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by

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REMARKS ON CLASSICAL UNSTABLE PARTICLES AND THE LONG-RANGE SCALAR FIELD

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ABSTRACT

A possible description of unstable particles, classically defined as those with a rest-mass depending on proper time, is examined. If one assumes the equality between inertial and gravitational masses valid for both stable and unstable particles, a universal interaction between a zero-mass scalar field and all particles, which would thus have a variable rest-mass, is allowed by this equality and has been proposed by Dicke in connection with Mach's principle.

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1. CLASSICAL EQUATION OF AN UNSTABLE PARTICLE

A typical example of an unstable system described by classical theory is the Lorentz model of the hydrogen atom. As this system emits radiation continuously, its energy decays in time.

Let us now consider an unstable elementary particle, for instance, a neutron. Its decay into a proton with the emission of leptons is usually pictured as a transition of a nucleon from a neutron to a lower rest-mass proton state. Quantum mechanically, as is well known, this transition is a result of the Fermi coupling between the nucleon and lepton fields, which allows the difference in rest-energy between the neutron and proton to be transformed away as an electron-antineutrino pair. Classically, this picture may be translated in the statement that the nucleon rest-energy decreases in time and is radiated away.

One is thus led to examine the classical definition of an unstable particle as one, the rest-mass of which, μ_o, is not a constant but depends on the particle's proper time s:

$$\mu_o = \mu_o(s) \tag{1}$$

The equation of motion of a free stable particle:

$$m_o c \frac{du^\alpha}{ds} = 0$$

where $z^\alpha = z^\alpha(s)$ is its world-line and

$$u^\alpha = \frac{dz^\alpha}{ds}, \; ds^2 = dz^\alpha dz_\alpha = (dz^o)^2 - (dz^k)^2 \tag{2}$$

will be replaced, for an unstable free particle, by the phenomenological equation:

$$\frac{d}{ds}(\mu_o \, cu^\alpha) = D^\alpha \tag{3}$$

where D^α - the disintegration damping force - is the four-force that accounts for the particle's decay.

The equation (3) and the normalisation:

$$u^\alpha u_\alpha = 1 \tag{4}$$

lead to:

$$u^\alpha \frac{du_\alpha}{ds} = 0 \tag{5}$$

hence:

$$u_\alpha D^\alpha = c \frac{d\mu_o}{ds} . \tag{6}$$

In the case of the neutron beta-decay, if one were to ascribe this transformation to such a force D^α, the rate of work of this force would have to be equal to a radiated energy of about $\frac{c^2}{\tau}(m_N - m_H)$ MeV/sec., where m_N and m_H are the neutron and the hydrogen atom rest-masses and τ is the neutron lifetime.

If the unstable particle is electrically charged its equation of motion will be

$$\frac{d}{ds}(\mu_o \, cu^\alpha) = \frac{e}{c} F^{\alpha\beta} u_\beta + D^\alpha \tag{7}$$

where $F^{\alpha\beta}$ is the electromagnetic field. As a result of the antisymmetry of $F^{\alpha\beta}$, the relation (6) still holds in the case where D^α is defined by equation (7).

2. UNSTABLE PARTICLE IN A GRAVITATIONAL AND ELECTROMAGNETIC FIELD

Let us now consider an unstable particle in a gravitational field. The equation (3) can be written:

$$d(\mu_o \; cu^\alpha) = D^\alpha ds$$

It is natural to generalise this equation into the following one:

$$\Delta(\mu_o \; cu^\alpha) = D^\alpha ds \qquad (8)$$

where the symbol Δ stands for the operator of covariant or absolute differentiation, and:

$$ds^2 = g_{\lambda\nu} \, dx^\lambda \, dx^\nu$$

$g_{\lambda\nu}(x)$ is the gravitational tensor. The equation (8) reads:

$$\mu_o \, c \, \Delta u^\alpha + cu^\alpha \, \Delta \, \mu_o = D^\alpha ds$$

As μ_o is a scalar function of s, $\Delta\mu_o$ is identical to $d\mu_o$. We thus have, if one takes into account the well known expression for Δu^α:

$$\mu_o \, c \left\{ \frac{du^\alpha}{ds} + \Gamma^\alpha_{\lambda\nu} u^\lambda \, u^\nu \right\} + cu^\alpha \frac{d\mu_o}{ds} = D^\alpha \qquad (9)$$

where:

$$\Gamma^\alpha_{\lambda\nu} = \frac{1}{2} g^{\alpha\varepsilon} \left(\frac{\partial g_{\varepsilon\lambda}}{\partial x^\nu} + \frac{\partial g_{\varepsilon\nu}}{\partial x^\lambda} - \frac{\partial g_{\lambda\nu}}{\partial x^\varepsilon} \right)$$

are the Christoffel symbols.

The equation:

$$g_{\lambda\nu} u^\lambda \frac{du^\nu}{ds} + \frac{1}{2} \frac{\partial g_{\lambda\nu}}{\partial x^\beta} u^\lambda \, u^\nu \, u^\beta = 0 \qquad (10)$$

which is a consequence of the normalisation:

$$g_{\lambda\nu}\, u^{\lambda}\, u^{\nu} = 1$$

leads, when combined with the equation (9), to the relation (6), where now:

$$u_{\alpha} = g_{\alpha\beta} u^{\beta} .$$

The equation (6) may be replaced into the equation of motion (9) to give:

$$\mu_o\, c \left\{ \frac{du^{\alpha}}{ds} + \Gamma_{\lambda\nu}\, u^{\lambda}\, u^{\nu} \right\} = (\delta^{\alpha}_{\eta} - u^{\alpha}\, u_{\eta}) D^{\eta} . \qquad (9a)$$

In the presence of an electromagnetic and a gravitational field, the equation of motion of a particle with variable rest-mass is, therefore:

$$\mu_o\, c \left\{ \frac{du^{\alpha}}{ds} + \Gamma^{\alpha}_{\lambda\nu}\, u^{\lambda}\, u^{\nu} \right\} - \frac{e}{c} F^{\alpha}_{\nu}\, u^{\nu} = (\delta^{\alpha}_{\eta} - u^{\alpha}\, u_{\eta})\, D^{\eta} \qquad (11)$$

where:

$$g_{\lambda\nu}\, u^{\lambda}\, D^{\nu} = c\, \frac{d\mu_o}{ds} . \qquad (12)$$

According to our assumption, the self-force D^{η} characterises classically an unstable particle and vanishes for a stable one.

The equations of the electromagnetic and the tensor gravitational fields are known. To have a meaning, the equation (9) must be supplemented by equations which determine the force D^{α} or μ_o as a function of s.

If this force is assumed to derive from a scalar field $\phi(x)$:

$$D^{\alpha} = \frac{\partial \phi}{\partial Z_{\alpha}} \qquad (13)$$

the equation (12) for D^{α} obtains the form:

$$\frac{d}{ds}(\mu_o c - \phi) = 0 \tag{14}$$

This means that the scalar field ϕ, at the particle's world-line, would determine the mass of the particle. Let m_o be a constant mass; we have, from (14):

$$\mu_o(s) = m_o + \frac{1}{c}\phi\,(Z(s)) \tag{15}$$

<u>The variable mass of an unstable particle is equivalent to a particle with a constant mass in interaction with a sclar field</u>.

3. DICKE'S EQUATION OF MOTION

It is of interest to consider now the long range scalar field φ whose existence has been assumed by Dicke [1] in order to overcome, at least in part, the absolute space-time character of Einstein's relativistic theory of gravitation. The properties of this field are:

a) the source of the field is a scalar mesure of the mass density of the universe, T; a simplified equation satisfied by the field may be of the form:

$$\Box\varphi = -4\pi f\, T \tag{16}$$

where f is a coupling constant;

b) the scalar field gives rise to an attractive force between all bodies;

c) the scalar field coupling is weak, of the order of the gravitational coupling;

d) the interaction of Dicke's field with a particle cannot occur unless the mass of the particle is a function of this field.

Dicke's equation of motion for a particle of rest-mass μ_o is, in our notation:

$$\frac{d}{ds}(\mu_o\, cu_\alpha) - \frac{1}{2}\mu_o\, c\, \frac{\partial g_{\lambda\nu}}{\partial Z^\alpha}\, u^\lambda u^\nu + c\, \frac{d\mu_o}{d\varphi}\, \frac{\partial\varphi}{\partial Z^\alpha} = 0$$

This equation is equivalent to equation (9), if one sets:

$$D^\alpha = -\, c\, \frac{d\mu_o}{d\varphi}\, \frac{\partial\varphi}{\partial Z_\alpha}\, .$$

Indeed, we can write (9) in the following form:

$$\mu_o\, c\, g^{\alpha\eta}\, \frac{du_\eta}{ds} + \mu_o\, cu_\eta\, \frac{\partial g^{\alpha\eta}}{\partial Z^\lambda}\, u^\lambda + \frac{1}{2}\mu_o\, c\, g^{\alpha\lambda}\left(\frac{\partial g_{\lambda\eta}}{\partial Z} + \right.$$

$$\left. + \frac{\partial g_{\ \nu}}{\partial Z^\eta} - \frac{\partial g_{\eta\nu}}{\partial Z^\lambda}\right) u^\eta u^\nu + c\, g^{\alpha\eta}\, u_\eta\, \frac{d\mu_o}{ds} = D^\alpha$$

or:

$$g^{\alpha\eta}\frac{d}{ds}(\mu_o\, cu_\eta) - \frac{1}{2}\mu_o\, c\, g^{\alpha\eta}\, \frac{\partial g_{\lambda\nu}}{\partial Z^\eta}\, u^\lambda u^\nu + \mu_o\, c\, g_{\eta\beta}\, \frac{\partial g^{\alpha\eta}}{\partial Z^\lambda}\, u^\beta u^\lambda +$$

$$+ \mu_o\, cg^{\alpha\beta}\, \partial g_{\eta\beta}\, u^\beta u^\lambda = D^\alpha$$

hence:

$$\frac{d}{ds}(\mu_o\, cu_\eta) - \frac{1}{2}\mu_o\, c\, \frac{\partial g_{\lambda\nu}}{\partial Z^\eta}\, u^\lambda u^\nu = D_\eta\, .$$

The difference between ours and Dicke's equation lies in the significance of the force D^α. Whereas we tried to introduce such a force to distinguish, in the real m of classical physics, an unstable from a table particle, Dicke introduce it as an additional gravitational force, satisfying the item b) above.

4. THE EOTVOS EXPERIMENT AND DICKE'S UNIVERSAL INTERACTION

In Dicke's theory, therefore, equation (9) is valid for all particles. The equality between the inertial and the gravitational masses, assumed to hold for all particles, imposes a condition on the variability of the mass μ_o and on the force D^α.

If the equation (9a) is assumed to be valid for all particles:

$$\frac{du^\alpha}{ds} + \Gamma^\alpha_{\lambda\nu} u^\lambda u^\nu = \frac{1}{\mu_o c} (\delta^\alpha_\eta - u^\alpha u_\eta) D^\eta, \qquad (9a)$$

the second-hand side of this equation will be independent of the particle if we postulate, as already pointed out by Dicke, that:

a) the variable mass μ_o be equal to a constant λ_o - presumably characteristic of the particle - multiplied by a universal function of s, the same for all particles, V(s):

$$\mu_o(s) = \lambda_o V(s); \qquad (17)$$

b) the scalar field ϕ, as defined by equation (13), be equal to the same constant λ_o, which depends on the particle, multiplied by a universal function φ:

$$\phi(x) = \lambda_o \varphi(x) . \qquad (18)$$

If one identifies the constant λ_o with the constant mass m_o given in (15), one sees that the universal function V(s) is given in terms of φ by:

$$V = 1 + \frac{\varphi}{c}, \quad \lambda_0 = m_0 \qquad (19)$$

The relations (13), (15), (17) and (18) transform this equation into:

$$\frac{du^\alpha}{ds} + \Gamma^\alpha_{\lambda\nu} u^\lambda u^\nu = \frac{1}{V}\left\{ \frac{\partial V}{\partial Z_\alpha} - u^\alpha \frac{dV}{ds} \right\} . \qquad (20)$$

The equality between the inertial and the gravitational masses requires, therefore, that the scalar field generate a universal interaction among all particles. We emphasize that Dicke's field being produced, by hypothesis, by the matter in universe, it is supposed to act on all particles, including the stable ones like the electron - the scalar field would be essentially a part of the gravitational field, the other part being the tensor field. This is best seen when one examines the problem of a particle moving in a weak, static gravitational field.

We can write equation (20) in another form, of the geodesic type, if we transform the metric by means of the relation:

$$\bar{g}_{\mu\nu} = V^2 g_{\mu\nu}, \quad \bar{g}^{\mu\alpha} \bar{g}_{\alpha\nu} = \delta^\mu_\nu \qquad (21)$$

and define the new variables:

$$d\bar{s}^2 = V^2 ds^2, \quad \bar{u}^\alpha = \frac{dz^\alpha}{ds} = V^{-1} u^\alpha. \qquad (22)$$

The equation (20) goes over into the following one:

$$\frac{d\bar{u}^\alpha}{ds} + \bar{\Gamma}^\alpha_{\lambda\nu} \bar{u}^\lambda \bar{u}^\nu = 0 \qquad (23)$$

In the limit of a weak, static gravitational field, one writes:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \varepsilon \gamma_{\mu\nu}$$

where ε is a small parameter and $g^{(o)}_{\mu\nu}$ is the Lorentz metric tensor:

$$g^{(o)}_{oo} = - g^{(o)}_{11} = - g^{(o)}_{22} = - g^{(o)}_{33} = 1; \; g^{(o)}_{\mu\nu} = 0, \quad \mu \neq \nu .$$

$\bar{g}_{\mu\nu}$ becomes, according to (21) and (19):

$$\bar{g}_{\mu\nu} \cong g^{(o)}_{\mu\nu} + \eta_{\mu\nu}, \; \eta_{\mu\nu} = \varepsilon \gamma_{\mu\nu} + \frac{2\varphi}{c} g^{(o)}_{\mu\nu}$$

if the function φ is also treated as a small perturbation:

$$\varphi \ll 0$$

In the first approximation in ε, φ and the particle's velocity, one therefore obtains for the equation (23):

$$\frac{d^2 z^{\alpha}}{dt^2} + \bar{\Gamma}^{\alpha}_{oo} c^2 = 0$$

where, in this approximation:

$$\Gamma^{\alpha}_{oo} \cong \frac{1}{2} g^{(o)\alpha\lambda} \; 2 \frac{\partial \eta_{\lambda o}}{\partial zo} - \frac{\partial \eta_{oo}}{\partial z^{\lambda}}$$

The consistency condition $\bar{\Gamma}^{o}_{oo} = 0$ requires that $\varepsilon \gamma_{oo} + \frac{2\varphi}{c}$ be time-independent. If this dependence also holds for the other components of $\gamma_{\alpha\beta}$, one obtains Newton's equation of motion for a particle moving in a static potential $U = \frac{c^2}{2} \varepsilon \gamma_{oo} + c\,\varphi$.

The scalar field φ is thus seen to be amalgamated with the zero-zero component of $\gamma_{\mu\nu}$ to give the observable potential U- in fact, according to the definition (21), the scalar field V hides itself in the new metric tensor $\bar{g}_{\mu\nu}$. Its existence would presumably be revealed in observables depending on μ_o.

5. SPECIFIC DECAY INTERACTIONS

We may, still, wish to distinguish, in the real m of classical physics, unstable particles like neutrons, from stable ones, like electrons. If one maintains the definition of an unstable particle as one with a decreasing rest-mass, we may be led to distinguish two components in the force D^{α} which occurs on the right-hand-side of equation (11):

$$D^{\alpha} = D_{o}^{\alpha} + D_{1}^{\alpha}$$

D_{o}^{α} is the force derived from Dicke's universal scalar field, acting on all particles; D_{1}^{α} is the decay force, acting on unstable particles but vanishing for stable particles. Equation (9) now is:

$$\mu_{o}\, c \left\{ \frac{du^{\alpha}}{ds} + \Gamma^{\alpha}_{\lambda\nu}\, u^{\lambda}\, u^{\nu} \right\} + u^{\alpha}\, c\, \frac{d\mu_{o}}{ds} = D_{o}^{\alpha} + D_{1}^{\alpha} \qquad (24)$$

and the relation (6) has the form:

$$u_{\alpha}(D_{o}^{\alpha} + D_{1}^{\alpha}) = c\, \frac{d\mu_{o}}{ds} \quad . \qquad (25)$$

We see, however, that the occurence of the force D_{1}^{α} seems artificial. For in the same way that charged particles have a universal interaction with the electromagnetic field, one would prefer to state that the mass variation of all particles would result from an universal interaction with the scalar field such as defined in the preceding paragraph. But this interaction, if it exists, does not correspond to any instability of particles - since it occurs for all them - but rather to a scalar gravitational interaction, in addition to the tensor field interaction;

and the mass variation would perhaps correspond to a cosmological variation of the gravitational constant. In fact, one has:

$$\mathcal{G}\mu_p^2/\hbar c \cong 10^{-40}$$

where μ_p is the proton mass. If μ_p is given by the equation (17) where λ_o is the proton rest-mass m_p one obtains:

$$\mathcal{G}'\, m_p^2\, /\hbar c \cong 10^{-40}$$

where $\mathcal{G}' = \mathcal{G}\, V^2(s)$ varies with time.

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