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A QUANTUM CONTOUR FORMULATION FOR YANG-MILLS THEORY AND QUANTUM CHROMODYNAMICS

by

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Abstract

We propose a formulation for the confining phase of Yang--Mills Theory and Quantum Chromodynamics (QCD) in the Contour Space Leading to a Dynamics of Interacting Quantized Strings.

Key-words: Quantum contour formulation; Yang-Mills theory; Quantum chromodynamics.

In the last years a new formulation of Yang-Mills gauge fields has been pursued by several authors and seems appropriate for handling the compound hadron structure in the Q.C.D. model for strong interactions. It makes use of the Ordered Phase Factor as dynamical variable [2]-[15], which takes into account more explicitly the geometrical setting of the Gauge Theory than its description by Gauge Potentials.

On the other hand, the strong coupling expansion from lattice gauge theories shows that the fundamental concept is the notion of lattice surfaces (sets of plaquettes) [3], thus raising hopes that the continuous limit of the lattice, at the confining phase, leads to a dynamical description of Gauge Fields by a Theory of Quantized Interacting Strings [3],[6],[7], [8],[10],[13].†

In this letter we present an attempt to implement these ideas. Our program aims at defining another continuous limit of the lattice: the space of all possible closed contours, instead of the usual Euclidean space, and propose a quantum dynamical system described by an action with the dynamical variable being the Ordered Phase Factor. These two formulations of Continuous Gauge Theories are analogous to the well-known two phase-dependent lagrangeans which describe the continuous limit of Spin systems [10].

We begin our analysis by considering the 4-dimensional lattice action for U(N) Gauge Fields in a form closely analogous to the 2-dimensional lattice chiral action [12]:

† It is assumed throught the Euclidean Formulation of Gauge Theories.

$$S_0[U[\Gamma]] = \frac{1}{4g^2N} \sum_{(\Gamma)((x_\ell);(\mu,\nu))} \sum$$

$$.Tr\{U^{-1}[\Gamma]\, U[\Gamma + \Pi_{((x_\ell);(\mu,\nu))}] + h.c\} \qquad (1)$$

where Tr indicates the trace operation over U(N) indices, $U[\Gamma]$ is the Ordered Phase Factor around a simple closed contour Γ in lattice; $\sum_{\{\Gamma\}}$ is the sum over all these simple closed contours and $\Gamma + \Pi_{((x_\ell);(\mu,\nu))}$ is the contour obtained from Γ by addition of a "Tooth" in the ν-direction at the link $\Gamma_{((x_\ell);(\mu))}$. Finally N is a normalization factor which counts the number of times a given plaquette appears in the sum $\sum_{\{\mu\}}$.

The equation of motion for the action (1) takes the form [8]:

$$\sum_{(\nu)} \{U[\Gamma + \Pi_{((x_\ell);(\mu,\nu))}] \cdot U^{-1}[\Gamma] = \sum_{(\nu)} U[\Gamma].U^{-1}\, [\Gamma - \Pi_{((x_\ell);(\mu,\nu))}] \qquad (2)$$

with $x_\ell \in \Gamma$.

The Quantum Lattice Theory, in the Feynmann functional integral formalism, is described by the partition functional:

$$Z_0 = \int \prod_{\Gamma_{((x_\ell);(\mu))}} dU[\Gamma_{((x_\ell);(\mu)}] . e^{-S_0[U[\Gamma]]} \qquad (3)$$

where $\prod_{(\Gamma_{(x_\ell)};(\mu))} dU[\Gamma_{(x_\ell)};(\mu)]$ denotes the normalized Haar measure in the manifold of configuration $\{U[\Gamma_{(x_\ell)};(\mu)\}$ of the lattice Gauge Theory.

The continuous formulation of (3), related to the non-confining phase and describing interacting gauge particles, is given by the Yang-Mills partition functional in Euclidean Space R^4:

$$Z = \int_{M} D[A_\mu(x)] \exp\{-\int d^4x \ S[A_\mu](x)\} \tag{4}$$

$$S[A_\mu] = \frac{1}{4g^2} Tr\{F_{\mu\nu}F_{\mu\nu}\} \tag{5}$$

Here M is the "Infinite Dimensional Manifold" of Yang-Mills Potentials with the appropriate normalized Functional Haar measure $D[A_\mu(x)]$.

Let us now introduce the Contour Space, denoted by L^4, formed by all closed oriented contours in R^4 $\Gamma = \{x_\mu(\tau)\}$; $0 \leq \tau \leq T$ and $\mu = 1;\ldots,4$. We can suppose this space as another continuous limit of the lattice, relevant to the confining phase of gauge theories. The dynamical variable in the L^4-space is taken as the Ordered Phase Factor

$$W[\Gamma] = \frac{1}{N} P\{e^{-\oint_\Gamma A_\mu dx_\mu}\} \tag{6}$$

with $\Gamma \in L^4$ and $A_\mu \in M$. We represent the "Infinite Dimensional Manifold" of these $W[\Gamma]$ by $\mathbb{C}^4$. After defining the "Physical Space" L^4 and the dynamical variable (6) we have to construct an action in $\mathbb{C}^4$-space. As the first step, we should define a notion of integration in L^4-space: the continuous limit of $\sum_{\{\Gamma\}}$ in (1). The introduction of fermions in the Contour Space will suggest a natural one. The symbolic sum

$$\sum_{\substack{\{x_\mu(\tau)\} \\ x_\mu(o) = x_1 \\ x_\mu(T) = x_2}} \Phi[\Gamma_{x_1 x_2}] \tag{7}$$

here $\Phi[\Gamma]$ denotes an arbitrary functional in the L^4-space, should be understood as

$$\int_0^\infty \frac{dT}{T} \int d\mu[\Gamma_{x_1 x_2}]\,\Phi[\Gamma_{x_1 x_2}] \tag{8}$$

with

$$\int d\mu[\Gamma_{x_1 x_2}] = \int_{\substack{x(0)=x_1 \\ x(T)=x_2}} \prod_{(\tau)} dx(\tau)\, d\,e(\tau)\, \exp\{-\frac{1}{2}\int_0^T \frac{\dot{x}^2(\tau)}{e(\tau)} d\tau\} \tag{9}$$

We observe in (8) the use of reparametrization invariant action for a massless particle in the Brink-Vecchia-Howe formulation [17] and it is used here as a purely geometrical object [13].

Introducing the dual string operator in $\mathbb{C}^4$-space

$$\Delta_{(\Gamma)} = \frac{1}{|x'(\tau)|^2} \frac{\delta^2}{\delta x_\mu(\tau)^2} \tag{10}$$

the $\mathbb{C}^4$-continuous version of the lattice operations $\sum_{((x_\ell);(\mu,\nu))}$ and $\Gamma \to \Gamma + \Pi_{((x_\ell);(\mu;\nu))}$, we obtain the following formal $\mathbb{C}^4$-limit action of (1):

$$S[W[\Gamma]] = \frac{1}{4g^2} \int d^4x \int_0^\infty \frac{dT}{T} \int d\mu[\Gamma_{xx}]\ \mathrm{Tr}\{W^{-1}[\Gamma].(\Delta_{(\Gamma)} W[\Gamma])\} \tag{11}$$

and hence the first quantized Quantum Contour Formulation for Yang-Mill Theory is given by the partition functional

$$Z_{(\mathbb{C}^4)} = \int \prod_{(\Gamma)} dW[\Gamma] \exp\{-S[W[\Gamma]]\} \tag{12}$$

with $\prod_{(\Gamma)} dW[\Gamma]$ being the U(N)-Functional Haar Measure in $\mathbb{C}^4$.[††]

The formal $\mathbb{C}^4$-continuous limit of equation of motion (2) is given by:

$$\frac{1}{4g^2}\left(\frac{1}{|x'(\tau)^2}\frac{\delta^2}{\delta x_\mu(\tau)^2} W[\Gamma]\right).W[\Gamma]^{-1}$$

$$= \frac{1}{4g^2} W[\Gamma]\left(\frac{1}{|x'(-\tau)|^2}\frac{\delta^2}{\delta x_\mu(-\tau)^2} W[\Gamma]^{-1}\right) \tag{13}$$

with the reparametrization invariant condition

$$\frac{dx_\mu}{d\tau}.\frac{\delta}{\delta x_\mu} W[\Gamma] = 0 \tag{14}$$

We note that all solutions of the Evolution Equation of Dual Quantized Strings,

$$\frac{1}{|x'(\tau)|^2}\frac{\delta^2}{\delta x_\mu(\tau)^2} W[\Gamma] = 4g^2\beta^4 W[\Gamma] \tag{15}$$

$$\frac{dx_\mu}{d\tau}\frac{\delta}{\delta x_\mu} W[\Gamma] = 0$$

where β is a mass parameter, are also solutions for evolution equation (13).

In order to explain the proposed treatment of fermions in the L^4-space we consider the effective partition functional for the composite mesons relevant for scattering process involving the vacuum polarization by quarks

[††] It is useful compare with the Feynmann loop measure for Wilson loops analised in Ref. [14].

$$Z_{\mathbb{C}^4}[J_\mu(x);\Gamma] = \int \prod_{(\Gamma)} dW[\Gamma] e^{-S[W[\Gamma]]}$$

$$\mathrm{Det}[(i\gamma_\mu \mathbb{D}_\mu + i\gamma_\mu J_\mu)](W[\Gamma]) \quad (16)$$

where in (16) the functional fermionic determinant should be expressed in terms of $W[\Gamma]$. For this task we make use of the Proper-Time formalism as in [1],[13]. Nevertheless, care should be taken in applying straightforwardly the Feynmann integral to represent the propagator of a particle possessing fermionic degrees in the presence of an external gauge field [16]. We avoid this complication by squaring the fermionic determinant and making use of the Proper-Time formalism for bosonic coloured particles in the framework of [17],[18], I E:

$$\mathrm{Det}(i\gamma_\mu \mathbb{D}_\mu + i\gamma_\mu J_\mu) = \mathrm{Det}(\mathbb{D}_\mu \mathbb{D}_\mu)^{1/2}\ \mathrm{Det}(\mathbb{1} - \frac{1}{2}[\gamma^\mu, \gamma^\nu](\mathbb{D}_\mu \mathbb{D}_\mu)^{-1} F_{\mu\nu}) \quad (17)$$

and

$$(\mathbb{D}_\mu \mathbb{D}_\mu)^{-1}(x_1, x_2) = N \int_0^\infty dT \int d\mu[\Gamma_{x_1 x_2}]\ \mathrm{Tr}\{W[\Gamma_{x_1 x_2}]\Phi[\Gamma_{x_1 x_2}, J_\mu]\} \quad (18)$$

where $\Phi[\Gamma_{x_1 x_2}, J_\mu] = \exp\{-\int_{\Gamma_{x_1 x_2}} J_\mu dx_\mu\}$ is the abelian phase factor, defined by the external mesons source $J_\mu(x)$. Further, the colour degrees of freedom of the bosonic particles were integrated out in (18) producing the contour ordenation of the term $\exp\{-\oint_\Gamma A_\mu dx_\mu\}$.

The final expression for the fermionic determinant reads:

$$\mathrm{Det}(\mathbb{D}_\mu \mathbb{D}_\mu)^{1/2} = \exp\{-\frac{1}{2} N \int_0^\infty \frac{dT}{T} \int_{-\infty}^{+\infty} d^4x_1 \int d\mu[\Gamma_{x_1x_1}]$$

$$\mathrm{Tr}\{W[\Gamma_{x_1x_1}]\}\Phi[\Gamma_{x_1x_1}, J_\mu] \tag{19}$$

$$\mathrm{Det}(\mathbb{1} - \frac{1}{2}[\gamma^\mu,\gamma^\nu](\mathbb{D}_\mu \mathbb{D}_\mu)^{-1} F_{\mu\nu}) = \exp\{-\sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} \frac{N^n}{n}$$

$$\int_{-\infty}^{+\infty} d^4x_1 \ldots d^4x_n \int_0^\infty dT_1 \ldots dT_n \int d\mu[\Gamma_{x_1x_2}] \ldots d\mu[\Gamma_{x_nx_1}]$$

$$.\mathrm{Sp}([\gamma_{\mu_1},\gamma_{\nu_1}]\ldots[\gamma_{\mu_n},\gamma_{\nu_n}])\mathrm{Tr}\{\frac{\delta}{\delta\sigma_{\mu_1\nu_1}(x_1)}(W[\Gamma_{x_1x_2}]\Phi[\Gamma_{x_1x_2},J_\mu])$$

$$\ldots \frac{\delta}{\delta\sigma_{\mu_n\nu_n}(x_n)}(W[\Gamma_{x_nx_1}]\Phi[\Gamma_{x_nx_1}, J_\mu]) \tag{20}$$

where Sp denotes the trace over Dirac matrices and

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x(s))} = \lim_{\varepsilon\to 0} \int_{-\varepsilon}^{\varepsilon} d\tau.\tau \frac{\delta^2}{\delta x_\mu(s+\frac{\tau}{2})\,\delta x_\nu(s-\frac{\tau}{2})} \tag{21}$$

is the Polyakov's analytical expression for the Mandelstan Path Derivative. As the operation $\frac{\delta}{\delta\sigma_{\mu\nu}(x)}$ divides the contour $\Gamma_{x_1x_1}$ into two pieces through the point x, we have that the C^4-interaction action for fermions is closely anologous to the joining and splitting picture of the Theory of Interacting Dual Strings. Defining $g^2N = \lambda^2$, this L^4-contour QCD has as non - perturbative expansion parameter the "coupling constant" $\frac{1}{N}$ similar to the t'Hooft topological expansion of Euclidean QCD [19].

Introducing the contour currents

$$j_\alpha(\Gamma,z) = \oint_\Gamma \delta^{(4)}(x(\tau)-z)\,dx_\alpha(\tau) \tag{22}$$

and making the approximation $N\to\infty$ in $\mathbf{C}^4$, we have the following expression for the meson propagator

$$D_{\alpha\beta}(x-y) = \frac{1}{N^2} \left.\frac{\delta^2 Z_{\mathbf{C}}[J_\mu(x),\Gamma]}{\delta J_\alpha(x)\ \delta J_\beta(y)}\right|_{J_\mu(x)=0}$$

$$= \frac{1}{4}\int_0^\infty \frac{dT}{T}\int_0^\infty \frac{ds}{s}\int_{-\infty}^{+\infty} d^4x_1\, d^4x_2 \int d\mu[\Gamma_{x_1x_1}]d_\mu[\Gamma'_{x_2x_2}]$$

$$j_\alpha(\Gamma,x)j_\beta(\Gamma',y)\,(\mathrm{Tr}W[\Gamma_{x_1x_1}])(\mathrm{Tr}W[\Gamma'_{x_2x_2}]) \tag{23}$$

with $W[\Gamma]$ satisfying (13).

Formally $D_{\alpha\beta}(x-y)$ is proportional to the Free Contour Propagator (first quantized) connecting the initial contour Γ to the final one Γ' if we restrict ourselves to the solutions (14), [20],[21].

So, our conclusion is that an exact description of Gauge Theories in terms of Quantized Strings might be possible.

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