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GENERATION AND PROPAGATION OF SYNCHRO - CHERENKOV RADIATION

by

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Abstract:

Particles moving along the magnetic field lines emit under favorable conditions Cherenkov radiation in a cold, rarefied plasma. A peculiar phenomenon occurs for curved magnetic fields: in for example a toroidal magnetic field the radiation spirals inward and approaches a resonance. Both the generation and the study of the propagation of these Cherenkov modes appear to be within reach of present technology.

It is well know that electrons radiate in vacuum only if they are accelerated (and the generation of these waves is "instantaneous"), whereas in a dielectric medium they already emit radiation, if $\varepsilon(\omega) > 1$, for constant rectilinear motion (Cherenkov radiation). This, however, is a cumulative effect which requires $a\omega/c \gg 1$ if a is the distance traversed in the medium and ω the frequency emitted.

Cherenkov radiation can also be generated by electrons moving through a cold, magnetized plasma where, however, only the case of a homogeneous magnetic field and rectilinear motion has been investigated so far in literature [1] . Here we report some results for circular motion in a toroidal field (plus numerical results for a dipole field) which may be of astrophysical relevance (pulsar magnetosphere), but we hope that the main conclusions of this letter can be tested in the laboratory with present technology.

We describe the properties of matter by a phenomenological dielectric tensor $\overleftrightarrow{\varepsilon}$. A standard manipulation of Maxwell's equations gives for the induced magnetic field $\delta\vec{B}$ due to a small external current $\delta\vec{j}$ (see e.g. [1])

$$\mathrm{rot}\, \overleftrightarrow{\varepsilon}^{-1}\, \mathrm{rot}\, \delta\vec{B}_\omega = (\omega/c)^2\, \delta\vec{B}_\omega + (4\pi/c)\, \mathrm{rot}\, \overleftrightarrow{\varepsilon}^{-1}\, \delta\vec{j}_\omega \qquad (1)$$

where the subscript ω indicates Fourier transformation with respect to time.

Our final aim is to solve equation (1) for circular motion of charges in an anisotropic medium. We proceed by first solving the problem for rectilinear motion in the given medium and then generalizing the results to circular motion.

We consider here a cold, one-component magnetized plasma. Its dielectric properties are conveniently described [2] by the two parameters $Y: = \Omega/\omega$, $\Omega = eB/mc$ and $X: = (\omega_p/\omega)^2$, $\omega_p^2 = 4\pi e^2 n/m$. We shall confine our discussion to the case

Y>>X>1 and neglect terms of order X/Y. The dielectric tensor $\overleftrightarrow{\varepsilon}$ then takes the form

$$\varepsilon_{ab} = \varepsilon_{\perp}\, \delta_{ab} + (\varepsilon_{\parallel} - \varepsilon_{\perp})\, Y_a Y_b \tag{2}$$

Here $Y_a = B_a/(\vec{B}^2)^{1/2}$ is the unit vector along the magnetic field. The dielectric coefficients are connected with the quantity X by $\varepsilon_{\parallel} = 1-X$, $\varepsilon_{\perp} = 1$. We note that $\varepsilon_{\parallel}$ is negative according to the above inequality $X > 1$ and that we are in a sense dealing with a uniaxial medium with one negative index.

For rectilinear motion one can obtain an exact solution to (1). For the sake of simplicity we choose a coordinate system in such a way that the background magnetic field coincides with the z-axis. The dielectric tensor (2) is then diagonal, and equation (1) takes the form

$$\varepsilon_{\parallel}^{-1}(\partial^2/\partial x^2 + \partial^2/\partial y^2) + \varepsilon_{\perp}^{-1}\, \partial^2/\partial z^2 \quad \delta\vec{B}_{\omega} = -4\pi/c \ \mathrm{rot}\ \overleftrightarrow{\varepsilon}^{-1}\ \delta\vec{j}_{\omega} \tag{3}$$

The solution can be found by a coordinate transformation [1] . We obtain the Green's function to (3) from the vacuum Green's function through the substitution $\bar{x} = \sqrt{\varepsilon_{\parallel}}\, x$, $\bar{y} = \sqrt{\varepsilon_{\parallel}}\, y$ and $\bar{z} = \sqrt{\varepsilon_{\perp}}\, z$

$$G(\vec{r},\vec{r}) = \sqrt{\varepsilon_{\perp}}\ \varepsilon_{\parallel}/\hat{r} \quad \exp(i\omega\hat{r}/c) \tag{4}$$

where

$$\hat{r} = \varepsilon_{\parallel}(x-x')^2 + \varepsilon_{\parallel}(y-y')^2 + \varepsilon_{\perp}(z-z')^{1/2}$$

After some calculations (see eq. III.18 ref [1]) we get for rectilinear motion of an electron along the magnetic field

$$\delta\vec{B}_\omega = ie/\varepsilon_\perp \varepsilon_\parallel f(s)\ (\vec{r}\times\vec{v})\ \exp(i\omega r g(\theta)/c)\ /\ (2\pi c r^2 g^2(\theta)) \quad (5)$$

with

$$g(\theta) = (\varepsilon_\perp \cos^2\theta + \varepsilon_\parallel \sin^2\theta)^{1/2}$$

$$s = \varepsilon_\perp \cos\theta/g(\theta) - c/v$$

$$f(s) = \begin{cases} 2(\sin(sa\omega/c))/s & , a\omega/c \text{ finite} \\ 2\pi\delta(s) & , a\omega/c \to \infty \end{cases}$$

As in the case of an isotropic medium, Cherenkov radiation is emitted for $s \approx 0$ and $a\omega/c >> 1$. However, the radiation pattern is much more complicated, as can be seen by making a comparison with the isotropic case.

The radiated power per unit frequency is

$$i_\omega = e^2\omega/c((\varepsilon_\perp\beta^2)^{-1} - 1) = e^2\omega\beta/(c^2\gamma^2) \quad (6)$$

As a consequence, the radiation becomes smaller with a higher Lorentz factor γ of the electrons.

We now treat the radiation from circular motion along a toroidal magnetic field. It is not possible to find a Green's function to (1) since for a toroidal magnetic field the dielectric tensor becomes space dependent in a cartesian frame. Therefore we have to rely on approximate methods. In the sense of a WKB approximation our solution for rectilinear motion should still hold locally. One learns from an analysis of circular motion in an isotropic medium that the formulas for the radiated power are even the same for rectilinear and circular motion, so that we are quite sure that our local solutions (5), (6) are good approximations. However, to draw conclusions about the fields far from the charge we have to study the propagation of the wave fields given by (5) through the medium. For this we use the approach described in Ref. [3] and construct the ray

path given by $d\vec{x}/d\tau = \nabla_{\vec{k}} D$, $\delta\vec{k}/d\tau = -\nabla_{\vec{x}} D$ where D is the dispersion relation which can be expressed as
$D = \varepsilon_{\parallel} k_{\parallel}^2 + \varepsilon_{\perp} k^2 - \varepsilon_{\parallel}\varepsilon_{\perp}(\omega/c)^2$, and $d/d\tau$ means the covariant differentiation along the ray path with respect to the affine parameter τ. The dispersion relation is a first integral to the ray equations. In cyclindrical coordinates we obtain

$$\begin{aligned} \dot{r} &= 2\varepsilon_r k_r & \dot{k}_r &= k_\phi \dot{\phi} \\ r\dot{\phi} &= 2\varepsilon_\phi k_\phi & \dot{k}_\phi &= -k_r\dot{\phi} - 2(\varepsilon_r - \varepsilon_\phi) k_r k_\phi / r \\ \dot{z} &= 2\varepsilon_z k_z & \dot{k}_z &= 0 \end{aligned} \tag{7}$$

These equations can be easily integrated. To simplify matters we put $k_z(0) = 0$. With the help of the dispersion relation the ϕ-component of the wave vector can be eliminated from (7). Taking the coordinate ϕ as a new affine parameter of the ray trajectories, the differential equation for k_r can immediately be solved. We find that for large values of $\phi - \phi_o$ the wave vector grows exponentially as

$$k_r \simeq k_{ro} \exp\left[(-\varepsilon_{\perp}/\varepsilon_{\parallel})^{1/2} \quad (\phi - \phi_o)\right] \tag{8}$$

The ray trajectory is obtained by a further integration. Asymptotically we get

$$r \simeq r_o \exp\left[-(-\varepsilon_{\perp}/\varepsilon_{\parallel})^{1/2} \quad (\phi - \phi_o)\right] \tag{9}$$

The waves therefore propagate inwards as stated above and approach a resonance condition. What happens at this resonance can again be studied by means of a homogeneous, anisotropic medium. Let $\vec{B}$ point in the z-direction and $\varepsilon_{\parallel} = \varepsilon(z)$, $\varepsilon_{\perp} = 1$. We then obtain for $B = B_y = b(z) \exp(i\kappa x)$

$$-b'' + \left[\kappa^2/\varepsilon(z) - k_o^2\right] b = o \tag{10}$$

so that in WKB approximation

$$b = \text{const. } f(z)^{-1/4} \quad \exp\left[\pm i \int_{z_o}^{z} f(z')^{1/2}\, dz'\right], \qquad (11)$$

$$f(z) = k_o^2 - \kappa^2/\varepsilon(z)$$

At the resonance ($\varepsilon(z) = o$) we find total reflection with zero amplitude in WKB approximation. A fuller treatment still gives total reflection but inside the region $\varepsilon(z)>o$.

An experimental verification of our conclusion could be done in the laboratory with a rarefied plasma satisfying the condition $Y>>X>1$, which implies, normalized to a magnetic field of 100 KG: $\omega \simeq 10^{11}\ B_5[s^{-1}]$, $\omega_p \simeq 10^{11}\ B_5[s^{-1}]$ and a plasma pressure of $p \simeq 10^{-3}\ B_5^2$ Torr in a vessel of linear dimension $L>>2.B_5^{-1}$ cm. The contribution of the ions is negligible.

We conclude with the remark that Cherenkov radiation could also be of importance for the pulsar magnetosphere. In the canonical picture charges move along a magnetic dipole field through a plasma, which in certain regions satisfies the Cherenkov conditions. The ray equations for the Cherenkov waves can no longer be integrated analytically for a dipole field. A computer calculation, however, shows the same features as for circular motion. In addition, we remark that the Cherenkov radiation would also be coherent if the normal coherence condition is satisfied, so that curvature - and Cherenkov-radiation can be of comparable importance in certain regions of the canonical pulsar magnetosphere.

References

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