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# A MARIONETTE UNIVERSE

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ABSTRACT

We present a study on the non-minimal coupling of a vector field and gravity in the context of a Weyl integrable spacetime (WIST). The dynamics for this system is shown to allow a cosmic solution in which this vector field, responsible for the evolution of the metric properties of the Universe, can undergo unrestricted fluctuations. In a sense, the physical causes of the evolution of the cosmos seem to be uncontrollable themselves — a feature that can be thought of as representing a *marionette Universe*.

Key-words: Non-minimal coupling; Weyl-integrable spacetimes; Arbitrary causes of cosmic curvature.

# SUMMARY

## I. Introduction

Since its first appearance[1], in 1917, up to present times, the cosmological constant $\Lambda$ introduced by Einstein in order to organize the gravitational field at large scales, when cosmic dimensions are involved, has provoked a series of speculations concerning its physical origin — still mysterious today. From the practical point of view, the modifications its introduction determines, in the context of the equations of gravitation, are seen to be rather wide, ranging from altering the inevitability of gravitational colapse (like in the De Sitter case), to the elimination of primordial cosmic irregularities, as for instance in the work of Starobinsky[2]. Recently, particle physicists made effort pro the rehabilitation of $\Lambda$, after a long period in the 70's when it had been almost forgotten — or better, put apart of the principal lines of cosmological investigations.

Its origin has been considered intriguing for some sixty years. However, since the application of Quantum Field Theory methods to cosmological questions it became apparent that, contrary to one's first belief, it is not the presence of $\Lambda$ that should cause troublesome, but rather the opposite: its absence. In effect, though some cosmologists have proposed arguments according to which one obtains $\Lambda < 10^{-55}cm^{-2}$, the physics of elementary particles offers so many processes capable of inducing the occurrence of a non-null (and even very great) constant $\Lambda$ that we are lead to consider the opposite question: why should $\Lambda$ be almost exactly zero ?

Just as $\Lambda$ emerged as an inscrutable object aggreagated to

Einstein's equations of gravitation, along recent years several alterations of these equations were idealized, according to diverse conceptions anchored at cosmological reasonings. Among these, a proposal that in a certain period enjoyed great respectability was that of the introduction of a scalar field $\phi$ in order to provide for a covariant formulation of a (somewhat "Pythagoric") idea aimed at explaining the existence of some numbers, constructed with adimensional ratios of well-known quantities that appear in distinct areas of physics, which are extremely large (Dirac's Large Number Hypothesis[3]). Thus, for instance, $e^2/Gm_e m_p \sim 10^{39}$ — which measures the ratio of electric and gravitational strenghts among protons and electrons; also, the lifetime of the Universe (in the standard model) when measured in microscopic time units ($e^2/m_e c^3 \sim 10^{-23}$sec) yields the same number, $10^{39}$; and the number of particles existing inside our horizon is thought to be the order of $10^{80}$ — which is approximately the square of the previous number. For conciliating these numerical coincidences with a evolutionary (i. e., non-stationary) Universe scenario, it was suggested that possibly some of the so-called "fundamental constants" were not truly constant, but would rather vary with cosmic time.

The most abrangent accomplishment of this trend was achieved by the "gauge-covariant" theory of gravitation of Canuto and collaborators[4], in which a conformal function was introduced to correlate cosmic and atomic units, in the spirit of the above--mentioned LNH of Dirac, in order to establish a conformally scale--covariant set of Einstein-type equations, to be valid in general units and thus displaying, besides usual coordinate covariance, also a scale or "gauge" covariance. However, due to the presence of the conformal function as a fundamental feature of the theory, the structure of spacetime did not result to be Riemannian, but rather conformally-Riemannian or, as is more commonly aknowledged, a Weyl-integrable spacetime (WIST) structure.

What seems to have striked a deadly blow on this theory was the observation that the world we live in is not conformally invariant ! In other words, if perchance we exist in a Weyl-integrable space domain, we could for better calibrate our physics in such a way as to express it in a Riemannian fashion — unless this should appear to be impossible, in virtue of dynamical hinders. For instance, what would happen if the system of equations describing, on a cosmic scale, what we call gravitational processes were to develop a WIST structure ? From the outset we would inherit, besides a metric structure tensor $g_{\mu\nu}(x)$ (to be used for measuring lenghts), also a scalar function $\phi(x)$ (to be used for measuring variations of lenghts). Could we always disregard this function, for example carrying it away through some unit transformation procedure, as in the standard theory ? This possibility is, of course, intimately dependent on the nature of the dynamics obeyed by those processes, that is, on the way this field $\phi$ becomes dynamically activated.

We shall see in the present work that the theory we propose here for describing gravitational processes in a WIST leads to a dynamical separation of a cosmic "scalar" field, which passes to hover beyond any determination. Although we do not claim that this is a general property of gravitational systems, it means that it is possible to find and exhibit, as we do later on, a special configuration in which the metric structure of spacetime has as source of curvature a scalar function of unaccesible origin, as if for every point of spacetime were ascribed a distinct local cosmological constant, each of equally indeterminate provenance. The reader might ask himself, at this point, if such solution is not just a consequence of a bad dynamics, and so should be neglected. We show in the following that the internal coherence of our set of dynamical equations does not allow one to get rid so swiftly of this conundrum.

Before emprehending such task, however, we shall begin by discussing some fundamental theoretical issues and defining the basic objects of the theory.

## II. Non-minimal Coupling with Gravitation

Let us suppose that we are interested in the description of physical processes involving a vector field $A^{\mu}$ in curved space. In his article "Die Grundlagen der Allgemeinen Relativitäts Theorie" (Annalen der Physik, 1916), Einstein describes in a simple and direct way the coupling between a vector field and gravitation. His procedure — which rests at the basis of all subsequent generalizations of theories firstly written in Minkowskian spacetime and then extended for curved spacetimes — consists simply in a straightforward use of tensor theory in order to obtain equations that are valid in any system of coordinates, whatever the state of motion of the observers concerned with this interaction. In this procedure, it is assumed that functions of spacetime curvature do not enter in the picture of the interaction. Such approach is sometimes identified as constituting an expression of the strong principle of equivalence of gravitation. Though this minimal coupling indeed possessed special atractives (among which the issue of introducing less arbitrariness, that is, of introducing a small number of arbitrary conditions), not always have the physicists made indiscriminate use of it. To quote only one remarkable example, in recent times, we have just to recall the important rôle that conformal coupling (which is non-minimal) of a scalar field with gravitation has performed in some modern investigations. Beyond that simplist formal choice, some authors have produced a new, particularly attracting, reasoning for the adoption of non-minimal couplings of physical fields with gravitation. This subject leads us to the question of the cosmic singularity, of gravitational collapse in general.

## III. The Cosmic Singularity

At the end of a long debate concerning the inevitability or not of the existence of a true singularity in the gravitational field, it seems that the great majority of physicists were led to accept the arguments suggested by some authors[5], according to which it is not possible, for any gravitational field having well-behaved matter as source, to exhibit an absolute regularity. Somewhere, sometime, some singular region shall be encountered, such that possible paths of actual observers may desappear of our spatiotemporal representation of the world. This conclusion became consubstantiated in a series of mathematical theorems endeavouring the irrevocable elimination of the world's regularity, foreseeing in some domain of spacetime a singular region. Unfortunately, one may say, the sucess of these theorems was so great that alternative critical approaches were inhibited. In fact, almost all of the theorems then demonstrated rely upon some hypotheses that are not so that easy to support, if we were to mantain the highly critical standards that are commonplace in scientific literature today. Among these basic hypotheses, two are particularly fragile and of acceptance, at least, doubtful: (i) the existence of a global Cauchy surface, and (ii) the condition $(R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}) V^\mu V^\nu \leqslant 0$, for arbitrary timelike observers with velocity $V^\mu$.

It is a hard matter to conceive any actual observation or evidence for the first one. However, it allows one to grip firmly to the old classical determinism, and so, it seems, only those who put at risk their respectability, or else some scientists in their vague metaphysical moments dare to restrain their adhesion. The second condition, on the other hand, is far more objective and at the same time more dramatic. Its validity, in the context of Einstein's theory of gravitation, is linked to the condition of positivity of energy. In effect, $T_{\mu\nu}V^\mu V^\nu \geqslant 0$ implies, through Einstein's equations, $(R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}) V^\mu V^\nu \leqslant 0$.

Precisely here enters the stage the new characteristic, allowed for by non-minimal coupling, that we have mentioned above: it can produce a splitting between those two conditions - energy positivity and the relation $(R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}) V^\mu V^\nu \leqslant 0$ - hence disabling the application of the theorems and conveying the question on the existence or not of a singularity to a frontal examination of each solution. Thus, for example, owing to this circunvention Novello and Salim produced a model of an Eternal Universe[6] (further explored in Ref.7), in which a spatially homogeneous and isotropic (Friedman-like) universe without singularity is presented. It could be conjectured, therefore, that a good systematic procedure for inhibiting the appearance of disagreable singular regions would be to promote a non-minimal coupling of gravitation with some physical field. We shall return to these generic questions elsewhere.

## IV. A Vector Field coupled non-minimally to Gravity

This rather long introductory discussion was made necessary in order to lead a more critical reader to regard with some sympathy the issue of non-minimal coupling — which is at the basis of the present work. In effect, we consider the dynamics of processes involving a vector field $A^{\mu}$ and gravitation as given by the Lagrangian

$$\mathcal{L} = \sqrt{-g}\ [\ \frac{1}{k}\ R - \frac{1}{4}\ f^{\mu\nu}f_{\mu\nu} + \beta\ RA_{\mu}A^{\mu} + \mathcal{L}_{mat}\ ] \qquad (1)$$

where $f_{\mu\nu} \equiv A_{[\mu,\ \nu]}$ and $\beta$ is an adimensional number of the order of unity; since we shall not deal with matter in this article, we take $\mathcal{L}_{mat} = 0$ .

The first step is to obtain the dynamics, via a variational principle, from this Lagrangian. Before this, however, we must specify the way we shall treat the fundamental variables to be varied. It has been common, since Palatini[8], to vary independently, as geometric variables, both the metric tensor $g_{\mu\nu}$ and the connection $\Gamma^{\alpha}_{\mu\nu}$ . In Einstein's theory (for the vacuum), with $\mathcal{L}_E = \sqrt{-g}\, R$, one obtains the beautiful result that the variation $\delta\Gamma^{\alpha}_{\mu\nu}$ of the connection gives rise naturally to a Riemannian structure for spacetime. This result is quite general and can be obtained also when one couples gravity to matter, provided the coupling be minimal ! This small feature seems to have passed unnoticed to a majority of authors. Recently, Novello and Heintzmann[9] have demonstrated that performing a Palatini variation when matter is described by a vector field coupled non-minimally to gravity conduces in a straightfoward way to the result that spacetime structure is not Riemannian, but rather a WIST structure, as is the case also in this work.

Indeed, a Weyl space[10], in which by definition holds

$$g_{\mu\nu;\lambda} = g_{\mu\nu}\,\phi_\lambda \quad , \tag{2}$$

is called a WIST when the vector $\phi_\lambda$ is irrotational, that is, when

$$\phi_\lambda = \nabla_\lambda \phi \quad , \tag{3}$$

for some scalar field $\phi$. Now, using Palatini's variational principle the following equations are implied for the case of our Lagrangian eq(1):

i) From $\delta\Gamma^{\alpha}_{\mu\nu}$ :

$$g_{\mu\nu;\alpha} = g_{\mu\nu}\,\omega_\alpha \quad , \tag{4}$$

where

$$\omega_\alpha = \nabla_\alpha \left[-\ln\left(\frac{1}{k} + \beta A_\mu A^\mu\right)\right] \quad , \tag{5}$$

and where symbol ; denotes covariant differentiation generated by WIST connection $\Gamma^\alpha_{\mu\nu}$, which is given, according to Weyl space theory, by

$$\Gamma^\alpha_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} - \frac{1}{2}\left(\omega_\mu \delta^\alpha_\nu + \omega_\nu \delta^\alpha_\mu - \omega^\alpha g_{\mu\nu}\right) \quad , \tag{6}$$

$\left\{ {\alpha \atop \mu\nu} \right\}$ being the usual Christoffel symbols of General Relativity.

ii) <u>From $\delta A_\mu$</u> :

$$f^{\mu\nu}{}_{||\nu} = -\beta R A^\mu \tag{7}$$

where the double bar represents covariant differentiation in the Riemannian sense, that is, making use only of Christoffel symbols $\left\{ {\alpha \atop \mu\nu} \right\}$ .

iii) <u>From $\delta g^{\mu\nu}$</u> :

$$\left(\frac{1}{k} + \beta A_\alpha A^\alpha\right) G_{\mu\nu} = -E_{\mu\nu} - \beta R A_\mu A_\mu \tag{8}$$

where

$$E_{\mu\nu} \equiv f^\alpha_\mu f_{\alpha\nu} + \frac{1}{4} f_{\alpha\beta} f^{\alpha\beta} g_{\mu\nu} \ . \tag{9}$$

Three important remarks: firstly, spacetime is WIST--type naturally, due to non-minimal coupling, and this result is not altered by the introduction of (minimally coupled) matter.

Secondly, eq(7) for the vector field is non-linear due to the right-hand side, in which a complicate functional of $A_\mu$ is contained in the scalar of curvature R. Finally, we observe the renormalization of constant k, depending on the factor $A_\mu A^\mu$, and implying a possible weakening of the usual positivity (i.e., atractiveness) of gravitational interaction, and so the viability of a gravitational behaviour far more complex than and quite distinct from that of the conventional theory of General Relativity.

## V. The Marionette Universe

In order to exhibit the property of generalization of $\Lambda$ mentioned at the Introduction, we proceed to elaborate a special solution for the set eqs(5, 7, 8) of dynamical equations. We begin by asking for an homogeneous and isotropic spacetime metric structure. By the well-known Robertson-Walker arguments, we can write the line element in the form

$$ds^2 = dt^2 - S^2(t)[d\chi^2 + \sigma^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)] \quad . \tag{10}$$

From the expression eq(6) for the WIST connection $\Gamma^\alpha_{\mu\nu}$ in terms of Christoffel symbols $\left\{ {\alpha \atop \mu\nu} \right\}$ and vector $\omega_\alpha$ we can write for the contracted curvature tensor $R_{\mu\nu}$ the expression

$$R_{\mu\nu} = \overset{(\circ)}{R}_{\mu\nu} - \frac{3}{2}\,\omega_{\mu||\nu} + \frac{1}{2}\,\omega_{\nu||\mu} - \frac{1}{2}\,\omega^\alpha{}_{||\alpha}\, g_{\mu\nu} - \frac{1}{2}\omega_\mu\omega_\nu + \frac{1}{2}\,\omega_\varepsilon\omega^\varepsilon g_{\mu\nu} \,, \tag{11}$$

and accordingly, for the trace R,

$$R = \overset{(o)}{R} + 3\omega_\mu \omega^\mu - 3\omega^\mu{}_{||\mu} \quad , \tag{12}$$

where symbol $^{(o)}$ refers to Riemann spacetime (RST) structure. Adopting the ansatz

$$R = 0 \tag{13a}$$

$$A_\mu \equiv (\psi(t), 0, 0, 0) = \psi(t)\delta^0_\mu \tag{13b}$$

it follows that eq(7) for the field $A_\mu$ is automatically satisfied, since field $f_{\mu\nu}$ is null. Einstein's equations eq(8) turn out to be just

$$G_{\mu\nu} = 0 \ . \tag{14}$$

Using expressions eqs(11), (12) into this equation, we have

$$\overset{(o)}{G}_{\mu\nu} = \omega_{\mu||\nu} + \frac{1}{2}\omega_\mu\omega_\nu - (\omega^\alpha{}_{||\alpha} - \frac{\omega^2}{4})\ g_{\mu\nu} \tag{15}$$

It is worth at this point to observe that the reader well acquainted with scalar-tensor theories of gravitation[11] would have noticed the similarity of the above equation to those occurring in these theories. Consider, for example, the most popular of them, the Brans-Dicke theory, whose equations are

$$\left\{ \begin{array}{ll} \overset{(o)}{G}_{\mu\nu} = \frac{m}{\phi^2}\,(\phi_{|\mu}\,\phi_{|\nu} - \frac{1}{2}\,g_{\mu\nu}\,\phi_{|\lambda}\,\phi^{|\lambda}) & \\ + \frac{1}{\phi}\,(\phi_{|\mu||\nu} - g_{\mu\nu}\,\Box\phi) \ , & (16a) \\ \Box\phi = 0 \ . & (16b) \end{array} \right.$$

The resemblance among eqs(15) and (16a) is transparent. However, a relation such eq(16b), that restricts B-D field $\phi$ and thus allows one to generate independent but interlinked dynamics for the scalar and the tensor fields, does not happen in our theory. This feature is related to the elected ansatz, since as eq(7) is identically satisfied, a suplementary equation for our "scalar" field $A^2$ results eliminated.

Here rests also the reason for an interesting phenomenon: the physical cause of an expanding Universe is misteriously projected into an arbitrary function that determines the degree of Weylization of spacetime. In effect, eq(15) reduces to the expressions for the components (0-0) and (1-1):

$$\begin{cases} 2\,\dfrac{\ddot{S}}{S} = \ddot{a} + \dot{a}\,\dfrac{\dot{S}}{S} & \text{(17a)} \\[2ex] \dfrac{\ddot{S}}{S} + 2\left(\dfrac{\dot{S}}{S}\right)^2 - \dfrac{2\sigma''}{S^2\sigma} = \dfrac{1}{2}\,\ddot{a} - \dfrac{1}{2}\,\dot{a}^2 + \dfrac{5}{2}\,\dfrac{\dot{S}}{S}\,\dot{a}\ , & \text{(17b)} \end{cases}$$

where $a(t) \equiv -\,\ell n\,(\frac{1}{k} + \beta\psi^2(t))$,

a system that, as should be expected for coherence, reduces further to just one condition correlating the functional dependence of S(t) and a(t):

$$\dot{a} = \frac{2\dot{S}}{S} \pm \frac{\sqrt{\varepsilon}}{S}\ , \tag{18}$$

in which we have set, for compatibility of the other spatial components with (1-1),

$$\frac{\sigma''}{\sigma} = \frac{\varepsilon}{4}\ , \tag{19}$$

for $\varepsilon$ = constant = (0, ±1), and the prime denotes differentiation with respect to coordinate $\chi$.

It is then possible to integrate eq(18), yielding

$$S(t) = \frac{1}{(\frac{1}{k} + \beta A_\mu A^\mu)^{1/2}} \left[ \pm \frac{\sqrt{\varepsilon}}{2} \int (\frac{1}{k} + \beta A_\mu A^\mu)^{1/2} \, dt + \text{const.} \right] . \quad (20)$$

Let us examine in some detail this relation. Before aught else, we point out to the reader that eq(20) is the final and complete solution of our set of dynamical equations. Its indeterminacy, comprised in the arbitrariness of $A_\mu A^\mu$, enables one to generate particularly attracting metric structures, given the characteristic of spacetime departure from the Riemannian regime (when $A_\mu \neq$ const.).

A very interesting situation occurs when we demand the De Sitter condition $\frac{\dot{S}}{S} = H =$ const. This happens when

$$a(t) = 2Ht \pm \frac{\sqrt{\varepsilon}}{H} e^{-Ht} \quad . \quad (21)$$

A question that immediately arises is: where does this field $\psi$ come from ? For the present solution this question should have an answer analogous to that concerning the singular origin of the Friedman model: it is an <u>initial datum</u> of the theory. Just like $\Lambda$ would have a misterious global origin, our function $\psi$ would equally represent a sort of variable cosmic influence on gravitational phenomena.

From a wider point of view, we see that in this scheme the form of the evolution of the cosmic function $S(t)$ becomes dependent of an entirely uncontrollable function $\psi(t)$, whose presence charactherizes the Weyl structure of spacetime. It is a remarkable property of the dynamics generated by Lagrangian eq(1) the fact that such arbitrariness in the cosmical context is allowed. Since we can prescribe at will function $\psi(t)$ (which can be understood as the real cause of cosmic curvature), we are quite naturally induced to name this solution a *Marionette Universe*.

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