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MAGNETOHYDRODYNAMIC COSMOLOGIES WITH A BERTOTTI-ROBINSON LIMIT

by

R. Portugal and I. Damião Soares

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

We present a class of cosmological solutions of Einstein-Maxwell equations, which have the Bertotti-Robinson model as an asymptotic configuration. The novel feature of the models is the presence of a conductivity current in Maxwell equations characterizing a regime of magnetohydrodynamics. Exact analytical solutions are exhibited and the solutions may be used as the interior model for the collapse of a self-gravitating bounded fluid with electric conductivity.

Key-words: Cosmology; Kantowski-Sachs models; Magnetohydrodynamic cosmologies; Bertotti-Robinson models.

1 INTRODUCTION

The purpose of this paper is to examine a class of Kantowski-Sachs [1,2] (KS) cosmological solutions of Einstein-Maxwell equations, which have the static Bertotti-Robinson model [3,4] as a limiting configuration. In spite of Robertson-Walker-Friedmann models fairly describe the present geometry of the Universe, some astrophysical data - e.g. the intergalactic magnetic field [5] sugggesting the existence of a primordial magnetic field, and the amount of Helium production [6] - received a more accurate explanation in the background of anisotropic big bang models. In this context, KS models have been used as a possible class of big-bang models mainly including those with an anisotropic expansion [7].

On the other hand, it is known from the thermal history of the Universe that in eras prior to recombination the material content of the Universe was primarily ionized hydrogen and electromagnetic radiation [8]. If in these phases a non-null electromagnetic field was also present (whose remnant could be the intergalatic magnetic field), the collisions of the accelerated electrons with matter would then produce a net temperature-dependent electrical conductivity. In fact, as well known from plasma theory the conductivity of a fully ionized plasma depends on the temperature as a power law [9] implying that the conductivity is time dependent, and changes as the Universe evolves in this era before recombination.

The presence of electric conductivity could be of importance to the theory of galaxy formation - actually Fennelly [10] showed that an exponential growth of the contrast density is possible in a Bianchi I model with electric conductivity.

In this paper we present a class of universe models which incorporate the above discussed features: they may have anisotropic or isotropic expansion, electromagnetic fields together with a time-dependent electric conductivity. The geometry is taken to be of the Kantowski-Sachs type with topology $RxRxS^2$. The models are to be solutions of the Einstein-Maxwell equations with the cosmological constant term, and a perfect fluid (electrically neutral in average) satisfying the equation of state $p = \lambda\rho$, $-1/3 \leq \lambda \leq 1$. The source of Maxwell equations is a space-like four current in the direction of the fluid. In other words the fluid is said to be in a magnetohydrodynamic [11] regime with a conductivity current. Since the space-like four current is parallel to the electric field, the conductivity parameter is defined as the ratio current/electric field and it is a time-dependent scalar [12].

We program the paper as follows. In Section 2 we give the dynamics of the models according to Einstein-Maxwell equations. In Section 3 we examine the restrictions we must impose on the solutions in order that they are physically meaningful. The differential equation for the scale function B(t) is analysed as the 1-dim motion of a particle in a potential V(B). In Section 4 exact analytical solutions are given and their properties analyzed. In the final Section we summarize our results and discuss some possible physical applications.

2 THE DYNAMICS OF THE MODELS

In the four-dimensional space-time with structure $R\times R\times S^2$ we introduce the Kantowski-Sachs geometry given by

$$ds^2 = dt^2 - A^2(t)d\chi^2 - B^2(t)(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.1}$$

The matter content of the models is a perfect fluid plus electromagnetic fields. We assume that observers comoving with the fluid have four velocity $u = \partial/\partial t$, and we denote by ρ and p the matter-energy density and pressure respectively, as measured locally by the comoving observers. The equation of state for the fluid is assumed [13]

$$p = \lambda\rho \quad , \quad -1/3 \leq \lambda \leq 1 \tag{2.2}$$

Electromagnetic fields satisfy Maxwell equations with source. From spatial homogeneity and the existence of a preferred spatial direction determined by the Killing field $\partial/\partial\chi$, we restrict the electromagnetic tensor

$$\begin{aligned} F_{01} &= AE(t) \\ F_{23} &= B^2\sin\theta H(t) \end{aligned} \tag{2.3}$$

all other components zero. This is actually the unique possibility, as can be shown from purely algebraic considerations in Einstein-Maxwell equations for (2.1).

The electric four current is parallel to the electric field, as it follows from Maxwell equations for (2.1) and (2.3), and it can therefore be given in the general covariant form of Ohm's Law [11]

$$j^{\alpha} = \sigma E^{\alpha} = \sigma F^{\lambda\alpha} u_{\lambda} \tag{2.4}$$

The space-like character of the four-current (2.4) implies that the density of electric charge of the fluid must be zero. In other words the fluid is said electrically neutral in average and said to be in a magnetohydrodynamic regime with a conductivity current. Since the four current is parallel to the electric field, the scalar σ is identified as the electric conductivity of the fluid. As we have mentioned earlier, σ depends on the cosmological time and this dependence must be prescribed by Maxwell equations. To completely specify the dynamics we assume here a relation between A and B, and restrict ourselves to a class of models such that

$$A = B^{r} \tag{2.5}$$

with r a real parameter.

Einstein-Maxwell equations [14] for (2.1)-(2.5) yield then

$$\sigma = - \frac{d}{dt} \ell n(\dot{E}B^{2}) \quad , \quad H = \frac{H_o}{B^{2}} \tag{2.6}$$

$$E^{2} = (1-r)\frac{\ddot{B}}{B} + (1-r^{2})\frac{\dot{B}^{2}}{B^{2}} + \frac{1}{B^{2}} - \frac{H_o^{2}}{B^{4}} \tag{2.7}$$

$$\rho = \frac{r-1}{2}\frac{\ddot{B}}{B} + \frac{r^{2}+4r+1}{2}\frac{\dot{B}^{2}}{B^{2}} + \frac{1}{2B^{2}} - \Lambda \tag{2.8}$$

$$\frac{(1+\lambda)r+3-\lambda}{2}\frac{\ddot{B}}{B}+\frac{(1+\lambda)r^2+4\lambda r+1+\lambda}{2}\frac{\dot{B}^2}{B^2}+\frac{1+\lambda}{2B^2}-\Lambda(1+\lambda)=0 \qquad (2.9)$$

where H_0 is a constant and the cosmological constant Λ is positive. Equations (2.6)-(2.8) are considered as defining the electric conductivity σ, the matter-energy density ρ and the square of the electric field (in a local Lorentz frame) E^2. In the domain of solutions of (2.9) we must then guarantee the positiveness of σ, ρ and E^2 in order to have physically admissible solutions.

Introducing a new time variable η defined by

$$d\eta = B^{-\alpha}dt \qquad (2.10)$$

where

$$\alpha = \frac{(1+\lambda)r^2+4\lambda r+1+\lambda}{(1+\lambda)(r-r_0)} \qquad (2.11)$$

with $r_0 = (\lambda-3)/(\lambda+1)$ [15], equation (2.9) can be reexpressed

$$B'' + \frac{1-2\Lambda B^2}{r-r_0}B^{2\alpha-1} = 0 \qquad (2.12)$$

where a prime denotes η-derivate.

In the space of solutions of (2.12), the *point*

$$B = \frac{1}{\sqrt{2\Lambda}} \quad , \quad B' = 0 \qquad (2.13)$$

is a solution with $\rho = 0$, $\sigma = 0$ and

$$E^2 + H^2 = 2\Lambda \tag{2.14}$$

This corresponds to the Bertotti-Robinson (BR) solution with topology $R\times R\times S^2$ [16].

Equation (2.12) has the first integral

$$B'^2 + \left(\frac{1}{\alpha} - \frac{2\Lambda}{\alpha+1} B^2\right)\frac{B^{2\alpha}}{r-r_o} = C \tag{2.15}$$

where C is an integration constant. For the BR solution C must assume the value

$$C = \frac{1}{\alpha(\alpha+1)(r-r_o)(2\Lambda)^{\alpha}} \tag{2.16}$$

In the remaining of the paper, we shall restrict ourselves to solutions of (2.15) for C assuming the above value [17]. Using equations (2.12), (2.15) and (2.16) we obtain

$$\rho = \frac{2\Lambda}{(1+\lambda)(r-r_o)}\left[\frac{2(r-1)}{\alpha+1} - \frac{r}{\alpha\Lambda B^2} + \frac{2(r+1)(2r+1)}{\alpha(\alpha+1)(r-r_o)(2\Lambda)^{\alpha+1}B^{2\alpha+2}}\right]. \tag{2.17}$$

and

$$E^2 = \frac{2\Lambda}{r-r_o}\left[\frac{(1-r)(r+2)}{\alpha+1} + \frac{r(r-r_1)}{\alpha\Lambda B^2} + \frac{2(2r+1)(1-r)(1-\lambda)}{\alpha(\alpha+1)(r-r_o)(1+\lambda)(2\Lambda)^{\alpha+1}B^{2\alpha+2}}\right] - \frac{H_o^2}{B^4} \tag{2.18}$$

where $r_1 = -2\lambda/(\lambda+1)$. Also we have from (2.6)

$$\sigma = \frac{2\Lambda}{E^2 B^{1+\alpha}(r-r_o)}\left[\frac{2(r-1)(r+2)}{\alpha+1} - \frac{r(r-r_1)}{\alpha\Lambda B^2} + \frac{2(\alpha-1)(2r+1)(1-r)(1-\lambda)}{\alpha(\alpha+1)(r-r_o)(1+\lambda)(2\Lambda)^{\alpha+1}B^{2\alpha+2}}\right] B' \tag{2.19}$$

We remark from the above expressions that $B = 0$ is a physical singularity of the models since the relevant physical quantities diverge as $B \to 0$. However, for the cases $H_0 = r = 0$, $\lambda = 1$ and $H_0 = 0$, $r = -1/2$, $\lambda = 1/3$ the electric field is a constant, namely $E^2 = 2\Lambda$.

3 THE PHYSICAL SOLUTIONS

The range of admissible values of the exponent r must be restricted so that the solutions satisfy the physical conditions of positiveness of ρ , σ and E^2. Indeed these conditions are fullfilled only if

$$-\frac{1}{2} \leq r \leq 1 \tag{3.1}$$

To see this, first we observe from (2.11) that if $-1/3 \leq \lambda \leq 1$ than $\alpha(r-r_0) \geq 0$. Second we interpret equation (2.15) as the conserved Hamiltonian of a 1-dim system with the effective potential

$$V(B) = \left(\frac{1}{\alpha} - \frac{2\Lambda}{\alpha+1} B^2\right) \frac{B^{2\alpha}}{r-r_0} \tag{3.2}$$

The BR solution is an extremum of V(B), and must be an absolute maximum in order to exist time-dependent solutions. By plotting the possibles curves of V(B) for differents values of α, we can see that α must be positive and $r > r_0$ [18]. Under these restrictions it follows that the positiveness of ρ and E^2 -

given in (2.17) and (2.18) - as $B \to 0$ or $B \to \infty$ necessarily implies (3.1).

Concerning the sign of ρ, we note that ρ is null exactly at the BR configuration $B_{BR} = 1/\sqrt{2\Lambda}$, as expected (cf. (2.17)). For $B > 1/\sqrt{2\Lambda}$ we have $\rho < 0$. The positiveness of ρ restricts then the range of B to

$$0 < B \leq \frac{1}{\sqrt{2\Lambda}} \tag{3.3}$$

and two possible classes of evolving cosmological models arise: first, the universe starts from the singularity $B = 0$ and expands towards the configuration of the BR solution ($B = 1/\sqrt{2\Lambda}$, $B' = 0$) in an infinite period of time; second, the universe departs from the BR configuration (for example, by a perturbation in the model) and contracts to the singularity $B = 0$.

The analysis of the sign of E^2 is a little more complicated. The parameter α plays an important role in this analysis. The restriction on λ and r imposed by (2.2) and (3.1) imply that $0 \leq \alpha \leq 2$. We distinguish two cases:

(i) $0 \leq \alpha < 1$ or $\alpha = 2$

A necessary condition for E^2 to be always positive is that $H_0 = 0$. Further, taking into account (2.2) and (3.1) the parameters must be restricted to the undotted region of Fig. 1.

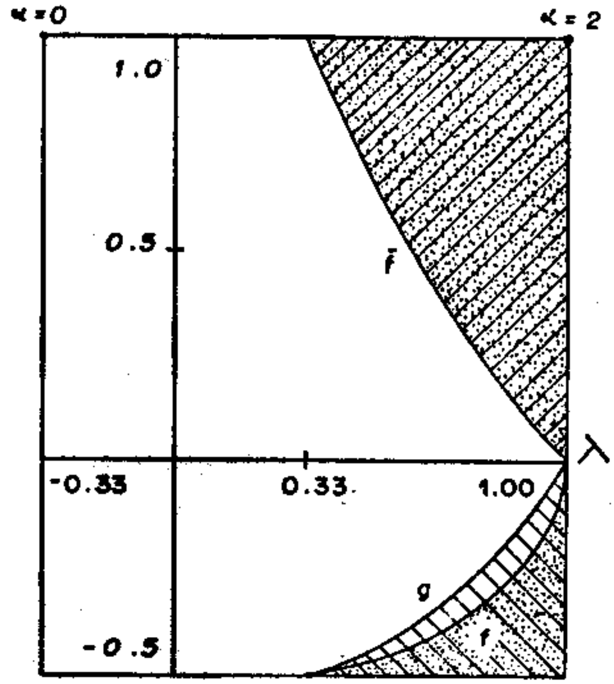

Fig. 1: Domain of positiviness of ρ, E^2 and σ.

For (λ,r) in the undotted region, E^2 is positive over the range (3.3). The solid curve $\bar{f}$ is defined by $\alpha = 1$ and the solid curve f is defined by the equation

$$\left[\frac{2r(r-r_1)}{(r-1)(r+2)}\right]^{\alpha+1} - \frac{2(1-\lambda)(2r+1)}{(1+\lambda)(r+2)(r-r_0)} = 0 \qquad (3.4)$$

The curve f represents the points (λ,r) where the function $E^2(B)$, given by (2.18), evaluated in its minimum, is zero. For (λ,r) below the curve f (see Fig. 1) it follows that E^2 is negative for B in a certain sub-interval of $(0,B_{BR})$.

(ii) $1 \leq \alpha < 2$

We have two possibilities here: (1) If $H_0 = 0$, E^2 is positive (modulo the range (3.3) for (λ,r)) in the dashed region of Fig. 1 and above the curve $\bar{f}$. The points on the curve $\bar{f}$ must also be included. (2) If $H_0 \neq 0$ one further (sufficient) condition appears relating λ, r and ΛH_0^2, and will be analyzed only for the value $\alpha = 1$. As we shall see, this is the unique physical case of (ii).

We finally discuss the sign of σ. We start by noting that the expansion parameter θ of the models (associated to the four-velocity field $\partial/\partial t$) is given by

$$\theta = \frac{(2+r)B'}{B^{\alpha+1}} \tag{3.5}$$

From equation (2.19) it is immediate to see that the sign of σ changes whenever θ changes sign. For physical arguments connected to Ohm's Law (2.4) and the interpretation of σ as the electric conductivity of the fluid, we will only retain the solutions for which the conductivity parameter is positive in the range (3.3) [19].

For $1 < \alpha < 2$, $\lambda \neq 1$ and $r \neq 1$, we can verify that σ/B' has not a definite sign in (3.3). Therefore these values of α must be excluded.

For $0 \leq \alpha \leq 1$, σ is positive if the universe is contracting ($\theta < 0$) and if the values of (λ, r) lie in the undashed region of Fig. 1. The curve g in Fig. 1 is defined by the equation

$$\left[\frac{r(r-r_1)}{(r-1)(r+2)}\right]^{\alpha+1} - \frac{(1-\alpha)(1-\lambda)(2r+1)}{(1+\lambda)(r+2)(r-r_0)} = 0 \tag{3.6}$$

and is obtained in a similar way as the curve f, i.e., if B_c

is the point where the function $\sigma(B)E^2B^{1+\alpha}/2\Lambda B'$ - given by (2.19)- has a minimun, then the curve g represents the points (λ, r) where $\sigma(B_c) = 0$. We note that the points on the curve g belong to the physical region.

The cases $(\lambda = 1,\ 0 < r \leq 1)$ and $(r = 1,\ 1/3 < \lambda \leq 1)$ must also be included and correspond to contracting solutions with $\sigma > 0$.

The undashed region of the square of Fig. 1 - including the boundaries plus the points $(\lambda = 1,\ 0 < r \leq 1)$ and $(r = 1, 1/3 < \lambda \leq 1)$ - is then the physical region of parameters for this case and corresponds to cosmological solutions which start from the static BR configuration (for example, by a perturbation in the density ρ, or in the electromagnetic fields, etc.) and contracts towards the singularity $B = 0$. The latter can be used as the interior model of a collapsing self-gravitating bounded fluid with electric conductivity, which we shall discuss elsewhere [20].

4 EXACT ANALYTICAL SOLUTIONS

Equation (2.15) can in general be integrated and for several values of α the solutions can be expressed in terms of Jacobian elliptic functions. For the cases (2.16) considered here some of them reduce to elementary functions. We obtain here explicit analytical solutions and discuss their properties for the following values of α:

(i) $\alpha = 1/2$

For this case we have two possibilities: first $r=(1-3\lambda)/(1+\lambda)$ so that the range of (λ,r) is $0 \leq \lambda \leq a$ and $b \leq r \leq 1$ where the values of (a,b) are approximately $(0.55, -0.42)$, and correspond to the intersection point of the curves $\alpha = 1/2$ and g(cf. Fig. 1). The second possibility is $r=-1/2$ and $-1/3 \leq \lambda \leq 1/3$. The solution of (2.15)/(2.16) is

$$B = \frac{1}{\sqrt{2\Lambda}}\left[3\,\mathrm{tgh}^2\,\frac{\sqrt{3}K_o}{2\sqrt{2}}(\eta-\eta_o) - 2\right] \qquad (4.1)$$

where $K_o = \sqrt{\frac{(1+\lambda)\sqrt{2\Lambda}}{6(1-\lambda)}}$ for the first possibility and $K_o = \sqrt{\frac{2(1+\lambda)\sqrt{2\Lambda}}{5-3\lambda}}$ for the second. The value of η_o is $\frac{-2\sqrt{2}}{\sqrt{3}K_o}$ arctgh $\frac{\sqrt{2}}{\sqrt{3}}$ for both possibilities.

Expression (4.1) describes a class of contracting solutions with equation of state $p = \lambda\rho$, $0 \leq \lambda \leq a$. These contracting solutions have zero magnetic field and positive definite conductivity σ. The rate of contraction depends on λ and Λ through K_o, as can be seen from (4.1). The BR configuration occurs at $\eta = \infty$ and the cosmological singularity $B = 0$ occurs at $\eta = 0$. The electric field is real and the mass-energy density positive, as we have discussed, the latter being small near the BR configuration and diverging as $B \to 0$.

(ii) $\alpha = 1$

We have the relation $r = 2(1-\lambda)/(1+\lambda)$ so that $1/3 \leq \lambda \leq 1$ and $0 \leq r \leq 1$ (cf. also Fig. 1). If $H_o \neq o$ the additional restriction must be satisfied

$$\Lambda H_o^2 \leq \frac{(3\lambda-1)(1-\lambda)}{2(5-3\lambda)(1+\lambda)} \tag{4.2}$$

We obtain the solution

$$B = \frac{1}{\sqrt{2\Lambda}} \operatorname{tgh} L_o(\eta-\eta_o) \tag{4.3}$$

where $L_o = \sqrt{\lambda+1}/\sqrt{2(5-3\lambda)}$. This also describes a class of contracting solutions with equation of state $p = \lambda\rho$, $1/3 \leq \lambda \leq 1$. The rate of contraction depends on λ through L_o, as can be seen from (4.3), but contrary to the case $\alpha = 1/2$ it does not depend on the cosmological constant. The solution may present a non-null magnetic field, but the magnetic parameter H_o (cf. (2.6)) must then satisfy the inequality (4.2) for a given λ in the interval $[1/3,1]$. The BR configuration is attained as $\eta \to \infty$.

(iii) $\alpha = 2$

From eq. (2.11) and considering Fig. 1, we are restricted in this case to $r = \lambda = 1$. The solution is given by

$$B = \frac{1}{\sqrt{2\Lambda}} \left[\frac{\sinh^2 N_o\eta - 6\cosh N_o\eta + 6}{\sinh^2 N_o\eta - 24}\right]^{1/2} \tag{4.4}$$

where $N_o = 1/\sqrt{2\Lambda}$. In this case we must have $H_o = 0$ in order that E^2 be positive. The BR configuration is attained at $\eta = \infty$ and the cosmological singularity at $\eta = 0$. Since we have $r = 1$ only, these contracting solutions are isotropic, namely the shear of the fluid four-velocity $\partial/\partial t$ is null.

5 CONCLUSIONS

We have examined here a class big-bang models - of the Kantowski-Sachs type and solutions of Einstein-Maxwell equations - which have the static Bertotti-Robinson model as a limiting configuration. The novel feature of the models is the presence of a conductivity current in Maxwell equations, characterizing these models in a magnetohydrodynamic regime and possibly contributing to a more accurate description for the matter content of the universe in phases prior to recombination. The models are in general anisotropic but isotropic solutions are also present in the class, for values of the parameter $r = 1$. The conductivity is defined through Ohm's Law and Maxwell equations over the cosmological background.

From the energy conditions and from physical assumptions connected to the interpretation of Ohm's Law in flat space-time, we demand the positiveness of the matter-energy density ρ, the conductivity σ and the square of the electric field E^2. It follows that all physical solutions must evolve from the Bertotti-Robinson configuration ($B = B_{BR}$) to cosmological singularity ($B = 0$), in an infinite period of time.

The solutions depend on two parameters (λ, r) whose domain is described in Fig. 1. If the magnetic field is null, the physsical region of the parameters is the undashed region of Fig. 1, including the boundaries and the points ($\lambda = 1$, $0 < r \leq 1$) and ($r = 1$, $1/3 \leq \lambda \leq 1$). If the magnetic field is non-null, the physical region is the curve $\bar{f}$ (defined by $\alpha = 1$) excluded the end points. We remark that the positiveness of σ restricts the solution to contracting ones. The solutions may be used as the interior model for the collapse of a self-gravitating bounded fluid with electric conductivity.

Exact analytical solutions are exhibited for the cases $\alpha = 1/2$, 1 and 2 in terms elementary functions.

We must finally comment about the sign of the conductivity and the distinction between contracting and expanding solutions. Let us for simplicity restrict ourselves to the cases $r = 0$ and $r = 1$. From equation (2.19) we obtain by a straightforward calculation that the sign of σ is opposite to the sign of the expansion parameter θ given by (3.5), namely σ is positive in the contracting phase of the model. There is however a bold distinction between the solutions $r = 0$ and $r = 1$ concerning the interpretation of σ. For a *closed system* in flat space-time it can be shown [21] that σ must be positive in order that the entropy of the system increases. In the curved space-time of a cosmological model the concept of entropy of a closed system is not in general well defined. If we adhere to the orthodox principle that locally the entropy of any system must increase and assume that the sign of the conductivity σ is related to the local rate of change of entropy, then σ must be greater than zero. For the anisotropic models $r = 0$, this view can actually be sustained by local thermodynamic considerations: from the local conservation of $T_{\mu\nu}$ we can derive [22] that the time derivate of the specific entropy is given by $\phi = \pi_{\mu\nu}\sigma^{\mu\nu}$, where $\pi_{\mu\nu}$ is the traceless anisotropic pressure tensor, and $\sigma^{\mu\nu}$ is the shear of the matter velocity fluid $\partial/\partial t$, and a simple calculations yields $\phi = -2E^2B'/3B^{\alpha+1}$. Using the expression (2.19) we show that the sign of σ is equal to the sign of ϕ.

For the isotropic solutions $r=1$ however $\sigma_{\mu\nu}=0$. We have obviously $\phi=0$ and the above interpretation fails. It then remains to be given a physical criterion for defining the sign of σ if the concept of increasing entropy (even from a local point of view) has any meaning at all for a macroscopic system in interaction with the cosmological background.

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[15] The case $r = r_0$ can be integrated without change of variable but it gives no physical solutions.

[16] In the denomination of Bertotti [3], the geometry of the solution is constituted of two blades (2-dim sections), one plane (RxR) and the other with positive curvature (S^2).

[17] For other values of C, solutions will be discussed in another publication.

[18] The case $\alpha = 0$ admits also one solution, for $r = 0$, $\lambda = -1/3$ and $H_0 = 0$. The corresponding expressions for $V(B)$, ρ, E^2 and σ must be calculated separately and will not be given here.

[19] In fact, from the second law of thermodynamics and for a closed system in Minkowski space-time it can be shown that σ must be positive in order that the entropy of the system increases [21]. In Section 5 we discuss the possible extension of this interpretation to curved spaces and show that it cannot always be sustained.

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