NOTAS DE FÍSICA

VOLUME XII

Nº 18

ALGEBRA OF CURRENT AND $\rho^{0} \longrightarrow \pi^{+} \pi^{-} \gamma$ DECAY MODE

by

Prem P. Srivastava

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1967

Notas de Física - Volume XII - Nº 18

ALGEBRA OF CURRENT AND $\rho^0 \longrightarrow \pi^+ \pi^- \gamma$ DECAY MODE *

Prem P. Srivastava **
Physics Department, Syracuse University
Syracuse, New York

(Received May 4, 1967)

ABSTRACT

Current algebra technique is applied to the decay $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ to relate it to the decay mode $\rho^0 \rightarrow \ell^+ \ell^-$. The branching ratio $\Gamma(\rho^0 \rightarrow \pi^+ \pi^- \gamma)/\Gamma(\rho^0 \rightarrow \mu^+ \mu^-)$ is found to be ~ 16.

---

* Supported in part by the U. S. Atomic Energy Commission.

** On leave of absence from "Centro Brasileiro de Pesquisas Físicas" and "Universidade do Brasil", Rio de Janeiro, Brazil.

The algebra of current commutation relations [1] and the hypothesis of partially conserved axial vector current [2] have been applied recently to several decay processes with impressive successes. [3]

We use the same technique for calculating the decay $\rho^0 \longrightarrow \pi^+\pi^-\gamma$ and relate it to the decay mode $\rho^0 \longrightarrow \ell^+ \ell^-$. The branching ratio of ρ^0 into a muon pair [4] compared to the dominant decay mode has been obtained experimentally [5] to be $0.44^{+0.21}_{-0.09} \times 10^{-4}$.

The two pions in the decay under consideration must be produced in state of even angular momentum because of charge conjugation invariance. Sakurai [6] has pointed out that many of the results derived from current algebra assuming octet dominance together with unsubtracted dispersion relations can also be derived by assuming the vector meson (or current) dominance model. It is interesting that in the decay mode under consideration even through the two pions go out say, in s-state, the vector current dominance is again emphasized by the current algebra technique.

Let us consider the quantity

$$M^{\mu\nu} = \iint d^4x\, d^4y\, e^{iq\cdot x}\, e^{ip\cdot y} \langle 0|T(A_+^{\mu}(x)\, A_-^{\nu}(y)\, J^{\lambda}(0))|\rho^0\rangle$$

where $A_{\pm}^{\mu}$ are the members of an octet of axial currents and J^{λ} is the electromagnetic current which under SU_3 transforms as

$$J^{\lambda} = e(V_3^{\lambda} + \frac{1}{\sqrt{3}} V_8^{\lambda})$$

$v_i (i = 1, 2, \dots 8)$ being the octet of vector currents.

Following Weinberg's discussion,[7] and on removing[8] all the pion poles from $q_\mu p_\nu M^{\mu\nu}$ we find

$$C_\pi^2 \langle \pi^+ \pi^- | J^\mu(0) | \rho^0 \rangle$$

$$= e \langle 0 | V_3^\mu(0) | \rho^0 \rangle$$

$$- \frac{i}{2}(q-p)_\nu \int e^{i(q+p)\cdot x} \langle 0 | T(V_3^\nu(x) J^\mu(0)) \rho^0 \rangle d^4x$$

$$- i\, e\, q_\nu \int e^{iq\cdot x} \langle 0 | T(A_+^\nu(x)\; A_-^\mu(0)) | \rho^0 \rangle d^4x$$

$$+ i\, e\, p_\nu \int e^{ip\cdot y} \langle 0 | T(A_-^\nu(y)\; A_+^\mu(0)) | \rho^0 \rangle d^4y$$

+ terms involving scalar field operator σ

$$+ q_\mu\, p_\nu\, N^{\mu\nu}$$

where the last terms is $q_\mu\, p_\nu\, M^{\mu\nu}$ with all the pion ooles removed Here we have used the commutation relations[9] suggested by quark model:

$$\delta(x^0)\left[A_+^0(x),\; A_-^\mu(0)\right] = - V_3^\mu(0)\, \delta^4(x)$$

$$\delta(x^0)|A_\pm^0(x),\; J^\mu(0)| = \mp\, e\, A_\pm^\mu(0)\, \delta^4(x)$$

$$\delta(x^0)|A_i^0(x),\; \varphi_j(0)| = \delta_{ij}\, \sigma(0)\, \delta^4(x) + \text{Schw. terms.}$$

σ being the field operator corresponding to a scalar meson field.[10] We also made use of the PCAC hypothesis[2]:

$$\partial_\mu\, A_\pm^\mu = C_\pi\, m_\pi^2\, \varphi_{\pi\pm}$$

with

$$C_\pi = \sqrt{2}\, M_N\, g_A(0)/g_V\, K_{NN\pi}(0)$$

We are now interested in the limit in which q^μ and $p^\mu \to 0$. The second term drops out due to charge conjugation invariance. Considering only the single particle intermediate states we see that the remaining terms involved scalar σ field or are of higher order in momenta and thus may be dropped in our case.[11]

Thus we obtain:

$$C_\pi^2 \langle \pi^+\pi^- | J^\mu(0) | \rho^o \rangle = e \langle 0 | V_3^\mu(0) | \rho^o \rangle$$

Hence

$$\langle \pi^+\pi^-\gamma | \rho^o \rangle = i\,(2\pi)^4 \;\delta^4(P - p - q - k)\,\frac{e}{C_\pi^2}\,\langle 0 | V_3^\mu(0) | \rho^o \rangle\, \varepsilon_\mu$$

where $\varepsilon_\mu = \varepsilon_\mu(k,\lambda)$ is the polarization vector of photon ($\lambda = 1,2$).

The ρ dominance of vector current $V_i (i = 1, 2, 3)$ allows us to write:

$$\langle 0 | V_3^\mu(0) | \rho^o \rangle = \frac{m_\rho^{\;2}}{f_{\rho\pi\pi}}\,\rho^\mu$$

where $f_{\rho\pi\pi}$ is $\rho\pi\pi$ coupling constant [12] and ρ^μ is the polarization vector of ρ^o meson. Thus [13]

$$\langle \pi^+\pi^-\gamma | \rho^o \rangle = i(2\pi)^4\;\delta^4(P - q - p - k)\,\frac{e}{C_\pi^2}\,\frac{m_\rho^2}{f_{\rho\pi\pi}}\,\varepsilon_\mu\,\rho^\mu$$

For the leptonic decay mode of ρ^o we have

$$\Gamma(\rho^o \to \ell^+\ell^-) = \left(\frac{e^2}{4\pi}\right)^2 \left(\frac{f_{\rho\pi\pi}^2}{4\pi}\right)^{-1} \frac{m_\rho}{3} \left(1 + \frac{2m_\ell^2}{m_\rho^2}\right)\left(1 - \frac{4m_\ell^2}{m_\rho^2}\right)^{\frac{1}{2}}$$

The branching ratio is independent of $f_{\rho\pi\pi}$ and is found to be

$$\frac{\Gamma(\rho^o \to \pi^+\pi^-\gamma)}{\Gamma(\rho^o \to \mu^+\mu^-)} \sim 16$$

Thus the modes $\rho^{0} \longrightarrow \ell^{+} \ell^{-}$ and $\rho^{0} \longrightarrow \pi^{+}\pi^{-}\gamma$ have comparable decay rates analogous to the case of $\eta^{0} \longrightarrow \gamma\gamma$ and $\eta^{0} \longrightarrow \pi^{+}\pi^{-}\gamma$ decay modes.[14] Also the angular distribution of the final particles is essentially that given by the phase space factor. The predicted branching ratio for the decay rate of $\rho^{0} - \pi^{+}\pi^{-}\gamma$ compared to that of the dominant mode thus turns out to be $\sim 7 \times 10^{-4}$, which does not contradict the present experimental results.

* * *

ACKNOWLEDGEMENTS

The author is thankful to Professor N. Ginsburg for the hospitality extended to him at the Syracuse University and also to Professor E. C. G. Sudarshan and the colleagues of the High Energy Group for discussions.

He also acknowledges thankfully a travel grant awarded by C.A.P.E.S. of Brazil and the John Simon Guggenheim Memorial Foundation for a Fellowship.

REFERENCES

1. M. GELL-MANN, Physics 1, 63 (1964); Phys. Rev. 125, 1067 (1962).

2. M. GELL-MANN and M. LEVY, Nuovo Cimento 16, 705 (1960); Y. NAMBU, Phys. Rev. Letters 4, 380 (1960); J. BERNSTEIN, S. FUBINI, M. GELL-MANN and W. THIRRING, Nuovo Cimento 17, 757 (1960).

3. See for example references cited in B. Renner Lectures on Current Algebra-Rutherford Laboratory Report, RHELIR 126-1966.

4. J. K. DEPAGTER, J. I. FRIEDMAN, G. GLASS, R. C. CHASE, M. GETTNER, E. VON GOELER, R. WEINSTEIN and A. M. BOYARSKI, Phys. Rev. Letters 16, 35 (1966); See also R. A. ZDAIS, L. MADANSKY, R. W. KRAEMER, S. HERTZBACH and R. STRAND, Phys. Rev. Letters 14, 721 (1965).

5. See comments on this branching ratio in reference 23 of J. J. SAKURAI, Phys. Rev. Letters 17, 1021 (1966).

6. J. J. SAKURAI, invited paper presented at the V^{th} Annual Eastern Theoretical Physics Conference, Brown University, November, 1966.

7. S. WEINBERG, Phys. Rev. Letters, 16, 879 (1966).

8. H. D. I. ABARBANEL, preprint, The Structure of K_3 Decay.

9. $A^{\mu}_{\pm} = \mp \frac{1}{\sqrt{2}} (A^{\mu}_{1} \pm iA^{\mu}_{2})$.

10. M. GELL-MANN and M. LEVY, Nuovo Cimento 16, 705 (1960); M. LEVY, to be published.

11. See for example reference 7 and 8.

12. Y. NAMBU and J. J. SAKURAI, Phys. Rev. Letters 8, 79 (1962); J. J. SAKURAI, Phys. Rev. Letters 17, 1021 (1966).

13. This result is gauge invariant in the limit $q^{\mu}, p^{\nu} \rightarrow 0$.

14. M. ADEMOLLO and R. GATTO, Nuovo Cimento 44, 282 (1966); R. E. MARSHAK and J. PASUPATHY, Phys. Rev. Letters 17, 888 (1966); A. M. POLYAKOV, Soviet Physics JEPT Letters 4, 74 (1966).