000 - Belém, PA - Brasil

ABSTRACT

An analytic expression for the gauge independent transition probability amplitudes is found for a one-dimensional Dirac-delta function potential at Strong Electric Fields.

Key-words: Photoionization; Strong electric fields; One-dimensional Dirac-delta function potential.

PACS NUMBERS: 32.80 Fb

We will examine the case in which the optical radiation intensity is of such magnitude that the usual perturbative treatment can not be employed. Therefore we will deal with the construction of a non-perturbative expansion in an electromagnetic field.

The Schrödinger's equation for an hydrogen-like atom in the presence of a spatially homogeneous electromagnetic field can be written as:

$$\hat{H}|\psi_{(t)}\rangle \equiv \left[\frac{1}{2m}(\hat{p} + e/c\vec{A}(t))^2 + V(\hat{r})\right]|\psi_{(t)}\rangle = i\hbar\frac{d}{dt}|\psi_{(t)}\rangle \qquad (1)$$

The solution of this equation in the integral form is

$$|\psi_{(t)}\rangle = T_{(t,t_o)}|\psi_{(t_o)}\rangle \qquad (2)$$

where

$$T(t,t_o) = T_o(t,t_o) - 1/\hbar \int_{t_o}^{t} dt' T_o(t,t')\left[\hat{H} - \hat{H}_o\right]T(t',t_o) \qquad (3)$$

is the temporal evolution operator associated to the operator H and $T_o(t,t_o)$ is associated to H_o. The non-perturbative expansion in the electromagnetic field consists in setting $\hat{H}_o \equiv \hat{h}_o = \frac{1}{2m}(\hat{p} + \frac{e}{c}\vec{A}_{(t)})^2$ and the temporal evolution operator associated to this operator has the form:

$$T_o(t,t_o) = \exp\left\{- i/\hbar \int_{t_o}^{t} dt' \frac{1}{2m}(\hat{p} + \frac{e}{c}\vec{A}(t'))^2\right\} \qquad (4)$$

In the expression (3) an iterative process results in an expansion in terms of the intra-atomic potential $V(\hat{r})$ which is useful for the case of interaction of strong fields with the atomic system. Taking this expansion in expression (2) and using the coordinate representation we obtain:

$$\psi(\vec{r},t) = \int d^3r_1 g_o(\vec{r},t|\vec{r}_1,0)\psi_b(\vec{r}_1) -$$

$$- \frac{i}{\hbar}\int_0^t dt' \iint d^3r_1 d^3r_2\, g_o(\vec{r},t|\vec{r}_1,t')V(\vec{r}_1)g_o(\vec{r}_1,t'|\vec{r}_2,0)\psi_b(r_2) + \ldots \quad (5)$$

where it is assumed that in $t_o = 0$ the atomic system is in the ground state $|\psi_{b(o)}\rangle$ and $g_o(\vec{r},t|\vec{r}_1,t')$ is given by:

$$g_o(\vec{r},t|\vec{r}_1,t') = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3p_1 \exp\left\{-\frac{i}{\hbar}\int_{t'}^{t} dt'' \frac{1}{2m}(\vec{p}_1 + \frac{e}{c}\vec{A}(t''))^2\right\}\exp\left\{\frac{i}{\hbar}\vec{p}_1.(\vec{r}-\vec{r}_1)\right\} \quad (6)$$

The transition amplitude to an eigenstate of the atomic hamiltonian is:

$$M_{ji}(t) = \langle\psi_j|\exp\left\{\frac{ie}{\hbar c}\vec{A}(t).\hat{r}\right\}|\psi(t)\rangle \quad (7)$$

and the appearance of the gauge term $\exp\left\{-\frac{ie}{\hbar c}\vec{A}(t).\hat{r}\right\}$ in this expression preserves the gauge invariance of the transition probability [1].

We will now apply this to the case in which the atomic system is represented by a particle bound by a one-dimensional Dirac-delta function potential $V(x) = -B\delta(x)$ [2]. As this potential admits only one bound state, the time dependent ionization probability can be written: (Atomic unit are now used).

$$P(t) = 1 - |M_{ii}(t)|^2 \tag{8}$$

where $M_{ii}(t) = \langle\psi_b|\exp\left\{\frac{i}{c}A(t).x\right\}|\psi(t)\rangle$ represents the probability amplitude for the particle to remain in a bound state. Using in this expression the expansion (5) we obtain:

$$M_{ii}(t) = M_{ii}^{(0)}(t) + M_{ii}^{(1)}(t) + \ldots \tag{9}$$

where

$$M_{ii}^{(0)}(t) = \int dx\phi_b^*(x)\exp\left\{\frac{i}{c}A(t)x\right\}\int dx_1 g_o(x,t|x_1,0)\phi_b(x_k) \tag{10}$$

$$M_{ii}^{(1)} = -i\int dx\phi_b^*(x)\exp\left\{\frac{i}{c}A(t)x\right\}\int_o^t dt'\int dx_1 g_o(x,t|x_1,t')V(x_1) \tag{11}$$

$$\int dx_2 g_o(x_1,t_1|x_2,0)\phi_b(x_2)$$

and so on.

At strong electric fields, the main contribution to the transition probability comes from the first term of the expansion (9) which can be interpreted as the evolution of the initial packet in the presence of the electric field [3]. Thus, we can describe:

$$M_{ii}^{(0)}(t) = \exp(-i\beta(t))\int dp\exp(-i\frac{p^2}{2}t)\exp(ip\alpha(t))U_b(p)\bar{U}_b(p) \tag{12}$$

where $\bar{U}_{b(p)}$ is the Fourier transform of the bound state modified by the presence of the electric field

$$\bar{U}_n(p) = \sqrt{\frac{2}{\pi}}\frac{B^{3/2}}{B^2+(p+\frac{1}{c}A(t))^2} \quad ; \quad U_b(p) = \sqrt{\frac{2}{\pi}}\frac{B^{3/2}}{B^2+p^2}$$

and
$$\begin{cases} \beta(t) = \frac{1}{2c^2}\int_0^t dt' A^2(t') \\ \alpha(t) = -\frac{1}{c}\int_0^t A(t')dt' \end{cases}$$

Let us introduce the adimensional variables

$$\tau = B^2 t;\ \bar{A}(\tau) = \frac{A(\tau)}{c};\ \bar{\alpha}(\tau) = B\alpha(\tau);\ \bar{\beta}(\tau) = B^2\beta(\tau)$$

Thus the term $M_{ii}^{(0)}(\tau)$ can be expressed in a closed form at cost of noticeable manipulations on the integrals involved:

$$M_{ii}^{(0)}(\tau) = \frac{\exp(-i\bar{\beta}(\tau))}{1+\frac{\bar{A}^2}{4}}\Big\{(1/2 + i/\bar{A}(\tau))\Big[m(-\bar{\alpha}(\tau),-i,\tau) + m(\bar{\alpha}(\tau),-iF^*,\tau)\Big] +$$
$$+ (1/2 - i/\bar{A}(\tau))\Big[m(\bar{\alpha}(\tau),-i,\tau) + m(-\bar{\alpha}(\tau),-iF,\tau)\Big]\Big\} \qquad (13)$$

where $m(x,k,t)$ are the Moshinsky's function [4] given by:

$$m(x,k,t) = \frac{1}{2}\exp\Big[i(kx - \frac{k^2 t}{2})\Big]\mathrm{erfc}\Big[W\exp(-i\pi/4)\Big]$$

with $W = (x - kt)/(2t)^{1/2}$; $\mathrm{Erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty dt\exp(-t^2)$

and $F = 1 - i\bar{A}(\tau)$.

If one puts $A(t) = -cE_o t$ in the expression (13), one finds the results obtained by Arrighini et al. [5]. An extensive discussion about the problem analized here but without considering the gauge term $\exp\left\{-\frac{i}{c}A(t)x\right\}$ is found in [6]. We present numerical results as a check of the quality of our approximate ionization probability $P_0(\tau) = 1 - |M_{ii}^{(0)}(\tau)|^2$. As in references [3] and [6] we consider two fundamentals parameters in the behavior of $P_0(\tau)$: I_0/ω and E_0/E_c, where E_c is the critical field and I_0 is the ionization energy. The case examined here, with $I_0/\omega = 20$ for all curves and $E_0/E_c = 10^3; 10^2; 10$ and 1 for figures (1), (2), (3) and (4) respectively, correspond to an ionization by a statical field. Since the ionization time is shorter than the period of the oscillating fixed [see Ref. [3] and [6]], in terms of the "adiabaticity parameter" [2] $\gamma = \frac{1}{2}\left(\frac{E_0}{E_c}\right)^{-1}\left(\frac{I_0}{\omega}\right)^{-1}$, this situation corresponds to the "tunnelling regime" $\gamma << 1$.

As expected, our approximation converges for the exact solution in the super-strong field regime $\left(\frac{E_0}{E_c} >> 1\right)$ and presents some discrepances in the moderate regime $\left(\frac{E_0}{E_c} \lesssim 1\right)$.

We recall that our result is obtained with a correct gauge invariant procedure while in references [3] and [6] a hybrid procedure [1] was used. However the comparation between figures (1) and (3) in the present work with figures (a) and (b) in [6] doesn't enable us to prefer any one of these two procedures. This fact reserves a deeper analysis in a future publication.

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and Coordenação de Aperfeiçoamento do Pessoal de Ensino Superior (CAPES).

REFERENCES

[1] R.R. Schlicher; W. Bercker; J. Bergou M.O. Scully, in Quantum Electrodyamics and Quantum Optics, edited by A.O. Barut (plenun, New York, 1984) pp. 405-448.

D.H. Kobe and K.H. Yang - Phys. Rev. A 32, 952 (1985).

[2] S. Geltman, J. Phys. B-10, 831 (1977).

[3] H.S. Antunes Neto. L. Davidovich and D. Marchesin, in Coference and Quantum Optics V, Edited by L. Mandel and E. Wolf (Plenum, N.Y. In press).

[4] M. Moshinsky, Phys. Rev. 84, 525 (1951).

H.M. Nussenzweig - "Causality and Dispersion Relations" (Academic, NY, 1972).

[5] E.P. Arrighini and M. Gavarini, Lett. Nuovo Cimento 33, 353 (1982).

[6] H.S. Antunes Neto and L. Davidovich, Phys. Rev. Lett. 53 2238 (1984).