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ON THE CONSTRUCTION OF THE U-MATRIX FROM DIRAC BRACKETS IN Q.C.D.

by

M.A. dos Santos*, J.C. de Mello and F.R.A. Simão

Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Departamento de Física
Universidade Federal de Juiz de Fora
Cidade Universitária
36100 - Juiz de Fora, MG - Brasil

ABSTRACT

We obtain, formally, the U-matrix in Q.C.D. from Dirac brackets.

Key-words: Dirac brackets; U-matrix; Quantum chromodynamics.

Recently, a general procedure for the construction of the U-matrix from Dirac brackets has been obtained by Kiefer and Rothe (K-R) [1]. There, as an example, the U-matrix for Q.E.D. was obtained in the temporal and Coulomb gauges. This comment was originated by the possibility of applying this procedure to Q.C.D.

For the application of the Dirac bracket formalism (DBQP) one has as a necessary condition the non-vanishing of the Faddeev--Popov determinant (det $Q\neq 0$), as it is well-known [2]. However, it is well established that in compactified Q.C.D. theory this condition is never verified, i.e., it is impossible to find a set of gauge conditions that satisfies det $Q\neq 0$ [3]. Nevertheless, the Coulomb gauge has been used in Q.C.D. via Dirac brackets [4], ignoring the above mentioned difficulties.

As a first approach in the construction of the U-matrix from Dirac brackets in Q.C.D. we will also ignore these difficulties. Working in the Coulomb gauge and following K-R, we obtain, in analogy to Eq. (2.8) of reference [1]

$$[H_{in}^{(0)}(t),\psi_{in}(x)] = -i\gamma^0(\gamma^k\partial^k - im)\psi_{in}(x) + \frac{1}{2}\int d^3z\,[\pi_{in}^{2\,i,a}(z),\psi_{in}(x)] \,. \tag{1}$$

The additional trouble in this case comes from the tentative of transforming expression $\frac{1}{2}\int d^3z\,[\pi_{in}^{2\,i,a}(z),\psi_{in}(x)]$ into $[H_I^{(2)}(\tau),\psi_{in}(x)]$ because there exists no analytical expression for the QCD propagator $K^{a,b}(z,x)$ at our disposal. However, by the use of the power series expansion in g [5] for this propagator:

$$K^{a,b}(\vec{x},\vec{z}) = -\frac{\delta_{ab}}{4\pi|\vec{x}-\vec{z}|} - g\int d^3y\,\frac{1}{4\pi|\vec{x}-\vec{z}|}\,\varepsilon_{abc}\,A_c^i(y)\,\frac{\partial}{\partial y^i}\,\frac{1}{4\pi|\vec{y}-\vec{z}|} + \ldots \tag{2}$$

we find that

$$\frac{1}{2}\int d^3z\,[\pi_{in}^{2\,i,a}(z)\,,\psi_{in}(x)] = [H_I^{(2)}(\tau),\psi_{in}(x)] \tag{3}$$

where $H_I^{(2)}(\tau)$ is given by:

$$H_I^{(2)}(\tau) = \frac{g^2}{4}\int d^3z\; d^3y\;\; D_{in}^{o,a}(z)\;\; K^{a,b}(z,y)\; D_{in}^{o,b}(y) \tag{4}$$

with $D^{o,a}(z) = \varepsilon_{abc}\pi_{jin}^{c}(z)\,.A_{jin}^{b}(z) + \frac{i}{2}\pi_{\psi in}(z)\,.\,\tau^{a}\psi_{in}(z)$.

To obtain the U-matrix one follows the same steps of K-R from Eq. (2-13) to eq. (2-17). In this way we get

$$i\,\frac{dU}{dt} = H_I U \tag{5}$$

where

$$H_I = H_I^{(1)} + H_I^{(2)} \tag{6}$$

with $H_I^{(2)}$ given by (4) and

$$H_I^{(1)} = -\frac{ig}{2}\int d^3z\;\pi_{\psi in}(z)\,\gamma^0\gamma^k A_{in}^{k,a}(z)\,\tau^a\psi_{in}(z)] \tag{7}$$

Therefore, aside those restrictions, we conclude that the K-R procedure can be applied to Q.C.D. An analysis of the consequences of these results will be the subject of a forthcoming work.

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